Discrete *R* symmetries in the MSSM and its extensions

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We study possible anomaly-free discrete *R* symmetries that avoid the μ problem and the dangerous $D \leq 5$ operators considering charge assignments that do not commute with the traditional grand unifying simple groups, such as $SU(5)$ or $SO(10)$, but commute instead with the so-called flipped $SU(5)$, with or without the operation of the Green-Schwarz (GS) mechanism. We find $Z_N^{(R)}$ symmetries with $N = 3, 6$ in the anomaly-free case or $N = 3, 4, 6, 8, 12, 24$ in the case of anomaly cancellation through the Green-Schwarz mechanism. Nonunified cases (Z_5) are also discussed. We also confront the construction of a 4D grand unified flipped model endowed with $Z_N^{(R)}$ and find phenomenologically acceptable solutions with $N = 2k + 7$ and $N = 2k + 10$.

I. INTRODUCTION AND GENERAL FRAMEWORK

Supersymmetric unified models, such as the minimal supersymmetric standard model (MSSM) or extensions of it, have been proposed mainly as a framework that eliminates, at least, the technical aspects of the hierarchy problem. Apart from that and various other attractive ingredients, such as the unification of gauge couplings and by default, the existence of dark matter candidates, have a number of problems of their own. These are the so-called μ problem and the need to eliminate dangerous $D \leq 5$ baryon- and lepton-violating operators. *Discrete symmetries* have been introduced as a means to control these potential problems [1–5]. In addition to that, discrete symmetries, and in particular *discrete R* symmetries might play a role with respect to the scale of supersymmetry breaking, parametrized by the gravitino mass, since this can be controlled by existing discrete *R* symmetries of the superpotential. Therefore, if supersymmetry is going to be interesting for low-energy physics, it should be important to investigate the role of such discrete *R* symmetries. Such a discrete symmetry has been shown to avoid the μ problem and protect the electroweak scale in singlet extensions of MSSM [6]. Nevertheless, there exist convincing arguments that a discrete symmetry should be embeddable in an anomaly-free gauge symmetry, otherwise quantum gravitational effects would violate it [7,8]. Thus, an anomaly-free discrete *R* symmetry should be employed. Cancellation of anomalies may be direct [9] or proceed through the *Green-Schwarz (GS) mechanism*. Such a discrete *R* symmetry, commuting with $SO(10)$ was shown to avoid the μ problem in MSSM and proton decay through $D = 5$ operators [10,11]. In the present article we extend the investigation of possible anomaly-free discrete *R* symmetries that avoid the μ problem and the dangerous $D \leq 5$ operators by considering charge assignments that do not commute with the traditional grand unifying simple groups, such as $SU(5)$ or $SO(10)$, but commute instead with the so-called flipped $SU(5)$, with or without the

operation of the GS mechanism. We stay within the framework of MSSM, discussing briefly possible singlet or multidoublet extensions. We find that, in the anomaly-free case, the symmetries $Z_3^{(R)}$, $Z_6^{(R)}$ with flipped assignments are possible and present sets of phenomenologically acceptable charges. In the case of GS-anomaly cancellation, we recover $Z_N^{(R)}$ symmetries with flipped assignments for $N = 3, 4, 6, 8, 12, 24$ and present sets of phenomenologically acceptable charges. In addition, we discuss a singlet extension of MSSM with $Z_5^{(R)}$ symmetry as an example of nonunified charge assignments. Finally, we confront the issue of constructing a 4D grand unified theory endowed with such symmetry and arrive at an extended flipped $SU(5) \times U(1)_X$ model endowed with an anomaly-free (through GS) symmetry $Z_N^{(R)}$ for $N = 7$ and $N \geq 9$. Sets of phenomenologically allowed charge assignments are listed.

Next, we proceed to briefly review the general framework. Consider an *R*-symmetry Z_N, under which chiral superfields transform as $\Phi \longrightarrow e^{i(l2\pi q/N)}\Phi$, where the integer q is the *charge* of Φ under it. We are free to choose the superpotential charge to have a specific value and we take $q_{\mathcal{W}} = 2$. The *anomaly coefficients* corresponding to $(\text{gauge})^2 Z_N$ and $(\text{grav})^2 Z_N$ anomalies, denoted respectively by $\mathcal{A}$ and $\mathcal{A}_0$, are

$$\mathcal{A} = \sum_f \ell_f(q_f - 1) + \ell(\text{adj}),$$

$$\mathcal{A}_0 = -21 + \sum \ell(\text{adj}) + \#(U(1)) + \sum_a (q_a - 1),$$

where, in $\mathcal{A}$, ℓ_f is the Dynkin index of the fermion representation f with Z_N charge q_f, $\ell(\text{adj})$ is the contribution of the gauginos, and in $\mathcal{A}_0$, -21 is the contribution of the gravitino, $\sum \ell(\text{adj}) + \#(U(1))$ is the contribution of the gauginos, and q_a sums over all remaining fermion charges. The Z_N^3 anomaly coefficient is

not considered, since the corresponding condition can be interpreted as an *embedding condition*. In the case of the MSSM, we have

$$\mathcal{A}_3 = 3 + \frac{N_f}{2}(2q_Q + q_{d^c} + q_{u^c} - 4),$$

$$\mathcal{A}_2 = 2 + \frac{N_f}{2}(3q_Q + q_\ell - 4) + \frac{1}{2}(q_{h_u} + q_{h_d} - 2),$$

$$\mathcal{A}_1 = \frac{N_f}{10}(q_Q + 2q_{d^c} + 8q_{u^c} + 3q_\ell + 6q_{e^c} - 20)$$
$$+ \frac{3}{10}(q_{h_u} + q_{h_d} - 2),$$

$$\mathcal{A}_0 = -9 + (-1) + N_f(6q_Q + 3q_{u^c} + 3q_{d^c} + 2q_\ell + q_{e^c}$$
$$+ q_{N^c} - 16) + 2(q_{h_u} + q_{h_d} - 2) + \sum_s (q_s - 1). \quad (1)$$

In the gravitational coefficient we have included the contribution of the *right-handed neutrino* $N_f(q_{N^c} - 1)$ and the contribution of the *dilatino (axino)* (-1). We have also left room for the contribution of an unspecified number of singlets with charges q_s.

For $\mathcal{Z}_N$ charges commuting with $SU(5)$, the coefficients are

$$\mathcal{A}_3 = 3 + \frac{N_f}{2}(3q_{10} + q_{\bar5} - 4),$$

$$\mathcal{A}_2 = 2 + \frac{N_f}{2}(3q_{10} + q_{\bar5} - 4) + \frac{1}{2}(q_{h_u} + q_{h_d} - 2),$$

$$\mathcal{A}_1 = \frac{N_f}{2}(3q_{10} + q_{\bar5} - 4) + \frac{3}{10}(q_{h_u} + q_{h_d} - 2),$$

$$\mathcal{A}_0 = -9 + (-1) + N_f(10q_{10} + 5q_{\bar5} + q_1 - 16)$$
$$+ 2(q_{h_u} + q_{h_d} - 2) + \sum_s (q_s - 1). \quad (2)$$

II. NONANOMALOUS DISCRETE
R SYMMETRIES

One possibility to obtain an anomaly-free theory in MSSM is to impose

$$\mathcal{A}_3 = \mathcal{A}_2 = \mathcal{A}_1 = 0 \bmod(\eta), \quad (3)$$

where the condition on $\mathcal{A}_1$ is required to be met by at least on a shifted set of charges. The parameter η is $\eta = N/2$ for $N = 2k$ and $\eta = N$ for $N = 2k + 1$. These are considered independently of the gravitational coefficient, which in general depends on an unknown set of gauge singlets. The conditions, in the case of $SU(5)$-invariant charges (2), are equivalent to

$$\mathcal{A}_3 = 0 \bmod(\eta),$$

$$\mathcal{A}_2 - \mathcal{A}_3 = \frac{1}{2}(q_{h_u} + q_{h_d} - 4) = 0 \bmod(\eta),$$

$$\mathcal{A}_1 - \mathcal{A}_3 = \frac{3}{10}(q_{h_u} + q_{h_d} - 4) - \frac{12}{5} = 0 \bmod(\eta). \quad (4)$$

Imposing as a constraint the existence of the Yukawa couplings necessary for the fermion masses,[1]

$$2q_{10} + q_{h_u} = q_{10} + q_{\bar5} + q_{h_d} = q_{\bar5} + q_1 + q_{h_u}$$
$$= 2 \bmod(N), \quad (5)$$

the conditions (4) are equivalent to

$$3 - 2N_f = 0 \bmod(\eta), \qquad (q_{h_u} + q_{h_d} - 4) = 0 \bmod(N),$$
$$12 = 0 \bmod(\eta). \quad (6)$$

For $N_f = 3$, this restricts the possible $\mathcal{Z}_N$ R symmetries to

$$N = 3, 6. \quad (7)$$

Note that the μ term is absent,[2] since $q_u + q_d = 4 \neq 2 \bmod(N)$. Dangerous $D = 5$ baryon- and lepton-violating operators are also absent, since $QQQ\ell$, $u^c u^c e^c d^c \rightarrow 3q_{10} + q_{\bar5} = 4 - q_{h_u} - q_{h_d} = 0 \bmod(N)$ and $u^c d^c d^c N^c \rightarrow 2q_{\bar5} + q_{10} + q_1 = 4 - q_{h_u} - q_{h_d} = 0 \bmod(N)$. Nevertheless, the potentially dangerous $D = 3, 4$ operators transform as $\ell h_u \rightarrow q_{\bar5} + q_{h_u}$, $Q\ell d^c$, $d^c d^c u^c \rightarrow q_{10} + 2q_{\bar5}$ and a choice of charge assignments is required.

In the $N = 3$ case, since $q_{h_u} + q_{h_d} = 1 \bmod(3)$, for $q_{h_u} = 0$ and $q_{h_d} = 1$, we can have $q_{10} = 1$, $q_{\bar5} = 0$, and $q_1 = 2$. Then, the $D = 3, 4$ matter-parity violating operators cannot be present. Note however that the right-handed neutrino Majorana mass is not possible. The vanishing of the coefficient $\mathcal{A}_1$ can be met for the shifted charges $\tilde{q}_{h_u} = 12$, $\tilde{q}_{h_d} = 10$, $\tilde{q}_{10} = 4$.

In the $N = 6$ case, we have $q_{h_u} + q_{h_d} = 4 \bmod(6)$, which can be met only with even charges due to the mass relations (5). The choice $q_{h_d} = q_{h_u} = 2$ is $SO(10)$-invariant. The phenomenologically acceptable charges, for which the above $D = 3, 4$ operators are absent, are listed in the following table (note that in Table I, $\tilde{q}_a$ stands for shifted charges and the last entry refers to the right-handed neutrino Majorana mass operator).

In the above search for a nonanomalous $\mathcal{Z}_N$ R symmetry it has been assumed that it commutes with $SU(5)$ [9]. This need not be necessarily the case. Let us consider the following *"flipped"* assignments:

$$q_Q = q_{d^c} = q_{N^c} = q_{10}, \qquad q_\ell = q_{u^c} = q_{\bar5},$$
$$q_{e^c} = q_1, \quad (8)$$

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[1]These can be "*solved*" $\bmod(N/2)$ as

$$q_{10} = 1 - q_{h_u}/2, \qquad q_{\bar5} = 1 + q_{h_u}/2 - q_{h_d},$$
$$q_1 = 1 - 3q_{h_u}/2 + q_{h_d}.$$

Any allowed sets of charges should obey these relations.

[2]Here, as in all subsequent cases, it is assumed that a μ term of the correct order of magnitude is generated through nonperturbative effects [12] or through another indirect mechanism [13].

TABLE I.   Allowed $SU(5)$-invariant charges for $\mathcal{Z}_6^{(R)}$.

q_{h_u}	q_{h_d}	q_{10}	$q_{\bar5}$	q_1	$\tilde{q}_{h_u}$	$\tilde{q}_{h_d}$	$(N^c)^2$
0	4	1	3	5	12	10	−
0	4	4	0	2	12	10	−
4	0	5	3	1	10	12	+
2	2	3	3	3	14	8	−

TABLE II.   Allowed flipped charges for $\mathcal{Z}_3^{(R)}$ and $\mathcal{Z}_6^{(R)}$.

$\mathcal{Z}_N$	q_Q	q_{d^c}	q_{N^c}	q_ℓ	q_{u^c}	q_{e^c}	q_{h_u}	q_{h_d}	$\tilde{q}_{e^c}$	$\tilde{q}_{h_u}$	$\tilde{q}_{h_d}$
$N = 3$	2	2	2	0	0	1	0	1	4	12	10
$N = 6$	5	5	5	3	3	1	0	4	7	12	10
$N = 6$	2	2	2	0	0	4	0	4	4	12	10
$N = 6$	1	1	1	3	3	5	4	0	−1	10	12
$N = 6$	3	3	3	3	3	3	2	2	3	14	8

which commute with the so-called *"flipped"* $SU(5) \times U(1)$. The corresponding gauge anomaly coefficients are identical to those of Eq. (2) with the exception of

$$\mathcal{A}_1 = \frac{N_f}{10}(3q_{10} + 11q_{\bar5} + 6q_1 - 20) + \frac{3}{10}(q_{h_u} + q_{h_d} - 2). \quad (9)$$

However, note that when we impose the mass relations

$$2q_{10} + q_{h_d} = q_{10} + q_{\bar5} + q_{h_u} = q_{\bar5} + q_1 + q_{h_d}$$
$$= 2 \, \mathrm{mod}(N), \quad (10)$$

we can prove[3]

$$3q_{10} + 11q_{\bar5} + 6q_1 = 5(3q_{10} + q_{\bar5}),$$

leading to

$$\mathcal{A}_1 = \frac{N_f}{2}(3q_{10} + q_{\bar5} - 4) + \frac{3}{10}(q_{h_u} + q_{h_d} - 2), \quad (11)$$

which is identical to the corresponding expression in Eq. (2). Thus, in both cases of straight and flipped charge assignments, all gauge anomaly coefficients are identical, provided we invoke the conditions for the existence of mass Yukawa couplings. Nevertheless, it should be reminded that the anomaly cancellation condition on $\mathcal{A}_1$ rests on the existence of at least one set of shifted charges that satisfy the corresponding condition. Note also, even in the case of nonunified assignments that will follow, we assume that the $U(1)_Y$ hypercharge factor is normalized in the standard E_6 fashion.

The allowed cases for a nonanomalous discrete R symmetry are still $N = 3, 6$. Nevertheless, the resulting models need not be the same. The μ term is still absent, since $q_{h_u} + q_{h_d} = 4 \neq 2 \, \mathrm{mod}(N)$. Similarly, for the $D = 5$ baryon- and lepton-violating operators, we have $QQQ\ell$, $d^c d^c N^c u^c \to 3q_{10} + q_{\bar5} = 4 - (q_{h_u} + q_{h_d}) = 0 \neq 2 \, \mathrm{mod}(N)$ and $u^c u^c d^c e^c \to 2q_{\bar5} + q_{10} + q_1 = 4 - (q_{h_u} + q_{h_d}) = 0 \neq 2 \, \mathrm{mod}(N)$. Thus, these operators are absent. In order to conclude whether the $D = 3, 4$ dangerous operators are

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[3]These are now *"solved"* by $q_{10} = 1 - q_{h_d}/2$, $q_{\bar5} = 1 + q_{h_d}/2 - q_{h_u}$ and $q_1 = 1 - 3q_{h_d}/2 + q_{h_u}$.

allowed, we must proceed further with the charge assignments for each value of N.

In the $N = 3$ case, we can take $q_{h_d} = 1$, $q_{h_u} = 0$. Then, we can have $q_{10} = 2$, $q_{\bar5} = 0$, $q_1 = 1$. The $D = 3$ operator $\ell h_u \to q_{\bar5} + q_{h_u} = 0$ cannot be present. The $D = 4$ operators $Q\ell d^c$, $d^c d^c u^c \to 2q_{10} + q_{\bar5} = 1$ cannot be present either. Thus, the $\mathcal{Z}_3$ R symmetry in this case of flipped assignments is also phenomenologically feasible. The $N = 6$ case proceeds in an analogous fashion. Taking $q_{h_d} = 4$, $q_{h_u} = 0$, we are led to $q_{10} = 5$, $q_{\bar5} = 3$, $q_1 = 1$, which disallows ℓh_u and $Q\ell d^c$, $d^c d^c u^c$. Similarly, for the rest of the flipped charge assignments shown in Table II. Shifted charges are denoted as $\tilde{q}$.

If the model is extended beyond MSSM by the introduction of a singlet S coupled to the Higgs fields through a term $S h_u h_d$, the required charge of the new field has to be $q_S = 2 - q_{h_u} - q_{h_d} = -2 \, \mathrm{mod}(N)$, i.e., $q_S = 1$ for $\mathcal{Z}_3$ or $q_S = 4$ for $\mathcal{Z}_6$. Nevertheless, in both cases the term S^2 is allowed, corresponding to a large mass for the singlet and making this extension phenomenologically uninteresting.

$\tilde{N} > 1$ *pairs of isodoublets.* Another possible extension of the MSSM to be considered is the case of extra Higgs isodoublets with identical charges. The anomaly coefficients for $\tilde{N} \geq 1$ pairs of isodoublets take the form

$$\mathcal{A}_3 = 3 + \frac{N_f}{2}(3q_{10} + q_{\bar5} - 4),$$

$$\mathcal{A}_2 = 2 + \frac{N_f}{2}(3q_{10} + q_{\bar5} - 4) + \frac{\tilde{N}}{2}(q_{h_u} + q_{h_d} - 2),$$

$$\mathcal{A}_1 = \frac{N_f}{2}(3q_{10} + q_{\bar5} - 4) + \frac{3\tilde{N}}{10}(q_{h_u} + q_{h_d} - 2). \quad (12)$$

The corresponding conditions $\mathcal{A}_i = 0 \, \mathrm{mod}(\eta)$ in the case $N_f = 3$, after we enforce the common in flipped or $SU(5)$ assignments, mass relation $3q_{10} + q_{\bar5} - 4 = -(q_{h_u} + q_{h_d}) \, \mathrm{mod}(N)$, amount to

$$\mathcal{A}_3 \to 3(q_{h_u} + q_{h_d} - 2) = 0 \, \mathrm{mod}(N),$$

$$\mathcal{A}_2 - \mathcal{A}_3 \to \tilde{N}(q_{h_u} + q_{h_d} - 2) = 2 \, \mathrm{mod}(N),$$

$$\mathcal{A}_1 - \mathcal{A}_3 \to \frac{3\tilde{N}}{5}(q_{h_u} + q_{h_d} - 2) = 6 \, \mathrm{mod}(N) \leftrightarrow 24$$

$$= 0 \, \mathrm{mod}(N). \quad (13)$$

From the first two equations we obtain in the familiar case $\tilde{N} = 1$ the standard nontrivial solutions $N = 3, 6$ with

$q_{h_u} + q_{h_d} = 4 \mod(N)$ discussed previously. In an analogous manner, for $\tilde{N} = 2$ we again obtain $N = 3, 6$ for the allowed symmetries but with a different relation for the Higgs charges, namely $q_{h_u} + q_{h_d} = 0 \mod(N)$. In contrast, the $\tilde{N} = 3$ case has no solution for nontrivial symmetries ($N \geq 3$).

III. DISCRETE R SYMMETRIES WITH ANOMALY CANCELLATION THROUGH THE GS MECHANISM

Another possibility for the realization of discrete R symmetries is when gauge and gravitational anomalies are canceled through the operation of the *Green-Schwarz mechanism*.[4] The corresponding conditions on the anomaly coefficients (2) read

$$\mathcal{A}_3 = \mathcal{A}_2 = \mathcal{A}_1 = \rho \mod(\eta),$$
$$\mathcal{A}_0 = 24\rho \mod(\eta). \quad (14)$$

From the explicit expressions of the coefficients (2) for the MSSM content, enforcing the mass conditions, we obtain[5] for Z_N charges commuting with $SU(5)$ or flipped,

$$\mathcal{A}_3 = \rho \rightarrow \rho = -3 \mod(\eta),$$
$$\mathcal{A}_2 - \mathcal{A}_3 = 0 \rightarrow q_{h_u} + q_{h_d} = 4 \mod(N),$$
$$\mathcal{A}_1 - \mathcal{A}_3 = 0 \rightarrow 12 = 0 \mod(\eta),$$
$$\mathcal{A}_0 - 24\mathcal{A}_3 = 0 \rightarrow 18 + \sum_s (q_s - 1) = 0 \mod(\eta). \quad (15)$$

Note that the first three of these conditions, for $\rho = 0$ (i.e., $N = 3, 6$), coincide with the conditions in Eq. (4).

The last two conditions can be reconciled without the use of extra singlets in the case of $N = 3, 4, 6, 12$. Nevertheless, they can be compatible for all possible N if we extend MSSM introducing extra singlets. A most straightforward possibility is that of a singlet S coupled to the Higgses as $Sh_u h_d$. Then its required charge would be $q_S = -2 \mod(N)$. Two such singlets reconcile the last two conditions.[6] Thus, the allowed discrete R symmetries would be those corresponding to $N = 3, 4, 6, 8, 12, 24$. Note that in the case $N = 3, 6$ only the quadratic term S^2 is allowed. In the $N = 4$ case, only a linear and a cubic term are allowed. Finally, in the $N = 8$ case, only a cubic term is

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[4]We assume universality in the anomaly cancellation conditions instead of more complicated alternatives [14].

[5]We have included in $\mathcal{A}_0$ the contribution of the right-handed neutrino and that of the dilatino (axino). Enforcing the mass conditions on it we obtain $\mathcal{A}_0 = -14 - 10(q_{h_u} + q_{h_d}) + \sum_s (q_s - 1)$. The difference with $\mathcal{A}_3 = 3 - \frac{3}{2}(q_{h_u} + q_{h_d})$ is $\mathcal{A}_0 - 24\mathcal{A}_3 = 18 + 26(q_{h_u} + q_{h_d} - 4) + \sum_s (q_s - 1)$.

[6]Of course, such a "solution" is not unique. Six singlets of zero charge could do the job as well. Note also that, in the case of $N = 3, 6$, these conditions are met with an arbitrary number of such singlets, since $\sum_s (q_S - 1) = N_s(-3 \mod(N))$, which is a subset of $0 \mod(3)$.

allowed. Thus, in the $N = 8$ case the singlet extension coincided with the standard next to minimal supersymmetric standard model, in the $N = 4$ case it is a modified next to minimal supersymmetric standard model with an additional linear term. Finally, the cases $N = 12, 24$, where no singlet self term is present, correspond to what has been termed an alternative next to minimal supersymmetric standard model [6].

Allowed Z_N R symmetries commuting with $SU(5)$ or $SO(10)$ have been studied [11]. We shall go further in our analysis considering charge assignments that do not commute with these symmetries. More specifically, we shall consider "*flipped*" assignments

$$q_Q = q_{d^c} = q_{N^c} = q_{10}, \qquad q_\ell = q_{u^c} = q_{\bar 5}, \qquad q_{e^c} = q_1, \quad (16)$$

which commute with the so-called "*flipped*" $SU(5) \times U(1)$. The anomaly coefficient conditions considered above are identical for all these cases, provided the mass relations are invoked. Nevertheless, the final charge assignments for matter correspond to distinct models. Turning now to the phenomenological features of the allowed models, we see that the condition on the Higgs charges (15) is sufficient to forbid the μ term. In addition, dangerous $D = 5$ baryon and lepton number-violating operators are absent as well, due to Eqs. (10) and (15). Indeed, we have

$$QQQ\ell, \quad d^c d^c u^c N^c \rightarrow 3q_{10} + q_{\bar 5}$$
$$= 4 - q_{h_u} - q_{h_d} = 0 \neq 2 \mod(N),$$
$$u^c u^c d^c e^c \rightarrow 2q_{\bar 5} + q_{10} + q_1$$
$$= 4 - q_{h_u} - q_{h_d} = 0 \neq 2 \mod(N).$$

In order to conclude whether dangerous $D = 3, 4$ operators are present, we need to proceed further with the matter and Higgs charge assignments for each particular value of N.

$N = 3, 6$. In these cases the first of the conditions in Eq. (15) implies that $\rho = 0$. Thus, for these cases the analysis will coincide with the one carried on previously in anomaly-free case without the operation of the GS mechanism.

$N = 4$. The condition $q_{h_u} + q_{h_d} = 0 \mod(4)$ can only be satisfied with even charges. These are the choices $q_{h_u} = q_{h_d} = 0$ and $q_{h_u} = q_{h_d} = 2$. The first choice leads to two $SO(10)$-invariant solutions with identical phenomenology. Note that these solutions allow for a right-handed neutrino Majorana mass. All constraints for the absence of $D = 3, 4$ operators are met. The second choice is discarded due to the presence of unwanted $D = 3, 4$ operators. The allowed cases are listed in Table III.

$N = 8, 12, 24$. The condition $q_{h_u} + q_{h_d} = 4 \mod(N)$ allows again only even charges and can in principle be satisfied for various sets of Higgs charges. Among them, two distinct general solutions with $D = 3, 4$ operators

TABLE III. Allowed flipped charges for $\mathcal{Z}_4^{(R)}$.

N	q_Q	q_{d^c}	q_{N^c}	q_ℓ	q_{u^c}	q_{e^c}	q_{h_u}	q_{h_d}	$\tilde{q}_{h_u}$	$\tilde{q}_{h_d}$
4	1	1	1	1	1	1	0	0	4	8
4	3	3	3	3	3	3	0	0	4	8

TABLE V. $\mathcal{Z}_5^{(R)}$ charges.

$N = 5$	q_Q	q_ℓ	q_{d^c}	q_{u^c}	q_{e^c}	q_{N^c}	q_S	q_{h_u}	q_{h_d}
	4	4	1	1	1	1	3	2	2

absent, are listed in Table IV, where, of course, these charges can always be modulo shifted to lie in the first modulo of the given symmetry. The charge $\tilde{q}_{N^c} = 1$ of the second row allows always for a right-handed neutrino Majorana mass term.

A "*nonunified*" case ($\mathcal{Z}_5^{(R)}$).—The above found set of allowed $\mathcal{Z}_N$ does not exclude other "*nonunified*" possibilities. The anomaly coefficients for nonunified charges are

$$\mathcal{A}_3 = 3\left(q_Q + \frac{1}{2}q_{d^c} + \frac{1}{2}q_{u^c} - 2\right) + 3,$$

$$\mathcal{A}_2 = 3\left(\frac{3}{2}q_Q + \frac{1}{2}q_\ell - 2\right) + \frac{\tilde{N}}{2}(q_{h_d} + q_{h_u} - 2) + 2,$$

$$\mathcal{A}_1 = \frac{9}{5}\left(\frac{1}{6}q_Q + \frac{1}{3}q_{d^c} + \frac{4}{3}q_{u^c} + \frac{1}{2}q_\ell + q_{e^c} - \frac{10}{3}\right)$$
$$+ \frac{3\tilde{N}}{10}(q_{h_d} + q_{h_u} - 2), \tag{17}$$

where $\tilde{N}$ is the number of Higgs isodoublets. A particularly interesting case is that of a $\mathcal{Z}_5$ discrete R symmetry, motivated by the solution to the μ problem in terms of a minimal singlet extension of MSSM. The assigned charges [6] are shown in Table V.

For these charges the anomaly coefficients are

$$\mathcal{A}_3 = \rho = 2\,\mathrm{mod}(5), \qquad \mathcal{A}_2 = \tilde{N}\,\mathrm{mod}(5),$$
$$\mathcal{A}_1 = \frac{24}{5} + \frac{3N_\sigma}{5}(q_\sigma + q_{\bar\sigma} - 2)\,\mathrm{mod}(5), \tag{18}$$

where we have introduced tentatively N_σ pairs of hypercharge ± 1 singlets. Finally, the condition on $\mathcal{A}_0 = 70 + N_\sigma(q_\sigma + q_{\bar\sigma} - 2) + \sum_s(q_s - 1)$ reads $2 + N_\sigma(q_\sigma + q_{\bar\sigma} - 2) + \sum_s(q_s - 1) = 0\,\mathrm{mod}(5)$. The anomaly conditions corresponding to Eq. (18) can be met for $\tilde{N} = 2$, $N_\sigma = 1$ and the shifted charges[7] $q_\sigma = q_{\bar\sigma} = 7$. The gravitational anomaly condition can also be met if, apart from the charge-3 singlet S, we also introduce an additional neutral singlet. Note that for these charges all possible dangerous terms are disallowed ($\sigma\bar\sigma$, $e^c\sigma$, $\ell\bar\sigma h_d$, $h_d d_d\bar\sigma$, $h_u h_u\sigma$). Note also that a term $\sigma\bar\sigma S$ is allowed. Apart from being anomaly-free, this model allows for all

TABLE IV. Allowed (shifted) flipped charges for $\mathcal{Z}_8^{(R)}$, $\mathcal{Z}_{12}^{(R)}$, $\mathcal{Z}_{24}^{(R)}$.

N	$\tilde{q}_Q$	$\tilde{q}_{d^c}$	$\tilde{q}_{N^c}$	$\tilde{q}_\ell$	$\tilde{q}_{u^c}$	$\tilde{q}_{e^c}$	$\tilde{q}_{h_u}$	$\tilde{q}_{h_d}$
8, 12, 24	−1	−1	−1	3	3	−5	24	28
8, 12, 24	1	1	1	−3	−3	5	28	24

standard terms, including neutrino Majorana masses, and forbids all unwanted $D \leq 5$ terms. The extra charged singlets introduced here for the sake of anomaly cancellation can obtain a mass through the vacuum expectation value (VEV) of the singlet S.

IV. UNIFICATION

Can these discrete symmetries be incorporated in a 4D grand unified theory? There are convincing arguments that, for a simple gauge group and MSSM particle content at low energies, this is not possible [15]. We shall depart from both of these assumptions and allow on the one hand, additional matter at low energies, beyond MSSM, and on the other hand go beyond simple unifying groups. Specifically, we shall consider $SU(5) \times U(1)$ and use "*flipped*" $\mathcal{Z}_N$-charge assignments commuting with it. One of the motivations for this model is that it is accompanied by an elegant mechanism for *triplet-doublet splitting*. Note that this is one of the main problems that one has to face in promoting MSSM to a GUT endowed with the discussed discrete R symmetries.

Minimal case.—The standard matter content of $SU(5) \times U(1)$ comes in three copies of $F(10, 1)$, $f^c(\bar{5}, -3)$, $\ell^c(1, 5)$ with corresponding $\mathcal{Z}_N$ charges q_{10}, $q_{\bar 5}$, q_1. The standard Higgs content is $h(5, -2; q_h)$, $h^c(\bar{5}, 2; q_{h^c})$, $H(10, 1; 0)$, $\bar{H}(\overline{10}, -1; 0)$. The last pair, through its nonzero VEV, will achieve the breaking down to MSSM. Obviously, it has to be neutral under the discrete symmetry. The standard matter couplings are

$$FFh + Ff^c h^c + f^c \ell^c h^c. \tag{19}$$

Nevertheless, in order to realize triplet-doublet splitting, the couplings $HHh + \bar{H}\bar{H}h^c$ are necessary. These couplings force the Higgs charges

$$q_h = q_{h^c} = 2\,\mathrm{mod}(N). \tag{20}$$

Note that these charges do not allow a μ term. Nevertheless, enforcing the mass conditions

$$2q_{10} + q_h = q_{10} + q_{\bar 5} + q_{h^c} = q_{\bar 5} + q_1 + q_h$$
$$= 2\,\mathrm{mod}(N), \tag{21}$$

along with the absence of the potentially dangerous operators hh^c, $H\bar{H}$, $F\bar{H}$, FHh, $Hf^c h^c$, we are led to the $SO(10)$-invariant assignments

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[7]Another working choice is $N_\sigma = 2$ and $q_\sigma = q_{\bar\sigma} = 4$.

$$q_{10} = q_{\bar{5}} = q_1 = \frac{N}{2} \bmod(N)(N > 4). \tag{22}$$

Note that, since the charges have to be integers, only even values of N are allowed. Note also that the case $N = 4$, in which all matter fields have charge 2 mod(4), is excluded since it allows the operator $F\bar{H}$. The anomaly coefficient conditions for general even $N > 4$ are

$$\mathcal{A}_5 = 3N - 3 = \rho \bmod(N/2),$$
$$\mathcal{A}_X = 3N - 3 - \frac{5}{2} = \rho \bmod(N/2), \tag{23}$$

and anomaly cancellation cannot be met without introducing extra matter. It can also be checked that anomaly cancellation cannot be saved by shifted charges.

Extra fields.—In what follows, we shall introduce extra matter and at the same time avoid the $SO(10)$-symmetric assignment enforced by Eq. (20) in order to realize the split assignments found in the previous section. In order to avoid Eq. (20), we introduce an extra pair of Higgs fields $H'(10, 1; q_{H'})$, $\bar{H}'(\overline{10}, -1; q_{\bar{H}'})$. Assuming that the only nonzero VEVs are those[8] of $\langle H \rangle = \langle \bar{H} \rangle$ and assuming that the couplings $HH'h$, $\bar{H}\bar{H}'h^c$ are present, after symmetry breaking, the pairs $d_h(3, 1, -1/3; q_h)$, $d_{H'}^c(\bar{3}, 1, 1/3; q_{H'})$, and $d_{h^c}^c(\bar{3}, 1, 1/3; q_{h^c})$, $d_{\bar{H}'}(3, 1, -1/3; q_{\bar{H}'})$ will obtain a large mass and will be removed from the spectrum. In addition, the fields Q_H, $Q_{\bar{H}}^c$, $N_H^c - N_{\bar{H}}^c$ will be "higgsed away." The surviving fields, apart from the standard matter and a neutral Higgs singlet, will be a "hybrid" pair $10' + \overline{10}'$ composed out of the $H'(\bar{H}')$ with their $d_{H'}^c(d_{\bar{H}'})$ replaced by $d_H^c(d_{\bar{H}})$. The conditions on the charges resulting from the existence of the standard couplings

$$FFh + Ff^c h^c + f^c \ell^c h^c + HH'h + \bar{H}\bar{H}'h^c \tag{24}$$

are given by Eq. (21) and

$$q_{H'} + q_h = q_{\bar{H}'} + q_{h^c} = 2 \bmod(N). \tag{25}$$

Nevertheless, a number of unwanted terms are still allowed by gauge symmetry; namely, the terms

$$hh^c + H'\bar{H} + \bar{H}'H + \bar{H}'H' + F\bar{H} + F\bar{H}' + H^2 h$$
$$+ \bar{H}^2 h^c + FHh + FH'h + Hf^c h^c + H'f^c h^c, \tag{26}$$

which are expected to be removed by the discrete symmetry.

Next let us consider the anomaly coefficients. The gauge anomaly coefficients are

$$\mathcal{A}_5 = 5 + 3\left(\frac{3}{2}q_{10} + \frac{1}{2}q_{\bar{5}} - 2\right) + \frac{3}{2}(q_{H'} + q_{\bar{H}'} - 4)$$
$$+ \frac{1}{2}(q_h + q_{h^c} - 2),$$

$$\mathcal{A}_X = \frac{3}{40}(10q_{10} + 45q_{\bar{5}} + 25q_1 - 80)$$
$$+ \frac{1}{4}(q_{H'} + q_{\bar{H}'} - 4) + \frac{1}{2}(q_h + q_{h^c} - 2), \tag{27}$$

or, applying the conditions (21) and (25),

$$\mathcal{A}_5 = 4 - \frac{5}{2}(q_h + q_{h^c}) = \rho \bmod(\eta),$$
$$\mathcal{A}_X = -1 - \frac{5}{4}(q_h + q_{h^c}) = \rho \bmod(\eta). \tag{28}$$

Taking the familiar condition

$$q_h + q_{h^c} = 4 \bmod(N), \tag{29}$$

we obtain

$$\mathcal{A}_5 = \mathcal{A}_X = -6, \tag{30}$$

which satisfies the anomaly condition for any N.

The gravitational coefficient is[9] $\mathcal{A}_0 = -7 - 17(q_h + q_{h^c}) + \sum_s (q_s - 1)$ and the corresponding condition becomes

$$\mathcal{A}_0 - 24\mathcal{A}_5 = 69 + \sum_s (q_s - 1) = 0 \bmod(\eta) \tag{31}$$

and can be satisfied, among other choices, for any N with a single neutral singlet of shifted charge $\tilde{q}_s = -68$. For all symmetries which are divisors of 138, this extra singlet is not required.

Note that the $D = 3, 4$ baryon- and lepton-violating operators of the standard field content ℓh_u, $Q\ell d^c$, $u^c d^c d^c$, although not directly present due to the gauge symmetry, may appear through higher-dimensional operators.[10] Thus, their absence will eventually be determined by their respective $\mathcal{Z}_N$ charges. On the other hand, the analogous $D = 5$ operators $FFFf^c$, $f^c f^c F\ell^c$ are always absent since they have charges $3q_{10} + q_{\bar{5}}$ and $2q_{\bar{5}} + q_{10} + q_1$, both equal to $4 - q_{h^c} - q_h = 0 \bmod(N)$, due to Eq. (29).

Taking Eq. (29) as our starting point, we proceed to consider phenomenologically allowed charge assignments. For even N, a general assignment that satisfies Eq. (29) and is compatible with Eqs. (21) and (25) of *shifted charges* is

$$q_h = -2, \qquad q_{h^c} = 6, \qquad q_{H'} = 4, \qquad q_{\bar{H}'} = -4,$$
$$q_{10} = 2 + N/2, \qquad q_{\bar{5}} = -6 + N/2, \qquad q_1 = 10 + N/2. \tag{32}$$

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[8]Thus, assuming a $\mathcal{Z}_N^{(R)}$-invariant vacuum.

[9]The contribution of a dilatino (axino) is also considered.
[10]$f^c h^c$, FFf^c are not gauge singlets but $Hf^c h^c$, $HFFf^c$ are.

TABLE VI. Allowed $\mathcal{Z}_{10}^{(R)}$, $\mathcal{Z}_{12}^{(R)}$ charges for $SU(5) \times U(1)_X$.

N	q_{10}	$q_{\bar{5}}$	q_1	q_{h^c}	q_h	$q_{H'}$	$q_{\bar{H}'}$
10	7	9	5	6	8	4	6
12	8	0	4	6	10	4	8

TABLE VII. Allowed $\mathcal{Z}_7^{(R)}$, $\mathcal{Z}_9^{(R)}$ charges for $SU(5) \times U(1)_X$.

N	q_{10}	$q_{\bar{5}}$	q_1	q_{h^c}	q_h	$q_{H'}$	$q_{\bar{H}'}$
7	4	2	6	3	1	1	6
9	5	3	7	3	1	1	8

The anomaly constraints are readily satisfied[11] $\mathcal{A}_5 = \mathcal{A}_X = 3N - 6$. Next, we demand that the dangerous terms (26) are absent. Their charges are

$$hh^c \to 4, \quad H'\bar{H} \to 4, \quad \bar{H}'H \to -4,$$
$$\bar{H}'H' \to 0, \quad F\bar{H} \to 2 + N/2, \quad F\bar{H}' \to -2 + N/2,$$
$$H^2h \to -2, \quad \bar{H}^2h^c \to 6, \quad FHh \to N/2,$$
$$FH'h \to 4 + N/2, \quad Hf^ch^c \to N/2,$$
$$H'f^ch^c \to 4 + N/2, \tag{33}$$

and they should be $\neq 2\,\mathrm{mod}(N)$. The absence of these dangerous operators leads to the constraints $N \neq 4; 6; 8$, while the third does not give any additional constraint. Therefore, the allowed even values of N are

$$N = 2k \geq 10. \tag{34}$$

Note that, since F^2 has charge $4 + N$, no right-handed neutrino Majorana mass is allowed. Furthermore, the dangerous $D = 3, 4$ operators are absent since

$$\ell h_u \to N/2, \quad Q\ell d^c, \quad d^cd^cu^c \to N/2 - 2 + N, \tag{35}$$

which are $\neq 2\,\mathrm{mod}(N)$ for the allowed symmetries. The corresponding charges for the cases $N = 10, 12$ are shown in Table VI.

Next, starting again from Eq. (29), we proceed to investigate possible odd values of N and consider a general assignment of *shifted* charges compatible with Eqs. (21) and (25)

$$q_h = 1, \quad q_{h^c} = 3, \quad q_{H'} = 1, \quad q_{\bar{H}'} = -1,$$
$$q_{10} = (N+1)/2, \quad q_{\bar{5}} = (N-3)/2, \quad q_1 = (N+5)/2. \tag{36}$$

The anomaly constraints are readily satisfied[12] $\mathcal{A}_5 = \mathcal{A}_X = 3N - 6$. Again, we demand that the dangerous terms (26) are absent. Their charges are

$$hh^c \to 4, \quad H'\bar{H} \to 1, \quad \bar{H}'H \to -1, \quad \bar{H}'H' \to 0,$$
$$F\bar{H} \to (N+1)/2, \quad F\bar{H}' \to (N-1)/2, \quad H^2h \to 1,$$
$$\bar{H}^2h^c \to 3, \quad FHh \to (N+3)/2, \quad FH'h \to (N+5)/2,$$
$$Hf^ch^c \to (N+3)/2, \quad H'f^ch^c \to (N+5)/2, \tag{37}$$

and they should be $\neq 2\,\mathrm{mod}(N)$. Absence of these dangerous operators leads now to the constraints $N \neq 3;5$, while the third does not supply us with any additional restriction. Thus, the allowed odd values of N are

$$N = 2k + 1 \geq 7. \tag{38}$$

Again, since F^2 has charge $N + 1$, no right-handed neutrino Majorana mass is allowed. The $D = 3,4$ operators are also absent. In Table VII we show the corresponding charges for the cases $N = 7, 9$.

It can also be checked that for any other Higgs charge assignments in the range $(0,7)$ satisfying $q_h + q_{h^c} = 4\,\mathrm{mod}(N)$, the corresponding phenomenologically viable models also forbid the symmetries $\mathcal{Z}_3$, $\mathcal{Z}_4$, $\mathcal{Z}_5$, $\mathcal{Z}_6$, $\mathcal{Z}_8$. This fact is sufficient to forbid these symmetries for all possible models with $q_h + q_{h^c} = 4\,\mathrm{mod}(N)$ although departing from this relation may in principle allow some of them.

Before closing this section it is interesting to note that the above list of symmetries does not exhaust all possible symmetries for the given gauge group. As an example, consider the model of Ref. [16] that is characterized by a radiative breaking of the $SU(5) \times U(1)$ symmetry. This model, having the same set of fields as the model considered above plus gauge singlets, possesses the discrete symmetry $\mathcal{Z}_3^{(R)} \times \mathcal{Z}_2$, which can be readily promoted to be anomaly-free at the expense of introducing a massive pair of hypercharge ± 1 singlets of $\mathcal{Z}_3^{(R)}$ charge equal to 2.

V. CONCLUSIONS

In the present article we have reconsidered the issue of possible anomaly-free discrete R symmetries $\mathcal{Z}_N^{(R)}$ that avoid the μ problem and the dangerous $D \leq 5$ operators within MSSM and extensions of it. Freedom from anomalies was considered either through strictly vanishing anomaly coefficients for the (gauge)$2\mathcal{Z}_N$ anomalies or through the operations of the *Green-Schwarz mechanism* for the former as well as the (grav)$^2\mathcal{Z}_N$ anomalies. We have extended known investigations by considering charge assignments, that do not commute with the standard $SU(5)$ or $SO(10)$ gauge groups but are, instead, compatible with a

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[11]Note that the anomaly coefficients for the low-energy spectrum with the new extra matter satisfy also $\mathcal{A}_3 = \mathcal{A}_2 = \mathcal{A}_1 = 3N - 6$.

[12]Again, the anomaly coefficients for the low-energy spectrum with the new extra matter satisfy also $\mathcal{A}_3 = \mathcal{A}_2 = \mathcal{A}_1 = 3N - 6$.

so-called flipped $SU(5)$ symmetry. Staying within the framework of MSSM, we have found that, in the anomaly-free case, the symmetries $\mathcal{Z}_3^{(R)}$, $\mathcal{Z}_6^{(R)}$ with flipped assignments are possible. We have also investigated the possibility of multidoublet extensions of MSSM in this case. Phenomenologically acceptable charge assignments have been listed. In the same framework, in the case of GS-anomaly cancellation, we have arrived at phenomenologically allowed $\mathcal{Z}_N^{(R)}$ symmetries with flipped assignments for $N = 3$, 4, 6, 8, 12, 24. Phenomenologically acceptable charges for these cases have been listed. As an example, discrete symmetries noncommuting with any of the above unifying symmetries, we have also considered a $\mathcal{Z}_5^{(R)}$ symmetry with nonunified charges in the framework of a singlet extension of MSSM. We have also considered the question of finding such symmetries for traditional 4D grand unified models. Having excluded simple gauge groups such as $SU(5)$ or $SO(10)$ or nonsimple groups like $SU(5) \times U(1)_X$ with MSSM-low-energy content, we arrived at an extended flipped $SU(5) \times U(1)_X$ model. For this model, $\mathcal{Z}_N^{(R)}$ symmetries were shown to be anomaly-free (through GS) and phenomenologically viable for $N = 2k + 7$ and $N = 2k + 10$.

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