THE SUPERSYMMETRIC SINGLET

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Here a model of family mixing and lepton-number breaking based on the existence of a singly-charged scalar field is extended in the supersymmetric case. As a result, the neutrinos acquire Majorana masses radiatively, while the introduction of the right-handed neutrino gives rise to a see-saw mechanism. Its predictions in $\mu \to e\gamma$ and $\mu \to \bar{e}ee$ flavour-violating processes appear to be opposite to those of the present superstring models.

In a previous paper [1] we have proposed a class of models of family and lepton-number breaking based on the existence of novel scalar fields which carry lepton and family quantum numbers. These models are based on the standard gauge symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$ but are grand-unifiable as SU(5), SO(10) or E_6 theories. Of particular simplicity is a model with an $SU(3)_c \times SU(2)_L$ singlet scalar field that has the quantum numbers of the right-handed electron [2]. In that model family mixing among leptons is automatic. Majorana masses for the left-handed neutrinos appear radiatively. In this short letter we shall have a fresh look at this model in the framework of $N=1$ supergravity [3]. At low energies such a theory is roughly a softly broken globally supersymmetric $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge theory [4] with a highly constrained spectrum.

Except the graviton–gravitino supermultiplet and the gauge-gaugino supermultiplet, the theory contains a set of chiral superfields classified in $SU(3)_c \times SU(2)_L \times U(1)_Y$ multiplets. Focusing on the leptonic sector, we shall have

$$\mathbf{L} = (\ell, \tilde{\ell}) = (1, 2, 1/2), \quad \mathbf{E}^c = (e^c, \tilde{e}^c) = (1, 1, -1) ,$$

and (perhaps)

$$\mathbf{N}^c = (N^c, \tilde{N}^c) = (1, 1, 0) .$$

In addition we must have at least two Higgs-doublet superfields

$$\mathbf{H} = (\tilde{H}, H) = (1, 2, 1/2), \quad \bar{\mathbf{H}} = (\tilde{\bar{H}}, \bar{H}) = (1, 2, -1/2) .$$

The crucial ingredient of the model is a pair of chiral superfields

$$\mathbf{S} = (1, 1, -1) = (\tilde{S}, S), \quad \mathbf{S} = (1, 1, +1) = (\tilde{\bar{S}}, \bar{S}) .$$

In the non-supersymmetric version of the model one singlet field S is necessary to realize family mixing. Nevertheless, here the fermionic components of chiral superfields can in general cause anomalies which are avoided if they enter in real reducible representations as $r + \bar{r}$. In addition to the above fields lepton number breaking and radiative left-handed neutrino Majorana masses require an additional pair of Higgs doublets $\mathbf{H}'$, $\bar{\mathbf{H}}'$.

In an SU(5) framework these superfields are contained in the $\mathbf{Q}_5$, Q_{10}, $\mathbf{H}_5$, $\mathbf{H}_5$ matter and Higgs superfields together with S_{10} and $\bar{S}_{10}$. The allowed couplings in the superpotential are

$$w = \mathbf{Q}_5 Q_{10} \bar{\mathbf{H}}_5 + Q_{10} Q_{10} H_5 + \bar{\mathbf{H}}_5 H_5 + ... + \mathbf{Q}_5 \mathbf{Q}_5 S_{10} + S_{10} \bar{S}_{10} + \bar{\mathbf{H}}_5 \bar{\mathbf{H}}'_5 S_{10} + H_5 H'_5 \bar{S}_{10} + ...$$

Nevertheless in SO(10) one has to introduce in addition to Q_{10} and $\mathbf{H}_{10}$ the rather large representation S_{120} with couplings

$$w=Q_{16}Q_{16}(\mathbf{H}_{10}+S_{120})+S_{120}\ .$$

Additional $\mathbf{H}_{16}$, $\mathbf{H}'_{16}$ are needed for lepton-number violating couplings of the type $\mathbf{HH}'$S.

In the superstring-inspired E_6 group all matter fields are contained in the Q_{27} representation [5]. An additional $\mathbf{H}_{27}$ which includes the standard Higgses as well as the new fields leads to a coupling

$$Q_{27}Q_{27}\mathbf{H}_{27}\ ,$$

which does not contain the desired $\mathbf{Q}_5\mathbf{Q}_5\mathbf{H}_{10}$ coupling that in SO(10) requires the large representation S_{120}. This last representation is contained only in the 351 representation of E_6.

Returning to $SU(3)_c\times SU(2)_L\times U(1)_Y$, we write down the matter superpotential

$$w=\mathbf{L}_i\mathbf{E}^c_j(a_{ij}\mathbf{H}+b_{ij}\mathbf{H}')+c_{ij}\mathbf{L}_i\mathbf{L}_j\mathrm{S}+d\mathbf{HH}'\mathrm{S}+e\bar{\mathbf{H}}\bar{\mathbf{H}}'\bar{\mathrm{S}}+\mu\mathbf{H}\bar{\mathbf{H}}+\mu'\mathbf{H}'\bar{\mathbf{H}}'+M\mathrm{S}\bar{\mathrm{S}}$$
$$+[\mathbf{L}_i\mathbf{N}^c_j(f_{ij}\bar{\mathbf{H}}'+g_{ij}\bar{\mathbf{H}}')+\tfrac{1}{2}M_{ij}\mathbf{N}^c_i\mathbf{N}^c_j]\ . \qquad (1)$$

It is well known that the Yukawa lagrangian will simply be

$$L_\mu=\sum_{ij}\frac{\partial^2 w}{\partial\varphi^i\partial\varphi^j}\psi^i_{\mathrm{L}}\psi^j_{\mathrm{L}}+\text{c.c.}\ , \qquad (2)$$

while the potential will be

$$V=\sum_i\left|\frac{\partial w}{\partial\varphi_i}\right|^2+m^2_{3/2}\sum_i|\varphi^i|^2+Am_{3/2}(w+w^*)+Bm_{3/2}\left(\sum_i\varphi_i\frac{\partial w}{\partial\varphi_i}+\text{c.c.}\right)\ . \qquad (3)$$

The dimensionless parameters A, B depend on the details of the supergravity breaking and are both naturally of order 1.

The non-leptonic fermion mass terms derived from (1) are

$$\bar{\mathrm{S}}(M\mathrm{S}+e\bar{v}\bar{\mathbf{H}}'_-)+\mathrm{H}'_+(dv\mathrm{S}+\mu'\bar{\mathbf{H}}'_-)+\text{c.c.}\ ,$$

where $v\equiv\langle \mathrm{H}_0\rangle$ and $\bar{v}\equiv\langle\bar{\mathrm{H}}_0\rangle$. The fermionic mass matrix is more conveniently cast in the form

$$(\bar{\mathrm{S}},\bar{\mathrm{S}}^*,\mathrm{H}'^*_+,\bar{\mathbf{H}}'_-)^*\begin{pmatrix}0 & M & dv & 0\\ M & 0 & 0 & ev\\ dv & 0 & 0 & \mu'\\ 0 & e\bar{v} & \mu' & 0\end{pmatrix}\begin{pmatrix}\mathrm{S}\\ \bar{\mathrm{S}}^*\\ \mathrm{H}'^*_+\\ \bar{\mathbf{H}}'_-\end{pmatrix}, \qquad (4)$$

with eigenvalues

$$m^2=\tfrac{1}{2}\{M^2+\mu'^2+(dv)^2+(e\bar{v})^2\pm\{[(M+\mu')^2+(dv-e\bar{v})^2][(M-\mu'^2)+(dv+e\bar{v})^2]\}^{1/2}\}\ . \qquad (5)$$

The bosonic mass matrix is more complicated due to the supersymmetry breaking. Starting from the full scalar potential and neglecting D-terms we obtain ($\bar{A}=A+2B$, $\bar{B}=A+3B$)

$$V=|\tilde{e}^c_j(\mathrm{H}a_{ij}+\mathrm{H}'b_{ij})+\tilde{\ell}_j\mathrm{S}c_{ij}|^2+|\tilde{\ell}_i(\mathrm{H}a_{ij}+\mathrm{H}'b_{ij})|^2+|\tilde{\ell}_i\tilde{\ell}_jc_{ij}+d\mathrm{HH}'+M\bar{\mathrm{S}}|^2+|M\mathrm{S}+e\bar{\mathbf{H}}\bar{\mathbf{H}}'|^2$$
$$+|\tilde{\ell}_i\tilde{\ell}_jca_{ij}+d\mathrm{H}'\mathrm{S}+\mu\bar{\mathbf{H}}|^2+|\tilde{\ell}_i\tilde{e}^c_jb_{ij}+d\mathrm{HS}|^2+|\mu\mathrm{H}+e\bar{\mathbf{H}}'\bar{\mathrm{S}}|^2+|\mu'\mathrm{H}'+d\mathrm{HS}|^2$$
$$+m^2_{3/2}(|\tilde{\ell}_i|^2+|\tilde{e}^c_i|^2+|\mathrm{S}|^2+|\bar{\mathrm{S}}|^2+|\mathrm{H}|^2+|\mathrm{H}'|^2+|\bar{\mathbf{H}}|^2+|\bar{\mathbf{H}}'|^2)$$
$$+\bar{\mathbf{A}}m_{3/2}(\tilde{\ell}_i\tilde{e}^c_j(a_{ij}\mathrm{H}+b_{ij}\mathrm{H}')+\tilde{\ell}_i\tilde{\ell}_j\mathrm{S}c_{ij}+d\mathrm{HH}'\mathrm{S}+e\bar{\mathbf{H}}\bar{\mathbf{H}}'\mathrm{S}+\text{c.c.})+\bar{\mathbf{B}}m_{3/2}(\mu\mathrm{H}\bar{\mathbf{H}}+\mu'\mathrm{H}'\bar{\mathbf{H}}'+M\mathrm{S}\bar{\mathrm{S}}+\text{c.c.})\ . \qquad (6)$$

It is well known that radiative corrections due to these couplings as well as s quark couplings can generate a

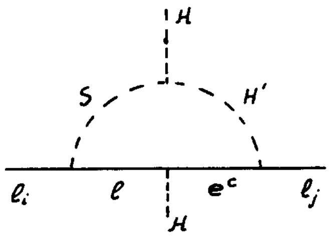

Fig. 1. Light neutrino Majorana masses.

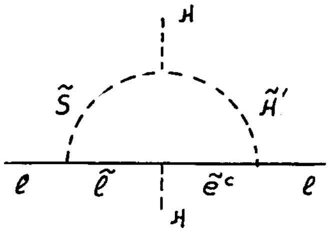

Fig. 2. Supersymmetric contribution to the neutrino mass matrix.

negative mass squared for a combination of Higgses which will cause non-zero vacuum expectation values and $SU(2)_L \times U(1)_Y$ breaking. We have denoted $\langle H_0 \rangle = v$ and $\langle \bar{H}_0 \rangle = \bar{v}$ while $|H'\rangle = \langle \bar{H}' \rangle = 0$.

Finally, the relevant quadratic part of the potential will be

$$\begin{aligned} V_{\text{QUAD.}} = {} & [m_{3/2}^2 + M^2 + (dv)^2]\,|S|^2 + [m_{3/2}^2 + M^2 + (e\bar{v})^2]\,|\bar{S}| + \bar{B} M m_{3/2}(\bar{S}S + \text{c.c.}) \\ & + [m_{3/2}^2 + (dv)^2 + \mu'^2]\,|H'_+|^2 + [m_{3/2}^2 + (e\bar{v})^2 + \mu'^2]\,|\bar{H}'_-|^2 + \bar{B}\mu' m_{3/2}(H'_+ \bar{H}'_+ + \text{c.c}) \\ & + (S\bar{H}_-'^* + \text{c.c.}) Mev + (SH'_+ + \text{c.c.}) dv m_{3/2} \bar{A} + (S\bar{H}_-'^* + \text{c.c.})(\mu v e + \bar{A} m_{3/2} e\bar{v}) + (S\bar{H}_+'^* + \text{c.c.})(dvM) . \end{aligned} \tag{7}$$

In the supersymmetric limit the bosonic mass matrix will be just the square of the fermion mass matrix.

The full bosonic mass matrix in the basis $(S, \bar{S}^*, H'^*, \bar{H}')$ is

$$\begin{pmatrix} m_{3/2}^2 + M^2 (dv)^2 & \bar{A} M m_{3/2} & (d\bar{B} m_{3/2} v + \mu\bar{v} d) & \mu' dv + Me\bar{v} \\ \bar{A} M m_{3/2} & M^2 + (e\bar{v})^2 + m_{3/2}^2 & \mu' e\bar{v} + Mdv & \bar{B}\bar{v} e m_{3/2} + \mu v \\ (\bar{B} m_{3/2} v + \mu\bar{v}) d & \mu' e\bar{v} + Mdv & m_{3/2}^2 + (dv)^2 + \mu'^2 & \bar{A}\mu' m_{3/2} \\ \mu' dv + Me\bar{v} & (\bar{B} m_{3/2} \bar{v} + \mu v) e & \bar{A}\mu' m_{3/2} & \mu'^2 + m_{3/2}^2 + e^2 \bar{v}^2 \end{pmatrix} . \tag{8}$$

The above matrix is simplified considerably if we neglect the direct mass $MS\bar{S}$. Thus, for $M = 0$ and $\mu' \ll \mu$, v, $\bar{v}$, we obtain

$$\begin{pmatrix} \kappa & 0 & \xi & 0 \\ 0 & \lambda & 0 & \nu \\ \xi & 0 & \kappa & 0 \\ 0 & \nu & 0 & \lambda \end{pmatrix} , \tag{9}$$

where

$$\kappa = m_{3/2}^2 + (dv)^2 \quad \lambda = m_{3/2}^2 + (e\bar{v})^2, \quad \xi = d(\bar{B} m_{3/2} v + \mu\bar{v}) \quad \nu = e(\bar{B}\bar{v} m_{3/2} + \mu v) .$$

The resulting eigenvalues and eigenfields are

$$m_{1,2}^2 = \kappa \pm \xi, \quad m_{3,4}^2 = \lambda \pm \nu, \quad \phi_{1,2} \equiv 1/\sqrt{2}(S \pm H'^*), \quad \phi_{3,4} = 1/\sqrt{2}(\bar{S}^* \pm \bar{H}') .$$

However, in the general case M, $\mu' \neq 0$, if we assume that $M > v$, $\bar{v}$, then we estimate that $M_{\bar{S}, S}^2 \sim Q(M^2 + m_{1,2})$ and $M_{H', \bar{H}'} \sim O(\mu'^2 + m_{3,4})$.

Neutrino masses and lepton flavor violation. The lepton-number violating vertices $HH'S$ and $\bar{H}\bar{H}'S$ generate one-loop Majorana masses for the left-handed neutrinos through the diagrams of figs. 1 and 2. The rough order of magnitude of these masses is

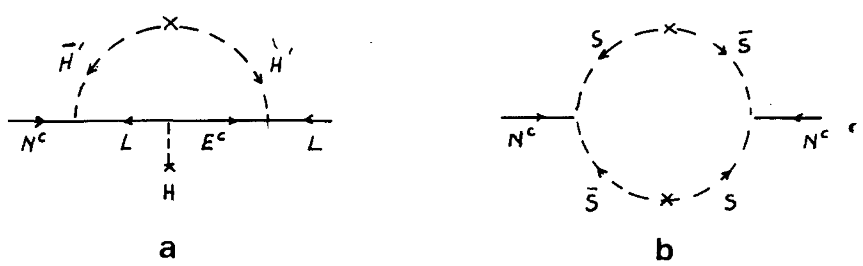

Fig. 3. Dirac and Majorana neutrino masses for the right-handed neutrino.

$$m_\nu \simeq \frac{dA}{16\pi^2}\frac{v\bar{v}}{m_{\mathrm{H'}}^2} C_{ij}\frac{\mu m_{3/2}}{m_{\mathrm{S}}^2 - m_{\mathrm{H'}}^2}\ln\frac{m_{\mathrm{S}}^2}{m_{\mathrm{H'}}^2}\,(m_{ej}-m_{ei})\,. \tag{10}$$

We can find the eigenvalues here in a similar way to the non-supersymmetric case. Thus

$$m_1 = -m_0\rho\,\sin 2\varphi,\quad m_{2,3}\approx m_0\rho(\pm 1+\tfrac{1}{2}\rho\,\sin\varphi)\,, \tag{11}$$

where

$$m_0 = \left(\frac{dA}{\cos\varphi}\frac{C_{\tau e}}{16\pi^2}\frac{v\bar{v}}{m_{\mathrm{H'}}^2}\frac{\mu m_{3/2}}{m_{\mathrm{S}}^2 - m_{\mathrm{H'}}^2}\ln\frac{m_{\mathrm{S}}^2}{m_{\mathrm{H'}}^2}\right) m_\tau\,, \tag{12}$$

and

$$\rho = \frac{C_{\mu e}}{C_{\tau e}}\frac{m_\mu}{m_\tau},\quad \tan\varphi = \frac{C_{\tau\mu}}{C_{\tau e}}(1-m_\mu/m_\tau)\,. \tag{13}$$

The limits on neutrino masses from ββ-decay [6] can be converted into limits for the singlets in a similar way to the non-supersymmetric case [1].

Let us now discuss the possibility of introducing the right-handed neutrino. Then one can have a radiative contribution to the Dirac masses of the neutrinos. The relevant terms shown explicitly in the superpotential can generate the graph of fig. 3a. Then one gets the contribution

$$(m_{\mathrm{D}})_{ij} \approx \frac{\bar{\mathbf{B}}}{16\pi^2}\frac{v m_{3/2}}{m_{\mathrm{H'}}^2}\mu'\frac{m_{ej}^2+m_{ei}^2}{m_{\mathrm{H'}}^2 - m_{\mathrm{H'}}^2}\ln\frac{m_{\mathrm{H'}}^2}{m_{\mathrm{H}}^2}\,. \tag{14}$$

We finally notice that a see-saw mechanism is possible here. If we consider the vertex $S\bar{S}\,N^c$ then the RH neutrinos get Majorana masses of order M_S through the diagram of fig. 3b.

Finally we discuss here in brief the $\mu\to e\gamma$ and $\mu\to 3e$ decays. The first one can occur through diagrams of fig. 4a. The branching ratio for $\mu\to e\gamma$ is calculated to be

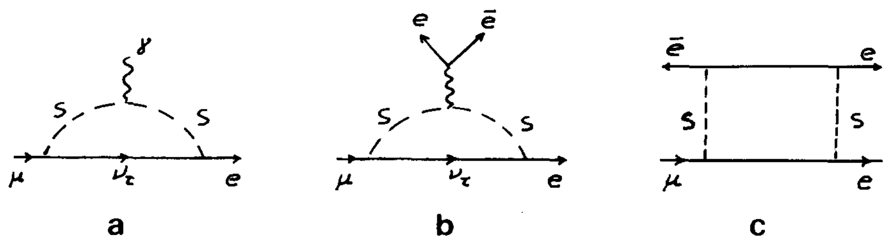

Fig. 4. Flavor-violating processes.

$$\mathrm{BR}(\mu\to e\gamma)\approx\frac{\sin^4\vartheta_W}{6\pi^2}\left(\frac{C_{\mu\tau}C_{\tau e}}{e}\right)^2\left(\frac{m_W}{m_S}\right)^4\left(\frac{m_{\tilde{S}}-m_{\tilde{S}}^2\tilde{F}(\alpha)}{m_{\tilde{S}}^2}\right)^2, \tag{15}$$

where the function $F(\alpha)$ is given by

$$F(\alpha)=\frac{1}{1-\alpha}\left[1-3\frac{\alpha}{1-\alpha}-6\frac{\alpha^2}{(1-\alpha)^2}\left(1+\frac{\ln\alpha}{1-\alpha}\right)\right],\quad \alpha=\frac{m_{\tilde{\nu}}^2}{m_{\tilde{S}}^2}. \tag{16}$$

If $m_{\tilde{\nu}}\ll m_{\tilde{S}}$ then $F(\alpha)\sim 1$ and for $C_{\mu\tau}\sim C_{\tau e}\sim 10^{-1}$ we get

$$\mathrm{BR}(\mu\to e\gamma)\sim 0.97\times10^{-6}(m_W m_{3/2}/m_S m_{\tilde{S}})^4\,. \tag{17}$$

For $m_W\sim 80$ GeV, $m_S\sim 600$ GeV and $m_{\tilde{S}}\sim 400$ GeV we have

$$\mathrm{BR}(\mu\to e\gamma)\approx 1.2\times10^{-12}(m_{3/2}/100\,\mathrm{GeV})^4\,, \tag{18}$$

which is very close to the present experimental limit [7] $\mathrm{BR}_{\mathrm{exp}}\leqslant 4.9\times10^{-11}$). The second decay $\mu\to\bar{e}ee$ can occur through graphs like those of figs. 4b, 4c. In the case of fig. 4b the BR is at least two orders of magnitude smaller than that of $\mu\to e\gamma$ while the contribution of fig. 4c is even weaker. Indeed for fig. 4c we get

$$\mathrm{BR}(\mu\to 3e)\approx\left(\frac{C_{\mu\tau}C_{\tau e}^3}{g^2}\frac{1}{\sqrt{2}\cdot 16\pi^2}\right)^2\left(\frac{m_W}{m_S}\right)^4\left(\frac{m_{\tilde{S}}^2-\tilde{G}(\alpha)m_{\tilde{S}}^2}{m_{\tilde{S}}^2}\right)^2, \tag{19}$$

where $\tilde{G}(\alpha)\sim 1$ if $m_{\tilde{\nu}}\ll m_S$.
Substituting the same values for the masses as in (22) we get

$$\mathrm{BR}(\mu\to 3e)\sim 0.5\times10^{-13}\left(\frac{m_W m_{3/2}}{m_S m_{\tilde{S}}}\right)^4\sim 0.62\times10^{-22}\left(\frac{m^{3/2}}{100\,\mathrm{GeV}}\right)^4, \tag{20}$$

which is very small compared to that of (18). Thus, from the above analyis we conclude that the $\mu\to e\gamma$ decay is favoured in this model. For reasonable values of the gravitino mass ($\sim$O (100 GeV)) the theoretical prediction of this model comes very close to the experimental one. Thus, bearing in mind that:
(a) the conventional models with s-neutrino mixing predict very low branching ratios [8] for the above decay;
(b) in superstring models $\mu\to e\gamma$ is not favoured [9,10] while the $\mu\to 3e$ branching ratio is expected to be large,
we conclude that its possible observation would give a signature for the existence of Higgs particles beyond the standard doublet.

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