Effective weak mixing angle in the MSSM

A. Dedes,[1,*] A. B. Lahanas,[2,†] and K. Tamvakis[1,‡]

[1]*Division of Theoretical Physics, University of Ioannina, GR-45110, Greece*
[2]*Physics Department, Nuclear and Particle Physics Section, University of Athens, Athens 157 71, Greece*
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The predictions of the MSSM are discussed in the light of recent LEP and SLD precision data. The full supersymmetric one loop corrections to the effective weak mixing angle, experimentally determined in LEP and SLD experiments, are considered. It is demonstrated, both analytically and numerically, that potentially dangerous, large logarithmic sparticle corrections are cancelled. The relative difference factor Δk between the mixing angle defined as a ratio of couplings and the experimentally obtained angle is discussed. It is found that Δk is dominated by the oblique corrections, while the nonoblique overall supersymmetric EW and SQCD corrections are negligible. The comparison of the MSSM with radiative electroweak symmetry breaking to the LEP+SLD precision data indicates that rather large values of the soft breaking parameter $M_{1/2}$ in the region greater than 500 GeV are preferred. [S0556-2821(98)04713-4]

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I. INTRODUCTION

The electroweak mixing angle $\sin^2\theta_W$, defined as a ratio of gauge couplings, provides a convenient means to test unification in unified extensions of the standard model (SM) [1]. This quantity is not directly measured in experiments. Instead, LEP and SLD studies employ an effective coupling $\sin^2\theta_{\rm eff}^{\rm lepton}$ determined from on resonance asymmetries whose value is known with excellent accuracy [2–5]. This effective mixing angle has been studied in detail in the context of the SM at the one loop level in various renormalization schemes with the dominant two loop heavy top contributions and three loop QCD effects taken into account [4–8]. Due to large cancellations between fermion and boson contributions occurring at the one loop level, in the $\overline{\rm MS}$ scheme, these are the dominant contributions to the difference $\sin^2\theta_{\rm eff}^{\rm lepton} - \sin^2\theta_W \approx \mathcal{O}(10^{-4})$ which is less than the error quoted by the experimental groups. Therefore, although conceptually different the two angles are very close numerically. The mixing angle is sensitive on the values of the Higgs mass M_H and top mass m_t through the quantities Δr_W and $\Delta\rho$ and carries an uncertainty of about 0.1% from its dependence on the electromagnetic coupling $\alpha(M_Z)$. From the predictions of $\sin^2\theta_{\rm eff}^{\rm lepton}$ and $\Delta\rho$ one can draw useful theoretical conclusions concerning the Higgs and W-boson masses having as inputs the Z-boson mass, the value of the fine structure constant and the Fermi coupling constant which are experimentally known to a high degree of accuracy.

In the framework of supersymmetric extensions of the SM [9] the situation changes since $\sin^2\theta_W$ as well as $\sin^2\theta_{\rm eff}^{\rm lepton}$ receive contributions from the superparticles in addition to ordinary particles. Coupling unification at the grand unified

theory (GUT) scale in conjunction with experimental data for the strong coupling constant at M_Z and radiative breaking of the electroweak symmetry impose stringent constraints on the extracted value for $\sin^2\theta_W$. However, $\sin^2\theta_W$ is plagued by large logarithms $\log(M_Z/M_S)$, where M_S is the effective supersymmetry breaking scale.[1] Unlike $\sin^2\theta_W$ the experimentally determined $\sin^2\theta_{\rm eff}^{\rm lepton}$ is not plagued by such potentially dangerous large logarithms due to decoupling. Therefore, the difference of the two angles is not numerically small any more and $\sin^2\theta_W$ cannot be directly used for comparison with experimental data. Thus, in supersymmetric theories the precise relation between the two angles is highly demanded. The nondecoupled supersymmetric corrections to $\sin^2\theta_{\rm eff}^{\rm lepton}$ are expected to be small of order $(M_Z/M_S)^2$. However small these contributions may be, they are of particular importance, since the experimental accuracy is very high, and these corrections can be larger than the SM corrections occurring beyond the one loop order. Moreover the effect of the one loop supersymmetric corrections may not be necessarily suppressed in some sectors, such as the neutralino and chargino sectors, which are characterized by a relatively small effective supersymmetry breaking scale for particular inputs of the soft supersymmetry (SUSY) breaking parameters. Motivated by this we undertake a complete one loop study of the supersymmetric corrections to the effective mixing angle in the context of the MSSM which is the simplest supersymmetric extension of standard theory.

Although there are several studies [10] in the literature concerning the value of the weak mixing angle $\sin^2\theta_W$ in the MSSM and other unified supersymmetric extensions of the SM, only a few have tackled the problem of calculating the complete supersymmetric corrections $\mathcal{O}(M_Z/M_S)^2$ to the experimentally measured angle $\sin^2\theta_{\rm eff}^{\rm lepton}$. In Ref. [11] the ef-

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*E-mail: adedes@cc.uoi.gr

†E-mail: alahanas@atlas.uoa.gr

‡E-mail: tamvakais@cc.uoi.gr

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[1]See, for instance, P. Chankowski, Z. Plucienic, and S. Pokorski in Ref. [10].

fective mixing angle is calculated in particular cases and the decoupling of large logarithms is numerically shown. In that calculation all the one-loop corrections, including the non-universal supersymmetric vertex and external fermion corrections, for the leptonic effective mixing angle were considered. The nonuniversal corrections were found to be small. In other studies [12–14], the serious constraints imposed by unification and radiative electroweak symmetry breaking [15] have not been considered. Instead the MSSM parameters are considered as free parameters chosen in the optimal way to improve the observed deficiencies of the SM in describing the data.

In the present article we show explicitly how the cancellation of potentially dangerous logs takes place and perform a systematic numerical study by scanning the entire parameter space having as our main outputs the effective weak mixing angle, the values of the on Z-resonance asymmetries measured in experiments, as well as the value of the strong coupling constant at M_Z. In each case we also give the theoretical prediction for the W-boson mass through its relation to the parameter rho and the weak mixing angle.

It is perhaps worth noting that nonuniversal corrections, claimed to be small, are dominated by large logs. These logs cancel at the end, as expected. Nevertheless, their presence dictates that nondecoupled terms of order $(M_Z/M_S)^2$ may be of the same order of magnitude as the corresponding terms stemming from the universal corrections and cannot be *a priori* omitted. Knowing from other studies that universal corrections tend to decrease the value of the effective mixing angle by almost six standard deviations from the experimental central value it is important to see what is the effect of the nonuniversal contributions. We take into account all constraints from unification and radiative EW symmetry breaking. These constraints, along with the experimental bounds for the strong coupling constant and $\sin^2\theta_{\text{eff}}^{\text{lepton}}$, may restrict further the allowed parameter space.

II. FORMULATION OF THE PROBLEM

The value of the weak mixing angle, defined as the ratio of the gauge couplings, is

$$\hat{s}^2(Q) = \frac{\hat{g}'^2(Q)}{\hat{g}^2(Q) + \hat{g}'^2(Q)}, \qquad (1)$$

where $\hat{g}$ and $\hat{g}'$ are the SU(2) and U(1)$_Y$ gauge couplings. Throughout this paper the hat refers to renormalized quantities in the modified $\overline{\text{DR}}$ scheme [16,17]. These couplings are running in the sense that they depend on the scale Q. Particularly for the electroweak processes, Q is chosen to be M_Z. There are many sources for the determination of the $\hat{s}^2$. From muon decay, for instance, and knowing that $M_Z = 91.1867 \pm 0.0020$ GeV, $\alpha_{EM} = 1/137.036$, and $G_F = 1.16639(1) \times 10^{-5}$ GeV^{-2}, we get in the $(\overline{\text{DR}})$ scheme

$$\hat{s}^2 \hat{c}^2 = \frac{\pi \alpha_{EM}}{\sqrt{2} M_Z^2 G_F (1 - \Delta\hat{\alpha})\hat{\rho}(1 - \Delta\hat{r}_W)}, \qquad (2)$$
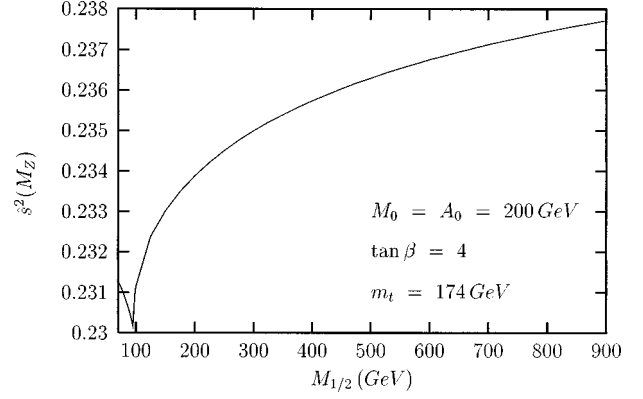

FIG. 1. The values of the running weak mixing angle $\hat{s}^2$ at M_Z in the $\overline{\text{DR}}$ scheme defined as a ratio of gauge couplings for various input universal soft gaugino masses $M_{1/2}$ for particular input of M_0, A, $\tan\beta$, and m_t. The strong dependence of $\hat{s}^2$ on $M_{1/2}$ near M_Z is due to the presence of sparticle thresholds.

where

$$\hat{\rho}^{-1} = 1 - \Delta\hat{\rho} = 1 - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\Pi_{WW}(M_W^2)}{M_W^2}, \qquad (3)$$

$$\Delta\hat{r}_W = \frac{\Pi_{WW}(0) - \Pi_{WW}(M_W^2)}{M_W^2} + \hat{\delta}_{VB}, \qquad (4)$$

$$\hat{\alpha} = \frac{\alpha_{EM}}{1 - \Delta\hat{\alpha}}. \qquad (5)$$

Π's are the transverse gauge bosons self energies evaluated in the $\overline{\text{DR}}$ scheme. Explicit forms for these self-energies can be obtained from Ref. [11]. The weak mixing angle obtained from Eq. (2) although it plays a crucial role in the analysis of grand unification, is *not* an experimental quantity. Actually, it is obtained after fitting experimental observations with α_{EM} and G_F as accurately known parameters (for more details see Ref. [18]). The radiative corrections on $\hat{s}^2$ involve two subtleties: (i) the renormalization scheme dependence[2] and (ii) the dependence on the mass of the top quark, Higgs boson, masses, and superparticle masses which depend on the supersymmetric breaking parameters $M_{1/2}$, M_0, and A_0.

As we can see from Fig. 1, $\hat{s}^2$ takes large values when we increase the masses of the soft breaking parameters. In other words, the soft breaking parameters do not decouple from $\hat{s}^2$. This is due to the fact that the net effect of the contributions (3), (4), (5) to Eq. (2), contains large logarithms of the form $\log(M_{\text{SUSY}}/M_Z)$. On the other hand, the LEP Collaboration [2] employs an *effective weak mixing angle* $\sin^2\theta_{\text{eff}}^f \equiv s_f^2$, first introduced by the authors of Ref. [4], which is not plagued by large logarithms due to decoupling. It is a com-

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[2]We are working on the modified $\overline{DR}$ scheme of Ref. [17] which preserves supersymmetry up to two loops.

mon belief among GUT theorists that these two angles $\hat{s}^2$ and s_f^2, although different conceptually, are very close numerically [5]. Nevertheless, this is not true in the MSSM since there are large logarithmic dependencies of the weak mixing angle $\hat{s}^2$.

The tree level Lagrangian associated with the $Zf\bar{f}$ can be written in the form

$$\mathcal{L}_{\text{tree}}^{Zf\bar{f}} = \frac{\hat{e}}{2\hat{c}\hat{s}} Z_\mu \bar{f}\gamma^\mu [(T_3^f - 2\hat{s}^2 Q^f) - \gamma_5 T_3^f] f, \qquad (6)$$

where Q^f is the electric charge and T_3^f is the third component of isospin of the fermions f. Electroweak corrections in Eq. (6) yield the effective Lagrangian

$$\mathcal{L}_{\text{eff}}^{Zf\bar{f}} = (\sqrt{2} G_F M_Z) \rho_f^{1/2} Z_\mu \bar{f}\gamma^\mu [(T_3^f - 2\hat{s}^2 \hat{k}_f Q^f) - \gamma_5 T_3^f] f, \qquad (7)$$

which is relevant to study neutral current processes on the Z-resonance. Then, the effective weak mixing angle is simply defined from Eq. (7) as

$$s_f^2 \equiv \hat{s}^2 \hat{k}_f = \hat{s}^2 (1 + \Delta\hat{k}_f). \qquad (8)$$

The angle s_f^2 can be compared directly with experiment while $\hat{s}^2$ can be predicted from a grand unification analysis. The LEP and SLD average gives the value 0.23152 ± 0.00023 [3] for the $s_l^2 \equiv \sin^2\theta_{\text{eff}}^{\text{lepton}}$. Since

$$c_f^2 = \hat{c}^2 \left(1 - \frac{\hat{s}^2}{\hat{c}^2} \Delta k_f \right),$$

one obtains by making use of Eqs. (2) and (8)

$$s_f^2 c_f^2 = \frac{\pi \alpha_{EM} (1 + \Delta\hat{k}_f)[1 - (\hat{s}^2/\hat{c}^2) \Delta\hat{k}_f]}{\sqrt{2} M_Z^2 G_F (1 - \Delta\hat{\alpha}) \hat{\rho} (1 - \Delta\hat{r}_W)}, \qquad (9)$$

where

$$\Delta\hat{k}_f = \frac{\hat{c}}{\hat{s}} \frac{\Pi_{Z\gamma}(M_Z^2) - \Pi_{Z\gamma}(0)}{M_Z^2} + \frac{\hat{\alpha}\hat{c}^2}{\pi \hat{s}^2} \log\left(\frac{M_W^2}{M_Z^2} \right)$$

$$- \frac{\hat{\alpha}}{4\pi \hat{s}^2} V_f(M_Z^2) + \delta k_f^{\text{SUSY}}. \qquad (10)$$

The function $V_f(M_Z^2)$ can be obtained from Ref. [4].[3] δk_f^{SUSY} denotes the nonuniversal supersymmetric self-energies and vertex corrections to s_f^2.

In order to study MSSM (or SM) corrections to s_f^2, we need calculate first the Z and W gauge boson self-energy corrections which contribute to $\hat{\rho}$ and $\Delta\hat{r}_W$. Our expressions

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[3]In the case f=bottom, the important top quark corrections to $Zb\bar{b}$ vertex should be added to V_f.

agree with those of Ref. [11][4] and [19]. We need also calculate the Z-γ propagator corrections, the wave function renormalization of external fermions as well as the $Zf\bar{f}$ vertex corrections which contribute to $\Delta\hat{k}_f$. The supersymmetric contributions to last two were found to be negligible, for the leptonic case, in the minimal supergravity model studied in Ref. [11]. Including all these corrections in Eq. (9), we expect that the effective weak mixing angle s_f^2, does not suffer from potentially large logarithms $\sim\log(M_{\text{SUSY}}^2/M_Z^2)$.

At this point we should say that when the electroweak symmetry is broken by radiative corrections, the value of the parameter μ, which specifies the mixing of the two Higgs multiplets within the superpotential, turns out to be of the order of the supersymmetry breaking scale in most of the parameter space. Under these circumstances it is not only the large logarithms $\log(M_{\text{SUSY}}^2/M_Z^2)$ which should be cancelled but also logarithms involving the parameter μ.

III. DECOUPLING OF $\log(M_{\text{SUSY}}/M_Z)$ IN THE EFFECTIVE MIXING ANGLE

In this section we will first show how the potentially dangerous $\sim\log(M_{1,2}/M_Z), \log(\mu/M_Z)$ from the contributions of the neutralinos and charginos are cancelled in the expression for $\sin^2\theta_{\text{eff}}^f$ when the soft SUSY breaking parameter $M_{1/2}$ is large ($M_{\text{SUSY}}^2 \gg M_Z^2$). There are three sources of large logarithms which affect the value of the weak mixing angle $\sin^2\theta_{\text{eff}}^f$: (i) Gauge boson self-energies which feed large logs to the quantities $\Delta\hat{r}_W$, $\hat{\rho}$, and $\Delta\hat{k}_f$; (ii) vertex, external wave function renormalizations, and box corrections to muon decay which affect $\Delta\hat{r}_W$ through $\delta_{VB}^{\text{SUSY}}$; (iii) Nonuniversal vertex and external fermion corrections to $Z\bar{f}f$ coupling which affects $\Delta\hat{k}_f$. We shall see that the corrections (i) are cancelled against large logs stemming from the electromagnetic coupling $\hat{\alpha}(M_Z)$. (ii) and (iii) are cancelled against themselves.

In order to prove the cancellation of the large $\log(M_{\text{SUSY}}/M_Z)$ terms among the dimensionless quantities $\Delta\hat{r}_W$, $\hat{\rho}$, $\Delta\hat{k}_f$, and $\hat{\alpha}(M_Z)$, through which $\sin^2\theta_{\text{eff}}^f$ is defined, it suffices to ignore the electroweak symmetry breaking effects, e.g., $\langle H_1^0 \rangle = \langle H_2^0 \rangle = 0$. In this case the masses of charginos and neutralinos take the simple form

$$m_{\chi_i^0} = M_1, \quad M_2, \quad \mu, \quad -\mu, \qquad (11)$$

$$m_{\chi_i^+} = M_2, \quad \mu. \qquad (12)$$

A. Vector boson self-energy corrections

The contributions from the chargino/neutralino sector to the vector bosons self-energies are

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[4]To match our conventions with those of Ref. [11] we have to replace their matrices with the following: $N \to \mathcal{O}^T$ and $U \to U^*$.

$$\Pi_{ZZ}^{\chi_i^0/\chi_i^+} = \frac{\hat{g}^2}{16\pi^2}\left\{\frac{1}{2}H(\mu,\mu)\left[\frac{1}{\hat{c}^2}+4\left(\hat{c}-\frac{1}{2\hat{c}}\right)^2\right]\right.$$

$$+\mu^2 B_0(\mu,\mu)\left[\frac{1}{\hat{c}^2}+4\left(\hat{c}-\frac{1}{2\hat{c}}\right)^2\right]$$

$$\left.+2\hat{c}^2 H(M_2,M_2)+4\hat{c}^2 M_2^2 B_0(M_2,M_2)\right\},$$

$$(13)$$

$$\Pi_{WW}^{\chi_i^0/\chi_i^+} = \frac{\hat{g}^2}{16\pi^2}[H(\mu,\mu)+2\mu^2 B_0(\mu,\mu)$$

$$+2H(M_2,M_2)+4M_2^2 B_0(M_2,M_2)], \qquad (14)$$

$$\Pi_{Z\gamma}^{\chi_i^0/\chi_i^+} = \frac{\hat{e}\hat{g}\hat{c}_{2\theta_W}}{16\pi^2\hat{c}}[4\tilde{B}_{22}(\mu,\mu)+p^2 B_0(\mu,\mu)]$$

$$+\frac{2\hat{e}\hat{g}\hat{c}}{16\pi^2}[4\tilde{B}_{22}(M_2,M_2)+p^2 B_0(M_2,M_2)],$$

$$(15)$$

where $\hat{g}=\hat{e}/\hat{s}$ is the running $\overline{\text{DR}}$ SU(2) gauge coupling.

In order to calculate the dependence of s_f^2 on $M_{1,2}/\mu$ we make use of Eqs. (3–5) and reduce all functions appearing in the expressions for the two point functions above in terms of the basic integrals A_0,B_0 (see Appendix B). Isolating the logarithmic dependencies on $M_{1,2}/\mu$ we find that

$$\Delta\hat{r}_W = \frac{\hat{\alpha}}{4\pi}\frac{2}{3\hat{s}^2}\left[1+2\log\left(\frac{M_2^2}{Q^2}\right)+\log\left(\frac{\mu^2}{Q^2}\right)\right], \qquad (16)$$

$$\Delta\hat{\rho} = \frac{\hat{\alpha}}{4\pi}\frac{2}{9\hat{c}^2}\left[1+2\hat{c}_{2\theta_W}+6\hat{c}^2\log\left(\frac{M_2^2}{Q^2}\right)\right.$$

$$\left.+3\hat{c}_{2\theta_W}\log\left(\frac{\mu^2}{Q^2}\right)\right], \qquad (17)$$

$$\Delta\hat{k}_f = -\frac{\hat{\alpha}}{4\pi}\frac{2\cot\theta_W}{9\hat{s}^2}\left[\hat{s}_{2\theta_W}+\hat{c}_{2\theta_W}\tan\theta_W+3\hat{s}_{2\theta_W}\log\left(\frac{M_2^2}{Q^2}\right)\right.$$

$$\left.+3\hat{c}_{2\theta_W}\tan\theta_W\log\left(\frac{\mu^2}{Q^2}\right)\right], \qquad (18)$$

$$\Delta\hat{\alpha} = -\frac{\hat{\alpha}}{3\pi}\left[\log\left(\frac{M_2^2}{Q^2}\right)+\log\left(\frac{\mu^2}{Q^2}\right)\right]. \qquad (19)$$

The angle $\hat{\theta}_W$ is the weak mixing angle defined through ratios of couplings in the $\overline{\text{DR}}$ scheme and θ_W is the on shell mixing angle defined by $\sin^2\theta_W=1-M_W^2/M_Z^2$. In the equations above $\hat{c}_{2\theta_W}\equiv\cos(2\hat{\theta}_W)$, $\hat{s}_{2\theta_W}\equiv\sin(2\hat{\theta}_W)$ with similar definitions for $c_{2\theta_W},s_{2\theta_W}$.

Plugging all that into Eq. (9), we find that $s_{\text{eff}}^2 c_{\text{eff}}^2$ is corrected as

$$\Delta(s_{\text{eff}}^2 c_{\text{eff}}^2) = \frac{\pi\alpha_{EM}}{\sqrt{2}M_Z^2 G_F}\frac{8}{9}\left(\frac{\hat{\alpha}}{4\pi}\right), \qquad (20)$$

which at one loop order is independent of large logs. It must be noted that this result is also independent of the sign of μ. However, this finite correction vanishes when the next to leading terms in the expansion of B_0 are considered.

B. Vertex and box corrections from muon decay

The nonuniversal contribution to $\Delta\hat{r}_W$, which contains vertex and box as well as external wave function renormalization corrections, is divided into two parts

$$\delta_{VB} = \delta_{VB}^{\text{SM}} + \delta_{VB}^{\text{SUSY}}. \qquad (21)$$

The standard model part appears in Ref. [4]. The supersymmetric contributions can be found in Refs. [11,20]. We reproduce the results of Ref. [11] for the wavefunction and vertex corrections here,

$$\delta_{VB}^{\text{SUSY}} = -\frac{\hat{s}^2\hat{c}^2}{2\pi\hat{\alpha}}M_Z^2\text{Re}\,a_1+\left(\delta v_e+\frac{1}{2}\delta Z_e+\frac{1}{2}\delta Z_{\nu_e}\right)$$

$$+\left(\delta v_\mu+\frac{1}{2}\delta Z_\mu+\frac{1}{2}\delta Z_{\nu_\mu}\right), \qquad (22)$$

where the wave-function and vertex corrections are

$$16\pi^2\delta Z_{\nu_e} = -\sum_{i=1}^{2}|a_{\tilde{\chi}_i^+\nu_e\tilde{e}_L}|^2 B_1(0,m_{\tilde{\chi}_i^+},m_{\tilde{e}_L})-\sum_{j=1}^{4}|a_{\tilde{\chi}_j^0\nu_e\tilde{\nu}_e}|^2 B_1(0,m_{\tilde{\chi}_j^0},m_{\tilde{\nu}_e}), \qquad (23)$$

$$16\pi^2\delta Z_e = -\sum_{i=1}^{2}|a_{\tilde{\chi}_i^+ e\tilde{\nu}_e}|^2 B_1(0,m_{\tilde{\chi}_i^+},m_{\tilde{\nu}_e})-\sum_{j=1}^{4}|a_{\tilde{\chi}_j^0 e\tilde{e}_L}|^2 B_1(0,m_{\tilde{\chi}_j^0},m_{\tilde{e}_L}), \qquad (24)$$

$$16\pi^2\,\delta v_e=\sum_{i=1}^{2}\sum_{j=1}^{4}a_{\tilde\chi_i^+\nu_e\tilde e_L}a^*_{\tilde\chi_j^0 e\tilde e_L}\left\{-\sqrt{\frac{2}{g}}a_{\tilde\chi_j^0\tilde\chi_i^+ W}m_{\tilde\chi_i^+}m_{\tilde\chi_j^0}C_0(m_{\tilde e_L},m_{\tilde\chi_i^+},m_{\tilde\chi_j^0})\right.$$

$$\left.+\frac{1}{\sqrt{2}g}b_{\tilde\chi_j^0\tilde\chi_i^+ W}\left[B_0(0,m_{\tilde\chi_i^+},m_{\tilde\chi_j^0})+m_{\tilde e_L}^2 C_0(m_{\tilde e_L},m_{\tilde\chi_i^+},m_{\tilde\chi_j^0})-\frac{1}{2}\right]\right\}$$

$$-\sum_{i=1}^{2}\sum_{j=1}^{4}a_{\tilde\chi_i^+ e\tilde\nu_e}a_{\tilde\chi_j^0\nu_e\tilde\nu_e}\left\{-\sqrt{\frac{2}{g}}b_{\tilde\chi_j^0\tilde\chi_i^+ W}m_{\tilde\chi_i^+}m_{\tilde\chi_j^0}C_0(m_{\tilde\nu_e},m_{\tilde\chi_i^+},m_{\tilde\chi_j^0})\right.$$

$$\left.+\frac{1}{\sqrt{2}g}a_{\tilde\chi_j^0\tilde\chi_i^+ W}\left[B_0(0,m_{\tilde\chi_i^+},m_{\tilde\chi_j^0})+m_{\tilde\nu_e}^2 C_0(m_{\tilde\nu_e},m_{\tilde\chi_i^+},m_{\tilde\chi_j^0})-\frac{1}{2}\right]\right\}$$

$$+\frac{1}{2}\sum_{j=1}^{4}a^*_{\tilde\chi_j^0 e\tilde e_L}a_{\tilde\chi_j^0\nu_e\tilde\nu_e}\left[B_0(0,m_{\tilde e_L},m_{\tilde\nu_e})+m_{\tilde\chi_j^0}^2 C_0(m_{\tilde\chi_j^0},m_{\tilde e_L},m_{\tilde\nu_e})+\frac{1}{2}\right],\tag{25}$$

and the nonvanishing couplings are given by[5]

$$a_{\tilde\chi_1^+\nu_e\tilde e_L}=a_{\tilde\chi_1^+ e\tilde\nu_e}=\frac{\hat e}{\hat s},\tag{26}$$

$$a_{\tilde\chi_1^0\nu_e\tilde\nu_e}=a_{\tilde\chi_1^0 e\tilde e_L}=-\frac{\hat e}{\sqrt{2}\hat c},\tag{27}$$

$$a_{\tilde\chi_2^0\nu_e\tilde\nu_e}=-a_{\tilde\chi_2^0 e\tilde e_L}=\frac{\hat e}{\sqrt{2}\hat s}.\tag{28}$$

In all expressions above the functions $B_{0,1},C_0$ are considered with vanishing momenta squared and their analytic expressions in terms of the masses involved are given in Appendix B. We recall that we have ignored EW symmetry breaking effects so that $m_{\tilde\chi_1^0}=M_1$, $m_{\tilde\chi_2^0}=M_2$, and $m_{\tilde\nu_e}=m_{\tilde e_L}=M_{\tilde L}$. We have compared these results with those of Ref. [20] and we have found agreement. Dangerous large log corrections are contained only in the second and third part of the Eq. (22). For these terms we obtain

$$\left(\delta v_e+\frac{1}{2}\delta Z_e+\frac{1}{2}\delta Z_{\nu_e}\right)=-\frac{1}{\hat s^2}\left(\frac{\hat\alpha}{4\pi}\right)\left\{2M_2^2 C_0(M_{\tilde L},M_2,M_2)-M_{\tilde L}^2 C_0(M_{\tilde L},M_2,M_2)+\frac{1}{4}M_2^2 C_0(M_2,M_{\tilde L},M_{\tilde L})\right.$$

$$-\frac{1}{4}M_1^2\frac{\hat s^2}{\hat c^2}C_0(M_1,M_{\tilde L},M_{\tilde L})-B_0(0,M_2,M_2)+\frac{1}{4}\left(1-\frac{\hat s^2}{\hat c^2}\right)B_0(0,M_{\tilde L},M_{\tilde L})+\frac{1}{2}+\frac{1}{8}\left(1-\frac{\hat s^2}{\hat c^2}\right)$$

$$\left.+\frac{3}{2}B_1(0,M_2,M_{\tilde L})+\frac{\hat s^2}{2\hat c^2}B_1(0,M_1,M_{\tilde L})\right\}.\tag{29}$$

Using Eqs. (B7)–(B10) we find that the expression above involves no large logarithms. Also as said before the first term (Re α_1) in Eq. (22) contains finite parts which go as $\sim M_Z/M_{\rm SUSY}$. Thus no large logarithmic terms arising from the wave function and vertex corrections of the muon decay and the decoupling of large logarithms in s_f^2 appear.

C. Nonuniversal corrections to $\Delta\hat k_f$

The $Zf\bar f$ vertex corrections can be written as

$$i\frac{\hat e}{2\hat s\hat c}\gamma^\mu(F_V^{(f)}-\gamma_5 F_A^{(f)}),\tag{30}$$

where $F_V^{(f)},F_A^{(f)}$ denote the vector and axial couplings, respectively. Incorporating the tree level couplings we can write this vertex in a slightly different form as[6]

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[5]To conform with the notation of Ref. [11] we use the couplings $a_{\tilde\chi_a^0\tilde\chi_i^+ W}\equiv g\mathcal{P}_{ai}^L$, $b_{\tilde\chi_a^0\tilde\chi_i^+ W}\equiv g\mathcal{P}_{ai}^R$. $\mathcal{P}_{ai}^L$ and $\mathcal{P}_{ai}^R$ are given in Appendix A [see Eqs. (A13)]. Also the lepton, slepton, chargino (or neutralino) couplings in Eqs. (25)–(28) differ in sign from those given in Eq. (A20).

[6]We follow the notation of Ref. [13] which will be useful in what follows.

$$i \; \frac{\hat{e}}{\hat{s}\hat{c}} \gamma^{\mu}(u'_L \mathcal{P}_L + u'_R \mathcal{P}_R), \qquad (31)$$

where

$$u'_L = u_L + \frac{F_L^{(f)}}{16\pi^2}, \qquad (32)$$

$$u'_R = u_R + \frac{F_R^{(f)}}{16\pi^2}. \qquad (33)$$

In the equations above u_L, u_R are the tree level left and right handed couplings respectively related to the vector v_f and axial a_f tree level couplings by $v_f = u_L + u_R$, $a_f = u_L - u_R$. $F_{L,R}^{(f)}$ denote the corresponding one loop corrections to the aforementioned couplings, with the coefficient $1/16\pi^2$ factored out for convenience. These are related to $F_V^{(f)}, F_A^{(f)}$ of Eq. (30) by

$$F_V^{(f)} \equiv \frac{1}{16\pi^2} (F_L^{(f)} + F_R^{(f)}), \qquad (34)$$

$$F_A^{(f)} \equiv \frac{1}{16\pi^2} (F_L^{(f)} - F_R^{(f)}). \qquad (35)$$

As a result the corrections to $\Delta\hat{k}_f$ are given by

$$\Delta\hat{k}_f = -\frac{1}{16\pi^2} \frac{1}{\hat{s}^2 Q_f (u_L - u_R)} (u_L F_R^{(f)} - u_R F_L^{(f)}), \quad (36)$$

and are equivalent to the well known expression

$$\Delta\hat{k}_f = -\frac{1}{2\hat{s}^2 Q_f} \left(F_V^{(f)} - \frac{v_f}{a_f} F_A^{(f)} \right). \qquad (37)$$

In Eq. (10) we have denoted by $\delta k_f^{\mathrm{SUSY}}$ the supersymmetric contributions to $\Delta\hat{k}_f$. Here we consider the example of the decoupling of large logs in $\delta k_f^{\mathrm{SUSY}}$ in the case where the fermion f stands for a ''down'' quark denoted by b being in the same isospin multiplet with the ''up'' quark denoted by t. In this case we have

$$u_L = -\frac{1}{2} + \frac{1}{3}\hat{s}^2, \qquad (38)$$

$$u_R = \frac{1}{3}\hat{s}^2, \qquad (39)$$

$$Q_f = -\frac{1}{3}. \qquad (40)$$

The cases of other fermion species are treated in a similar manner. In what follows we will consider only the chargino corrections to vertices and external fermion lines. The decou-
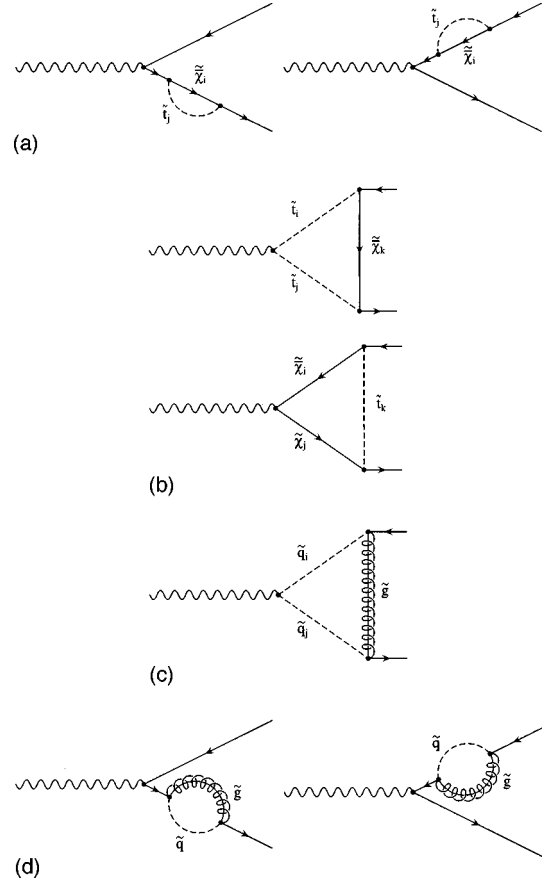

(a)

(b)

(c)

(d)

FIG. 2. (a) Self-energy chargino and squark corrections to the $Zb\bar{b}$ vertex. (b) Chargino and squark contributions to the $Zb\bar{b}$ vertex. (c) Supersymmetric QCD corrections to the $Zq\bar{q}$ vertex from gluino and squark contributions. (d) Self-energy gluino and squark contributions to the $Zq\bar{q}$ vertex.

pling of large logarithmic terms arising from the neutralinos exchanges proceeds in exactly the same manner.

We will first discuss the self energy corrections to $Zb\bar{b}$ vertex. From the diagrams of the Fig. 2(a), we obtain,[7] in an

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[7] The functions b_1, c_0 used throughout this section which are defined below should not be confused with the Passarino-Veltman functions [21] which are commonly denoted by capital letters. These are actually the reduced Passarino-Veltman functions [22] defined as

$$b_1(m_1, m_2, q)$$

$$\equiv \int_0^1 dx \, x \log \frac{xm_1^2 + (1-x)m_2^2 - q^2 x(1-x) - i\epsilon}{Q^2},$$

$$c_0(m_1, m_2, m_3)$$

$$\equiv \int_0^1 dx \int_0^{1-x} dy$$

$$\times \log \frac{(1-x-y)m_1^2 + xm_2^2 + ym_3^2 - (1-x-y)(x+y)m_b^2 - xyP^2}{Q^2}.$$

obvious notation,

$$F_L^{(b)} = \sum_{i=1,2} \sum_{j=1,2} b_1(m_{\tilde{t}_j}, m_{\tilde{\chi}_i}, m_b) u_L |a_{ij}^{b\tilde{t}\tilde{\chi}}|^2, \qquad (41)$$

$$F_R^{(b)} = \sum_{i=1,2} \sum_{j=1,2} b_1(m_{\tilde{t}_j}, m_{\tilde{\chi}_i}, m_b) u_R |b_{ij}^{b\tilde{t}\tilde{\chi}}|^2. \qquad (42)$$

From Appendix A [see the discussion following Eqs. (A19)] we get $a_{11}^{b\tilde{t}\tilde{\chi}} = g$, $a_{22}^{b\tilde{t}\tilde{\chi}} = -h_t$, and $b_{21}^{b\tilde{t}\tilde{\chi}} = -h_b$. All other couplings vanish when the electroweak symmetry breaking effects are ignored. Thus we get

$$F_L^{(b)} = u_L[g^2 b_1(m_{\tilde{t}_L}, M_2, m_b) + h_t^2 b_1(m_{\tilde{t}_R}, \mu, m_b)], \qquad (43)$$

$$F_R^{(b)} = u_R[h_b^2 b_1(m_{\tilde{t}_L}, \mu, m_b)]. \qquad (44)$$

On the other hand, from the first triangle graph of Fig. 2(b) we obtain

$$F_L^{(b)} = \sum_{i,j,k=1,2} c_0(m_{\tilde{\chi}_k}, m_{\tilde{t}_i}, m_{\tilde{t}_j})$$
$$\times \left(\frac{2}{3} \hat{s}^2 \delta_{ij} - \frac{1}{2} K_{i1}^{\tilde{t}*} K_{j1}^{\tilde{t}} \right) a_{ki}^{b\tilde{t}\tilde{\chi}} a_{kj}^{b\tilde{t}\tilde{\chi}*}, \qquad (45)$$

$$F_R^{(b)} = \sum_{i,j,k=1,2} c_0(m_{\tilde{\chi}_k}, m_{\tilde{t}_i}, m_{\tilde{t}_j})$$
$$\times \left(\frac{2}{3} \hat{s}^2 \delta_{ij} - \frac{1}{2} K_{i1}^{\tilde{t}*} K_{j1}^{\tilde{t}} \right) b_{ki}^{b\tilde{t}\tilde{\chi}} b_{kj}^{b\tilde{t}\tilde{\chi}*}, \qquad (46)$$

which, when the electroweak effects are ignored, have the following form:

$$F_L^{(b)} = \left(\frac{2}{3} \hat{s}^2 - \frac{1}{2} \right) g^2 c_0(M_2, m_{\tilde{t}_L}, m_{\tilde{t}_L})$$
$$+ \frac{2}{3} \hat{s}^2 h_t^2 c_0(\mu, m_{\tilde{t}_R}, m_{\tilde{t}_R}), \qquad (47)$$

$$F_R^{(b)} = \left(\frac{2}{3} \hat{s}^2 - \frac{1}{2} \right) h_b^2 c_0(\mu, m_{\tilde{t}_L}, m_{\tilde{t}_L}). \qquad (48)$$

The calculation of the second diagram of Fig. 2(b) gives[8]

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[8] $[c_2, c_6](m_1, m_2, m_3)$

$$= \int_0^1 dx \int_0^{1-x} [1, x]$$

$$\times \frac{1}{(1-x-y)m_1^2 + xm_2^2 + ym_3^2 - (1-x-y)(x+y)m_b^2 - xyP^2 - i\epsilon}.$$

$$F_L^{(b)} = -\sum_{i,j,k=1,2} \left\{ \left[P^2 c_6(m_{\tilde{t}_k}, m_{\tilde{\chi}_i}, m_{\tilde{\chi}_j}) \right. \right.$$
$$\left. - \frac{1}{2} - c_0(m_{\tilde{t}_k}, m_{\tilde{\chi}_i}, m_{\tilde{\chi}_j}) \right] \mathcal{A}^L{}_{ij}$$
$$\left. + m_{\tilde{\chi}_i} m_{\tilde{\chi}_j} c_2(m_{\tilde{t}_k}, m_{\tilde{\chi}_i}, m_{\tilde{\chi}_j}) \mathcal{A}^R{}_{ij} \right\} a_{ik}^{b\tilde{t}\tilde{\chi}} a_{jk}^{b\tilde{t}\tilde{\chi}*}, \qquad (49)$$

$$F_R^{(b)} = -\sum_{i,j,k=1,2} \left\{ \left[P^2 c_6(m_{\tilde{t}_k}, m_{\tilde{\chi}_i}, m_{\tilde{\chi}_j}) \right. \right.$$
$$\left. - \frac{1}{2} - c_0(m_{\tilde{t}_k}, m_{\tilde{\chi}_i}, m_{\tilde{\chi}_j}) \right] \mathcal{A}^R{}_{ij}$$
$$\left. + m_{\tilde{\chi}_i} m_{\tilde{\chi}_j} c_2(m_{\tilde{t}_k}, m_{\tilde{\chi}_i}, m_{\tilde{\chi}_j}) \mathcal{A}^L{}_{ij} \right\} b_{ik}^{b\tilde{t}\tilde{\chi}} b_{jk}^{b\tilde{t}\tilde{\chi}*}, \qquad (50)$$

where P is the momentum carried by the Z boson. The couplings $\mathcal{A}^L{}_{ij}, \mathcal{A}^R{}_{ij}$ can be read from Appendix A [see Eqs. (A17)]. In the absence of electroweak symmetry breaking effects the only nonvanishing couplings are

$$\mathcal{A}^L{}_{11} = \hat{c}^2 = \mathcal{A}^R{}_{11}, \qquad (51)$$

$$\mathcal{A}^L{}_{22} = \hat{c}^2 - \frac{1}{2} = \mathcal{A}^R{}_{22}. \qquad (52)$$

Thus, we obtain

$$F_L^{(b)} = g^2 \hat{c}^2 c_0(m_{\tilde{t}_L}, M_2, M_2) + h_t^2 \left(\hat{c}^2 - \frac{1}{2} \right) c_0(m_{\tilde{t}_R}, \mu, \mu), \qquad (53)$$

$$F_R^{(b)} = h_b^2 \left(\hat{c}^2 - \frac{1}{2} \right) c_0(m_{\tilde{t}_L}, \mu, \mu). \qquad (54)$$

Summing up the diagrams of Figs. 2(a) and 2(b) we get

$$F_L^{(b)} = \left(-\frac{1}{2} + \frac{1}{3}\hat{s}^2 \right) [g^2 b_1(m_{\tilde{t}_L}, M_2, m_b) + h_t^2 b_1(m_{\tilde{t}_R}, \mu, m_b)]$$
$$+ \left(\frac{2}{3}\hat{s}^2 - \frac{1}{2} \right) g^2 c_0(M_2, m_{\tilde{t}_L}, m_{\tilde{t}_L})$$
$$+ \frac{2}{3}\hat{s}^2 h_t^2 c_0(\mu, m_{\tilde{t}_R}, m_{\tilde{t}_R}) + g^2 \hat{c}^2 c_0(m_{\tilde{t}_L}, M_2, M_2)$$
$$+ h_t^2 \left(\hat{c}^2 - \frac{1}{2} \right) c_0(m_{\tilde{t}_R}, \mu, \mu), \qquad (55)$$

$$F_R^{(b)} = \frac{1}{3}\hat{s}^2 h_b^2 b_1(m_{\tilde{t}_L}, \mu, m_b) + \left(\frac{2}{3}\hat{s}^2 - \frac{1}{2} \right) h_b^2 c_0(\mu, m_{\tilde{t}_L}, m_{\tilde{t}_L})$$
$$+ h_b^2 \left(\hat{c}^2 - \frac{1}{2} \right) c_0(m_{\tilde{t}_L}, \mu, \mu). \qquad (56)$$

In the limit of $q^2 = m_b^2 \simeq 0$, $P^2 = M_Z^2 \rightarrow 0$ or $M_Z^2 \ll M_{SUSY}^2$, the following useful relations hold:

$$c_0(m_1,m_2,m_3) = b_1(m_2,m_1,0), \qquad (57)$$

$$c_0(m_1,m_2,m_3) - b_1(m_1,m_2,0)$$

$$= \frac{1}{m_1^2 - m_2^2} \left[m_1^2 m_2^2 \log\left(\frac{m_1^2}{m_2^2}\right) - \frac{1}{2}(m_1^2 + m_2^2) \right]. \quad (58)$$

Using these we have for the expressions for $F_{L,R}^{(b)}$ above

$$F_L^{(b)} = h_t^2 O\left(\frac{m_{\tilde{t}_R}^2}{\mu^2}\right) + g^2 O\left(\frac{m_{\tilde{t}_L}^2}{M_2^2}\right) \qquad (59)$$

and

$$F_R^{(b)} = h_b^2 O\left(\frac{m_{\tilde{t}_L}^2}{\mu^2}\right), \qquad (60)$$

which is independent of large logs and the decoupling of terms $\log(M_{SUSY}/M_Z)$ is manifest.

So far we have considered the cancellation of potentially large logarithms involving the soft SUSY breaking scale $M_{1/2}$ and the mixing parameter μ which arise from the neutralino and chargino sectors when $M_{1/2} \gg M_Z$. A similar analysis can be repeated for the corresponding contributions of the squark and slepton sectors, whose masses depend also on the soft SUSY breaking parameters M_0, when M_0 gets large. We have carried out such an analysis and found that the decoupling of large logarithms does indeed occur when these parameters get large values. It is not necessary to present the details of such a calculation here. We merely state that large logarithms arising from the vector boson self-energy corrections which contribute to the quantities $\Delta\hat{r}_W$, $\hat{\rho}$, and $\Delta\hat{k}_f$ cancel against those from $\hat{\alpha}(M_Z)$. Also, the large log contributions from the muon decay amplitude, which affect the effective mixing angle through δ_{VB}^{SUSY}, cancel among themselves. As for the large logarithmic contributions to the weak mixing angle from the nonuniversal corrections to the factor $\Delta\hat{k}_f$, these are found to be cancelled in exactly the same way as in the case of the neutralinos and charginos.[9]

D. SQCD corrections to $\Delta\hat{k}_f$

The last corrections to be considered are the SQCD nonuniversal corrections [23] which, due the largeness of the strong coupling constant, are naively expected to yield contributions larger than those of the electroweak sector. This case is of relevance only when the external fermions in the $Zf\bar{f}$ vertex are quark fields and is of particular interest for the bottom case whose measurement of the forward-backward asymmetry $\mathcal{A}_b^{FB}$ yields the most precise individual measurement at LEP.

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[9] The logarithmic corrections of the Higgs sector to the $Zb\bar{b}$ vertex and external b lines are cancelled in exactly the same manner.

The one loop correction to $Zq\bar{q}$ vertex [see Fig. 2(c)] where two squarks, which are coupled to the Z boson, and a gluino are exchanged yields for the left and right handed couplings defined in Eqs. (30)–(33),

$$F_L^{(q)} = \frac{16}{3}(4\pi\alpha_s) \sum_{i=1,2} \sum_{j=1,2} K_{j1}^{\tilde{q}\star} K_{i1}^{\tilde{q}} A_{\tilde{q}}^{ji} C_{24}$$

$$\times (m_q^2, M_Z^2, m_q^2; M_{\tilde{g}}^2, m_{\tilde{q}_i}^2, m_{\tilde{q}_j}^2), \qquad (61)$$

$$F_R^{(q)} = \frac{16}{3}(4\pi\alpha_s) \sum_{i=1,2} \sum_{j=1,2} K_{j2}^{\tilde{q}\star} K_{i2}^{\tilde{q}} A_{\tilde{q}}^{ji} C_{24}$$

$$\times (m_q^2, M_Z^2, m_q^2; M_{\tilde{g}}^2, m_{\tilde{q}_i}^2, m_{\tilde{q}_j}^2). \qquad (62)$$

In these, the coupling $A_{\tilde{q}}^{ji}$ is given by

$$A_{\tilde{q}}^{ji} = u_L K_{j1}^{\tilde{q}\star} K_{i1}^{\tilde{q}} + u_R K_{j2}^{\tilde{q}\star} K_{i2}^{\tilde{q}},$$

with $K_{ij}^{\tilde{q}}$ the matrix diagonalizing the squark $\tilde{q}$ mass matrix. The function C_{24}, with momenta and masses as shown, is the coefficient of $g_{\mu\nu}$ in the tensor three point integral (this is denoted by C_{20} in Ref. [24]). The contribution of $F_{L,R}^{(q)}$ to the form factor $\Delta\hat{k}_q$ is free of large logarithms. In order to understand this consider the case of vanishing quark mass m_q. In this case the matrix $K_{ij}^{\tilde{q}}$ becomes the unit matrix. It is easy to see that the contribution to $\Delta\hat{k}_q$, as this is read from Eq. (36), is proportional to the difference

$$C_{24}(m_q^2, M_Z^2, m_q^2; M_{\tilde{g}}^2, m_{\tilde{q}_1}^2, m_{\tilde{q}_1}^2)$$

$$- C_{24}(m_q^2, M_Z^2, m_q^2; M_{\tilde{g}}^2, m_{\tilde{q}_2}^2, m_{\tilde{q}_2}^2).$$

In this difference the leading log terms cancel each other. Note that it would vanish if the left and right handed squark fields happened to be degenerate in mass. Due to their mass splitting, however, the result is not vanishing but at any rate small. In general the SQCD vertex corrections turn out to be smaller than the corresponding electroweak corrections, as we have verified numerically.

As for the external quark contributions [see Fig. 2(d)] we find

$$F_L^{(q)} = \frac{8}{3}(4\pi\alpha_s) u_L [c^2 B_1(m_q^2, M_{\tilde{g}}^2, m_{\tilde{q}_1}^2) + s^2 B_1(m_q^2, M_{\tilde{g}}^2, m_{\tilde{q}_2}^2)], \qquad (63)$$

$$F_R^{(q)} = \frac{8}{3}(4\pi\alpha_s) u_R [s^2 B_1(m_q^2, M_{\tilde{g}}^2, m_{\tilde{q}_1}^2) + c^2 B_1(m_q^2, M_{\tilde{g}}^2, m_{\tilde{q}_2}^2)]. \qquad (64)$$

In the equation above $c \equiv K_{11}^{\tilde{q}}$, $s \equiv K_{12}^{\tilde{q}}$. Their contribution to $\Delta \hat{k}_q$ is free of large logarithms and small due to cancellations of the leading terms exactly as in the case of the vertex corrections discussed previously. In fact in the limit of vanishing quark mass the self-energy corrections to $\Delta \hat{k}_q$ is proportional to the difference

$$B_1(m_q^2, M_{\tilde{g}}^2, m_{\tilde{q}_1}^2) - B_1(m_q^2, M_{\tilde{g}}^2, m_{\tilde{q}_2}^2)$$

which vanishes when the squark masses are equal. Therefore, following the same arguments as in the vertex case, we are led to the conclusion that SQCD contributions from the external quark lines are small.

In addition to the cancellations discussed above which lead to relatively small SQCD vertex and external fermion corrections, these two contributions tend to cancel each other since they contribute with opposite signs. This results in very small overall SQCD corrections to $\Delta \hat{k}_q$ almost one to two orders of magnitude smaller than the corresponding nonuniversal electroweak corrections. We shall come back to this point later when discussing our numerical results.

In the following section we shall discuss our numerical results concerning the predictions of the MSSM for the effective mixing angle and asymmetries. We will also present the corresponding theoretical predictions for the mass of the W-boson through its connection to the parameter rho and the effective mixing angle.

IV. NUMERICAL ANALYSIS AND RESULTS

For a given set of pole masses m_t^{pole}, m_b^{pole}, m_τ^{pole} we define the $\overline{\text{DR}}$ Yukawa couplings at M_Z. To start with, we set a test value for the $\hat{s}^2$ (i.e, $\hat{s}^2 = 0.2315$) and we define the $\overline{\text{DR}}$ gauge couplings $\hat{g}_1$ and $\hat{g}_2$ at M_Z. The numerical output is independent of the starting value for $\hat{s}^2$. For $\hat{s}^2$ around the value given above the number of iterations needed for convergence is minimized. Then we use the two-loop renormalization group equations [25] to run up to the scale M_{GUT} where $\hat{g}_1$ and $\hat{g}_2$ meet. At M_{GUT} we impose the unification condition

$$g_{\text{GUT}} \equiv \hat{g}_1 = \hat{g}_2 = \hat{g}_3. \tag{65}$$

Assuming universal boundary conditions for the soft breaking parameters M_0, $M_{1/2}$, and A_0, we run down to M_Z and find the couplings and the soft masses at M_Z which are inputs for the self-energies of the gauge bosons, wave functions and vertex corrections and they define the new $\hat{s}^2$. The whole procedure is iterated until convergence is reached satisfying the full one loop minimization conditions in order to have radiative symmetry breaking observing the experimental bounds on supersymmetric particles. For the calculation of the one loop integrals encountered we have made use of the FF library [26]. The conversion of the "theoretical" $\hat{s}^2$ to the experimental s_f^2 through Eq. (8) gives our basic output:

the effective weak mixing angle s_f^2. In addition, the value of the strong QCD coupling, as it is calculated in the $\overline{\text{MS}}$ scheme at M_Z, is among our outputs [27]. Note that we have used as inputs the parameters α_{EM}, M_Z, and G_F which are experimentally known to a high degree of accuracy, as well the masses of leptons and quarks.

The factor $\Delta \hat{k}_f$ needed to pass from the $\hat{s}^2(M_Z)$ to the effective angle $s_f^2(M_Z)$ receives universal corrections, from the γ-Z propagator, and nonuniversal corrections arising from vertices and external wave function renormalizations. We find that the nonuniversal electroweak supersymmetric corrections are very small. Although separately vertex and external fermion corrections are large they cancel each other yielding contributions almost two orders of magnitude smaller than the rest of the electroweak corrections. The nonuniversal SQCD contributions although *a priori* expected to to be larger than the Electroweak corrections turn out to be even smaller. The reason for this was explained in the previous section. In fact they are found to be one to two orders of magnitude smaller than the corresponding electroweak corrections. We conclude therefore that at the present level of accuracy one can safely ignore the supersymmetric nonuniversal corrections to the factors $\Delta \hat{k}_f$. The situation is very clearly depicted in Table I where for some characteristic input values we give the contributions of the various sectors to $\Delta \hat{k}_f$, as well as their total contributions, and also the corresponding predictions for the values of the effective mixing angle and the asymmetries. Concerning the values displayed in Table I, in a representative case, a few additional remarks are in order (i) The bulk of the supersymmetric corrections to $\Delta \hat{k}_f$ is carried by the universal corrections which are sizable, due to their dependence on large logarithmic terms. These cancel similar terms in $\hat{s}^2$.

(ii) The contribution of Higgs bosons, which is small, mimics that of the standard model with a mass in the vicinity of $\simeq 100$ GeV.

(iii) Gauge and Higgs boson contributions tend to cancel large universal contributions of matter fermions. Concerning the gauge boson contributions note that they are different for the different fermion species l, c, b. This is due to the fact that their nonuniversal corrections depend on the charge and weak isospin assignments of the external fermions and on the mass of the top for when the external fermion is a bottom.

(iv) The slepton universal corrections are suppressed relative to their corresponding squark contributions. This is due to the following reason. The couplings of the left and right handed sleptons to the neutral Z boson depend on the angle $\hat{s}^2$ and would be exactly opposite if $\hat{s}^2$ happened to be $\frac{1}{4}$. Thus their contributions to the γ-Z propagator would be exactly opposite if their masses were equal leading to a vanishing slepton contribution. The fact that $\hat{s}^2 \simeq 0.23$ is close to $\frac{1}{4}$ in conjunction with the fact that the left and right handed sleptons are characterized by small mass splittings leads to the conclusion that universal slepton contributions to $\Delta \hat{k}_f$ are small.

In Fig. 3, we display the effective weak mixing angle $s_l^2(M_Z)$, obtained from the vertex $Z - l^+ - l^-$, and the weak

TABLE I. Partial and total contributions to Δk_f ($f=$lepton,charm,bottom), for two sets of inputs shown at the top. Also shown are the predictions for the effective weak mixing angles and the asymmetries. In the first five rows we display the universal contributions to $10^3 \times \Delta k$ of squarks ($\tilde{q}$), sleptons ($\tilde{l}$), neutralinos and charginos ($\tilde{Z},\tilde{C}$), ordinary fermions and Higgs boson (the number shown in the middle below the ''charm'' column refers to ''lepton'' and ''bottom'' as well). In the next five rows we display the contributions of gauge bosons as well as the supersymmetric electroweak (EW) and SQCD vertex and external fermion wave function renormalization corrections to $10^3 \times \Delta k$.

	$M_0=200$ $m_t=175$	$M_{1/2}=200$ $\tan b=4$	$A_0=200$ $\mu>0$	$M_0=400$ $m_t=175$	$M_{1/2}=400$ $\tan b=4$	$A_0=500$ $\mu>0$
	lepton	charm	bottom	lepton	charm	bottom
$\tilde{q}$		-6.6206			-8.8188	
$\tilde{l}$		-0.3123			-0.3791	
$\tilde{Z},\tilde{C}$		-4.4892			-9.6138	
Fermions		4.6573			4.5103	
Higgs		-0.7312			-1.0113	
Gauge	-3.1782	-3.6272	2.2911	-3.1128	-3.5572	2.2822
Vertex (EW)	1.2324	3.5581	12.7641	2.2825	4.8539	18.1519
Wave (EW)	-1.3209	-3.5208	-12.9598	-2.2973	-4.8422	-18.0059
Vertex (SQCD)		0.2110	-1.1129		0.2361	-1.1166
Wave (SQCD)		-0.2103	1.1012		-0.2359	1.1135
$\Delta k(\times 10^2)$	-1.0763	-1.1085	-0.5412	-1.8440	-1.8858	-1.2888
$\sin^2 \theta_f$	0.23134	0.23126	0.23259	0.23145	0.23135	0.23276
$\mathcal{A}_{LR}^f$	0.1485	0.6684	0.9348	0.1476	0.6681	0.9347
$\mathcal{A}_{FB}^f$	0.0165	0.0744	0.1041	0.0163	0.0740	0.1035

mixing angle $\hat{s}^2(M_Z)$ as functions of the soft gaugino mass parameter $M_{1/2}$, the soft parameter M_0, A_0 as well as the parameters $\tan\beta$ and m_t inside the region which is indicated in the figure. Nonuniversal supersymmetric vertex and external fermion corrections have been taken into account. As is well known there is a discrepancy between the LEP and SLD experimental values of s^2_{eff}. The LEP average $s^2_{\text{eff}}=0.23199 \pm 0.00028$ differs by 2.9σ from the SLD value $s^2_{\text{eff}}=0.23055\pm0.00041$ obtained from the single measurement of left-right asymmetry [3]. The LEP+SLD average value is $s^2_{\text{eff}}=0.23152+0.00023$. We observe that $\hat{s}^2(M_Z)$ takes on the ''theoretical'' value $\hat{s}^2=0.2377$ for $M_{1/2}=900$ GeV and becomes larger and larger due to the fact that it contains large logarithms. Manifest cancellation of large logarithmic terms is obtained in the extracted value of the effective weak mixing angle as we have analytically demonstrated in the previous chapter. In Fig. 3, the dispersion of the values of $s^2_l(M_Z)$ in the lower region of $M_{1/2}$ is caused by the presence of the finite parts of order $\mathcal{O}(M_Z/M_{\text{SUSY}})$ in the expression (8), which become very important when $M_{1/2}\sim M_0\sim M_Z$ (case which is preferred by SLD data) and contribute positively to Δk. When $M_{1/2}\rightarrow 900$ GeV (a case which is rather preferred by LEP data) then $s^2_l(M_Z)\rightarrow0.23145$ indepen-

dently of the value of M_0. It must be noted that, when $M_{1/2}=M_Z$ and $M_0\simeq200$ GeV, the values of the two angles are equal, i.e., $s^2_l(M_Z)=\hat{s}^2(M_Z)=0.2310$. We have also explored the case where the Higgs boson mixing parameter is negative ($\mu<0$). In this case, as $M_{1/2}$ tends to larger values $M_{1/2}\rightarrow900$ GeV, $s^2_l(M_Z)$ approaches the value 0.23145 which means that, for large values of $M_{1/2}$, $s^2_l(M_Z)$ is independent of the sign of μ as it is expected from the decoupling shown in Sec. III. The sign of μ does not affect either the $\hat{s}^2(M_Z)$ value for large $M_{1/2}$. The effect of the sign appears in the lower values of $M_{1/2}$. In the region $M_{1/2}\rightarrow M_Z$, the value of Δk_l ($l=e$, μ, τ) is always negative in the case $\mu<0$ and thus $s^2_l<\hat{s}^2$. There is no possibility of equality between the two angles in this case. The largest value of $s^2_l(M_Z)=0.2315$ [$\hat{s}^2(M_Z)\rightarrow0.238$] is reached when $M_{1/2}\rightarrow1200$ GeV. Just above this value no radiative symmetry breaking occurs. The lowest value of $s^2_l(M_Z)=0.2305$ [$\hat{s}^2(M_Z)=0.2302$], for $\mu>0$, and $s^2_l(M_Z)=0.2309$ [$\hat{s}^2(M_Z)=0.2316$] for $\mu<0$, is bounded by the new experimental limit on the chargino mass which is around $\sim84-86$ GeV [28].

In Fig. 4, we plot the values of s^2_f for the fermions $f=c,b$. In the large SUSY breaking limit, where all superpar-
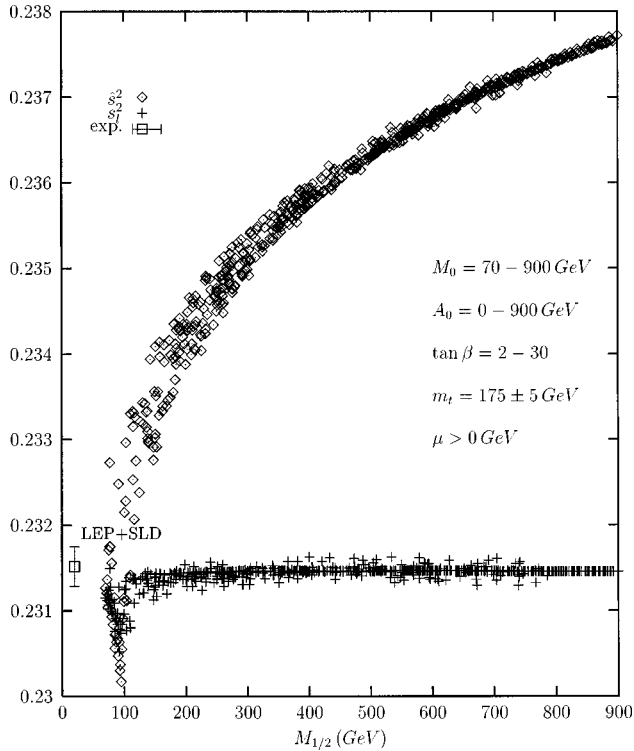
FIG. 3. The effective weak mixing angle $s_l^2(M_Z)$ in comparison to the weak mixing angle $\hat{s}^2(M_Z)$ versus $M_{1/2}$ when the soft parameters M_0, A_0 vary in the indicated regions. The width of each branch is due mainly to the variation on M_0, for low $M_{1/2} < 200$ GeV, and to the variation of the top mass for $M_{1/2} > 200$ GeV. The effect of the variation of $A_0 = 0 - 900$ GeV and of $\tan\beta = 5 - 28$ on s_l^2 is negligible. The error bar show the measured value of $s_l^2 = 0.23152 \pm 0.00023$, obtained at LEP and SLD. The MSSM value is in agreement with the LEP+SLD data for the bulk of the values in the soft parameter space.

ticles are quite massive ($M_{1/2} \to 900$ GeV), we obtain for the central values $s_b^2 = 0.2330$ and $s_c^2 = 0.2314$. In the light limit, $M_{1/2} \simeq M_0 \simeq M_Z$, they take on the values, $s_b^2 = 0.2298$ and $s_c^2 = 0.2308$. The main effect in the extracted values of the effective angle s_l^2 is coming dominantly from the variation of $M_{1/2}$ and secondly from M_0. If $M_{1/2}$ is kept constant, the variation of M_0 from 100 to 900 GeV, changes s_f^2 by $+0.0005$. In addition, the effect of A_0 on the effective angle is negligible. The effect of the independent parameter $\tan\beta$ is also negligible if it remains in the region $\tan\beta \simeq 5 - 28$. Large loop corrections to the b-Yukawa coupling (or to the bottom pole mass), which are proportional to the term $\mu\tan\beta$, affect the obtained values of s_f^2 in the large $\tan\beta$ region [29].

There is a strong correlation of the output value of the effective weak mixing angle with the top quark mass as it is shown in Fig. 5. In this figure, we have chosen two characteristic sets of input values $A_0 = M_0 = M_{1/2} = 600$ GeV and $A_0 = M_0 = M_{1/2} = 200$ GeV. It is clear that the first case is most preferable if one assumes the LEP+SLD data, where $s_l^2 = 0.23152 \pm 0.00023$. The present combined CDF/DØ [30] result for $m_t = 175 \pm 5$ GeV, is also compatible with the first
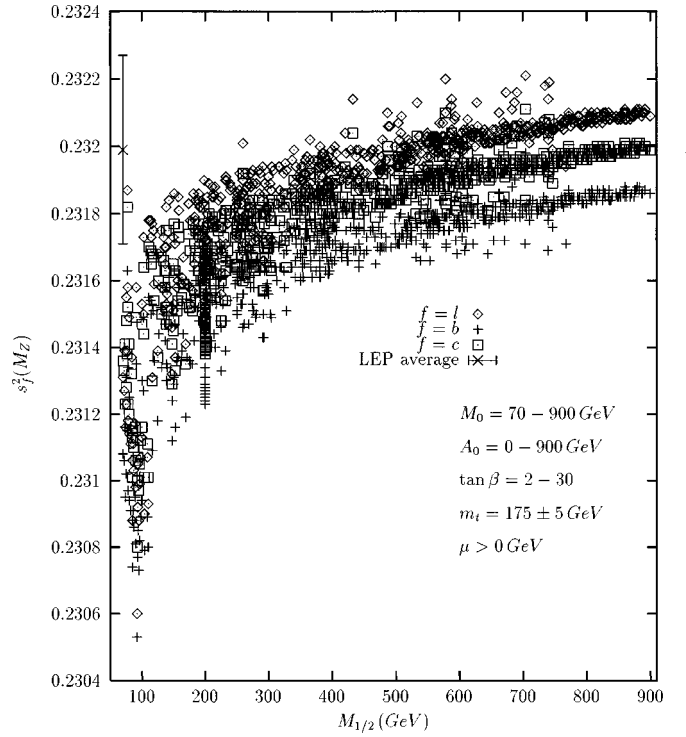

FIG. 4. The effective weak mixing angles s_c^2 and s_b^2. In the region $M_{1/2} \to 900$ GeV, the two angles are separated from each other. The dispersion of points around the central value for $M_{1/2} > 200$ GeV is due to the variation of the top mass. For the limiting behavior to be more clearly exhibited, in this figure and in Figs. 3, 7 and 8 we do not display the dispersion of points for $M_{1/2} > 800$ GeV.

case. Radiatively corrected light Higgs boson masses are also shown in Fig. 5. Figures 6 and 7 display the range of predictions for the mass of the W-gauge boson in the MSSM. As one can see, the W mass is in agreement with the presently experimentally observed value $M_W = 80.427 \pm 0.075$ ($M_W = 80.405 \pm 0.089$) GeV obtained from LEP (CDF,UA2,DØ)
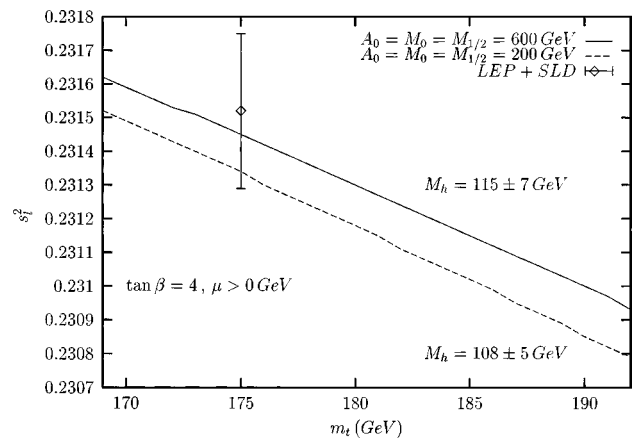

FIG. 5. MSSM predictions for s_l^2 as a function of m_t for two different characteristic values of the soft breaking parameters. The corresponding values of the light Higgs boson mass and their errors due to the variation of m_t are indicated.
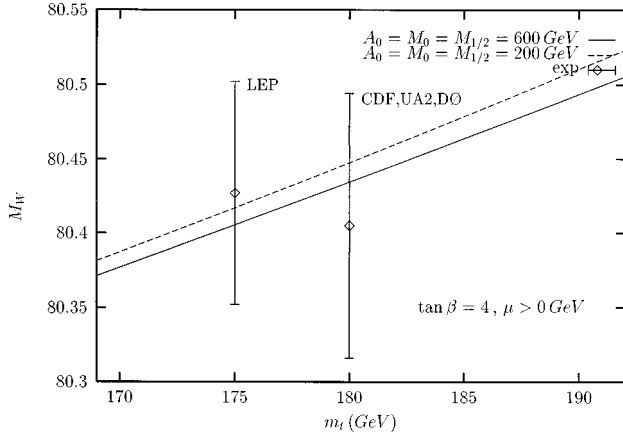
FIG. 6. MSSM predictions for physical mass of the W boson as a funcion of m_t for the same inputs as in Fig. 5.

experiments [2] for rather low (high) values of $M_{1/2}$ in the region of $m_t = 175 \pm 5$ GeV. Variation of m_t equal to $+5$ GeV leads to variation of M_W equal to $+0.032$ GeV while the effect on s_l^2 is -0.00017.

The *left-right* asymmetries are given by the effective Lagrangian (7) with

$$A_{LR}^f = \mathcal{A}^f = \frac{2v_{\text{eff}}^f/a_{\text{eff}}^f}{1 + (v_{\text{eff}}^f/a_{\text{eff}}^f)^2}, \tag{66}$$
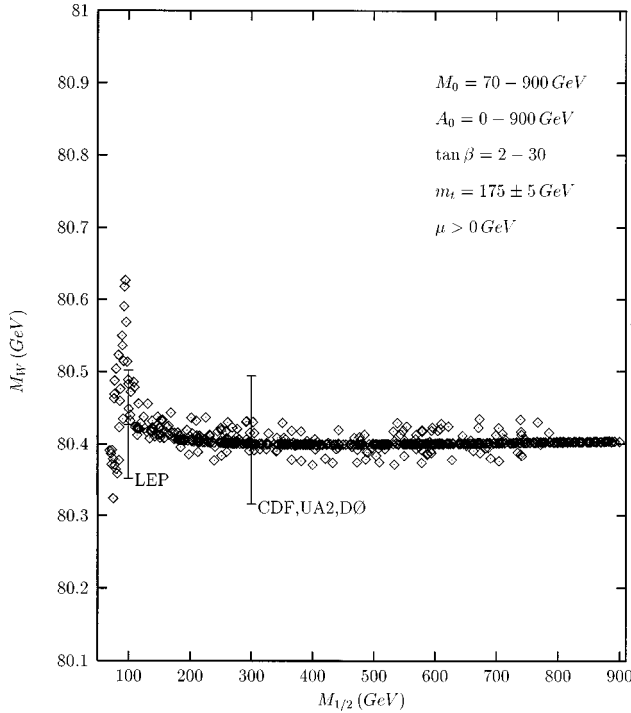
where

FIG. 7. MSSM prediction for the mass of the W-gauge boson as a function of the independent soft parameters $M_{1/2}$, M_0, and A_0. The experimental value $M_W = 80.427 \pm 0.075$ ($M_W = 80.405 \pm 0.089$) GeV obtained at LEP (CDF,UA2,D∅) is shown for comparison.
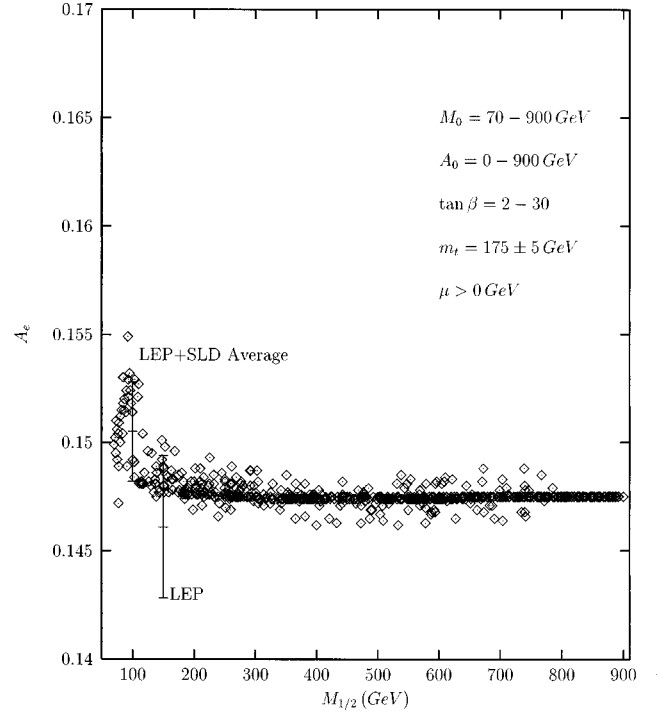

FIG. 8. The left-right asymmetry A_e in the MSSM as a function of $M_{1/2}$ when we vary M_0, A_0, $\tan\beta$, and m_t.

$$v_{\text{eff}}^f = T_3^f - 2s_f^2 Q^f,$$

$$a_{\text{eff}}^f = T_3^f. \tag{67}$$

As is depicted in Fig. 8, the MSSM prediction for $\mathcal{A}^e$ agrees with the LEP+SLD average value ($\mathcal{A}^e = 0.1505 \pm 0.0023$) when both $M_{1/2}$ and M_0 take on values around M_Z. In the heavy limit (large $M_{1/2}$), the MSSM agrees with the LEP value $\mathcal{A}^e = 0.1461 \pm 0.0033$. Note that as $M_{1/2} \to 900$ GeV, the value of $\mathcal{A}^e$ tends asymptotically [which means that large logarithms have been decoupled from the expression (66)] to the value 0.1476 corresponding to $s_l^2 \approx 0.23145$ (see also Fig. 4).

In the results shown in Figs. 3–8, we have not considered the constraint resulting from the experimental value of α_s. In Fig. 9, we have plotted the acceptable values of the soft breaking parameters $M_{1/2}$ and M_0,[10] which are compatible with the LEP+SLD ($\alpha_s = 0.119 \pm 0.004$, $s_{\text{eff}}^2 = 0.23152 \pm 0.00023$) [3] and the CDF/D∅ ($m_t = 175 \pm 5$ GeV) [30] data. The trillinear soft couplings as well as the parameter $\tan\beta(M_Z)$ are taken arbitrarily in the region (0 -900 GeV) and $(2-30)$, respectively. As we observe from Fig. 9, MSSM with radiative EW breaking is valid in the region $M_{1/2} \gtrsim 500$ GeV and $M_0 \gtrsim 70$ GeV.[11] In this region, the physical gluino mass is above 1 TeV, the LSP (one of the neutralinos) is $\gtrsim 200$ GeV, the chargino masses are $m_{\tilde{\chi}_{1,2}}$

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[10]We examine the region where A_0, M_0, $M_{1/2} \lesssim 900$ GeV.

[11]The requirement that the LSP is neutral puts this bound on M_0.
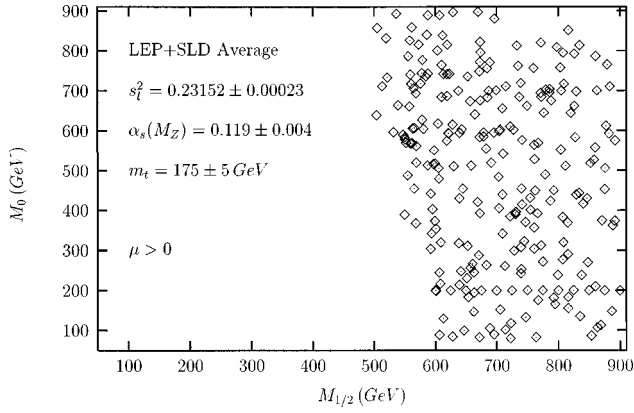
FIG. 9. Acceptable values in the $M_{1/2}$-M_0 plane according to the LEP+SLD data. The values of $\tan\beta$ and A_0 are taken in the region $2-30$ and $0-900$ GeV, respectively. Only large values of $M_{1/2}$ are acceptable.

$\gtrsim 650$, 370 GeV, the top squark masses are $m_{\tilde{t}_{1,2}} \gtrsim 1000$, 790 GeV, the sbottom masses are $m_{\tilde{b}_{1,2}} \gtrsim 1000$, 960 GeV, the slepton masses are $m_{\tilde{\tau}_{1,2}} \gtrsim 340$, 210 GeV, the sneutrinos are $m_{\tilde{\nu}} \gtrsim 330, GeV$ and the radiative one-loop corrected Higgs masses are M_h, $M_{A,H,H^\pm} \gtrsim 108$, 780 GeV, respectively. Thus, we conclude that the recent LEP+SLD and CDF/DØ data analysis favors the MSSM with radiative symmetry breaking only in the heavy limit of the sparticle masses.

V. CONCLUSIONS

We have considered the supersymmetric one loop corrections to the effective mixing angle s_f^2 which is experimentally determined in LEP and SLD experiments from measurements of on resonance left-right and forward-backward asymmetries. This effective angle differs from the corresponding mixing angle $\hat{s}^2$ defined as the ratio of couplings which is useful to test unification of couplings in unified schemes encompassing the standard model. The difference of the two angles, while very small in the standard model, is substantial in supersymmetric extensions of it due to large logarithmic $\log(M_{\text{SUSY}}^2/M_Z^2)$ dependences of $\hat{s}^2$. Thus, although $\hat{s}^2$ is a useful theoretical tool to test the unification of couplings, it is not the proper quantity to compare with experimental data which have already reached a high degree of accuracy. Therefore, the relation between the two definitions is of utmost importance for phenomenological studies of supersymmetric extensions of the standard model.

In this article we have calculated all corrections to the factor Δk_f relating the two angles s_f^2 and $\hat{s}^2$ including the nonuniversal corrections from vertices and external fermions. While Δk_f is plagued by large logarithms in the limit where the supersymmetry breaking scale is large, the effective weak mixing angle does not suffer from such large logarithms. In fact, we have proven that there are no dangerous logarithmic corrections $\log(M_{1/2}^2/M_Z^2)$ from the chargino-neutralino sector to the effective weak mixing angle. The decoupling of large logarithms involving the Higgsino mix-

ing parameter μ, which in the constrained MSSM with radiative symmetry breaking, is large, is obtained in the same manner. The cancellation of potentially dangerous terms also holds for the contributions of the squark and slepton sector. The cancellation of the $\log(M_{\text{SUSY}}^2/M_Z^2)$ terms in the $\overline{\text{DR}}$ scheme had been shown only numerically in previous studies.

It must be noted that there are large logarithmic terms in the ''nonoblique'' supersymmetric wave function renormalization of external fermions and vertex corrections of the vertex $Zf\bar{f}$. Nevertheless, we have analytically proven that they get decoupled from Δk_f and, hence, from the effective weak mixing angle itself. In addition to the analytical results described in Sec. III, we have also displayed representative numerical results in Table I in two particular cases of the MSSM.

We have also presented analytically, the decoupling of the large logarithmic terms from s_f^2 in the case of the nonuniversal SQCD corrections. Besides the self-cancellations of this terms from the relevant diagrams Figs. 2(c) and 2(d), there are additional cancellations from the summation of these diagrams due to their opposite sign. We have found that these corrections are very small and could be safely ignored from the analysis in the present experimental accuracy.

We have further proceeded to a numerical study of the one loop corrected effective mixing angle having as inputs the values of α_{EM}, M_Z, the Fermi coupling constant G_F, and the experimental values for the fermion masses. Assuming coupling constant unification and radiative breaking of the electroweak symmetry we have scanned the soft SUSY breaking parametric space and given theoretical predictions for the value of the effective mixing angles, the value of the strong coupling constant at M_Z and the value of the W-boson mass as this is determined from the parameter ρ and the effective weak mixing angle. We find that the large logarithmic corrections of the form $\log(M_{\text{SUSY}}^2/M_Z^2)$ indeed get decoupled from the extracted value of the effective weak mixing angle in the region of large $M_{1/2}$ and M_0 (Fig. 3) following our analytical calculations. The predicted MSSM values of the effective angles are in agreement with the LEP +SLD data (Fig. 4) as well as with the new CDF/DØ [30] results for the top mass $m_t = 175 \pm 5$ GeV (Fig. 5) in the region where all superparticles are quite massive. In this region, MSSM predicts values of the W-gauge boson mass which are in agreement with the new [2] CDF,UA2,DØ average value 80.405 ± 0.089 GeV (Figs. 6 and 7). Large logarithms are also decoupled from the *left-right* asymmetry value $\mathcal{A}^e$. MSSM seems to prefer the experimental LEP value of $\mathcal{A}^e$, rather than the average value from LEP+SLD (Fig. 8). Finally, values of $M_{1/2}$ which are greater than 500 GeV are favored by the MSSM if one assumes the present LEP and CDF/DØ data for s_l^2, α_s and m_t (Fig. 9).

Note added. After submitting this article for publication we became aware of the paper by P. Chankowski and S. Pokorski [31] where corrections to the leptonic mixing angle and predictions for the W boson mass are presented.

APPENDIX A: QUICK REFERENCE TO NEUTRALINO/CHARGINO AND THEIR INTERACTIONS

In the $\tilde{B}$, $\tilde{W}^{(3)}$, $i\tilde{H}_1^0$, $i\tilde{H}_2^0$, basis the neutralino mass matrix is

$$\mathcal{M}_N = \begin{pmatrix} M_1 & 0 & g'\dfrac{v\cos\beta}{2} & -g'\dfrac{v\sin\beta}{2} \\[2ex] 0 & M_2 & -g\dfrac{v\cos\beta}{2} & g\dfrac{v\sin\beta}{2} \\[2ex] g'\dfrac{v\cos\beta}{2} & -g\dfrac{v\cos\beta}{2} & 0 & -\mu \\[2ex] -g'\dfrac{v\sin\beta}{2} & g\dfrac{v\sin\beta}{2} & -\mu & 0 \end{pmatrix}. \tag{A1}$$

The mass eigenstates ($\tilde{\chi}_{1,2,3,4}^0$) of neutralino mass matrix $\mathcal{M}_N$ are written as

$$\mathcal{O}\begin{pmatrix} \tilde{\chi}_1^0 \\ \tilde{\chi}_2^0 \\ \tilde{\chi}_3^0 \\ \tilde{\chi}_4^0 \end{pmatrix} = \begin{pmatrix} \tilde{B} \\ \tilde{W}^{(3)} \\ i\tilde{H}_1^0 \\ i\tilde{H}_2^0 \end{pmatrix} \tag{A2}$$

and

$$\mathcal{O}^T \mathcal{M}_N \mathcal{O} = \text{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}), \tag{A3}$$

where $\mathcal{O}$ is a real orthogonal matrix. Note that when electroweak breaking effects are ignored $\mathcal{O}$ takes the form

$$\mathcal{O} = \begin{pmatrix} \mathbf{1_2} & \mathbf{0_2} \\[1ex] \mathbf{0_2} & \begin{matrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\[1.5ex] -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{matrix} \end{pmatrix}. \tag{A4}$$

The chargino mass matrix can be obtained from the following Lagrangian mass terms:

$$\mathcal{L}_{\text{charginos}}^{\text{mass}} = -(\tilde{W}^-, i\tilde{H}_1^-)\mathcal{M}_c\begin{pmatrix} \tilde{W}^+ \\ i\tilde{H}_2^+ \end{pmatrix} + (\text{H.c}), \tag{A5}$$

where we have defined $\tilde{W}^\pm \equiv \tilde{W}^{(1)} \mp i\tilde{W}^{(2)}/\sqrt{2}$ and

$$\mathcal{M}_C = \begin{pmatrix} M_2 & -g\dfrac{v\sin\beta}{\sqrt{2}} \\[2ex] -g\dfrac{v\cos\beta}{\sqrt{2}} & \mu \end{pmatrix}. \tag{A6}$$

Diagonalization of this matrix gives

$$U\mathcal{M}_c V^\dagger = \begin{pmatrix} m_{\tilde{\chi}_1} & 0 \\ 0 & m_{\tilde{\chi}_2} \end{pmatrix}. \tag{A7}$$

Thus,

$$\mathcal{L}_{\text{charginos}}^{\text{mass}} = -m_{\tilde{\chi}_1}\overline{\tilde{\chi}_1}\tilde{\chi}_1 - m_{\tilde{\chi}_2}\overline{\tilde{\chi}_2}\tilde{\chi}_2. \tag{A8}$$

The Dirac chargino states $\tilde{\chi}_{1,2}$ are given by

$$\tilde{\chi}_1 \equiv \begin{pmatrix} \lambda_1^+ \\ \overline{\lambda_1^-} \end{pmatrix}, \quad \tilde{\chi}_2 \equiv \begin{pmatrix} \lambda_2^+ \\ \overline{\lambda_2^-} \end{pmatrix}. \tag{A9}$$

The two component Weyl spinors $\lambda_{1,2}^\pm$ are related to $\tilde{W}^\pm$, $i\tilde{H}_1^-$, $i\tilde{H}_2^+$ by

$$V\begin{pmatrix} \tilde{W}^+ \\ i\tilde{H}_2^+ \end{pmatrix} \equiv \begin{pmatrix} \lambda_1^+ \\ \lambda_2^+ \end{pmatrix}, \quad (\tilde{W}^-, \ i\tilde{H}_1^-)U^\dagger \equiv (\lambda_1^-, \ \lambda_2^-). \tag{A10}$$

The gauge interactions of charginos and neutralinos can be read from the following Lagrangian.[12]

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[12]In our notation $\hat{e} \equiv$ electron's charge just opposite to that used in Ref. [24].

$$\mathcal{L}=\hat{g}(W^+_\mu J^\mu_- + W^-_\mu J^\mu_+)+\hat{e}A_\mu J^\mu_{em}+\frac{\hat{e}}{\hat{s}\hat{c}}Z_\mu J^\mu_Z. \quad \text{(A11)}$$

Also,

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \hat{c} & \hat{s} \\ -\hat{s} & \hat{c} \end{pmatrix} \begin{pmatrix} W^{(3)}_\mu \\ B_\mu \end{pmatrix}. \quad \text{(A12)}$$

The currents J^μ_+, J^μ_{em} and J^μ_Z are given by

$$J^\mu_+ \equiv \overline{\tilde{\chi}^0_a}\gamma^\mu[\mathcal{P}_L\mathcal{P}^L_{ai}+\mathcal{P}_R\mathcal{P}^R_{ai}]\tilde{\chi}_i \quad a=1\ldots 4, \quad i=1,2,$$
$$\text{(A13)}$$

where $\mathcal{P}_{L,R}=1\mp\gamma_5/2$ and

$$\mathcal{P}^L_{ai}\equiv +\frac{1}{\sqrt{2}}\mathcal{O}_{4a}V^*_{i2}-\mathcal{O}_{2a}V^*_{i1},$$

$$\mathcal{P}^R_{ai}\equiv -\frac{1}{\sqrt{2}}\mathcal{O}_{3a}U^*_{i2}-\mathcal{O}_{2a}U^*_{i1}. \quad \text{(A14)}$$

The electromagnetic current J^μ_{em} is

$$J^\mu_{em}=\bar{\tilde{\chi}}_1\gamma^\mu\tilde{\chi}_1+\bar{\tilde{\chi}}_2\gamma^\mu\tilde{\chi}_2. \quad \text{(A15)}$$

Finally, the neutral current J^μ_Z can be read from

$$J^\mu_Z\equiv\overline{\tilde{\chi}_i}\gamma^\mu[\mathcal{P}_L\mathcal{A}^L_{ij}+\mathcal{P}_R\mathcal{A}^R_{ij}]\tilde{\chi}_j$$
$$+\frac{1}{2}\overline{\tilde{\chi}^0_a}\gamma^\mu[\mathcal{P}_L\mathcal{B}^L_{ab}+\mathcal{P}_R\mathcal{B}^R_{ab}]\tilde{\chi}^0_b, \quad \text{(A16)}$$

with

$$\mathcal{A}^L_{ij}=\hat{c}^2\delta_{ij}-\frac{1}{2}V_{i2}V^*_{j2},$$

$$\mathcal{A}^R_{ij}=\hat{c}^2\delta_{ij}-\frac{1}{2}U_{i2}U^*_{j2},$$

$$\mathcal{B}^L_{ab}=\frac{1}{2}(\mathcal{O}_{3a}\mathcal{O}_{3b}-\mathcal{O}_{4a}\mathcal{O}_{4b}),$$

$$\mathcal{B}^R_{ab}=-\mathcal{B}^L_{ab}. \quad \text{(A17)}$$

Note that since $\mathcal{B}^R_{ab}=-\mathcal{B}^L_{ab}$ the neutralino contribution to J^μ_Z can be cast into the form

$$J^\mu_Z=-\frac{1}{2}\mathcal{B}^L_{ab}(\overline{\tilde{\chi}^0_a}\gamma^\mu\gamma^5\tilde{\chi}^0_b). \quad \text{(A18)}$$

For the supersymmetric external fermion corrections we need know the chargino and neutralino couplings to fermions and sfermions. The relevant chargino couplings are given by the following Lagrangian terms:

$$\mathcal{L}=i\bar{\tilde{\chi}}^c_i(\mathcal{P}_L a^{f'\tilde{f}}_{ij}+\mathcal{P}_R b^{f'\tilde{f}}_{ij})f'\tilde{f}^*_j$$
$$+i\bar{\tilde{\chi}}_i(\mathcal{P}_L a^{f\tilde{f}'}_{ij}+\mathcal{P}_R b^{f\tilde{f}'}_{ij})f\tilde{f}'^*_j+\text{H.c.} \quad \text{(A19)}$$

In this, $\chi_i(i=1,2)$ are the positively charged charginos and χ^c_i the corresponding charge conjugate states having opposite charge. f, f' are "up" and "down" fermions, quarks or leptons, while $\tilde{f}_i$, $\tilde{f}'_i$ are the corresponding sfermion mass eigenstates. The left- and right-handed couplings appearing above are given by

$$a^{f'\tilde{f}}_{ij}=gV^*_{i1}K^{\tilde{f}}_{j1}-h_fV^*_{i2}K^{\tilde{f}}_{j2}, \quad b^{f'\tilde{f}}_{ij}=-h_{f'}U^*_{i2}K^{\tilde{f}}_{j1},$$

$$a^{f\tilde{f}'}_{ij}=gU_{i1}K^{\tilde{f}'}_{j1}+h_{f'}U_{i2}K^{\tilde{f}'}_{j2}, \quad b^{f\tilde{f}'}_{ij}=h_fV_{i2}K^{\tilde{f}'}_{j1}.$$

In the equation above h_f, $h_{f'}$ are the Yukawa couplings of the up and down fermions, respectively. The matrices $K^{\tilde{f},\tilde{f}'}$ which diagonalize the sfermion mass matrices become the unit matrices in the absence of left-right sfermion mixings. For the electron and muon family the lepton masses are taken to be vanishing in the case that mixings do not occur. In addition the right-handed couplings, are zero. The corresponding neutralino couplings are given by

$$\mathcal{L}=i\bar{\tilde{\chi}}^0_a(\mathcal{P}_L a^{f\tilde{f}}_{aj}+\mathcal{P}_R b^{f\tilde{f}}_{aj})f\tilde{f}^*_j$$
$$+i\bar{\tilde{\chi}}^0_a(\mathcal{P}_L a^{f'\tilde{f}'}_{aj}+\mathcal{P}_R b^{f'\tilde{f}'}_{aj})f'\tilde{f}'^*_j+\text{H.c.} \quad \text{(A20)}$$

The left- and right-handed couplings for the up fermions, sfermions are given by

$$a^{f\tilde{f}}_{aj}=\sqrt{2}\left(gT^3_fO_{2a}+g'\frac{Y_f}{2}O_{1a}\right)K^f_{j1}+h_fO_{4a}K^f_{j2},$$

$$b^{f\tilde{f}}_{aj}=\sqrt{2}\left(-g'\frac{Y_{fc}}{2}O_{1a}\right)K^f_{j2}-h_fO_{4a}K^f_{j1},$$

while those for the down fermions and sfermions are given by

$$a^{f'\tilde{f}'}_{aj}=\sqrt{2}\left(gT^3_{f'}O_{2a}+g'\frac{Y'_f}{2}O_{1a}\right)K^{f'}_{j1}-h_{f'}O_{3a}K^{f'}_{j2},$$

$$b^{f'\tilde{f}'}_{aj}=\sqrt{2}\left(-g'\frac{Y'_{fc}}{2}O_{1a}\right)K^{f'}_{j2}+h_{f'}O_{3a}K^{f'}_{j1}.$$

APPENDIX B: PASSARINO-VELTMAN FUNCTIONS

All functions appearing in the propagator corrections in the main text can be expressed in terms of the basic Passarino-Veltman integrals A_0,B_0 in the following way:

$$A_0(m) = m^2 \left(\frac{1}{\hat{\epsilon}} + 1 - \ln\frac{m^2}{Q^2} \right), \qquad \text{(B1)}$$

$$B_0(p,m_1,m_2)$$

$$= \frac{1}{\hat{\epsilon}} - \int_0^1 dx \ln\frac{(1-x)m_1^2 + xm_2^2 - x(1-x)p^2 - i\epsilon}{Q^2}, \qquad \text{(B2)}$$

where $1/\hat{\epsilon} = 1/\epsilon - \gamma_E + \ln 4\pi$. This reduction can be done with the following identities (in what follows we made use of these functions only):

$$H(p,m_1,m_2) = 4\widetilde{B}_{22}(p,m_1,m_2)$$
$$+ (p^2 - m_1^2 - m_2^2)B_0(p,m_1,m_2), \qquad \text{(B3)}$$

$$\widetilde{B}_{22}(p,m,m) = -\frac{1}{12}p^2 B_0(p,m,m)$$
$$-\frac{1}{18}p^2 + \frac{1}{3}[m^2 + m^2 B_0(p,m,m) - A_0], \qquad \text{(B4)}$$

$$\widetilde{B}_{22}(0,m,m) = 0, \qquad \text{(B5)}$$

$$B_0[p,m,m] \overset{m^2 \gg p^2}{\longrightarrow} -\ln(m^2/Q^2). \qquad \text{(B6)}$$

For the vertex and box corrections to muon decay we need the functions B_0, B_1, C_0 at zero momenta. The following relations are useful in order to express the contributions to $\delta_{VB}^{\text{SUSY}}$ in terms of the masses of the particles in the loop

$$B_0(0,m_1,m_2) = \frac{1}{\hat{\epsilon}} + 1 + \ln\left(\frac{Q^2}{m_2^2}\right) + \frac{m_1^2}{m_1^2 - m_2^2}\ln\left(\frac{m_2^2}{m_1^2}\right), \qquad \text{(B7)}$$

$$B_1(0,m_1,m_2) = \frac{1}{2}\left[\frac{1}{\hat{\epsilon}} + 1 + \ln\left(\frac{Q^2}{m_2^2}\right) + \left(\frac{m_1^2}{m_1^2 - m_2^2}\right)^2 \right.$$
$$\left. \times \ln\left(\frac{m_2^2}{m_1^2}\right) + \frac{1}{2}\left(\frac{m_1^2 + m_2^2}{m_1^2 - m_2^2}\right) \right], \qquad \text{(B8)}$$

$$C_0(m_1,m_2,m_3) = -\int_0^1 dx \int_0^{1-x} dy$$
$$\times \frac{1}{m_1^2 x + m_2^2 y + m_3^2(1-x-y)}, \qquad \text{(B9)}$$

$$C_0(m_1,m_2,m_2) = \frac{1}{m_1^2 - m_2^2} + \frac{m_1^2}{(m_1^2 - m_2^2)^2}\ln\left(\frac{m_2^2}{m_1^2}\right). \qquad \text{(B10)}$$

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