COSMOLOGICAL INFLATION CRIES OUT FOR SUPERSYMMETRY

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We re-examine the inflationary scenario in the standard SU(5) model with Coleman–Weinberg symmetry breaking and point out difficulties which may be resolved in a broken supersymmetric model. Because of a partial cancellation at the one-loop level, the effective potential in a broken supersymmetric theory may be much flatter than in standard SU(5), thus permitting a greater amount of inflation.

The possibility that a phase transition during the Grand Unified Theory (GUT) epoch in the very early universe may have been responsible for the present high degree of isotropy and large-scale homogeneity of the universe, as well as the apparent suppression of magnetic monopoles, continues to make the GUT epoch one of the most intriguing in our past [1]. In suggesting that the phase transition from a GUT such as SU(5) to SU(3) × SU(2) × U(1) proceeded via radiatively induced one-loop corrections to the Higgs potential, Linde [2] presents a possible solution to several problems arising in the original inflationary scenario of Guth [1]. It had been argued previously that the cosmological observations mentioned above would be explained if the universe passed through a strong first-order phase transition when SU(5) broke down to SU(3) × SU(2) × U(1). During such a transition, the universe would have passed through a period during which the total energy density was dominated by the vacuum energy density of the Higgs potential, rather than by radiation. This would have caused an era of exponential expansion during which the universe passed from a Robertson–Walker–Friedman state to an approximate De Sitter state [3]. If the epoch of exponential expansion was long enough, the large-scale isotropy and homogeneity of the universe would perhaps be explained.

The main problem with the inflationary scenario proposed by Guth [1] was that obtaining sufficient expansion was incompatible with known mechanisms for completing the phase transition. Because of the exponential expansion, the bubble nucleation rate never becomes larger than the expansion rate and most of the universe gets hung up in the De Sitter phase. In his revised version, Linde [2] suggests that if the symmetry breaking proceeds à la Coleman–Weinberg [4], i.e., through one-loop corrections to the Higgs potential, the exponential expansion could have occurred after the phase transition started. Our entire universe could have arisen from a single bubble and the fate of the universe outside our bubble is irrelevant. Although the revised version of Linde [2] is extremely attractive, there are still some unanswered questions [‡1]. A principal difficulty is that of obtaining a long enough period of exponential expansion. In this paper we emphasize some of the difficulties and suggest that their resolutions may come from broken supersymmetric theories. These exhibit cancellations in the one-loop effective potential which enable one to construct flatter potentials which yield longer expansion time scales.

We begin with a brief review of the revised inflationary scenario. We assume that the early universe can be described by a Friedman–Robertson–Walker metric of the form

$$ds^2 = dt^2 - a^2(t) \times [dr^2/(1 - kr^2) + r^2(d\theta^2 + \sin^2\theta \, d\phi^2)] , \qquad (1)$$

[‡1] Some of these have also been realized by the participants in the Nuffield Workshop on the Very early universe [5].

where $a(t)$ is the Robertson–Walker scale factor and $k = 0, \pm 1$ is a measure of curvature. The Einstein field equations yield

$$H^2 = 8\pi\rho/3m_P^2 - k/a^2 + \tfrac{1}{3}\Lambda, \tag{2}$$

$$\dot{\rho}/\rho + 3(1 + p/\rho)H = 0, \tag{3}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, ρ is the mass energy density, p is the isotropic pressure, Λ is the cosmological constant and $m_P \equiv G_N^{-1/2}$ is the Planck mass. We use units such that $\hbar = c = k_B = 1$.

As we will see shortly, when SU(5) is broken down to SU(3) × SU(2) × U(1) the vacuum energy density of the universe changes by an amount $\Delta V \propto \sigma^4$ where $\sigma = \langle\phi\rangle$ is the vacuum expectation value of the adjoint **24** of Higgs. This constant energy density acts like a cosmological constant and must be cancelled at early times so that today $\Lambda \approx 0$. Thus we take an initial value for Λ in eq. (2): $(m_P^2/8\pi)\Lambda = -\Delta V$.

We suppose that the universe was radiation dominated at very early times so that eqs. (2) and (3) yield

$$\rho = N(T)\tfrac{1}{30}\pi^2 T^4, \tag{4}$$

$$a \sim T^{-1} \sim t^{1/2}, \tag{5}$$

where $N(T)$ is the number of bosonic and fermionic degrees of freedom $N(T) = N_B(T) + \frac{7}{8}N_F(T)$. When the temperature of the universe drops to a value such that $\rho \lesssim \Lambda$, the dynamics of the expansion are governed by Λ and we find from eq. (2) that the universe evolves towards an expansion

$$a \sim e^{Ht}, \tag{6}$$

characterizing the De Sitter phase and an exponentially expanding universe.

In order to break SU(5) down to SU(3) × SU(2) × SU(1), we consider one-loop corrections to the Higgs potential. In the region of interest $\phi < T \ll \sigma$, the potential may be expressed [4] as

$$V(\phi) = A\phi^4(\ln\phi^2/\sigma^2 - \tfrac{1}{2}) + D\phi^2, \tag{7}$$

where

$$A = \frac{1}{64\pi^2\sigma^4}\left(\sum_B g_B m_B^4 - \sum_F g_F m_F^4\right) \tag{8}$$

and $g_{B(F)}$ is the number of helicity states for the bosons (fermions) of mass $m_{B(F)}$ entering into the loop. For standard SU(5), the X and Y bosons dominate eq. (8) so that with $g_X = g_Y = 18$ and $m_X^2 = m_Y^2 = \frac{25}{8}g^2\sigma^2$, we have

$$A = (5625/1024\pi^2)g^4, \tag{9}$$

where g is the SU(5) gauge coupling. The $D\phi^2$ in eq. (7) serves as an effective mass term in the potential and can be expressed as

$$D = \tfrac{1}{2}(m_0^2 + cT^2 + bR - 3\lambda\langle\phi^2\rangle), \tag{10}$$

where m_0 is a bare mass, $c = \frac{75}{8}g^2$, b is an unknown parameter taking the value 1/6 for a conformal coupling of the scalar fields ϕ, $R = R^\mu_\mu$ is the scalar curvature, $-\lambda/4$ is the ϕ^4 self-coupling and $\langle\phi^2\rangle$ the quantum expectation value of ϕ^2. As we will see shortly, only for certain values of D can the inflationary scenario be realized.

The properties of the potential (7) and SU(5) breaking have been discussed in detail in the literature [6–13] and here we only point out some key features. The $D\phi^2$ term effectively serves as a barrier between the true and false vacua. At high temperatures, D is large and the field ϕ is confined near the origin. At lower temperatures it may become possible for ϕ to tunnel through the barrier to a point $\phi_1 \gtrsim O(T_1)$, the temperature at which tunnelling takes place. To get a lower bound on the tunnelling rate, we consider the proposal of Hawking and Moss [12] that a region of space larger than the De Sitter horizon volume jumps simultaneously to a local maximum V_1 of the potential. The rate for such a transition may be approximated by idealizing to a true De Sitter space and looking for classical solutions of the euclidean field equations. Hawking and Moss [12] estimated the resulting tunnelling probability

$$P \sim D^2 e^{-B}, \tag{11}$$

where

$$B = \tfrac{1}{8}m_P^4[1/V_0 - 1/V_1] \approx \tfrac{1}{8}m_P^4[(V_1 - V_0)/V_0^2], \tag{12}$$

in the limit where $V_1 - V_0 \ll V_0$. Here $V_0 = \frac{1}{2}A\sigma^4$ and at the local maximum

$$V_1 \approx V_0 + D^2/8A. \tag{13}$$

The Hawking–Moss action is therefore given by

$$B \approx m_P^4 D^2/16A^3\sigma^8. \tag{14}$$

We will use the expression (14) for B when the temperature of the universe has cooled to $T_H = H/2\pi$, the minimum temperature associated with the event horizon of De Sitter space [14]. The picture of a simultaneous flip of an entire horizon volume to the local maximum V of the potential only makes sense if $B \gg 1$. If $B = O(1)$ the universe could just as well jump to another value of ϕ which could easily be much larger than its value at the maximum, thus cutting down on the possible expansion time after the jump. Furthermore, if $B = O(1)$ the value of ϕ may be very inhomogeneous after the jump. There may well be other processes [15] which cause the universe to tunnel faster than the Hawking–Moss mechanism, so while $B \gg 1$ is a necessary condition for the revised inflationary scenario to work, it may not be sufficient.

The amount of inflation occurring after the tunnelling event is determined by the equation of motion of the Higgs field

$$\ddot{\phi} + 3H\dot{\phi} + 2D\phi = 0, \tag{15}$$

which gives a characteristic timescale $\tau = O(3H/2D)$ for the rollover to the SU(3) × SU(2) × U(1) minimum. In order to explain [1] the cosmological observations of large-scale isotropy and homogeneity we must require that the scale factor (6) a be inflated by a factor $O(10^{28})$. We must therefore require that the expansion factor $e^{H\tau} = \exp(3H^2/2D) > e^{65}$ or

$$3H^2/2D > 65. \tag{16}$$

When we combine this condition with the requirement that $B \gg 1$ we find from eq. (14) the double inequality

$$4A^{3/2}\sigma^4/m_P^2 < D < 3H^2/130. \tag{17}$$

We now argue that this cannot be satisfied by conventional Coleman–Weinberg SU(5). We take the following representative values for the GUT parameters: $\sigma = 3.8 \times 10^{14}$ GeV corresponding to $m_X = 3.7 \times 10^{14}$ GeV and $\Lambda_{SU(5)} = 1.5 \times 10^6$ GeV corresponding to $g^2 = 12\pi^2/[20 \ln T/m_X + (3\pi/\alpha_G)]$ with $\alpha_G = g^2/4\pi = 1/41$ at the scale m_X. From these we deduce

$$H = [(8\pi/3m_P^2)V_0]^{1/2} = (4\pi A\sigma^4/3m_P^2)^{1/2}$$

$$= 1.8 \times 10^{10} g^2. \tag{18}$$

Putting these values into the inequalities (17) we have

$$2.3 \times 10^{20} g^6 < D < 7.3 \times 10^{18} g^4 \tag{19}$$

in the De Sitter phase. The two conditions on D can only be reconciled if

$$g^2 < 3.2 \times 10^{-2}, \tag{20}$$

which is certainly not the case during the De Sitter phase when $g^2 > 4\pi\alpha_G = 0.3$.

Thus we see that the new inflationary scenario proposed by Linde [2] and developed by Hawking and Moss [12] still needs further improvement. While improvements might be found within the context of conventional GUTs, we prefer to re-examine these difficulties in a supersymmetric GUT [16]. It is well known by now that supersymmetry can in principle solve a number of outstanding problems found in ordinary GUTs, while leaving intact the benefits. Among the problems alleviated by supersymmetry are the hierarchy problem [16] and the strong CP problem [17]. In view of these assets, it is natural to consider what effect supersymmetry has on inflation. As we shall see, its net effect is again a positive one.

On a simple level, many of the problems found in GUTs are solved in supersymmetry by making use of the no-renormalization theorems [18]. In any theory in which supersymmetry remains unbroken, these theorems do away with many radiative corrections. In particular, in an exactly supersymmetric world there would be no Coleman–Weinberg [4] mechanism to govern inflation. This is clearly seen in eq. (8), since for any boson (fermion) of $m_{B(F)}$ supersymmetry tells us there is a corresponding partner with identical degrees of freedom and mass $m_{F(B)}$. Thus we have the identity $A \equiv 0$ and $V(\phi) = D\phi^2$. Hence one would have to revert back to the tree potential in order to break SU(5) in an exactly supersymmetric theory. In this case, the revised inflationary scenario [2] would not exist and we would have the old problems which faced Guth [1].

Supersymmetry, however, is not a good symmetry of nature. We know there is no degeneracy between bosons and fermions at present energies. Instead, there must exist some scale m_S at which supersymmetry is broken. In this case, typically we expect a mass splitting between bosons and fermions

$$m_B^2 - m_F^2 \sim m_S^2. \tag{21}$$

For the present discussion, we will not pin ourselves down to a specific supersymmetric model, but rather we will examine what effect the (at this point arbitrary) scale m_S has on the inflationary scenario. From present-day phenomenology, we only know that $m_S > O(100)$ GeV.

In a supersymmetric theory, as we have seen, there are no radiative corrections. When supersymmetry is broken at m_S, the one-loop potential exists and takes the form of eq. (7) with

$$A = (g_{B(F)}/32\pi^2\sigma^4)\, m_S^2 \left(\sum_B m_B^2\right), \tag{22}$$

where we have $m_S^2 \ll m_B^2, m_F^2$ so that $m_B^2 + m_F^2 \approx 2m_B^2$. In some cases there are still more cancellations so that $A = O((m_S/\sigma)^4)$. Let us suppose once more that the X bosons and their superpartners dominate eq. (22). Thus in analogy with eq. (9) we have

$$A = (75/32\pi^2\sigma^2) g^2 m_S^2. \tag{23}$$

The supersymmetric version of eq. (17) then becomes

$$0.46\, g^3 m_S^3 \sigma/m_P^2 < D < 2.3 \times 10^{-2} g^2 m_S^2 \sigma^2/m_P^2 \tag{24}$$

and puts the following constraint on m_S

$$g m_S < 5 \times 10^{-2} \sigma. \tag{25}$$

For typical values of $\sigma \approx 2 \times 10^{15}$ GeV in supersymmetric models the inequality (24) yields $g m_S < 10^{14}$ GeV, not a difficult constraint to satisfy.

To check the naturalness of such a model, let us consider what happens if $m_S = 10^{10}$ GeV [‡2]. Using $\sigma = 2 \times 10^{15}$ GeV we have $H \approx 1.6 \times 10^6\, g$ GeV and $T_H \approx 2.6 \times 10^5\, g$ GeV. [Note that the lower value of T_H requires us to lower the value of $\Lambda_{SU(5)}$ if we want to maintain the Hawking–Moss picture [12], in order that the transition does not take place via strong coupling phenomena [19]. In a supersymmetric theory, however, it is possible that $\Lambda_{SU(5)}$ is smaller than its standard SU(5) value.] To ensure that we have enough inflation we must require that $D \lesssim 6 \times 10^{10}\, g^2$ GeV^2 or

$$m_0^2 + 12bH^2 + cT_H^2 - 3\lambda\langle\phi^2\rangle \lesssim 10^{11}\, g^2\ \text{GeV}^2, \tag{26}$$

where we have used $R = 12H^2$ corresponding to De Sitter space. The term $\alpha\lambda\langle\phi^2\rangle$ is negligible [20] only if $m_0 = 0$ and $b = \frac{1}{6}$ – the conformal case. However, in this case the terms $12bH^2 + cT_H^2$ on the left-side of eq. (26) are already sufficient to violate the inequality. Linde [21] has argued that in general one must impose

$$\lambda \lesssim \tfrac{1}{1800}\pi^2 \sim 5 \times 10^{-3}. \tag{27}$$

One might have thought that the Coleman–Weinberg $A = O(g^4)$ could be sufficiently small to respect the condition (27). However, this is not the case, as eq. (9) tells us that $A = O(\frac{1}{20})$. However, the broken supersymmetric version (22) suggests that all is well if

$$g m_S < O(10^{-1})\, \sigma, \tag{28}$$

which is weaker than our previous condition (25).

The benefits of considering the one-loop potential in a broken supersymmetric theory have been threefold. First, it raised the barrier between the SU(5) and SU(3) × SU(2) × U(1) phases $V_1 - V_0 = D^2/8A$ by reducing the value of A by a factor $O(m_S/m_X)^2$, thus ensuring that the universe sits in the SU(5) phase until it falls into the essentially stationary De Sitter state with temperature T_H. Secondly, by lowering the quantity $\Delta V = \frac{1}{2}A\sigma^4$ again by $O(m_S/m_X)^2$, supersymmetry generates a flatter potential which facilitates inflation. Thirdly, supersymmetry allows [21] the growth of $\langle\phi^2\rangle$ to be sufficiently slow for the effective Higgs mass D to remain small enough to get a long timescale for the rollover and hence a large exponential growth (16). All of these advantages are directly traceable to the cancellations in A (8) which arise naturally in GUTs with broken supersymmetry, although the cancellation could be arranged ad hoc in a conventional GUT. It remains to be seen whether the new inflationary scenario remains intact when one considers a detailed supersymmetric GUT [22]. The indications are, however, that supersymmetry will indeed provide a more realistic inflationary scenario.

‡2 Presumably the value of m_S for the gauge supermultiplet must be much smaller than 10^{10} GeV, but other supermultiplets could have a supersymmetry breaking mass splitting of this order.

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