D-brane Standard Model variants and Split Supersymmetry: Unification and fermion mass predictions.

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Abstract

We study D-brane inspired models with $U(3)\times U(2)\times U(1)^N$ gauge symmetry in the context of split supersymmetry. We consider configurations with one, two and three ($N = 1, 2, 3$) abelian branes and derive all hypercharge embeddings which imply a realistic particle content. Then, we analyze the implications of split supersymmetry on the magnitude of the string scale, the gauge coupling evolution, the third family fermion mass relations and the gaugino masses. We consider gauge coupling relations which may arise in parallel as well as intersecting brane scenarios and classify the various models according to their predictions for the magnitude of the string scale and the low energy implications. In the parallel brane scenario where the $U(1)$ branes are superposed to $U(2)$ or $U(3)$ brane stacks, varying the split susy scale in a wide range, we find three distinct cases of models predicting a high, intermediate and low string scale, $M_S \sim 10^{16}$ GeV, $M_S \sim 10^7$ GeV and $M_S \sim 10^4$ GeV respectively. We further find that in the intermediate string scale model the low energy ratio m_b/m_τ is compatible with $b-\tau$ Yukawa unification at the string scale. Furthermore, we perform a similar analysis for arbitrary abelian gauge coupling relations at M_S corresponding to possible intersecting brane models. We find cases which predict a string scale of the order $M_S \geq 10^{14}$ GeV that accommodate a right-handed neutrino mass of the same order so that a see-saw type light left-handed neutrino component is obtained in the sub-eV range as required by experimental and cosmological data. Finally, a short discussion is devoted for the gaugino masses and the life-time of the gluino.

1 Introduction

Over the past few decades, low energy scale Supersymmetry (SUSY) has appeared as the most efficient mechanism to solve the hierarchy problem and protect the Higgs mass from unwanted large radiative corrections, in theories which attempt to unify the gauge couplings of the Standard Model (SM) at a high scale. Implementing SUSY in these theories, one avoids the fine-tuning of the parameters to 30 decimals thus, the criterion of naturalness of the theory is satisfied. However, there is a much more severe fine-tuning problem, i.e., that with regard to the Cosmological Constant which cannot be solved in the context of the existing theories like supersymmetry, technicolor etc. The solution of this problem would require new threshold dynamics at a scale as low as 10^{-3} eV.

It has been recently argued that the invention of a new mechanism which will solve the Cosmological Constant problem might offer a solution to the hierarchy problem as well. Bearing in mind that such a mechanism will also give an explanation to the hierarchy of the mass scales, recently, the scenario of split supersymmetry has been proposed [1] according to which supersymmetry could be broken at a high scale $\tilde{m}$ which can be even of the order of the GUT scale. In this scenario, squarks and sleptons obtain large masses of the order of the supersymmetry breaking scale $\tilde{m}$, while the corresponding fermionic degrees, gauginos and higgsinos, remain light with masses at the TeV scale. This splitting of the spectrum is possible when the dominant source of SUSY breaking preserves an R-symmetry which protects fermionic degrees to obtain masses at the scale $\tilde{m}$ [2]. Gauge coupling unification at a high scale M_S can be achieved, while $b-\tau$ Yukawa unification at M_S is compatible with the m_b/m_τ low energy ratio in split supersymmetry [3, 4]. In has been further argued that in certain D-brane constructions, the presence of internal magnetic fields [5] provide a concrete realization of split supersymmetry [6, 7], therefore, intermediate or higher string scales are also viable since there is no hierarchy problem in this case.

In a previous paper [8], based on D-brane constructions originally proposed in [9], we analyzed the gauge coupling evolution of D-brane inspired models with gauge symmetry $U(3)\times U(2)\times U(1)^N$ at the string scale. We restricted our analysis in non-supersymmetric configurations with two or three abelian branes ($N=2,3$) where only one Higgs doublet couples to the up quarks and a second one to the down quarks and leptons, while all fermion and Higgs fields are obtained from strings attached on different brane stacks. We examined six models which arise in the context of these two brane configurations due to the different hypercharge embeddings and different $U(1)$ brane orientations, and calculated the string scale in partial gauge coupling unification scenarios, where the $U(1)$ branes are aligned either to the $U(3)$ or to the $U(2)$ brane stacks. It was shown that the string scale depends strongly on the particular orientation of the $U(1)$ branes and the hypercharge embedding. There exist particular embeddings which allow a string scale of a few TeV, while in other D-brane configurations the string scale raises up to intermediate or even higher mass scales. We further investigated the possibility of obtaining the correct m_b/m_τ relation at M_Z for $b-\tau$ Yukawa equality at the string scale and found that this is possible for the class of models with string scale of the order 10^{6-7}GeV. Further interesting issues on the effective field theory of this class of models [9] have also been analyzed in detail by the authors of ref.[10].

In view of the interesting results obtained in split supersymmetry, as well as the possibility pointed out in [6], that the D-branes might provide a natural realization for the spectrum of split SUSY, in the present work, we wish to extend our previous analysis [8] (including also the models proposed in [6]), and work out various interesting phenomenological issues which reveal the fundamental differences for their non-supersymmetric, supersymmetric and split supersymmetry versions. We thus start with a classification of the various D-brane derived models with Standard Model gauge symmetry extended by $U(1)$ factors. In particular, we analyse various interesting phenomenological issues of the brane configurations with $U(3) \times U(2) \times U(1)^N$ symmetry, where the number of additional $U(1)$ branes is at most three, $N = 1, 2, 3$, so that an economical Higgs sector arises. We calculate the string unification scale, we discuss the successful $b - \tau$ equality of the traditional Grand Unified Theories (GUTs) and calculate the gaugino masses for a wide split-susy range and different initial conditions. Moreover, we seek preferable solutions that accommodate a right-handed neutrino ν^c with appropriate mass (which is usually of the order of the string scale), so that the 'see-saw' mechanism results to a sub-eV effective Majorana mass scale for the left-handed neutrino component ν_L, as required by experimental and cosmological data. We further examine how these models are discriminated by their different predictions for the gaugino masses and the life-time of the gluino.

The paper is organized as follows: In section 2 we present the general set up of the D-brane constructions based on the $U(3) \times U(2) \times U(1)^N$ symmetry. We consider all possible configurations with $N = 1, 2, 3$ abelian branes and find the different hypercharge embeddings compatible with the SM particle spectrum. In section 3 we perform a one-loop RG analysis to calculate the string scale for the models obtained in section 2, both for the case of parallel as well as intersecting brane scenarios. In section 4 we discuss the fermion mass relations of the third generation and in particular we examine the possibility of obtaining the correct bottom-tau mass ratio for $b - \tau$ Yukawa equality at the string scale. Gaugino masses and the lifetime of the gluino are discussed in section 5, while in section 6 we draw our conclusions.

2 Standard Model-like D-brane configurations with extra abelian factors

We consider models with SM gauge symmetry extended by several $U(1)$ factors which arise in the context of certain D-brane configurations. The basic ingredient of such D-brane constructions is the brane stack, i.e. a certain number of parallel D-branes which sit at the same point in the transverse space. A single D-brane carries a $U(1)$ gauge symmetry, while a stack of n parallel branes gives rise to a $U(n)$ gauge theory where its gauge bosons correspond to open strings having both their ends attached to the various brane stacks. In flat space D-branes lead to non-chiral matter whilst chirality arises when they are wrapped on a torus [11, 12]. Furthermore, in the case of intersecting branes chiral fermions sit in singular points in the transverse space while the number of fermion generations, and other fermions, are related to the two distinct numbers of brane wrappings around the two circles of the torus. We note in passing that the intersecting branes are the T-dual picture of D-brane case where one turns on magnetic fields [5] to stabilize the closed string moduli. For two stacks n_a, n_b, the gauge group is $U(n_a) \times U(n_b)$ while the fermions (which live in the intersections) belong to the bi-fundamental representations $(n_a, \bar{n}_b)$, or $(\bar{n}_a, n_b)$.

In our particular constructions the non abelian part of the gauge group is chosen to be the minimal one needed to accommodate the $SU(3)_C$ and $SU(2)_L$ gauge symmetries of the SM. We further assume the existence of N extra $U(1)$ abelian branes, thus the full gauge group is

$$G = U(3)_C \times U(2)_L \times U(1)^N \tag{1}$$

Since $U(n) \sim SU(n) \times U(1)$, the particular D-brane construction automatically gives rise to models with $SU(3) \times SU(2) \times U(1)^{N+2}$ gauge group structure, while SM fermions may carry additional quantum numbers under these extra $U(1)$'s. Thus, a general observation on the derivation of SM from D-branes, even in its simplest realization, is that, besides the linear combination related to the standard hypercharge factor, several $U(1)$ factors are involved. Many of these $U(1)$'s are anomalous, however, their anomalies are canceled by a generalized Green-Schwartz mechanism. The corresponding gauge bosons eventually become massive and the associated $U(1)$ gauge symmetries are broken. Some of these $U(1)$'s are associated with the conservation of fermion numbers. It can be seen that, when quarks and leptons belong to the bifundamentals, the abelian factor obtained from $U(3)_C \to SU(3)_C \times U(1)_C$ is related to the baryon number since all quarks, which transform non-trivially under the color gauge group, have the same 'charge' under $U(1)_C$. Even when the corresponding gauge symmetry is broken, a global symmetry remains at low energies which eliminates various baryon number violating operators [9, 10].

The existence of many $U(1)$ factors allows in principle various embeddings of the hypercharge, provided that the SM spectrum arises with the correct 'charges' under these embeddings. For the case considered in the present work, the most general hypercharge gauge coupling condition can be written as follows[1]

$$\frac{1}{g_Y^2} = \frac{6k_3^2}{g_3^2} + \frac{4k_2^2}{g_2^2} + 2\sum_{i=1}^{N} \frac{k_i'^2}{g_i'^2} \tag{2}$$

For a given hypercharge embedding the k_i''s can be determined and equation (2) relates the weak angle $\sin^2\theta_W = (1+k_Y)^{-1}$ with the gauge coupling ratios at the string scale (M_S) by:

$$k_Y \;\equiv\; \frac{\alpha_2}{\alpha_Y} \;=\; 6k_3^2\,\frac{\alpha_2}{\alpha_3} + 4k_2^2 + 2\sum_{i=1}^{N} k_i'^2\,\frac{\alpha_2}{\alpha_i'} \tag{3}$$

where $\alpha_i \equiv g_i^2/(4\pi)$. Note that, in a D-brane context the gauge couplings do not necessarily attain a common value at the string (brane) scale, so in general, the ratios α_2/α_3, α_2/α_i'

[1]We have used the traditional normalization $\mathrm{Tr}\, T^a T^b = \delta^{ab}/2, a, b = 1, \ldots, n^2$ for the $U(n)$ generators and assumed that the vector representation $(\mathbf{n})$ has abelian charge $+1$ and thus the $U(1)$ coupling becomes $g_n/\sqrt{2n}$ where g_n the $SU(n)$ coupling.

differ from unity there. However, in some classes of models several relations enter between the gauge couplings at M_S and a partial unification is feasible.

We start our investigation considering the case where some $U(1)$ branes are superposed with the $U(3)$ stack, whilst the remaining ones are aligned with the $U(2)$ stack. Therefore, if we assume that r ($r < N$) parallel $U(1)$ branes (with $\alpha'_{1,\ldots,r}$ couplings) are aligned with the $U(3)$ brane, while the remaining $N-r$ $U(1)$ branes (with $\alpha'_{r+1,\ldots,N}$ couplings) are superposed with the $U(2)$ brane, this implies at M_S that the couplings $\alpha'_{1,\ldots,r}$ are equal to α_3, while $\alpha'_{r+1,\ldots,N}$ are equal to α_2. We then can write k_Y at the string scale as

$$k_Y \equiv \frac{\alpha_2(M_S)}{\alpha_Y(M_S)} = n_1\,\xi + n_2, \tag{4}$$

where we have defined the gauge coupling ratio ξ entering in (4) by

$$\xi = \frac{\alpha_2(M_S)}{\alpha_3(M_S)} \tag{5}$$

and the coefficients n_1, n_2 by

$$n_1 = 6k_3^2 + 2\sum_{i=1}^{n} k_i'^2, \qquad n_2 = 4k_2^2 + 2\sum_{i=n+1}^{N} k_i'^2. \tag{6}$$

In our D-brane construction each SM particle corresponds to an open string stretched between pairs of $U(3)$, $U(2)$ and extra $U(1)'_i$ brane sets and belongs to some bi-fundamental representation of the associated unitary groups. Yet, higher antisymmetric or symmetric representations could also be obtained by considering strings with both ends on the same brane set. For example, when only one abelian brane is included beyond the non-abelian ones, some of the Standard Model fermions should be accommodated in these antisymmetric representations [6]. On the contrary for the cases $N = 2, 3$ we will see that all known fermions can be accommodated solely to bi-fundamental representations. More abelian branes could also be added, however, we restrict here only to $N \leq 3$ in order to ensure an economical Higgs sector and implement a $b-\tau$ Yukawa unification at M_S, where down quarks and leptons acquire their masses from a common Higgs.

2.1 SM D-Brane configurations with one $U(1)$ brane

We start our analysis with the minimal gauge symmetry obtained when only one abelian brane ($N = 1$) is included beyond the $U(3), U(2)$ stacks. Indeed, in [6, 13] it was shown that the embedding of the SM can be realized in a minimal set of only three brane-stacks. Three distinct models α_1, b_1, c_1 were constructed in the context of $U(3) \times U(2) \times U(1)$ symmetry, represented by the three brane configurations shown in Fig. 1. In all three cases, quark and lepton doublets are represented by open strings with ends attached appropriately on the corresponding brane-stacks. In model a_1 the u^c arises from a string with both ends attached to the $U(3)$ brane-stack, thus it belongs to the antisymmetric representation, while d^c is represented by a string stretched between the color and the $U(1)$ brane. In model b_1 the

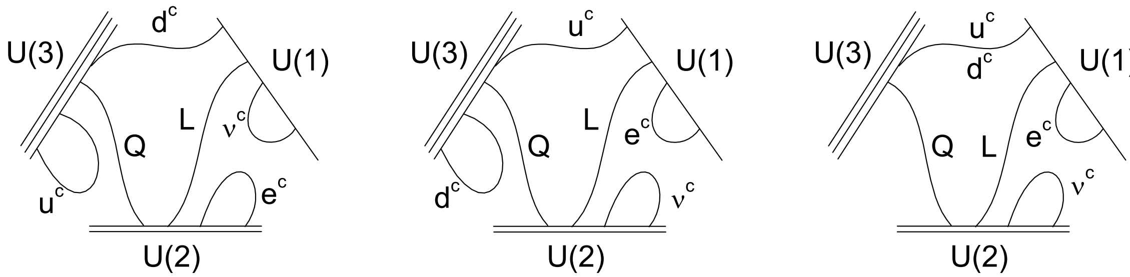

Figure 1: Standard Model D-brane configurations with one abelian brane ($N = 1$).

roles of u^c and d^c are exchanged, while in the last model c_1, both u^c and d^c originate from strings stretched between $U(3)$ and $U(2)$ stacks. Moreover, the right handed electron is obtained from strings with both ends attached on $U(2)$ or the $U(1)$ brane-stack as shown in the same Fig. 1. The quantum number of the SM fermion fields, for these configurations, are expressed in terms of the sign ambiguities ϵ_i ($\epsilon_i = \pm 1$, $i = 1, \cdots, 6$) and presented in Table 1. We stress here that the addition of the right handed neutrino (ν^c) forces some of the ϵ_i to take specific signs, however in what follows we solve the more general system without the right handed neutrino.

The hypercharge assignment conditions for these configurations determine the coefficients k_3, k_2, k_1' which subsequently determine k_Y given by equation (3). In particular, for the model a_1, these are

$$k_3 + \epsilon_1 k_2 = \frac{1}{6} \qquad -k_3 + \epsilon_2 k_1' = \frac{1}{3}$$
$$k_2\epsilon_4 + k_1'\epsilon_5 = -\frac{1}{2} \qquad 2k_3\epsilon_3 = -\frac{2}{3} \qquad 2\epsilon_6 k_2 = 1$$

Solving the above linear system of equations, we find $k_3 = \frac{1}{3}, k_2 = \frac{1}{2}, k_1' = 0$, thus $k_Y = \frac{2}{3}\xi + 1$ which further implies $\sin^2\theta_W(M_S) = \frac{3}{6+2\xi}$.

Similarly, for the models b_1, c_1, solving the corresponding linear system of hypercharge equations one obtains identical solutions $k_3 = \frac{1}{6}, k_2 = 0, k_1' = \frac{1}{2}$ while, $k_Y = \frac{1}{6}\xi + \frac{1}{2}\frac{\alpha_2}{\alpha_1'}$. In the parallel brane scenario, depending on the orientation of the $U(1)$ brane, k_Y may obtain two distinct values. Thus, if $\alpha_1' = \alpha_2$ at M_S, then $k_Y = \frac{\xi}{6} + \frac{1}{2}$, whilst, if $\alpha_1' = \alpha_3$, then $k_Y = \frac{2\xi}{3}$. The above results, are summarized in the first two rows of Table 2.

	Q	d^c	u^c	L	e^c
a_1:	$(3,2;1,\epsilon_1,0)$	$(\bar{3},1;-1,0,\epsilon_2)$	$(\bar{3},1;2\epsilon_3,0,0)$	$(1,2;0,\epsilon_4,\epsilon_5)$	$(1,1;0,2\epsilon_6,0)$
b_1:	$(3,2;1,\epsilon_1,0)$	$(\bar{3},1;2\epsilon_2,0,0)$	$(\bar{3},1;-1,0,\epsilon_3)$	$(1,2;0,\epsilon_4,\epsilon_5)$	$(1,1;0,0,2\epsilon_6)$
c_1:	$(3,2;1,\epsilon_1,0)$	$(\bar{3},1;-1,0,\epsilon_2)$	$(\bar{3},1;-1,0,\epsilon_3)$	$(1,2;0,\epsilon_4,\epsilon_5)$	$(1,1;0,0,2\epsilon_6)$

Table 1: The quantum numbers of the SM fermions in the three possible cases which arise in the $U(3) \times U(2) \times U(1)$ brane configuration.

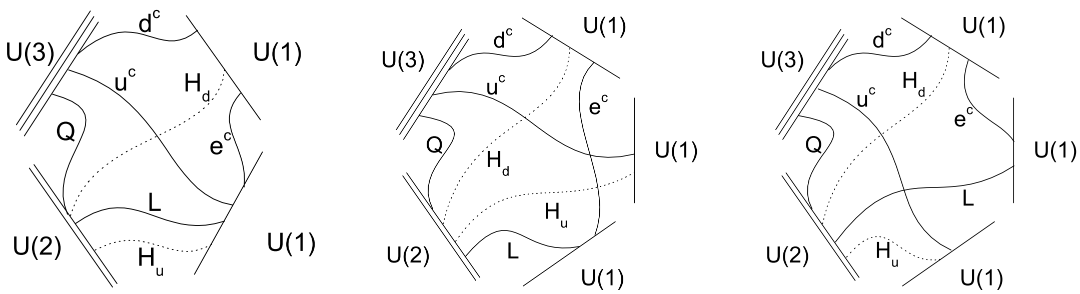

Figure 2: Configurations with two and three extra abelian branes $N = 2, 3$.

2.2 D-Brane configurations with $N = 2, 3$ abelian branes

We next study two more brane-configurations, namely those with $N = 2$ and 3 abelian branes in addition to the $U(3)$, $U(2)$ stacks, since as explained above, these are the only ones which share the property for a natural $b - \tau$ unification through an economical Higgs sector. The corresponding configurations are shown in Figure 2. As stated previously, the possible hypercharge embeddings for each configuration can be obtained by solving the hypercharge assignment conditions for SM particles. The SM particle quantum numbers under the full gauge group $SU(3)\times SU(2)\times U_3(1)\times U_2(1)\times U(1)'_1\times U(1)'_2$ are $Q(3, 2; +1, \varepsilon_1, 0, 0)$, $d^c(\bar{3}, 1; -1, 0, \varepsilon_2, 0)$, $u^c(\bar{3}, 1; -1, 0, 0, \varepsilon_3)$, $L(1, 2; 0, \varepsilon_4, 0, \varepsilon_5)$, $e^c(1, 1; 0, 0, \varepsilon_6, \varepsilon_7)$. A similar assignment can be written also for $N = 3$. Then, the hypercharge assignment equations for both brane configurations can be written

$$\begin{array}{ccc} k_3 + k_2\,\varepsilon_1 = \frac{1}{6} & -\,k_3 + k_1'\,\varepsilon_2 = \frac{1}{3} & \\ -k_3 + k\,\varepsilon_3 = -\frac{2}{3} & k_2\,\varepsilon_4 + k_2'\,\varepsilon_5 = -\frac{1}{2} & k_1'\,\varepsilon_6 + k_2'\,\varepsilon_7 = 1 \end{array} \tag{7}$$

where as previously $\varepsilon_i = \pm 1$, $i = 1, \cdots, 7$. Here we have used a compact notation where $k = k_2'$ for the $N = 2$ configuration and $k = k_3'$ for the $N = 3$ one. For the $N = 2$ case, the system (7) is satisfied by two sets of solutions

$$|k_3| = |\frac{2}{3} - k_2'| \ \ , \ \ |k_2| = |\frac{1}{2} - k_2'| \ \ , \ \ |k_1'| = |1 - k_2'| \ \ , \ \ k_2' \tag{8}$$

$$|k_3| = |\frac{2}{3} + k_2'| \ \ , \ \ |k_2| = |\frac{1}{2} + k_2'| \ \ , \ \ |k_1'| = |1 + k_2'| \ \ , \ \ k_2' \tag{9}$$

while for the $N = 3$ case, there are also two sets of solutions as well as one 'discrete' solution which cannot be obtained from the other two

$$|k_3| = |\frac{2}{3} - k_3'| \ \ , \ \ |k_2| = |\frac{1}{2} - k_3'| \ \ , \ \ |k_1'| = |1 - k_3'| \ \ , \ \ |k_2'| = |k_3'| \ \ , \ \ k_3' \tag{10}$$

$$|k_3| = |\frac{2}{3} + k_3'| \ \ , \ \ |k_2| = |\frac{1}{2} + k_3'| \ \ , \ \ |k_1'| = |1 + k_3'| \ \ , \ \ |k_2'| = |k_3'| \ \ , \ \ k_3' \tag{11}$$

$$|k_3| = \frac{5}{6} \ \ , \ \ |k_2| = 1 \ \ , \ \ |k_1'| = \frac{1}{2} \ \ , \ \ |k_2'| = \frac{1}{2} \ \ , \ \ k_3' = \frac{3}{2}. \tag{12}$$

N		$\lvert k_3\rvert$	$\lvert k_2\rvert$	$\lvert k'_1\rvert$	$\lvert k'_2\rvert$	$\lvert k'_3\rvert$
1	a_1	$\frac{1}{3}$	$\frac{1}{2}$	0	–	–
	b_1	$\frac{1}{6}$	0	$\frac{1}{2}$	–	–
2	a_2	$\frac{1}{6}$	0	$\frac{1}{2}$	$\frac{1}{2}$	–
	b_2	$\frac{2}{3}$	$\frac{1}{2}$	1	0	–
	c_2	$\frac{1}{3}$	$\frac{1}{2}$	0	1	–
3	a_3	$\frac{1}{6}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	b_3	$\frac{1}{3}$	$\frac{1}{2}$	0	1	1
	c_3	$\frac{2}{3}$	$\frac{1}{2}$	1	0	0

Table 2: The simplest hypercharge embeddings for SM D-brane configurations with $N = 1, 2, 3$ abelian brane.

As can be seen, solutions (8)-(11) are expressed in terms of one free parameter, namely k'_2 or k'_3 for $N = 2$, 3 respectively. Further, it can be checked that solution (12), leads to a physically unacceptable model and it will not be elaborated further.

To start our analysis, we pick up specific values for k'_2, k'_3 which imply the simplest solutions of equations (8)-(11). Hence, choosing $k'_2 = \frac{1}{2}, 0, 1$ and $k'_3 = \frac{1}{2}, 1, 0$, we end up with the models a_2, b_2, c_2 and a_3, b_3, c_3 presented in Table 2. Note that trivial values of k, k' indicate that the associated abelian factor does not contribute to the hypercharge. In the following sections we will analyze the predictions of these models for the string scale, the bottom-tau unification and the gaugino masses. We will further extend our analysis to more general gauge coupling relations at M_S which may occur in intersecting brane scenarios.

3 Unification and the String Scale

One of the most interesting properties of the SM gauge couplings, which led to the exploration of supersymmetric Grand Unified Theories (GUTs), is the fact that when they are extrapolated at high energies, they merge to a common coupling at a scale of the order 10^{16} GeV. In addition, traditional GUTs imply a value for the weak mixing angle, which gives low energy predictions in agreement with the experiment. In D-brane constructions however, the SM gauge couplings do not necessarily satisfy the usual unification condition. The reason is that in this case, the volume of the internal space enters between gauge and string couplings, thus the actual values of the SM gauge couplings may differ at the string scale. Nonetheless, in certain classes of D-brane configurations it is possible that some internal volume relations allow for a partial unification. Such configurations at the string level arise from the superposition of the associated parallel brane stacks. Other particular gauge coupling relations may also arise in classes of intersecting brane models.

Particularly, for D-brane models with split supersymmetry in the presence of internal magnetic fields $\mathcal{H}$, certain requirements for unification of α_2 and α_3 gauge couplings

	a_1	b_1	
$U'(1)_1$	$\mathbf{2}$ $\mathbf{3}$	$\mathbf{2}$	$\mathbf{3}$
$(n_1,\, n_2)$	$(\frac{2}{3},1)$	$(\frac{1}{6},\frac{1}{2})$	$(\frac{2}{3},0)$

	a_2			b_2		c_2	
$U'(1)_{\{1,2\}}$	$(\mathbf{2},\mathbf{2})$	$(\mathbf{3},\mathbf{3})$	$(\mathbf{2},\mathbf{3})$ $(\mathbf{3},\mathbf{2})$	$(\mathbf{2},\mathbf{2})$ $(\mathbf{2},\mathbf{3})$	$(\mathbf{3},\mathbf{3})$ $(\mathbf{3},\mathbf{2})$	$(\mathbf{2},\mathbf{2})$ $(\mathbf{3},\mathbf{2})$	$(\mathbf{2},\mathbf{3})$ $(\mathbf{3},\mathbf{3})$
$(n_1,\, n_2)$	$(\frac{1}{6},1)$	$(\frac{7}{6},0)$	$(\frac{2}{3},\frac{1}{2})$	$(\frac{8}{3},3)$	$(\frac{14}{3},1)$	$(\frac{2}{3},3)$	$(\frac{8}{3},1)$

	a_3				b_3			c_3	
$U'(1)_{\{1,2,3\}}$	$(\mathbf{2},\mathbf{2},\mathbf{2})$	$(\mathbf{2},\mathbf{3},\mathbf{2})$ $(\mathbf{3},\mathbf{2},\mathbf{2})$ $(\mathbf{2},\mathbf{2},\mathbf{3})$	$(\mathbf{3},\mathbf{3},\mathbf{3})$	$(\mathbf{3},\mathbf{2},\mathbf{3})$ $(\mathbf{3},\mathbf{3},\mathbf{2})$ $(\mathbf{2},\mathbf{3},\mathbf{3})$	$(\mathbf{2},\mathbf{2},\mathbf{2})$ $(\mathbf{3},\mathbf{2},\mathbf{2})$	$(\mathbf{2},\mathbf{3},\mathbf{3})$ $(\mathbf{3},\mathbf{3},\mathbf{3})$	$(\mathbf{2},\mathbf{2},\mathbf{3})$ $(\mathbf{2},\mathbf{3},\mathbf{2})$ $(\mathbf{3},\mathbf{2},\mathbf{3})$ $(\mathbf{3},\mathbf{3},\mathbf{2})$	$(\mathbf{2},\mathbf{2},\mathbf{2})$ $(\mathbf{2},\mathbf{2},\mathbf{3})$ $(\mathbf{2},\mathbf{3},\mathbf{2})$ $(\mathbf{2},\mathbf{3},\mathbf{3})$	$(\mathbf{3},\mathbf{3},\mathbf{3})$ $(\mathbf{3},\mathbf{2},\mathbf{2})$ $(\mathbf{3},\mathbf{2},\mathbf{3})$ $(\mathbf{3},\mathbf{3},\mathbf{2})$
$(n_1,\, n_2)$	$(\frac{1}{6},\frac{3}{2})$	$(\frac{2}{3},1)$	$(\frac{5}{3},0)$	$(\frac{7}{6},\frac{1}{2})$	$(\frac{2}{3},5)$	$(\frac{14}{3},1)$	$(\frac{8}{3},3)$	$(\frac{8}{3},3)$	$(\frac{14}{3},1)$

Table 3: Various $U(1)'$ alignments and corresponding $n_{1,2}$ values which determine $k_Y = n_1\xi + n_2$ for all models in Table 2. The first part of the Table refers to the case where only one $U(1)$ brane is included in the configuration. Similarly, the second and third parts of the Table correspond to configurations with two and three additional $U(1)$ branes.

have already been discussed in ref. [6]. More precisely, in this case the non-abelian four-dimensional α_2 and α_3 gauge couplings are given by

$$\alpha_i^{-1} \;=\; \frac{V^i}{g_s}\prod_{I=1}^{3}|n_I^i|\sqrt{1+(\mathcal{H}_I^i\alpha')^2} \tag{13}$$

where g_s is the string coupling, V^i is the compactification volume of the i-th stack, n_I^i is the number of wrappings around the I-th torus and $\mathcal{H}_I^i$ is the corresponding magnetic field component. Equality of α_2 and α_3 gauge couplings may occur when: (i) the compactification volume is independent of the particular brane stack, (ii) there is no multiple wrapping ($|n_I^i| = 1$) and (iii) magnetic fields ($\mathcal{H}_I^i\alpha' \ll 1$) are sufficiently weak.

In order to examine the more general case, when the magnetic fields are not weak, we relax the $\alpha_3 = \alpha_2$ condition and follow a different approach. At the string scale we assume that some $U(1)$ gauge couplings are equal to the $U(3)$ one, while the remaining $U(1)$ couplings are equal to the $U(2)$ one. A more general analysis, where the extra $U(1)$ gauge couplings α_i' take arbitrary values (corresponding to possible cases of intersecting branes), follows in the end of this section. We start our analysis by deriving the one-loop analytic expressions for the string scale in the context of split supersymmetry, considering various cases of partial gauge coupling unification at M_S. We express M_S in terms of the low energy experimentally measured values α_3, α_e, $\sin^2\theta_W$ and determine its range as a function of the split-supersymmetry scale $\tilde{m}$ (an average scale for the scalar supersymmetric spectrum) by imposing eq. (3). We discuss how the string scale predictions discriminate models with different numbers of $U(1)$ branes, as well as different hypercharge embeddings and gauge coupling relations at M_S. For the sake of completeness, we also compare the results with the supersymmetric and non-supersymmetric cases.

Following closely the analysis in ref. [8], we first concentrate on all possible gauge coupling relations arising from various models in the context of non-intersecting branes. Consequently, we are led to a discrete number of admissible cases which are presented in Table 3. Thus, when only one $U(1)$ brane is included in the configuration, this can be superposed either with the $U(2)$ or with the $U(3)$ brane-stack. In model a_1, since $k_1' = 0$, both cases lead to the same k_Y value. In model b_2, we have $k_1' = 1/2$, hence we obtain two distinct cases presented in the upper part of Table 3. A similar analysis results to the cases presented in the same Table when two and three abelian branes are included in the configuration. For clarity we stress here that the notation $(\mathbf{2},\mathbf{2},\mathbf{3})$ indicates the orientation of the extra $U(1)$ branes where $\mathbf{2}$ stands for the $U(2)$-direction and $\mathbf{3}$ stands for the $U(3)$ one. Hence $(\mathbf{2},\mathbf{2},\mathbf{3})$ means that the first two abelian branes in the three abelian brane scenario are aligned with the $U(2)$ stack, while the third is aligned with the $U(3)$ stack. We proceed now to analyze the cases of Table 3.

3.1 Correlation of the string and the split SUSY scale

In order to investigate qualitatively the influence of the split SUSY scale ($\tilde{m}$) on the string scale magnitude in various models of Table 2, it is sufficient to work out analytic expressions of the gauge coupling RGEs at the one-loop order (see Appendix). Above $\tilde{m}$, the beta function coefficients are those of the full supersymmetric theory, whereas below $\tilde{m}$ beta functions have contributions only from the fermion partners (SM fermions, gauginos and higgsinos) and one linear combination of the scalar Higgs doublets. We denote by b_i^{SU}, b_i the beta-coefficients valid in the energy range above and below $\tilde{m}$ respectively. Then, the one-loop RGEs lead to the following expression for the string scale

$$M_S \;=\; M_Z \left(\frac{\tilde{m}}{M_Z}\right)^{\frac{\mathcal{N}}{\mathcal{D}}} e^{f(n_1,n_2)} \tag{14}$$

where the numerator $\mathcal{N}$ and the denominator $\mathcal{D}$ of the exponent are

$$\mathcal{N} \;=\; n_1(b_3 - b_3^{SU}) + n_2(b_2 - b_2^{SU}) - (b_Y - b_Y^{SU}) \tag{15}$$

$$\mathcal{D} \;=\; -n_1 b_3^{SU} - n_2 b_2^{SU} + b_Y^{SU} \tag{16}$$

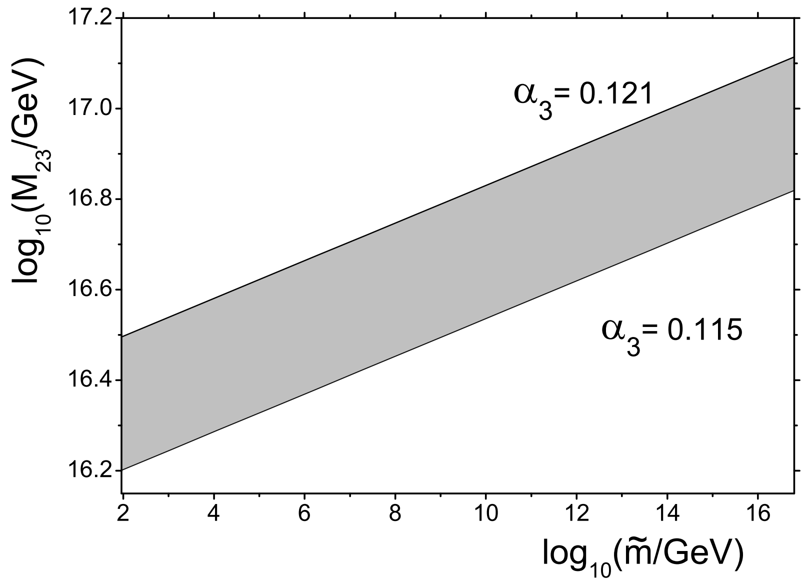

Figure 3: The one-loop unification range of $\alpha_{2,3}$ gauge couplings as a function of the split SUSY scale. The $\tilde{m}$-dependence appears due to the different number of Higgs doublets above and below $\tilde{m}$.

On the other hand the function $f(n_1, n_2)$ in (14) depends on the coefficients n_1, n_2 and the experimentally measured values for α_Y, α_2 and α_3 at M_Z. In particular[2],

$$f(n_1, n_2) \;=\; \frac{2\pi}{\mathcal{D}}\left(\frac{1}{\alpha_Y} - \frac{n_2}{\alpha_2} - \frac{n_1}{\alpha_3}\right) \tag{17}$$

Substituting the values of b_i, b_i^{SU} (see Appendix) in (15), (16) one finds that the ratio $\mathcal{N}/\mathcal{D}$ is given by

$$\frac{\mathcal{N}}{\mathcal{D}} = \frac{21 - 12n_1 - 13n_2}{6(11 + 3n_1 - n_2)} \tag{18}$$

Now, for all models of Table 2, the specific values of $n_{1,2}$ can be calculated from the coefficients k_i, k_i', using relations (6) (see Table 3). As can be seen from (14), for any model of Table 2, the string scale M_S could either increase or decrease as the split SUSY scale increases. The correlation between M_S and $\tilde{m}$ certainly depends on the sign of the exponent of the ratio $\tilde{m}/M_Z$ the latter being always greater that unity. For the models under consideration, the denominator $\mathcal{D}$ turns out to be always positive, therefore, the sign of the exponent depends on the sign of the numerator $\mathcal{N}$ which can be checked by substituting the values of n_1, n_2 obtained for the various models presented in Table 3. However, before we proceed further to classify the various models with respect to the string scale predictions,

[2]Recall that $\alpha_Y = \frac{\alpha_e}{1-\sin^2\theta_W}$ and $\alpha_2 = \frac{\alpha_e}{\sin^2\theta_W}$.

Case		Condition	$\tilde{m}$ bound
$\frac{\mathcal{N}}{\mathcal{D}} > \frac{1}{24}$	$f < f'$	$\tilde{m} \leq M_Z\, e^{\lvert\omega\rvert}$	$\leq M_Z\, e^{\omega}$
	$f > f'$	$\tilde{m} \leq M_Z\, e^{-\lvert\omega\rvert}$	$< M_Z$ (Unacceptable)
$\frac{\mathcal{N}}{\mathcal{D}} < \frac{1}{24}$	$f < f'$	$\tilde{m} \geq M_Z\, e^{-\lvert\omega\rvert}$	$\geq M_Z\ \max(1, e^{-\omega})$
	$f > f'$	$\tilde{m} \geq M_Z\, e^{\lvert\omega\rvert}$	

Table 4: Bounds imposed on $\tilde{m}$ from naturalness condition (21).

we discuss two naturalness conditions that should be satisfied. First of all, it is obvious that the relation (14) for the string scale is valid for $\tilde{m} \leq M_S$. On the other hand, as has already been noted in section 3, the non-abelian gauge couplings do not necessarily unify at the string scale M_S, hence for a given model naturalness imposes the condition $\alpha_2(\mu) \leq \alpha_3(\mu)$ for any scale $\mu \leq M_S$. If we denote by $M_{23}(\tilde{m}, \alpha_3)$ the scale where the $\alpha_2(\mu)$, $\alpha_3(\mu)$ gauge couplings merge to a common value, then for the models under consideration, the following condition should be imposed[3]

$$M_S \ \leq \ M_{23}(\tilde{m}, \alpha_3) = M_Z \left(\frac{\tilde{m}}{M_Z}\right)^{\frac{\mathcal{N}'}{\mathcal{D}'}} e^{f'} \quad \text{where} \quad f' = \frac{2\pi}{\mathcal{D}'}\left(\frac{1}{\alpha_2} - \frac{1}{\alpha_3}\right) \tag{19}$$

where $\mathcal{N}' = (b_2^{SU} - b_3^{SU}) - (b_2 - b_3)$ and $\mathcal{D}' = b_2^{SU} - b_3^{SU}$. In split SUSY with the SM fermion spectrum embedded in complete $SU(5)$ multiplets, $\mathcal{N}'$ is proportional to the difference of the Higgs doublets above and below the split SUSY scale $\tilde{m}$, thus we find $\frac{\mathcal{N}'}{\mathcal{D}'} = \frac{\delta n_H}{24} = \frac{1}{24}$.

In Figure 3 we plot M_{23} as a function of $\tilde{m}$, taking into account the experimental uncertainties of α_3; for given $\tilde{m}$ we can check the maximum allowed value of the string scale M_S satisfying the naturalness criterion $M_S \leq M_{23}$. We further infer from the same figure that, as $\tilde{m}$ varies from M_Z to M_S, the $\alpha_2 - \alpha_3$ unification point lies in the range (up to two-loop and threshold corrections),

$$M_{23} \ \approx \ [1.6 \times 10^{16} - 1.2 \times 10^{17}] \text{ GeV}. \tag{20}$$

For a more detailed analysis we can use eq. (14) and rewrite (19) as a constraint for $\tilde{m}$

$$\left(\frac{\tilde{m}}{M_Z}\right)^{\text{sign}\left(\frac{\mathcal{N}}{\mathcal{D}} - \frac{1}{24}\right)} \ \leq \ e^{\omega} \quad \text{where} \quad \omega = -\frac{f - f'}{\left|\frac{\mathcal{N}}{\mathcal{D}} - \frac{1}{24}\right|}. \tag{21}$$

All possible cases are summarized in Table 4. Let us further proceed to present the results obtained for some characteristic cases.

3.1.1 D-brane Standard Models with one abelian brane.

We consider first these models of Table 3 where only one abelian brane is included in the configuration. There are two different hypercharge embeddings in the parallel brane

[3] At this scale α_Y is smaller than the common $\alpha_{2,3}$ value for every $\tilde{m}$ which means that always α_2, α_3 couplings meet first.

scenario (first two rows of Table 2), which result to three distinct predictions for the string scale (see Table 5). Among them, the most interesting one is a_1 which predicts a string scale of the order

$$M_S = M_Z \exp\left\{\frac{\pi}{6}\left(\frac{1}{\alpha_Y} - \frac{1}{\alpha_2} - \frac{2}{3\alpha_3}\right)\right\} \approx 2\times 10^{16}\ \mathrm{GeV} \tag{22}$$

There are two noticeable points that should be mentioned here: firstly, it is a remarkable fact that in this minimal D-brane configuration, M_S coincides with the GUT scale obtained in traditional supersymmetric unification models[4]. Secondly, due to the fact that in this case $\mathcal{N} = 0$, the condition defining the string scale at the one-loop level is satisfied for any $\tilde{m}$ (a weak dependence is expected at two-loop level). However, the value of $\tilde{m}$ could be fixed if we impose a certain condition on α_2, α_3 at M_S. In particular, assuming complete unification of the non-abelian gauge couplings at the string scale we can use (19) and determine $\tilde{m}$ by the condition $M_{23}(\tilde{m}, \alpha_3) \equiv M_S$. It is easy to see that

$$\tilde{m} = M_Z \exp\left\{4\pi\left(\frac{1}{\alpha_Y} - \frac{4}{\alpha_2} + \frac{7}{3\alpha_3}\right)\right\}$$

with $M_Z \leq \tilde{m} \lesssim 6.26$ TeV provided that $\alpha_3 \leq 0.11694$.

The two cases of model b_1, for our particular parallel D-brane scenario lead to higher M_S scales ($M_S \geq M_{Pl}$) therefore, from condition (20) and the fact that they lead to a high see-saw suppression of the left handed neutrino components ($m_\nu^{eff} \leq 10^{-6}$eV), we infer that none of them can play the role of a viable low energy effective field theory in the context of the parallel brane scenario.

3.1.2 D-brane Standard Models with two abelian branes.

Three models a_2, b_2 and c_2 were analyzed for the D-brane configurations with two extra abelian branes. For the model a_2, depending on the orientation of the $U(1)'$ branes relative to $U(2)$, $U(3)$ stacks, we have three possible n_1, n_2 sets (see Table 3). In all these cases the string scale increases as $\tilde{m}$ attains higher values as well, which can be trivially inferred from the numerical values of the exponent $\mathcal{N}/\mathcal{D}$ namely 2/21, 7/87, 13/150. In the second case for example, the string scale

$$M_S = M_Z \left(\frac{\tilde{m}}{M_Z}\right)^{\frac{7}{87}} \exp\left\{\frac{4\pi}{29}\left(\frac{1}{\alpha_Y} - \frac{7}{6\alpha_3}\right)\right\} \tag{23}$$

turns out to be at least of the order of the Planck mass, even for a split SUSY scale comparable to the electroweak scale. Similar expressions can be derived for the other two cases of model a_2 (see Table 5). We should further note that the naturalness criterion (20) discussed in the previous section, is not satisfied in model a_2 since the non-abelian $\alpha_{2,3}$ gauge couplings meet at a scale lower than the one defined by condition (4).

[4] For this configuration the trivial hypercharge normalization $k_Y = 5/3$, $\xi = 1$ is recovered since from Table 3 we have $n_1 = 2/3$ and $n_2 = 1$, in accordance with ref. [6].

Models	(n_1, n_2)	M_S (GeV)	$\tilde{m}$ bound
a_1	$(\frac{2}{3}, 1)$	$[1.90 - 2.20] \times 10^{16}$	$\geq M_Z \max(1, \rho)$
b_1	$(\frac{2}{3}, 0)$	$[2.42 - 2.78] \times 10^{21}\ (\frac{\tilde{m}}{M_Z})^{\frac{1}{6}}$	Unacceptable
	$(\frac{1}{6}, \frac{1}{2})$	$[2.12 - 2.21] \times 10^{22}\ (\frac{\tilde{m}}{M_Z})^{\frac{25}{132}}$	Unacceptable
a_2	$(\frac{1}{6}, 1)$	$[2.84 - 2.96] \times 10^{19}\ (\frac{\tilde{m}}{M_Z})^{\frac{2}{21}}$	Unacceptable
	$(\frac{7}{6}, 0)$	$[3.60 - 4.48] \times 10^{18}\ (\frac{\tilde{m}}{M_Z})^{\frac{7}{87}}$	Unacceptable
	$(\frac{2}{3}, \frac{1}{2})$	$[8.57 - 9.90] \times 10^{18}\ (\frac{\tilde{m}}{M_Z})^{\frac{13}{150}}$	Unacceptable
b_2	$(\frac{8}{3}, 3)$	$[4.42 - 6.94] \times 10^{-1}\ (\frac{\tilde{m}}{M_Z})^{-\frac{25}{48}}$	$\geq M_Z$
	$(\frac{14}{3}, 1)$	$[1.46 - 2.47] \times 10^{5}\ (\frac{\tilde{m}}{M_Z})^{-\frac{1}{3}}$	$\geq M_Z$
c_2	$(\frac{2}{3}, 3)$	$[1.00 - 1.20] \times 10^{3}\ (\frac{\tilde{m}}{M_Z})^{-\frac{13}{30}}$	$\geq M_Z$
	$(\frac{8}{3}, 1)$	$[7.39 - 11.0] \times 10^{8}\ (\frac{\tilde{m}}{M_Z})^{-\frac{2}{9}}$	$\geq M_Z$
a_3	$(\frac{1}{6}, \frac{3}{2})$	$[1.96 - 2.05] \times 10^{16}\ (\frac{\tilde{m}}{M_Z})^{-\frac{1}{120}}$	$\geq M_Z \max(1, \rho)$
	$(\frac{2}{3}, 1)$	$[1.90 - 2.20] \times 10^{16}$	
	$(\frac{5}{3}, 0)$	$[1.81 - 2.41] \times 10^{16}\ (\frac{\tilde{m}}{M_Z})^{\frac{1}{96}}$	
	$(\frac{7}{6}, \frac{1}{2})$	$[1.85 - 2.32] \times 10^{16}\ (\frac{\tilde{m}}{M_Z})^{\frac{1}{168}}$	
b_3	$(\frac{2}{3}, 5)$	$[1.23 - 1.54] \times 10^{-17}\ (\frac{\tilde{m}}{M_Z})^{-\frac{13}{12}}$	$\geq M_Z$
	$(\frac{14}{3}, 1)$	$[1.46 - 2.47] \times 10^{5}\ (\frac{\tilde{m}}{M_Z})^{-\frac{1}{3}}$	$\geq M_Z$
	$(\frac{8}{3}, 3)$	$[4.42 - 6.94] \times 10^{-1}\ (\frac{\tilde{m}}{M_Z})^{-\frac{25}{48}}$	$\geq M_Z$
c_3	$(\frac{8}{3}, 3)$	$[4.42 - 6.94] \times 10^{-1}\ (\frac{\tilde{m}}{M_Z})^{-\frac{25}{48}}$	$\geq M_Z$
	$(\frac{14}{3}, 1)$	$[1.46 - 2.47] \times 10^{5}\ (\frac{\tilde{m}}{M_Z})^{-\frac{1}{3}}$	$\geq M_Z$

Table 5: The prediction of the string scale for the various models of Table 3 where $\ln \rho = 4\pi \left(\frac{1}{\alpha_Y} - \frac{4}{\alpha 2} + \frac{7}{3\alpha_3} \right)$ and $\rho \in [2.217 \times 10^{-4}, 68.636]$ due to α_3 uncertainties. Last column shows the $\tilde{m}$-bound due to the naturalness condition (19).

Likewise, there are two cases in model b_2. As can be checked from Table 5, the first case leads to an unacceptable small string scale, while the second one predicts a string scale at the TeV range, given by

$$M_S \;=\; M_Z \left(\frac{\tilde{m}}{M_Z} \right)^{-\frac{1}{3}} \exp\left\{ \frac{\pi}{12} \left(\frac{1}{\alpha_Y} - \frac{1}{\alpha_2} - \frac{14}{3\alpha_3} \right) \right\} . \tag{24}$$

In (24), the exponent of the mass ratio $\tilde{m}/M_S$ turns out to be negative, thus, as the scale

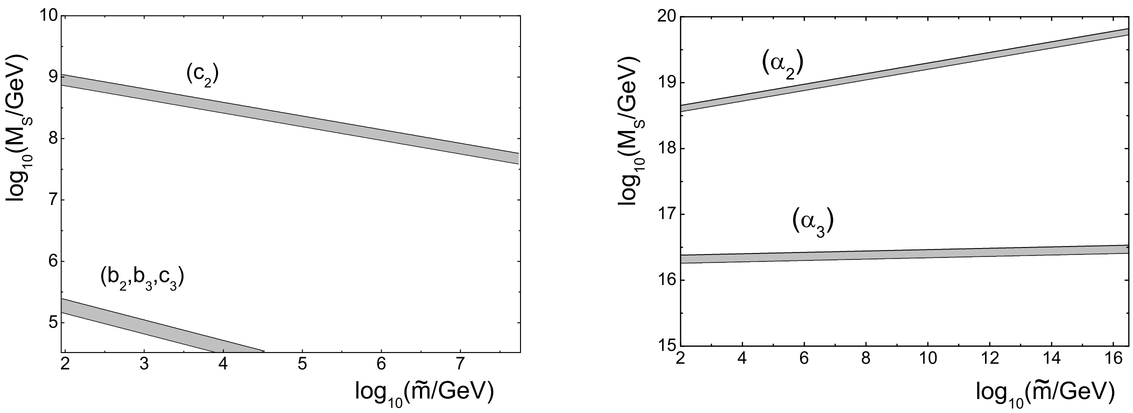

Figure 4: The string scale M_S vs the split susy scale $\tilde{m}$ in the parallel brane scenario for the four characteristic cases discussed in the text. The thickness of the M_S curves takes into account the experimental uncertainties of the strong gauge coupling at M_Z.

$\tilde{m}$ decreases, the string scale M_S increases. In the extreme case where $\tilde{m} \sim M_Z$, i.e., when the split susy scale is comparable to the electroweak scale, its highest value is $M_S \sim 100$ TeV, while for $\tilde{m} \sim M_S$, the string scale becomes as low as $M_S \sim 23.5$ TeV.

Two more cases arise in model c_2. Again, the first one predicts unacceptably small string scale. However, the second case of c_2 is more interesting. There M_S is given by the expression

$$M_S \;=\; M_Z \left(\frac{\tilde{m}}{M_Z}\right)^{-\frac{2}{9}} \exp\left\{\frac{\pi}{9}\left(\frac{1}{\alpha_Y} - \frac{1}{\alpha_2} - \frac{8}{3\alpha_3}\right)\right\}$$

which predicts an intermediate string scale of the order $\sim 10^8$ GeV (see Table 5). We will see in the next sections that in this model the low energy m_b/m_τ ratio is compatible with a $b-\tau$ Yukawa unification at M_S.

3.1.3 D-brane Standard Models with three abelian branes.

We focus on three classes of models a_3, b_3, c_3 which have the following features. Depending on the various orientations of the extra abelian branes, model a_3 occurs in four realizations and predicts a string scale compared to the traditional SUSY GUT scale 10^{16} GeV. In particular, the string scale for the second case of a_3 is given by eq. (22), leading to an identical prediction with the simpler model a_1. The remaining three cases of a_3 exhibit only a weak dependence on the scale $\tilde{m}$. Finally, models b_3, c_3 give similar values for M_S to those of model b_2. The results found for all cases, are summarized in Table 5.

In order to see some qualitative features of M_S we plot in Fig. 4 the string scale M_S versus the split-susy scale $\tilde{m}$ for four characteristic cases. We observe from this plot, that for brane configurations with certain $U(1)$ orientations, which allow for a low (TeV) or an intermediate string scale (left part of the graph), M_S decreases as $\tilde{m}$ increases. Models with

	a_1	b_1	a_2	b_2	c_2	a_3	b_3	c_3
n_1	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{8}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{8}{3}$
n_2	1	$\frac{\xi'}{2}$	ξ'	$1+2\xi'$	$1+2\xi'$	$\frac{3\xi'}{2}$	$1+4\xi'$	$1+2\xi'$

Table 6: Various $n_{1,2}$ values in the case of intersecting branes for the models of Table 2.

low M_S exhibit a stronger $\tilde{m}$-dependence compared to models with high M_S. The lower the string scale, the stronger its $\tilde{m}$-dependence. However, as can be seen from the right plot of figure 4, the $\tilde{m}$-dependence becomes almost irrelevant when the string scale is of the order 10^{16} GeV. Finally, cases with $M_S \gtrsim 2 \times 10^{17}$ GeV (right part of the graph) do not satisfy the naturalness criterion (20), thus, in the present context they cannot be considered as viable effective models. We stress here that in the limit $\tilde{m} \to M_Z$ the spectrum becomes fully supersymmetric for the whole energy range $M_Z - M_S$ and the results of the low energy supersymmetric models are recovered. We also observe a shift of the string scale to higher values relative to the non-supersymmetric case discussed in [8].

In summary, in this section we have presented a string scale classification of D-brane constructions with $U(3) \times U(2) \times U(1)^N$ ($N = 1, 2, 3$) gauge symmetry, in the parallel brane scenario and found the following interesting classes of models with respect to the string scale predictions:

(i) one class of models with $N = 1, 3$ abelian branes predicts a string scale of the order of the SUSY GUT scale i.e. $M_S \sim 10^{16}$ GeV. Interestingly these models also imply $\xi = 1$, to a good approximation, which means that the non-abelian gauge couplings α_2, α_3 do unify at M_S

(ii) in the case of $N = 2$ extra abelian branes, and for a specific $U(1)$ brane orientation we find a model with intermediate string scale $M_S \sim 10^7 - 10^8$ GeV

(iii) two cases in $N = 2, 3$ abelian brane scenarios predict a low M_S at the $10^4 - 10^5$ GeV range.

(iv) finally, in the $N = 2$ abelian case, models a_2 allow for a string scale of the order of the Planck mass, however, for the reasons explained above, these are not considered as viable possibilities.

3.2 The string scale at intersecting brane scenarios

We have presented above a classification of the models which arise from various superpositions of the abelian branes with the non-abelian ones. In these models the abelian gauge

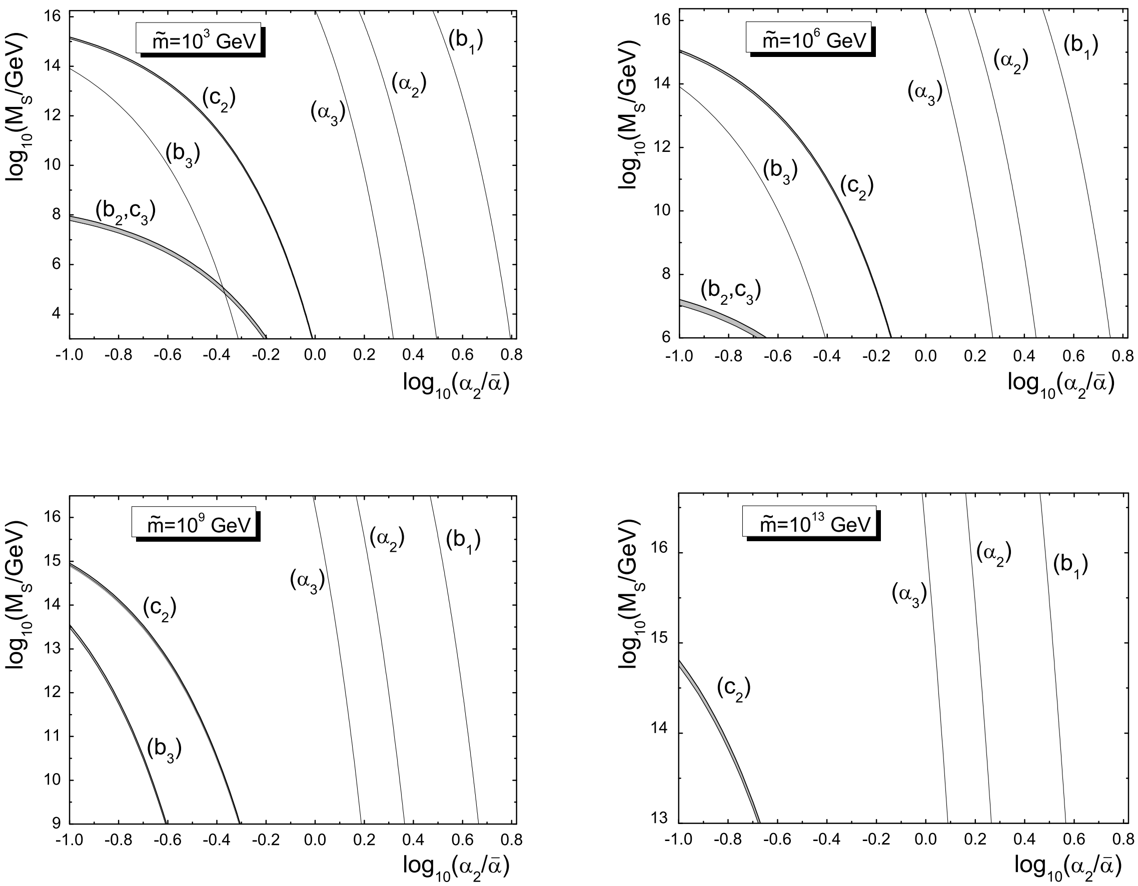

Figure 5: The string scale as a function of the parameter ξ' when the breaking of supersymmetry occurs at $\tilde{m} = 10^3, 10^6, 10^9, 10^{13}$ GeV. The thickness of the curves corresponds to the α_3 experimental uncertainties at M_Z.

couplings, at the string scale, have equal initial values with the α_2 or α_3 couplings. However, in the general case of intersecting branes the $U(1)$ gauge couplings α_i' are not necessarily equal to any of the non-abelian gauge couplings. Instead, depending on the details of the particular D-brane construction, they can take arbitrary values and as a consequence they imply different predictions for the string scale M_S. In this section, we wish to explore the possible variation of the string scale on a general perspective, i.e., without imposing specific brane orientations. In order to facilitate the subsequent analysis, we define a common average value ξ' for the gauge coupling ratios α_2/α_i' and express k_Y as

$$k_Y \;\; \equiv \;\; 6k_3^2\,\xi + 4k_2^2 + 2\xi' \sum_{i=1}^{N} {k_i'}^2 \qquad (25)$$

which means that the string scale depends now on two arbitrary parameters, namely ξ and ξ'. In Table 6 we present the values of n_1, n_2 for all models of Table 2.

Considering the models $a_{2,3}, b_{1,2,3}$ and c_2, we have plotted in Fig. 5 the variation of the string scale M_S as a function of the gauge coupling ratio $\log\xi' = \log(\alpha_2/\bar{\alpha})$ ($\bar{\alpha}$ stands for the average value of the abelian couplings α_i'), using four characteristic values of the split-susy scale namely $\tilde{m} = 10^3, 10^6, 10^9, 10^{13}$ GeV.

Starting our discussion with the case $\tilde{m} = 10^3$ GeV (upper left plot of Fig. 5) we see that in models $b_{2,3}$ and $c_{2,3}$, as ξ' varies the string scale takes values in a large range, however, it is always lower than the traditional SUSY GUT scale at least by one order of magnitude. More precisely, for $\tilde{m} \approx 10^3$ GeV as we vary ξ, ξ' model b_2 gives a string scale in the range $M_S = 10^3 - 10^8$ GeV. In addition, for a specific value of the gauge coupling ratio ξ' the b_2 and b_3 M_S-curves cross each other, thus they give an identical string mass prediction around 10^5 GeV. From Fig. 5 we also find that, for any value of M_S we have $\xi < 1$ and $\xi' \leq 1$ implying that[5] $\alpha_2 < \alpha_3$ and $\alpha_2 \leq \bar{\alpha}$. Of particular interest for low energy neutrino physics are the models b_3 and c_2 where M_S raises up to 10^{14} GeV and 10^{15} GeV respectively. Indeed, in these cases the mass of the right-handed neutrino can be of the same order with M_S, allowing thus a see-saw type Majorana mass for its left-handed component in the sub-eV range, consistent with the experimental value and the present cosmological bounds.

In a second class of models, namely a_2, a_3, b_1, the string scale-curves are steep thus, M_S is more sensitive to the variation of the ratio ξ' as compared to models discussed previously. As we move to higher scales $\tilde{m}$, the M_S curves become steeper. Moreover, in contrast to the previous cases, M_S can attain large values of the order $\sim 10^{16}$ GeV or higher. We also find that $\alpha_2 \geq \bar{\alpha}$, while for M_S of the order of the SUSY GUT scale and specific ξ'-values, which depend on the particular model, we may have complete unification ($\xi = 1$) of the non-abelian couplings $\alpha_{2,3}$ at M_S. In all cases, the highest M_S values (represented by the upper end points of the curves) correspond to the case of equal non-abelian couplings, i.e. $\alpha_3(M_S) = \alpha_2(M_S)$. Besides, for $\tilde{m} = 10^3$ GeV we find that $\xi' \sim 1, 1.5, 3$ for the models a_3, a_2, b_1 respectively, while the highest unification M_S values are in the range $[1.75 - 3.45] \times 10^{16}$ GeV. For $\tilde{m} = 10^{13}$ GeV, M_S raises up to $[4.5 - 9.0] \times 10^{16}$ GeV.

Similar conclusions can be extracted from the remaining two plots of the same figure which correspond to the cases of $\tilde{m} = 10^6$, 10^9 GeV. Note that in the third plot where $\tilde{m} = 10^9$ GeV, the b_2-curve does not exist since in this model the string scale cannot be higher than $\sim 10^8$ GeV. In the fourth plot ($\tilde{m} \sim 10^{13}$ GeV), the b_2, c_3 and b_3 M_S-plots are not present for the same reason. We finally note that models a_2, a_3, b_1, for appropriate gauge coupling values, can also accommodate a right-handed neutrino at the required mass scale $M_{\nu^c} \sim M_S \geq 10^{14}$ GeV.

4 b-τ unification

One of the most interesting features in traditional grand unified theories, is the relations they imply for the third generation Yukawa couplings. In particular, for a wide class of SUSY GUT models, the equality of the bottom – tau Yukawa couplings has been shown to be in accordance with the low energy m_b/m_τ mass ratio. In this section, in order to see

[5]The string gauge couplings α_i' are found in the perturbative region.

whether the present D-brane inspired models share this property, we use a renormalization group approach to examine the $b-\tau$ Yukawa coupling relation at the string scale.

Our procedure is the following. Using the experimentally determined values for the bottom, tau fermion masses (m_b, m_τ) we run the 2-loop $SU(3)_C \times U(1)_Y$ RGE system [8, 14] up to the weak scale (M_Z) and reconcile there the results with the well known experimental values of the weak mixing angle and the gauge couplings. For the scales above M_Z we consider a split supersymmetric theory [1, 2] where supersymmetry is broken at an energy scale $\tilde{m}$ generally far above the TeV scale. This means that the theory above $\tilde{m}$ is fully supersymmetric and the RGE system involves $\alpha_3, \alpha_2, \alpha_Y, Y_t, Y_b, Y_\tau$ couplings (see Appendix) while below $\tilde{m}$ the effective theory is obtained by removing squarks, sleptons, charged and pseudoscalar Higgs from the supersymmetric standard model. The spectrum of Split Supersymmetry $(\mu < \tilde{m})$ contains the Higgsino components $(\tilde{H}_{u,d})$, the gluino $(\tilde{g})$, the wino $(\tilde{W})$, the bino $(\tilde{B})$ and the SM particles with one Higgs doublet H. The relevant 1-loop RGE system involves $\alpha_3, \alpha_2, \alpha_Y, Y_t', Y_b', Y_\tau', \alpha_u, \alpha_u', \alpha_d, \alpha_d'$ where $Y'_{t,b,\tau}$ are the Yukawa couplings and $\alpha_{u,d}, \alpha'_{u,d}$ are the gaugino couplings (relevant equations are summarized in Appendix).

Once the Higgs doublet H is fine tuned to have small mass term, the matching conditions of the coupling constants at the scale $\tilde{m}$ become

$$
\begin{aligned}
Y_t' &= Y_t \sin^2\beta & \qquad \alpha_u' &= \alpha_Y \sin^2\beta \\
Y_b' &= Y_b \cos^2\beta & \qquad \alpha_u &= \alpha_2 \sin^2\beta \\
Y_\tau' &= Y_\tau \cos^2\beta & \qquad \alpha_d' &= \alpha_Y \cos^2\beta \\
& & \qquad \alpha_d &= \alpha_2 \cos^2\beta
\end{aligned}
$$

where the angle β is related to the vevs of the SUSY Higgs doublets by $\tan\beta = \upsilon_u/\upsilon_d$.

Let us briefly describe our strategy to solve the RGEs. In order to determine the string scale M_S, using the experimental values for $\alpha_3, \alpha_e, \sin^2\theta_W$, we evolve the gauge coupling RGEs above the electroweak scale until the condition

$$
\left[n_1 \frac{\alpha_2(\mu)}{\alpha_3(\mu)} + n_2 - \frac{\alpha_2(\mu)}{\alpha_Y(\mu)} \right]_{\mu = M_S} = 0 \tag{26}
$$

is satisfied. Our main objective, however, is the evaluation of the quark masses and the corresponding Yukawa couplings at M_S. For this reason we follow a top – bottom approach. The required quantities $m^S_{t,b,\tau} \equiv m_{t,b,\tau}(M_S)$, entering the RGE system as initial conditions, are considered to be unknown parameters. The RGE system is then evolved down to the scale $\tilde{m}$ where the matching conditions for the Yukawa and gaugino couplings are applied. We then continue the RGE evolution down to the electroweak scale and determine the unknown quark masses at M_S by solving numerically the following system of algebraic equations

$$
\left(\upsilon \sqrt{4\pi Y_t'(\mu)} - \frac{M_t}{1 + \frac{4\alpha_3(\mu)}{3\pi} - \frac{Y_t'(\mu)}{2\pi}} \right)_{\mu = M_t} = 0
$$

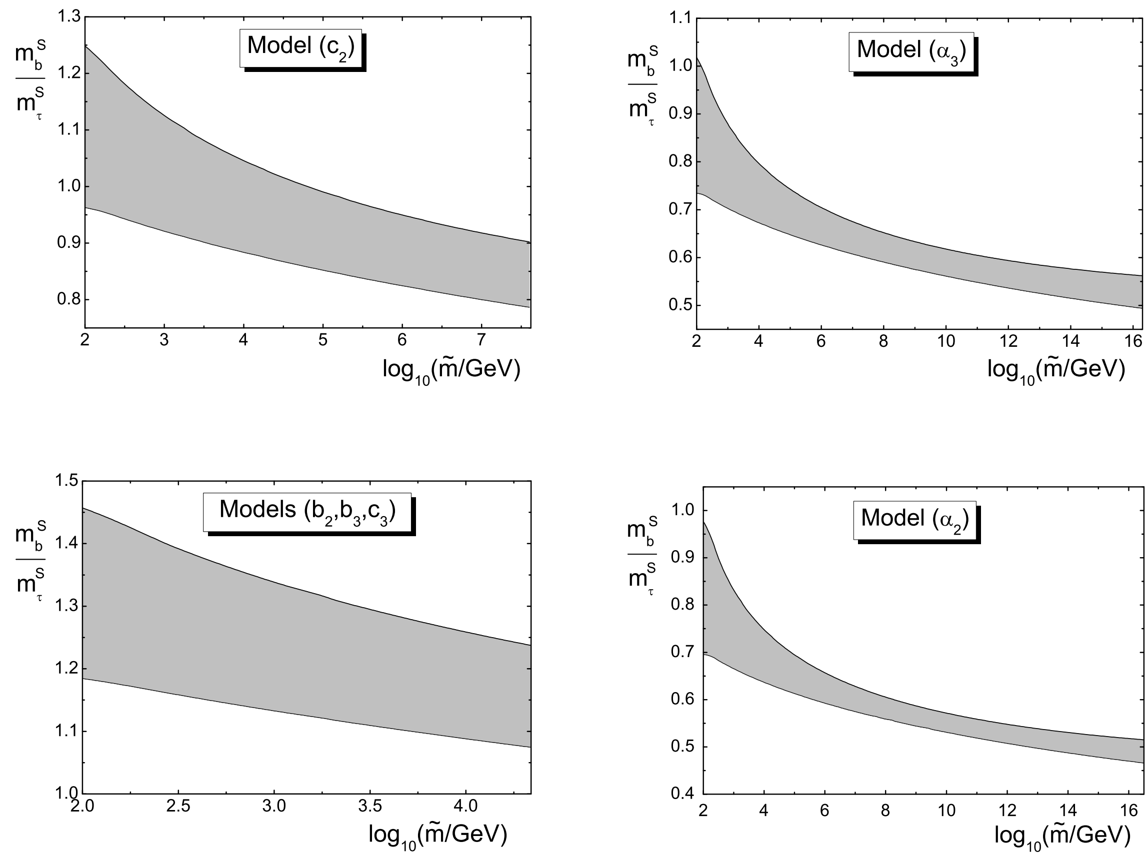

Figure 6: The ratio m_b/m_τ as a function of the supersymmetry breaking scale for models (c_2), (α_3), (b_2, b_3, c_3) and (α_2). The shaded region is due to uncertainties in $a_3(M_Z)$, $m_b(m_b)$ as well as M_t.

$$\begin{aligned} m_b(M_Z) - v\sqrt{4\pi Y_b'(M_Z)} &= 0 \\ m_\tau(M_Z) - v\sqrt{4\pi Y_\tau'(M_Z)} &= 0 \end{aligned} \tag{27}$$

where all Y' quantities[6] have an intrinsic dependence on $m^S_{t,b,\tau}$ while $v \approx 173.46$ GeV is the VEV of the Higgs field H related to the Z-boson mass by $M_Z = v\sqrt{2\pi(\alpha_Y + \alpha_2)}$. The experimental values we have used for the running quark masses are $m_\tau(m_\tau) = 1.777$ GeV, $m_b(m_b) = 4.25 \pm 0.15$ GeV while the top quark pole mass is $M_t = 172.7 \pm 2.9$ GeV [19].

In Fig. 6 we present our results for $b-\tau$ Yukawa unification versus the split supersymmetry scale for four characteristics classes of models in the parallel brane scenario discussed in previous sections. The lower curve of the shaded bands of the plots corresponds to $\tan\beta = 8.5$ and the upper one to $\tan\beta = 50$, while α_3 uncertainties are also included. In the right half of the same figure we show the $b-\tau$ relation at $M_S \sim \mathcal{O}(10^{16})$ GeV for the models a_2, a_3. We observe in these models that the string scale ratio m_b/m_τ is significantly

[6]When $\tilde{m} < M_t$ we should replace $Y' \to Y$ in eq. (27).

lower than unity for a wide range of $\tilde{m}$, except for low $\tilde{m} \sim \mathcal{O}(M_Z)$ values (i.e. when theory becomes supesymmetric), where it approaches unity. In the left half, we see that models $b_{2,3}, c_3$ predict a $b-\tau$ ratio larger than unity for any value of $\tilde{m}$. The most interesting result comes from the model c_2, where for a wide range of $\tilde{m} \sim [10^2 - 10^6]$ GeV, the $b-\tau$ ratio (at $M_S \sim 10^7$ GeV), is of order one. This result should be compared with the corresponding non-supersymmetric case where $b-\tau$ Yukawa equality at M_S holds for a string scale $M_S \sim 2 \times 10^6$ GeV.

5 Gaugino Masses and the lifetime of the gluino

In split supersymmetry, gaugino and higgsino masses, unlike their scalar superpartners, are protected by an R-symmetry and a PQ-symmetry, so that they are massless at tree-level. It is usually assumed that supersymmetry breaking occurs in the gravity sector, thus, R-symmetry is broken and gauginos obtain a light mass by some of the mechanisms described in refs. [1, 6]. At the same time, the PQ-symmetry, which protects higgsinos from being heavy, must be broken to generate the μB Higgs mixing term. This mass term together with the soft Higgs doublet masses are appropriately fine-tuned so that one linear Higgs combination is light, while the second one is of the order of the split susy scale $\sim \tilde{m}^2$. Thus, in this scenario all scalars, but one Higgs linear combination, are heavy with masses of the order $\tilde{m}$. This is an important advantage of split susy, since heavy scalars suppress remarkably the problematic flavor violating processes of the corresponding SUSY models.

The most distinct signature for a high scale split supersymmetry is a slow-decaying gluino. This important indication differentiates the various split supersymmetry models discussed so far in this work. The lifetime of the gluino, which is mediated by virtual squark exchange, is given by [6, 15]

$$\tau_{\tilde{g}} \;=\; 3 \times 10^{-2} \left(\frac{\tilde{m}_{sq}}{10^9 \mathrm{GeV}}\right)^4 \left(\frac{1\,\mathrm{TeV}}{m_{\tilde{g}}}\right)^5 \mathrm{sec} \qquad (28)$$

where $\tilde{m}_{sq}$ is the squark mass of the order of split-susy scale $\tilde{m}$ and $m_{\tilde{g}}$ is the TeV-scale gluino mass. The $\tau_{\tilde{g}}$ expression in eq. (28) depends on $\tilde{m}_{sq}$ and $m_{\tilde{g}}$ which are expressed in terms of $\tilde{m}$, thus, the experimental detection of a long-lived late-decaying gluino would determine the effective split supersymmetry scale. For small $\tilde{m}$ (i.e., less than a few hundreds TeV), gluinos will decay within the detectors, however, if the split SUSY scale is sufficiently large the gluinos can live long enough to travel distances longer than the size of the detector. In this case, they can form R-handrons that lose energy through ionization while a fraction of them will stop within the detector [16]. Several phenomenological studies explored the possibility of observing interesting signatures at LHC [15, 17].

Using the relevant equations given in the Appendix, we calculate the gluino pole mass and its lifetime as a function of $\tilde{m}$, assuming two starting values for the gaugino mass, $M_3 = 300$ GeV and $M_3 = 1$ TeV. In Fig. 7 we plot the lifetime of the gluino vs the split supersymmetry scale for the four distinct cases discussed in previous sections. In low string scale models b_2, b_3, c_3 the gluino lifetime is less than 10^{-18} sec, thus the gluinos will decay into the detectors. In model c_2, for $\tilde{m} \geq \mathcal{O}(10^6)$ GeV, one obtains $\tau_{\tilde{g}} \geq 10^{-12}$sec, meaning

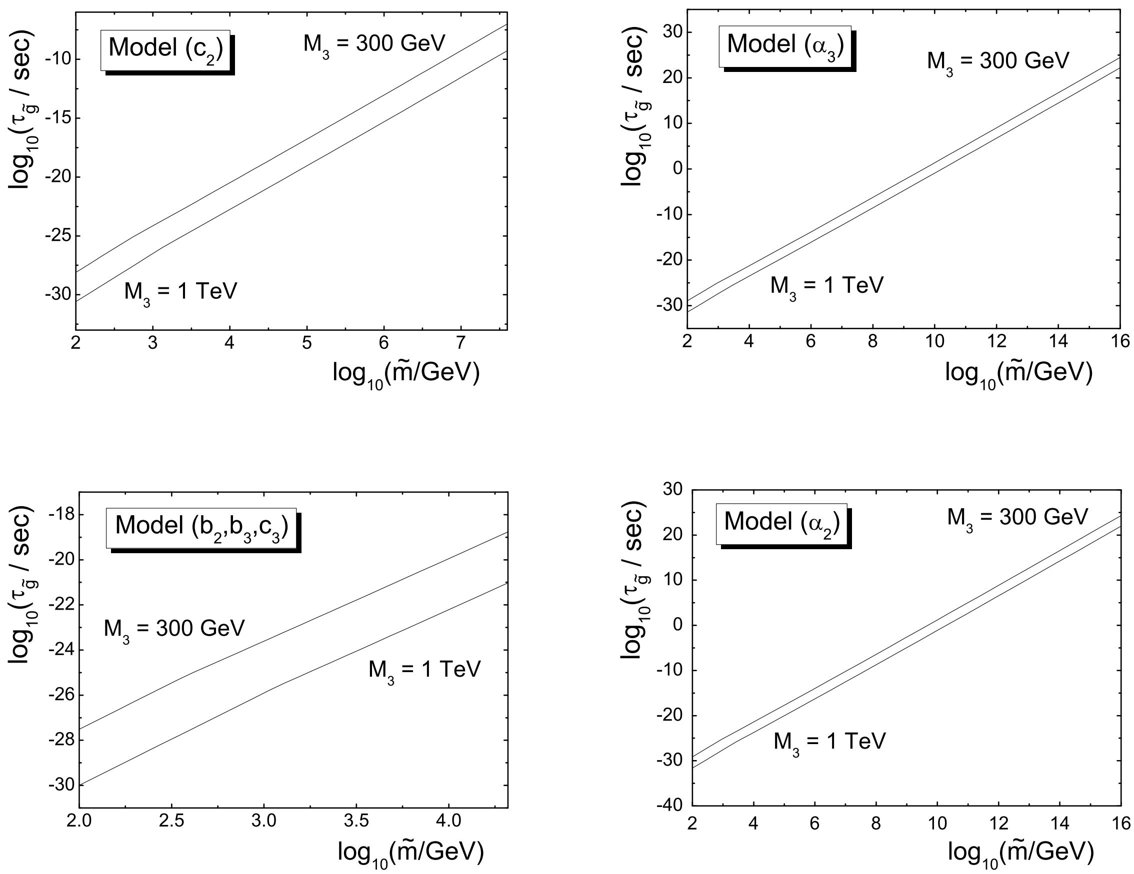

Figure 7: The gluino decay life time $\tau_{\tilde{g}}$ in four distinct cases for $M_3(M_S) = 0.3$ and 1 TeV.

that a vertex displacement can be observed in LHC and experimental measurements can determine the value of split susy scale.

Models a_2, a_3 predict a higher string scale ($M_S \sim 10^{16}$ GeV or so), allowing thus for the possibility of high $\tilde{m}$. Then, the squark mass, being of the order of the split-susy scale, i.e., $\tilde{m}_{sq} \sim M_S$, enhances dramatically the lifetime of the gluino. As can be seen from the corresponding plots in Fig. 7, a squark mass in the range 10^{13} to 10^{16} GeV, implies[7] a cosmologically stable gluino with a lifetime in the range of 10^{10} to 10^{26} sec. We should also note that there is a significant dependence of our M_3 and $\tau_{\tilde{g}}$ results, on the initial value of the gaugino mass M_3 at the string scale M_S. This dependence becomes more important for large $\tilde{m}$ values which shows that the renormalization effects below $\tilde{m}$ are substantial.

[7]For an incomplete list of works discussing other interesting implications of split susy, see [18].

6 Conclusions

Inspired by D-brane models, in the present work we studied extensions of the Standard Model gauge symmetry of the form $U(3) \times U(2) \times U(1)^N$, in the context of split supersymmetry. We considered configurations with one, two and three ($N = 1, 2, 3$) abelian branes, and made a complete classification of all models with regard to the various hypercharge embeddings which imply a realistic particle content.

We started our analysis with the implications of split supersymmetry on the string scale, considering first models which arise in parallel brane scenarios where the $U(1)$ branes are superposed with the $U(2)$ or $U(3)$ brane stacks. Varying the split susy scale in a wide range, we examined the evolution of the gauge couplings in the above context and found three distinct classes of models with the following characteristics: i) one class of models which arises in configurations with $N = 1$ and $N = 3$ abelian branes, predicts a string scale of the order of the SUSY GUT scale $M_S \sim 10^{16}$ GeV; interestingly, these models also imply that the non-abelian gauge couplings unify ($\alpha_2 = \alpha_3$) at M_S; ii) in a particular case of $N = 2$ abelian branes, corresponding to a specific $U(1)$ brane orientation we find a model with intermediate string scale $M_S \sim 10^7 - 10^8$ GeV, and iii) two cases in $N = 2$ and $N = 3$ abelian brane scenarios result to a low M_S at the TeV range. Moreover, we analyzed the third family fermion mass relations and found that in the intermediate string scale ($M_S \sim 10^8$GeV) model the low energy ratio m_b/m_τ is compatible with $b - \tau$ Yukawa unification at the string scale, for a wide range of split supersymmetry scale $\tilde{m} \sim [0.5 - 10^3]$ TeV.

We further performed a similar analysis considering arbitrary $U(1)_i$-gauge coupling relations corresponding to possible intersecting brane scenarios and classified the various models according to their predictions for the magnitude of the string scale, the gaugino masses and other low energy implications. We found that the main features observed in the case of parallel brane scenario persist, however, once we relax the conditions $\alpha_i' = \alpha_2$ and $\alpha_i' = \alpha_3$, models previously ruled out due to unacceptably large string scale ($M_S \geq M_{Pl}$), are now compatible with lower viable string scales $M_S \leq 10^{17}$ GeV for specific $\alpha_i' - \alpha_{2,3}$ gauge coupling relations at M_S. The D-brane configurations proposed here, under the specific charge assignments are also capable of accommodating a right-handed neutrino ν^c. In several viable cases, the string scale is found of the order $M_S \geq 10^{14}$ GeV, thus a ν^c mass of the same order arises so that a see-saw type light left-handed neutrino component is obtained in the sub-eV range as required by experimental and cosmological data. Finally, we calculated the gaugino masses and found how the lifetime of the gluino discriminates the various models discussed in this work.

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Appendix

In this appendix we collect the renormalization group equations for the split supersymmetry at one loop level that were used in the analysis of b-τ unification. The two loop equations can be found in [3], while a similar notation but with different conventions can be seen in [20]. In particular, the 1-loop RGEs for the gauge couplings are

$$\frac{d\tilde{\alpha}_i}{dt} = b_i \tilde{\alpha}_i^2 \tag{29}$$

where $i = Y, 2, 3$ and $\tilde{\alpha} = g^2/16\pi^2$. Below the scale $\tilde{m}$ where supersymmetry is broken the beta coefficients are $(b_Y, b_2, b_3) = (\frac{15}{2}, -\frac{7}{6}, -5)$, while above are $(b_Y^{\rm su}, b_2^{\rm su}, b_3^{\rm su}) = (11, 1, -3)$.

Below $\tilde{m}$ the equations that governs the Yukawa (h) and gaugino couplings ($\tilde{g}$) are also needed. For the Yukawa couplings we have

$$\frac{d}{dt}\ln \tilde{Y}_t' = -8\tilde{\alpha}_3 - \frac{9}{4}\tilde{\alpha}_2 - \frac{17}{4}\tilde{\alpha}_Y + \frac{9}{2}\tilde{Y}_t' + \frac{3}{2}\tilde{Y}_b' + \tilde{Y}_\tau' + \frac{3}{2}\tilde{\alpha}_u + \frac{3}{2}\tilde{\alpha}_d + \frac{1}{2}\tilde{\alpha}_u' + \frac{1}{2}\tilde{\alpha}_d' \tag{30}$$

$$\frac{d}{dt}\ln \tilde{Y}_b' = -8\tilde{\alpha}_3 - \frac{9}{4}\tilde{\alpha}_2 - \frac{5}{4}\tilde{\alpha}_Y + \frac{3}{2}\tilde{Y}_t' + \frac{9}{2}\tilde{Y}_b' + \tilde{Y}_\tau' + \frac{3}{2}\tilde{\alpha}_u + \frac{3}{2}\tilde{\alpha}_d + \frac{1}{2}\tilde{\alpha}_u' + \frac{1}{2}\tilde{\alpha}_d' \tag{31}$$

$$\frac{d}{dt}\ln \tilde{Y}_\tau' = -\frac{9}{4}\tilde{\alpha}_2 - \frac{15}{4}\tilde{\alpha}_Y + 3\tilde{Y}_t' + 3\tilde{Y}_b' + \frac{5}{2}\tilde{Y}_\tau' + \frac{3}{2}\tilde{\alpha}_u + \frac{3}{2}\tilde{\alpha}_d + \frac{1}{2}\tilde{\alpha}_u' + \frac{1}{2}\tilde{\alpha}_d' \tag{32}$$

where $\tilde{Y}' = h^2/16\pi^2$ and $\tilde{\alpha} = \tilde{g}^2/16\pi^2$. For the gaugino couplings the equations are

$$\begin{aligned}\frac{d\tilde{\alpha}_u}{dt} &= -3\tilde{\alpha}_u\left(\frac{11}{4}\tilde{\alpha}_2 + \frac{1}{4}\tilde{\alpha}_Y\right) + \frac{1}{4}\tilde{\alpha}_u(5\tilde{\alpha}_u - 2\tilde{\alpha}_d + \tilde{\alpha}_u') + (\tilde{\alpha}_u\tilde{\alpha}_d\tilde{\alpha}_u'\tilde{\alpha}_d')^{1/2} \\ &\quad + \frac{1}{2}\tilde{\alpha}_u(6\tilde{Y}_t' + 6\tilde{Y}_b' + 2\tilde{Y}_\tau' + 3\tilde{\alpha}_u + 3\tilde{\alpha}_d + \tilde{\alpha}_u' + \tilde{\alpha}_d')\end{aligned} \tag{33}$$

$$\begin{aligned}\frac{d\tilde{\alpha}_u'}{dt} &= -3\tilde{\alpha}_u'\left(\frac{3}{4}\tilde{\alpha}_2 + \frac{1}{4}\tilde{\alpha}_Y\right) + \frac{3}{4}\tilde{\alpha}_u'(\tilde{\alpha}_u' + 2\tilde{\alpha}_d' + \tilde{\alpha}_u) + 3(\tilde{\alpha}_u\tilde{\alpha}_d\tilde{\alpha}_u'\tilde{\alpha}_d')^{1/2} \\ &\quad + \frac{1}{2}\tilde{\alpha}_u'(6\tilde{Y}_t' + 6\tilde{Y}_b' + 2\tilde{Y}_\tau' + 3\tilde{\alpha}_u + 3\tilde{\alpha}_d + \tilde{\alpha}_u' + \tilde{\alpha}_d')\end{aligned} \tag{34}$$

$$\begin{aligned}\frac{d\tilde{\alpha}_d}{dt} &= -3\tilde{\alpha}_d\left(\frac{11}{4}\tilde{\alpha}_2 + \frac{1}{4}\tilde{\alpha}_Y\right) + \frac{1}{4}\tilde{\alpha}_d(-2\tilde{\alpha}_u + 5\tilde{\alpha}_d + \tilde{\alpha}_d') + (\tilde{\alpha}_u\tilde{\alpha}_d\tilde{\alpha}_u'\tilde{\alpha}_d')^{1/2} \\ &\quad + \frac{1}{2}\tilde{\alpha}_d(6\tilde{Y}_t' + 6\tilde{Y}_b' + 2\tilde{Y}_\tau' + 3\tilde{\alpha}_u + 3\tilde{\alpha}_d + \tilde{\alpha}_u' + \tilde{\alpha}_d')\end{aligned} \tag{35}$$

$$\begin{aligned}\frac{d\tilde{\alpha}_d'}{dt} &= -3\tilde{\alpha}_d'\left(\frac{3}{4}\tilde{\alpha}_2 + \frac{1}{4}\tilde{\alpha}_Y\right) + \frac{3}{4}\tilde{\alpha}_d'(\tilde{\alpha}_d' + 2\tilde{\alpha}_u' + \tilde{\alpha}_d) + 3(\tilde{\alpha}_u\tilde{\alpha}_d\tilde{\alpha}_u'\tilde{\alpha}_d')^{1/2} \\ &\quad + \frac{1}{2}\tilde{\alpha}_d'(6\tilde{Y}_t' + 6\tilde{Y}_b' + 2\tilde{Y}_\tau' + 3\tilde{\alpha}_u + 3\tilde{\alpha}_d + \tilde{\alpha}_u' + \tilde{\alpha}_d')\end{aligned} \tag{36}$$

Moving above $\tilde{m}$ up to the string scale M_S, the equations that describe the evolution of the Yukawa couplings (λ) are

$$\begin{aligned}
\frac{d}{dt}\ln\tilde{Y}_t &= -\frac{13}{9}\tilde{\alpha}_Y - 3\tilde{\alpha}_2 - \frac{16}{3}\tilde{\alpha}_3 + 6\tilde{Y}_t + \tilde{Y}_b \\
\frac{d}{dt}\ln\tilde{Y}_b &= -\frac{7}{9}\tilde{\alpha}_Y - 3\tilde{\alpha}_2 - \frac{16}{3}\tilde{\alpha}_3 + \tilde{Y}_t + 6\tilde{Y}_b + \tilde{Y}_\tau \\
\frac{d}{dt}\ln\tilde{Y}_\tau &= -3\tilde{\alpha}_Y - 3\tilde{\alpha}_2 + 3\tilde{Y}_b + 4\tilde{Y}_\tau
\end{aligned} \tag{37}$$

where $\tilde{Y} = \lambda^2/16\pi^2$. The RGE for the M_3 gaugino mass at one loop level below the split SUSY scale is

$$\frac{d}{dt}\ln M_3 = -9\tilde{\alpha}_3(1 + c_{\tilde{g}}\tilde{\alpha}_3), \tag{38}$$

where $c_{\tilde{g}} = 38/3$ in $\overline{MS}$ scheme and $c_{\tilde{g}} = 10$ in $\overline{DR}$. Above $\tilde{m}$ the previous equation becomes

$$\frac{d}{dt}\ln M_3 = b_3^{SU}\tilde{\alpha}_3 \tag{39}$$

References

[1] N. Arkani-Hamed and S. Dimopoulos, JHEP **0506** (2005) 073 [arXiv:hep-th/0405159].

[2] N. Arkani-Hamed, S. Dimopoulos, G. F. Giudice and A. Romanino, Nucl. Phys. B **709** (2005) 3 [arXiv:hep-ph/0409232].

[3] G. F. Giudice and A. Romanino, Nucl. Phys. B **699** (2004) 65 [Erratum-ibid. B **706** (2005) 65] [arXiv:hep-ph/0406088].

[4] A. Arvanitaki, C. Davis, P. W. Graham and J. G. Wacker, Phys. Rev. D **70** (2004) 117703 [arXiv:hep-ph/0406034].

[5] C. Bachas, [arXiv:hep-th/9503030].
C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnoti, Phys. Lett. **B489**(2000)209.

[6] I. Antoniadis and S. Dimopoulos, Nucl. Phys. B**715** (2005) 120 [arXiv:hep-th/0411032].

[7] I. Antoniadis, A. Delgado, K. Benakli, M. Quiros and M. Tuckmantel, [arXiv:hep-ph/0507192]. I. Antoniadis, K. Benakli, A. Delgado, M. Quiros and M. Tuckmantel, [arXiv:hep-th/0601003].

[8] D. V. Gioutsos, G. K. Leontaris and J. Rizos, Eur. Phys. J. C **45** (2006) 241 [arXiv:hep-ph/0508120].

[9] I. Antoniadis, E. Kiritsis and T. N. Tomaras, Phys. Lett. B **486** (2000) 186 [arXiv:hep-ph/0004214]. I. Antoniadis, E. Kiritsis and J. Rizos, Nucl. Phys. B **637** (2002) 92 [arXiv:hep-th/0204153]. I. Antoniadis, E. Kiritsis, J. Rizos and T. N. Tomaras, Nucl. Phys. B **660** (2003) 81 [arXiv:hep-th/0210263].

[10] C. Coriano, N. Irges and E. Kiritsis, Nucl. Phys. B **746**, 77 (2006) [arXiv:hep-ph/0510332].

[11] R. Blumenhagen, L. Goerlich, B. Kors and D. Lust, JHEP **0010** (2000) 006 [arXiv:hep-th/0007024]. R. Blumenhagen, B. Kors, D. Lust and T. Ott, Nucl. Phys. B **616** (2001) 3 [arXiv:hep-th/0107138].

[12] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan and A. M. Uranga, JHEP **0102** (2001) 047 [arXiv:hep-ph/0011132]. G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan and A. M. Uranga, J. Math. Phys. **42** (2001) 3103 [arXiv:hep-th/0011073].

[13] I. Antoniadis and J. Rizos, unpublished, 2003

[14] H. Arason, D. J. Castano, B. Keszthelyi, S. Mikaelian, E. J. Piard, P. Ramond and B. D. Wright, Phys. Rev. D **46**, 3945 (1992).

[15] P. Gambino, G. F. Giudice and P. Slavich, Nucl. Phys. B **726** (2005) 35 [arXiv:hep-ph/0506214].

[16] A. Arvanitaki, S. Dimopoulos, A. Pierce, S. Rajendran and J. Wacker, [arXiv:hep-ph/0506242].

[17] K. Cheung and W. Y. Keung, Phys. Rev. D **71** (2005) 015015 [arXiv:hep-ph/0408335]. L. Anchordoqui, H. Goldberg and C. Nunez, Phys. Rev. D **71**, 065014 (2005) [arXiv:hep-ph/0408284]. A. Arvanitaki, C. Davis, P. W. Graham, A. Pierce and J. G. Wacker, Phys. Rev. D **72** (2005) 075011 [arXiv:hep-ph/0504210]. M. Toharia and J. D. Wells, JHEP **0602**, 015 (2006) [arXiv:hep-ph/0503175]. J. G. Gonzalez, S. Reucroft and J. Swain, Phys. Rev. D **74**, 027701 (2006) [arXiv:hep-ph/0504260].

[18] B. Dutta and Y. Mimura, Phys. Lett. B **627** (2005) 145 [arXiv:hep-ph/0503052]. N. Haba and N. Okada, Prog. Theor. Phys. **114**, 1057 (2006) [arXiv:hep-ph/0502213]. K. S. Babu, T. Enkhbat and B. Mukhopadhyaya, Nucl. Phys. B **720** (2005) 47 [arXiv:hep-ph/0501079]. A. Masiero, S. Profumo and P. Ullio, Nucl. Phys. B **712** (2005) 86 [arXiv:hep-ph/0412058]. B. Kors and P. Nath, Nucl. Phys. B **711** (2005) 112 [arXiv:hep-th/0411201]. J. D. Wells, Phys. Rev. D **71** (2005) 015013 [arXiv:hep-ph/0411041].

[19] The CDF Collaboration, the D0 Collaboration, the Tevatron Electroweak Working Group, *Combination of CDF and D0 results on the top-quark mass*, [hep-ex/0507091].

[20] K. Huitu, J. Laamanen, P. Roy and S. Roy, Phys. Rev. D **72**, 055002 (2005) [arXiv:hep-ph/0502052].