Effective three-body interaction in ^{93}Tc, ^{93}Mo, and ^{94}Ru

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The influence of the effective three-body interaction on the spectra of ^{93}Tc, ^{93}Mo, and ^{94}Ru is studied. It is seen to give very small contributions for these nuclei.

[NUCLEAR STRUCTURE Calculation of the effective three-body interaction on the spectra of ^{93}Tc, ^{93}Mo, and ^{94}Ru.]

The importance of the effective three-body interaction was first discussed by Osnes[1] and Bertsch[2] in connection with the binding energies of nuclei of the oxygen and calcium regions. Recently Singh[3] and Dirim, Evans, and Elliott[4] investigated the effects of the three-body force on the spectra of $0p$ shell nuclei. In the present work we discuss the importance of the effective three-body interaction in the nuclei of the zirconium region. In particular we study the nuclei $^{93}_{43}$Tc, $^{93}_{42}$Mo, and $^{94}_{44}$Ru and examine the assumption made in Refs. 5 and 6 that to a good approximation the effective three-body interaction can be neglected for ^{94}Mo and ^{95}Tc, respectively.

There are two distinct types of three-body graphs. These are labeled by (a) and (b) in Fig. 1. One notes that for these two diagrams only one particle (hole) is outside the valence space whereas for the two-body diagrams 1(c), 1(d), and 1(e) two particles (holes) or one particle and one hole may be outside the valence space. This feature clearly suggests that the two-body diagrams are more important than 1(a) and 1(b).

The expression for graph 1(a) is

$$f(j_1,j_2,j_3,j_1',j_2',j_3';I,I',J) = -(-1)^{I+I'+j_3+j_3'}(\hat{I}\hat{I}')^{1/2}\eta_{j_1j_2}\eta_{j_1'j_2'} \times\sum_p \begin{Bmatrix} I & j_3 & J \\ I' & j_3' & p \end{Bmatrix} \frac{\langle j_1 j_2; I \mid V \mid p j_3'; I\rangle \langle j_1' j_2'; I' \mid V \mid p j_3; I'\rangle}{\Delta E} \eta_{pj_3}\eta_{pj_3'} , \quad (1)$$

where $\eta_{ab} = (1+\delta_{ab})^{1/2}$ while $\hat{I} = 2I+1$. The expression for graph 1(b) can be easily obtained from (1) by a change of sign and by replacing p by h. Graphs 1(a) and 1(b) each have eight exchange graphs not accounted for by the antisymmetrized matrix elements of expression (1). The nine graphs corresponding to 1(a) and 1(b) may all be expressed as

$$\langle (j_1 j_2)I, j_3; J \mid V \mid (j_1' j_2')I', j_3'; J\rangle = \sum_{K,K'} Q^{IKJ}_{j_1 j_2 j_3} Q^{I'K'J'}_{j_1' j_2' j_3'} f(j_1,j_2,j_3,j_1',j_2',j_3';K,K',J) \quad (2)$$

where the exchange operator $Q^{IKJ}_{j_1 j_2 j_3}$ is defined by

$$Q^{IKJ}_{j_1 j_2 j_3} = \delta_{IK} + (\hat{I}\hat{K})^{1/2} \begin{Bmatrix} j_1 & j_2 & I \\ j_3 & J & K \end{Bmatrix} \Delta_{j_1 j_3} - (-1)^{I+K+j_2+j_3}(\hat{I}\hat{K})^{1/2} \begin{Bmatrix} j_2 & j_1 & I \\ j_3 & J & K \end{Bmatrix} \Delta_{j_2 j_3} , \quad (3)$$

and the operator $\Delta_{j_1 j_2}$ interchanges particles j_1, j_2. The matrix element (2) has been written conveniently in the neutron-proton formalism since in our case protons and neutrons occupy different orbitals. It should be noted that expression (2) is not normalized if two or more of the particles are identical. If two of the particles are identical (the ^{93}Mo case) the normalization factor is equal to 0.5.

In the case of three identical particles, however, the basis states $|j^2 I, j; J\rangle$ are generally not orthogonal. In this case the normalization factor is evaluated by means of the Schmidt method which also serves to eliminate the redundant states from the basis. An expression equivalent to (1)–(3) has been given in Ref. 4 in terms of fermion creation and annihilation operators.

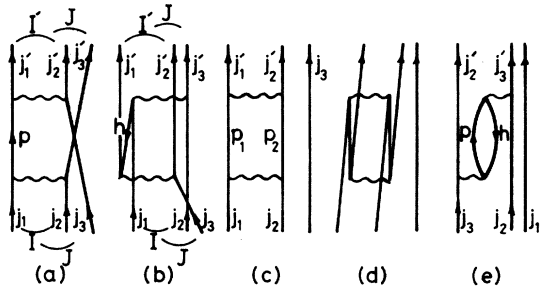

FIG. 1. Effective three-body and two-body diagrams discussed in the calculation. Exchange graphs accounted for by the antisymmetrized two-body matrix elements and topologically equivalent graphs are not shown.

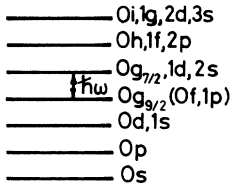

FIG. 2. Energy spacings used in the calculation.

For ^{93}Tc (^{94}Ru) our basis states are constructed among the $(0g_{9/2}p)^3$ $[(0g_{9/2}p)^4]$ configurations while for ^{93}Mo we similarly restrict the two protons to the $0g_{9/2}$ orbital but allow the valence neutron to be in any of the $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, and $0g_{7/2}$ orbitals. To be consistent with earlier related calculations[5-7] we assume that the $g_{9/2}$ orbital is depressed by $\hbar\omega$ with respect to the other states of the same oscillator energy. The unperturbed single-particle energies used in the calculation are shown in Fig. 2. The calculation has been performed with the Sussex matrix elements[8] for a b value of 2.1 fm.

The contribution of the two-body diagrams 1(c), 1(d), and 1(e) for all $2\hbar\omega$ excitations has already been reported[6,7] for the case of ^{93}Tc and ^{94}Ru. In Tables I and II we make a comparison of this contribution with the contributions of the three-body diagrams 1(a) and 1(b) calculated in the same space. Parity restrictions and conservation of charge imply that the p or h excitation in diagrams 1(a) and 1(b) must be an even parity proton. Thus only $1\hbar\omega$ excitations ($p = 0g_{7/2}, 1d, 2s$; $h = 0d, 1s$) contribute to these two diagrams. To test the effects of a larger space we have also calculated the three-body effects allowing for up to $3\hbar\omega$ excitations (p also $0i, 1g, 2d, 3s$; h also $0s$). The results of this enlarged space calculation are also shown in Tables I and II.

In Table III we make a similar comparison between the two-body and three-body contributions for ^{93}Mo. Here the protons are allowed $1\hbar\omega$ excitations and the neutrons $1+2\hbar\omega$ excitations ($p = 0i, 1g, 2d, 3s$; $h = 0g_{9/2}, 0d, 1s$). Due to the many configurations involved in this calculation we only list the $J = \frac{1}{2}$ case, but similar results have also been obtained for the other J states.

Like in earlier calculations,[3,4] the contribution of the effective three-body diagrams is found to be considerably smaller than that of the two-body diagrams in the case of three and four valence particle systems. This is partly because there is only one triad of valence particles in ^{93}Tc and ^{93}Mo and four triads for ^{94}Ru, while there are three and

TABLE I. Contribution of three-body diagrams to the ^{93}Tc spectrum.

J[a]	Two-body diagrams	Diagram 1(a) up to $1\hbar\omega$ exc.	Diagram 1(b) up to $1\hbar\omega$ exc.	Diagrams 1(a)+1(b) up to $1\hbar\omega$ exc.	Diagrams 1(a)+1(b) up to $3\hbar\omega$ exc.
$\frac{3}{2}^+$	−0.246	0.009	0.009	0.018	0.020
$\frac{5}{2}^+$	−0.592	0.005	0.002	0.007	0.012
$\frac{7}{2}^+$	−1.134	−0.001	0.001	−0.000	0.005
$\frac{9}{2}^+$ $\langle 1 \mid 1 \rangle$	−1.970	0.009	0.000	0.009	0.012
$\langle 1 \mid 2 \rangle$	−0.021	0.000	−0.001	−0.001	−0.000
$\langle 2 \mid 2 \rangle$	−0.071	0.007	−0.004	0.003	0.006
$\frac{11}{2}^+$	−0.348	0.010	−0.002	0.008	0.010
$\frac{13}{2}^+$	−0.423	0.012	0.000	0.012	0.010
$\frac{15}{2}^+$	0.330	0.003	−0.004	−0.001	0.001
$\frac{17}{2}^+$	0.302	0.002	0.000	0.002	0.001
$\frac{21}{2}^+$	0.625	−0.001	−0.004	−0.005	−0.006

[a]We list the matrix elements between orthogonal states of the $(g_{9/2})^3$ configurations.

TABLE II. Contribution of three-body diagrams to the ^{94}Ru spectrum.

J [a]		Two-body diagrams	Diagram 1(a) up to $1\hbar\omega$ exc.	Diagram 1(b) up to $1\hbar\omega$ exc.	Diagrams 1(a)+1(b) up to $1\hbar\omega$ exc.	Diagrams 1(a)+1(b) up to $3\hbar\omega$ exc.
0^+	$\langle 1\mid 1\rangle$	−3.941	0.036	0.001	0.037	0.046
	$\langle 1\mid 2\rangle$	−0.043	0.001	−0.004	−0.003	−0.001
	$\langle 2\mid 2\rangle$	−0.142	0.028	−0.017	0.011	0.024
2^+	$\langle 1\mid 1\rangle$	−1.172	0.032	0.003	0.035	0.043
	$\langle 1\mid 2\rangle$	0.574	−0.000	−0.003	−0.003	−0.001
	$\langle 2\mid 2\rangle$	−2.025	0.027	−0.002	0.025	0.036
3^+		−0.176	0.026	−0.005	0.021	0.031
4^+	$\langle 1\mid 1\rangle$	−0.626	0.027	0.009	0.036	0.042
	$\langle 1\mid 2\rangle$	−0.619	0.001	0.002	0.003	0.003
	$\langle 1\mid 3\rangle$	−0.145	−0.003	0.003	−0.000	−0.001
	$\langle 2\mid 2\rangle$	−1.217	0.024	−0.003	0.021	0.029
	$\langle 2\mid 3\rangle$	0.137	−0.009	−0.001	−0.010	−0.007
	$\langle 3\mid 3\rangle$	−1.237	0.024	0.000	0.024	0.031
5^+		−0.488	0.026	−0.001	0.025	0.030
6^+	$\langle 1\mid 1\rangle$	−0.171	0.022	−0.001	0.021	0.027
	$\langle 1\mid 2\rangle$	0.454	−0.000	0.000	0.000	−0.001
	$\langle 1\mid 3\rangle$	−0.207	0.002	0.000	0.002	0.002
	$\langle 2\mid 2\rangle$	−0.902	0.011	−0.001	0.010	0.020
	$\langle 2\mid 3\rangle$	0.166	−0.007	0.004	−0.003	−0.002
	$\langle 3\mid 3\rangle$	−1.147	0.022	−0.003	0.019	0.024
7^+		−0.451	0.018	−0.005	0.013	0.019
8^+	$\langle 1\mid 1\rangle$	−1.236	0.020	−0.003	0.017	0.019
	$\langle 1\mid 2\rangle$	−0.018	−0.001	0.002	0.001	0.001
	$\langle 2\mid 2\rangle$	0.186	0.021	−0.007	0.014	0.015
9^+		0.467	0.014	−0.010	0.004	0.005
10^+		0.240	0.019	−0.005	0.014	0.009
12^+		1.011	0.001	−0.011	−0.010	−0.013

[a] We list the matrix elements between orthogonal states of the $(g_{9/2})^4$ configurations.

six pairs, respectively, for these nuclei. A further reason why the three-body contribution is so small is, as stated before, that only a limited number of excitations contribute to the diagrams 1(a) and 1(b). In particular for ^{93}Mo there is an additional restriction. Thus there is no contribution from the direct graphs 1(a) and 1(b) but only from their eight exchange terms. Of these, four involve even parity proton, and four even parity neutron excitations.

As shown in Tables I and II of the two graphs 1(a) and 1(b) the former is generally the more important. This is expected since it involves more intermediate states than the latter. Another conclusion drawn from the results of Tables I and II is that the inclusion of the $3\hbar\omega$ excitations does not significantly affect the contributions of the three-body graphs.

It may be observed from Tables I–III that the three-body corrections generally have an opposite sign from the two-body contribution thus tending to decrease the two-body effects. On the other hand we find, as first suggested in Refs. 1 and 2, that there is an exact cancellation between the Pauli violating terms of diagrams 1(c) and 1(e)

TABLE III. Three-body as compared with two-body contributions to the spectrum of ^{93}Mo in the case $J=\frac{1}{2}$.

Matrix elements [a]	Two body	Three body
$\langle 1\mid 1\rangle$	−3.135	0.005
$\langle 1\mid 2\rangle$	0.563	−0.005
$\langle 1\mid 3\rangle$	−0.658	−0.003
$\langle 1\mid 4\rangle$	0.184	0.008
$\langle 2\mid 2\rangle$	−1.330	0.007
$\langle 2\mid 3\rangle$	0.270	−0.003
$\langle 2\mid 4\rangle$	−0.499	−0.004
$\langle 3\mid 3\rangle$	−1.217	−0.006
$\langle 3\mid 4\rangle$	0.176	0.001
$\langle 4\mid 4\rangle$	−1.133	−0.023

[a] $\mid 1\rangle = \mid (0g_{9/2}p)^2 0, (2s_{1/2}n)\rangle$; $\mid 2\rangle = \mid (0g_{9/2}p)^2 2, (1d_{3/2}n)\rangle$; $\mid 3\rangle = \mid (0g_{9/2}p)^2 2, (1d_{5/2}n)\rangle$; $\mid 4\rangle = \mid (0g_{9/2}p)^2 4, (0g_{7/2}n)\rangle$.

and 1(a) and 1(b). In particular the term with $j_3m_3 \equiv j_3'm_3'$ of diagram 1(a) cancels with the term of 1(c) where p_1m_{p1} (or p_2m_{p2}) is equal to j_3m_3. Similarly the term of 1(b) where $j_1m_1 \equiv j_1'm_1'$ cancels with the term of 1(e) where $pm \equiv j_1m_1$. The other type of Pauli violating terms of diagram 1(e) as, for example, when $j_2'm_2' \equiv j_2m_2$, cancel with appropriate one-body diagrams.

From the above it may be concluded that for few valence nucleons, the three-body graphs in the zirconium region are very small compared with the two-body graphs calculated in the same space. Thus the approximation made in Refs. 5 and 6 of neglecting the three-body graphs seems to be justified. However, from combinatorial arguments it is expected that these effects will become more important as one goes to heavier nuclei in this region. On the other hand, even for few-particle systems the three-body graphs ought to be included for an exact elimination of the Pauli violating terms.

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