

DISCRETE SYMMETRIES AND DIRAC-NEUTRINO MASS HIERARCHIES

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When the supersymmetric standard model is endowed with many Higgs isodoublets, as in compactified superstring models, suitable generation-dependent discrete symmetries specialize the form of the Higgs and neutrino Dirac mass matrices so that small eigenvalues arise. Models based on Z_3 and $Z_3 \times Z_3$ symmetries are constructed in which a two-step seesaw mechanism among Dirac masses in generation space results in hierarchically light neutrinos.

The various extensions of the supersymmetric standard model that are either inspired by or connected to compactified superstrings [1] with an E_6 visible-sector gauge symmetry [2-4], contain ordinary matter in N copies of the 27 representation of E_6 plus n , possibly incomplete, copies of $27 + \bar{27}$ representations. Quarks and leptons are accompanied by Higgs isodoublets as well as by new fields (D, D^c, η, ν^c) not contained in the minimal version of the standard model. More specifically, the decomposition of the 27 representation under $SU(3)_C \times SU(2)_L \times U(1)_Y$ is

$$\begin{aligned}
 & q(3, 2, \frac{1}{6}) + \ell(1, 2, -\frac{1}{2}) + d^c(\bar{3}, 1, \frac{1}{3}) \\
 & + u^c(\bar{3}, 1, -\frac{2}{3}) + e^c(1, 1, 1) + h(1, 2, -\frac{1}{2}) \\
 & + h^c(1, 2, \frac{1}{2}) + D(3, 1, -\frac{1}{3}) + D^c(\bar{3}, 1, \frac{1}{3}) \\
 & + \eta(1, 1, 0) + \nu^c(1, 1, 0). \tag{1}
 \end{aligned}$$

The superpotential obtained after flux-breaking is in general of the form

$$W(27, \bar{27}) = (27)^3 + (\bar{27})^3 + \frac{1}{M_c} (27)^2(\bar{27})^2 + \dots \tag{2}$$

with the Yukawa couplings of $SU(3)_C \times SU(2)_L \times U(1)_Y$ fields taking arbitrary values, i.e. not related by E_6 -symmetry, but related to the detailed topology of the compactification manifold. Among the topological properties of the compactifi-

cation manifold there may be discrete symmetries that can forbid some of the Yukawa couplings of the low-energy superpotential. Lack of knowledge about the dynamics of compactification as well as the existence of many possible compactification manifolds makes it difficult to actually evaluate the Yukawa couplings of light matter or at least uncover any existing global symmetries. For example, the simultaneous existence of all terms in (27)³,

$$\begin{aligned}
 & qd^c h + qu^c h^c + \ell e^c h + \ell \nu^c h^c + DD^c \eta + hh^c \eta \\
 & + qqD + q\ell D^c + u^c d^c D^c + u^c e^c D + d^c \nu^c D, \tag{3}
 \end{aligned}$$

would imply obvious phenomenological disasters, as baryon number and/or lepton number violation, associated directly to the D and D^c Yukawa couplings, which are usually avoided in the literature by imposing a suitable discrete symmetry and assuming that there exists a compactification manifold and scheme endowed with this property [5]^{#1}. In addition, there are other uncertainties related to whether the $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet fields η and ν^c acquire vacuum expectation values at an intermediate scale or not.

In the list of open problems of this class of models one should include the problem of obtaining light neutrinos in a natural way. The absence of a Majorana mass term for the right-handed neutrino ν^c leaves room only for a Dirac mass $\ell \nu^c \langle h^c \rangle$ which is not in

^{#1} For a review see ref. [6].

any obvious way smaller than the corresponding mass $\ell e^c \langle h \rangle$ for charged leptons. Among the solutions proposed, the most appealing one employs a discrete symmetry that forbids the neutrino Dirac mass altogether [7]. In case this discrete symmetry is broken by other terms, radiative corrections will generate a naturally small and phenomenologically acceptable Dirac mass for the neutrino. Other existing solutions to this problem employ singlet fields from $27 + \bar{27}$ incomplete representations that contribute with non-renormalizable terms and modify the structure of the neutral mass matrix so that small eigenvalues arise.

In models with a vacuum expectation value for the singlet η at an intermediate scale, Higgs mixing at that scale through the term $h h^c \langle \eta \rangle$ induces a large mass for h, h^c . That would deplete the theory from light Higgs isodoublets necessary for the electroweak breaking. On the other hand, a small, i.e. $O(100 \text{ GeV})$, Higgs mixing term is desirable in order to realize the $SU(2)_L \times U(1)_Y$ breaking. Therefore, this term should be present and small. At the same time we should keep in mind that it is through a similar term ($DD^c\eta$), that D-quarks get their mass.

Specific superstring inspired models can circumvent some of these problems in one way or the other, usually at the expense of assumptions or introduction of extra fields from $27 + \bar{27}$. In contrast, one could try to be general and constrain oneself within the known $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group and the particle content of the 27 representation in N copies, as the minimal set of surviving particles below the compactification scale. Having at hand more Higgs isodoublets that one needs for the electroweak breaking can be used to our advantage. Postulating suitable discrete symmetries that are generation-dependent one constrains the form of Higgs and standard fermion mass matrices so that, under certain assumptions, to a phenomenologically acceptable neutrino mass pattern can be obtained.

Let us consider a two-family model with the light-particle content of the 27 representation and the superpotential shown in (3). If we impose no extra symmetries, all family mixings are allowed but we can still make an assumption about their relative size. Let us assume that all off-diagonal couplings are at least an order of magnitude smaller than the diagonal ones. This is not an a priori unreasonable assumption, but within our framework, it can only be justified by its

consequences. For example, the Higgs mixing term is

$$\lambda_1 h_1 h_1^c \eta_1 + \lambda_2 h_1 h_1^c \eta_2 + \lambda_3 h_1 h_2^c \eta_1 + \lambda_4 h_1 h_2^c \eta_2 + \lambda_5 h_2 h_1^c \eta_1 + \lambda_6 h_2 h_1^c \eta_2 + \lambda_7 h_2 h_2^c \eta_1 + \lambda_8 h_2 h_2^c \eta_2. \tag{4}$$

Our assumption reads

$$\lambda_2, \lambda_3, \dots, \lambda_7 \lesssim \epsilon (\lambda_1, \lambda_8) \tag{5}$$

with $\epsilon \approx 0.1$.

If we demand now that the superpotential is invariant under a discrete symmetry, the allowed couplings are strongly restricted. For example, under a Z_2 symmetry $h_1, h_1^c, \eta_1, h_2, h_2^c, \eta_2 \rightarrow h_1, h_1^c, \eta_1, -h_2, -h_2^c, -\eta_2$, the allowed superpotential would be

$$\lambda_1 h_1 h_1^c \eta_1 + \lambda_4 h_1 h_2^c \eta_2 + \lambda_6 h_2 h_1^c \eta_2 + \lambda_7 h_2 h_2^c \eta_1. \tag{6}$$

Instead of Z_2 it will prove more interesting to postulate a larger symmetry. Let us introduce a Z_3 symmetry with the particle-transformation properties shown in table 1. Under Z_3 the Higgs mixing term is restricted to

$$\lambda_3 h_1 h_2^c \eta_1 + \lambda_5 h_2 h_1^c \eta_1 + \lambda_8 h_2 h_2^c \eta_2. \tag{7}$$

We shall assume that the singlet expectation values $\langle \eta_1 \rangle$ and $\langle \eta_2 \rangle$ are approximately equal and that they are at a scale neighboring to the scale of electroweak breaking. More specifically we shall assume that

Table 1

	Z_3
h_1	1
h_1^c	α
ℓ_1	α
e_1^c	α^2
ν_1^c	α
η_1	α
h_2	α
h_2^c	α^2
ℓ_2	1
e_2^c	α^2
ν_2^c	1
η_2	1
q_1, q_2	α
d_1^c, d_2^c	α^2
u_1^c, u_2^c	α
D_1^c, D_2^c	α
D_1, D_2	α^2

$\langle \eta_1 \rangle \simeq \langle \eta_2 \rangle \simeq 100$ TeV. Then, the Higgs mass matrix takes the form

$$\begin{pmatrix} 0 & m_1 \\ m_2 & M \end{pmatrix}, \quad (8)$$

with $m_1 = \lambda_3 \langle \eta_1 \rangle$, $m_2 = \lambda_5 \langle \eta_1 \rangle$ and $M = \lambda_8 \langle \eta_2 \rangle$. According to our assumption $\lambda_3 \approx \lambda_5 \approx \epsilon \lambda_8$. Consequently the approximate mass eigenvalues are

$$m_+(h) \simeq M, \quad m_-(h) \simeq \frac{m_1 m_2}{M} = \frac{\lambda_5 \lambda_3}{\lambda_8^2} M \simeq \epsilon^2 M.$$

The corresponding eigenfields are

$$h_+ \simeq h_2 + \epsilon h_1, \quad h_- \simeq h_1 - \epsilon h_2.$$

At least a pair of Higgs isodoublets must be light enough to affect the electroweak breaking i.e. its mixing mass must be present but no bigger than the order of magnitude of the effective supersymmetry breaking radiative corrections. In our case, we can naturally have $m_+ = M \approx 10$ TeV while $m_- \approx 100$ GeV. As a result, only h_- can obtain a vacuum expectation value. Any negative mass-square contribution by radiative corrections to h_+ will be overpowered by its large tree-level positive mass-square and therefore will stay at zero expectation value.

Hence, we conclude that

$$\langle h_+ \rangle = \langle h_+^c \rangle = 0, \quad \langle h_- \rangle = \langle h_-^c \rangle = V \quad (9)$$

or

$$\langle h_1 \rangle = \langle h_1^c \rangle \simeq V, \quad \langle h_2 \rangle = \langle h_2^c \rangle \simeq -\epsilon V. \quad (10)$$

Under the postulated Z_3 symmetry, the neutral and charged-lepton mass terms are

$$g_1 \ell_1 \nu_1^c h_1^c + g_4 \ell_1 \nu_2^c h_2^c + g_6 \ell_2 \nu_1^c h_2^c + f_1 \ell_1 e_1^c h_1 + f_3 \ell_1 e_2^c h_1 + f_c \ell_2 e_1^c h_2 + f_8 \ell_2 e_2^c h_2. \quad (11)$$

If we denote by g and f generic values for the diagonal Yukawa couplings, according to our assumptions the neutrino mass matrix will be of the form

$$gV \begin{pmatrix} 1 & -\epsilon^2 \\ -\epsilon^2 & 0 \end{pmatrix}, \quad (12)$$

with Dirac mass eigenvalues $m(\nu_+) \simeq gV$ and $m(\nu_-) \simeq -\epsilon^4(gV)$. The corresponding eigenfields are

$$(\nu_+, \nu_+^c) \simeq (\nu_1, \nu_1^c) + \epsilon^2(\nu_2, \nu_2^c),$$

$$(\nu_-, \nu_-^c) \simeq (\nu_2, \nu_2^c) - \epsilon^2(\nu_1, \nu_1^c).$$

On the other hand, the charged-lepton mass matrix

$$(fV) \begin{pmatrix} 1 & \epsilon \\ -\epsilon^2 & -\epsilon \end{pmatrix} \quad (13)$$

leads to approximate eigenvalues $m(e_+) \simeq fV$ and $m(e_-) \simeq -\epsilon(fV)$, which are dominantly e_1 , e_1^c and e_2 , e_2^c .

It is interesting that the neutrino Dirac mass hierarchy is three orders of magnitude larger than the charged lepton one,

$$\frac{m(\nu_-)}{m(\nu_+)} \simeq 10^{-4}, \quad \frac{m(e_-)}{m(e_+)} \simeq 10^{-1}, \quad (14)$$

although they are both in the same direction.

The model was constructed in order to illustrate how a discrete symmetry can affect favourably the lepton mass matrices so that light neutrinos arise. It can be completed by including quarks with transformation properties under Z_3 also shown in table 1. The allowed superpotential is of the form

$$\begin{aligned} & q_1 d_1^c h_1 + q_1 d_2^c h_1 + q_2 d_1^c h_1 + q_2 d_2^c h_1 \\ & + q_1 u_1^c h_1^c + q_1 u_2^c h_1^c + q_2 u_1^c h_1^c + q_2 u_2^c h_1^c \\ & + D_1 D_1^c \eta_2 + D_1 D_2^c \eta_2 + D_2 D_1^c \eta_2 + D_2 D_2^c \eta_2 \\ & + q_1 \ell_1 D_1^c + q_1 \ell_1 D_2^c + q_2 \ell_1 D_1^c + q_2 \ell_1 D_2^c. \end{aligned} \quad (15)$$

The ordinary quark mass matrices are of the form

$$\begin{pmatrix} 1 & \epsilon \\ \epsilon & \epsilon \end{pmatrix},$$

while all D-quarks get masses of order $\lambda \langle \eta_2 \rangle$. The baryon and lepton number is strictly conserved if we assign $B(D^c) = -B(D) = -\frac{1}{3}$ and $L(D^c) = -L(D) = -1$.

The key ideas illustrated above can be exploited further in models with an intermediate scale in order to obtain an even larger mass hierarchy. In the following we shall consider a four-family model and postulate a $Z_3 \times Z_3$ discrete symmetry. The transformation properties of the various particles are listed in table 2. Starting from the Higgs mixing term $\lambda_{ijk} h_i h_j^c \eta_k$ and applying invariance under $Z_3 \times Z_3$, we obtain the restricted couplings

Table 2

	$Z_3 \times Z_3$
h_1, h_1^c	$(1, \beta)$
h_2, h_2^c	$(1, \beta^2)$
h_3, h_3^c	(α, β)
h_4, h_4^c	$(\alpha^2, 1)$
η_1	(α^2, β^2)
η_2	(α, β)
η_3	(α, β^2)
η_4	$(\alpha^2, 1)$
ℓ_1, ν_1^c	(α^2, β)
ℓ_2, ν_2^c	$(1, 1)$
ℓ_3, ν_3^c	$(1, \beta^2)$
ℓ_4, ν_4^c	$(1, \beta)$
e_1^c, e_1^s	(α^2, β^2)
e_2^c, e_2^s	$(1, 1)$
e_3^c, e_3^s	$(1, \beta^2)$
e_4^c, e_4^s	$(1, \beta)$
q_1, d_1^c, u_1^c	$(1, \beta)$
q_2, d_2^c, u_2^c	$(1, \beta^2)$
q_3, d_3^c, u_3^c	(α, β)
q_4, d_4^c, u_4^c	$(\alpha^2, 1)$
D_i, D_i^c	(α^2, β^2)

$$\begin{aligned} & \lambda_1 h_1 h_4^c \eta_3 + \lambda_2 h_2 h_3^c \eta_4 + \lambda_3 h_2 h_4^c \eta_2 + \lambda_4 h_3 h_3^c \eta_4 \\ & + \lambda_5 h_3 h_3^c \eta_2 + \lambda_6 h_4 h_1^c \eta_3 + \lambda_7 h_4 h_2^c \eta_2 + \lambda_8 h_4 h_4^c \eta_4. \end{aligned} \quad (16)$$

When the $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet fields $\eta_1, \eta_2, \eta_3, \eta_4$ obtain a non-vanishing vacuum expectation value, the resulting mass matrix for the Higgs isodoublets is

$$\begin{pmatrix} 0 & 0 & 0 & \lambda_1 \langle \eta_3 \rangle \\ 0 & 0 & \lambda_2 \langle \eta_4 \rangle & \lambda_3 \langle \eta_2 \rangle \\ 0 & \lambda_4 \langle \eta_4 \rangle & \lambda_5 \langle \eta_2 \rangle & 0 \\ \lambda_6 \langle \eta_3 \rangle & \lambda_7 \langle \eta_2 \rangle & 0 & \lambda_8 \langle \eta_4 \rangle \end{pmatrix}. \quad (17)$$

Again, we assume that off-diagonal couplings are an order of magnitude smaller than diagonal ones. Further, we assume that couplings involving two generations are $O(\epsilon)$ smaller than those involving just one generation. Couplings involving three generations are even smaller by a factor of ϵ . Thus, we have

$$\lambda_1, \lambda_2, \lambda_4, \lambda_6 \lesssim \epsilon(\lambda_3, \lambda_5, \lambda_7) \lesssim \epsilon^2 \lambda_8,$$

with $\epsilon \approx 0.1$.

On the other hand it is reasonable to assume that

$$\langle \eta_1 \rangle = \langle \eta_2 \rangle = \langle \eta_3 \rangle = \langle \eta_4 \rangle$$

and consequently, the Higgs mass matrix is

$$\lambda_8 \langle \eta \rangle \begin{pmatrix} 0 & 0 & 0 & \epsilon_1^2 \\ 0 & 0 & \epsilon_2^2 & \epsilon_3 \\ 0 & \epsilon_4^2 & \epsilon_5 & 0 \\ \epsilon_6^2 & \epsilon_7 & 0 & 1 \end{pmatrix}, \quad (18)$$

where $\epsilon_i^2 = \lambda_i / \lambda_8$ ($i=1, 2, 4, 6$) and $\epsilon_i = \lambda_i / \lambda_8$ ($i=3, 5, 7$).

The eigenvalues of this matrix are, in an order of magnitude estimate, $1, \epsilon, -\epsilon^2, -\epsilon^5$.

If we take the singlet expectation value $\langle \eta \rangle$ at an intermediate scale $\langle \eta \rangle \approx 10^9$ GeV, then the resulting mass hierarchy of Higgses is for $\lambda \approx 0.01$.

$$10^7 : 10^6 : 10^5 : 10^2 \text{ GeV}$$

and only the eigenfield which is dominantly h_4, h_4^c and corresponds to the smallest eigenvalue can obtain a negative mass-square from the supersymmetry breaking and effect the electroweak breaking. It is not difficult to estimate that, neglecting $O(\epsilon^3)$ terms, we have for the Higgs eigenfields

$$\begin{aligned} h(1) & \simeq h_4 + \epsilon h_2 + \epsilon^2 h_1, \\ h(\epsilon) & \simeq -\epsilon^2 h_4 + h_3 + \epsilon h_2, \\ h(-\epsilon^2) & \simeq -\epsilon h_4 - \epsilon h_3 + h_2 + \epsilon h_1, \\ h(-\epsilon^5) & \simeq \epsilon^2 h_3 - \epsilon h_2 + h_1. \end{aligned} \quad (19)$$

Since $\langle h(1, \epsilon, -\epsilon^2) \rangle = \langle h^c(1, \epsilon, -\epsilon^2) \rangle = 0$ and $\langle h(-\epsilon^5) \rangle = \langle h^c(-\epsilon^5) \rangle = V \neq 0$, we can conclude that

$$\begin{aligned} \langle h_1 \rangle & = \langle h_1^c \rangle \simeq V \\ \langle h_2 \rangle & = \langle h_2^c \rangle \simeq -\epsilon V \\ \langle h_3 \rangle & = \langle h_3^c \rangle \simeq \epsilon^2 V \end{aligned} \quad (20)$$

and

$$\langle h_4 \rangle = \langle h_4^c \rangle \lesssim O(\epsilon^3) V.$$

If we postulate now that the lepton fields transform as in table 2 under $Z_3 \times Z_3$, the neutral lepton Dirac mass matrix in an ℓ, ν^c basis is

$$\begin{pmatrix} 0 & 0 & 0 & g_1 \langle h_3^c \rangle \\ 0 & 0 & g_2 \langle h_1^c \rangle & g_3 \langle h_2^c \rangle \\ 0 & g_4 \langle h_1^c \rangle & g_5 \langle h_2^c \rangle & 0 \\ g_6 \langle h_3^c \rangle & g_7 \langle h_2^c \rangle & 0 & g_8 \langle h_1^c \rangle \end{pmatrix}. \quad (21)$$

According to our assumptions, $g_1, g_2, g_4, g_6 \sim \epsilon^2 g$ and

$g_3, g_5, g_7, g_8 \sim \epsilon g$. As far as orders of magnitude are concerned the mass matrix is

$$\begin{pmatrix} 0 & 0 & 0 & \epsilon^4 \\ 0 & 0 & \epsilon^2 & \epsilon^2 \\ 0 & \epsilon^2 & \epsilon^2 & 0 \\ \epsilon^4 & \epsilon^2 & 0 & \epsilon \end{pmatrix}, \quad (22)$$

with eigenvalues of order

$$\epsilon(gV), \quad \epsilon^2(gV), \quad \epsilon^2(gV), \quad \epsilon^7(gV).$$

Choosing the coupling g to give $(gV) \sim O(1 \text{ GeV})$, we obtain the neutrino Dirac mass hierarchy

$$100 \text{ MeV} : 10 \text{ MeV} : 10 \text{ MeV} : 100 \text{ eV}. \quad (23)$$

On the other hand the charged lepton mass matrix takes the form

$$\begin{pmatrix} f_1 \langle h_4 \rangle & 0 & 0 & f_2 \langle h_3 \rangle \\ f_3 \langle h_3 \rangle & 0 & f_4 \langle h_1 \rangle & f_5 \langle h_2 \rangle \\ 0 & f_6 \langle h_1 \rangle & f_7 \langle h_2 \rangle & 0 \\ 0 & f_8 \langle h_2 \rangle & 0 & f_4 \langle h_1 \rangle \end{pmatrix} \quad (24)$$

leading to a hierarchy

$$\epsilon(fV), \quad \epsilon^2(fV), \quad \epsilon^2(fV), \quad \epsilon^4(fV).$$

The model is not realistic as it stands. It shows however how charged leptons could be differentiated in their hierarchical mass structure from neutral ones. It can be completed by including quarks which, in accordance with the $Z_3 \times Z_3$ assignments of table 2, will have a mass matrix

$$\begin{pmatrix} \lambda_1 \langle h_1 \rangle & 0 & 0 & 0 \\ 0 & \lambda_2 \langle h_2 \rangle & \lambda_3 \langle h_4 \rangle & \lambda_4 \langle h_3 \rangle \\ 0 & \lambda_5 \langle h_4 \rangle & \lambda_6 \langle h_3 \rangle & \lambda_7 \langle h_2 \rangle \\ 0 & \lambda_8 \langle h_3 \rangle & \lambda_9 \langle h_2 \rangle & \lambda_{10} \langle h_4 \rangle \end{pmatrix} \quad (25)$$

with an inverted eigenvalue hierarchy

$$(\lambda V)\epsilon^3, \quad (\lambda V)\epsilon^2, \quad (\lambda V)\epsilon, \quad (\lambda V).$$

The transformation properties of coloured fields dictate that there exist no terms of the form

$$qqD \text{ or } q^c q^c D^c$$

and therefore baryon number is conserved, since with terms of the type

$$q\ell D^c + q^c \ell^c D$$

we can take $B(D^c) = -\frac{1}{3}$ and $B(D) = \frac{1}{3}$. The D-quarks can obtain a large mass from the allowed coupling $D_i D_j^c \eta_1$.

Although the above constructed models are semi-realistic and the discrete symmetries imposed were introduced by hand, they serve the purpose of showing how discrete family-dependent symmetries and a multi-Higgs sector can induce hierarchically light neutrinos.

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