EVOLUTION OF THE HIDDEN SECTOR IN NO-SCALE MODELS

K. ENQVIST, E. PAPANTONOPOULOS [1] and K. TAMVAKIS [2]

CERN, CH 1211, Geneva 23, Switzerland

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We study the field equations of the simplest SU(1,1)/U(1) no-scale model of one chiral superfield z with and without gravity. We find that the evolution of Im z can have a non-trivial influence on the determination of the vacuum. In the presence of gravity, it is also possible that the minimum of the potential evolves together with the expansion of the universe which might give rise to a hierarchy of mass scales.

The search for a satisfactory explanation of the origin of the various mass hierarchies plays a central role in current theoretical physics. For instance, the notorious problem of the cosmological constant consists of the fact that the observed vacuum energy is smaller than any observed mass scale associated with massive particles by many orders of magnitude. Since any massive particles should induce contributions to the vacuum energy which are at least of the order of it (mass)^4, what is needed is a reason, based on either symmetry or dynamics, why such contributions should not arise or should cancel. Supergravity offers such a reason. On the one hand, supersymmetry keeps mass scales apart, suppressing radiative corrections by the supersymmetry breaking scale. On the other hand, in a class of supergravity models that are characterized by dynamical determination of both M_W and $m_{3/2}$, and which are known as no-scale models [1], symmetry arguments, based on the SU(N, 1)/SU(N) × U(1) structure of the Kähler manifold of the chiral superfields, have been invoked to show that a vanishing vacuum energy might persist also at the quantum level [2]. Independently, no-scale supergravity models have been shown to emerge as the low-energy limit of $E_8 \times E_8$ superstring theory which has excellent prospects as a finite unified theory of all known interactions [3].

The key feature of no-scale supergravity models is that in such models the scalar potential can vanish at tree level [1,2,4]. This is inseparable from the SU(1, 1)/U(1) or SU(N, 1)/SU(N) × U(1) structure of the Kähler manifold and the residual non-compact U(1) symmetry. Since the Kähler manifold has a nonlinear SU(1, 1)/U(1) structure, non-canonical kinetic energies of chiral superfields arise unavoidably. In fact, all known examples of supergravity models with naturally flat potentials go with non-compact global symmetries and non-canonical kinetic terms [5].

In realistic models, the supersymmetry breaking, or in other words the gravitino mass, is related to the vacuum expectation value of at least one such field with non-canonical kinetic term and vanishing potential along its axis in field space. Its non-zero vacuum expectation value arises from the overall minimization of the energy after the radiative corrections, weighted by the supersymmetry breaking scale, have been taken into account.

In the present paper we analyze the simplest case of an SU(1, 1)/U(1) no-scale model with one complex field z. We show how the effects of Im z, although not appearing in the potential, are transmitted to the observable sector through the non-trivial couplings in the kinetic energy. These effects can have a strong influence on the existence of classical vacua. We also point out some possible difficulties in the cosmolog-

[1] On leave from the National Technical University, Athens, Greece.

[2] Permanent address: Physics Department, University of Ioannina, Ioannina, Greece.

ical evolution of Re z towards the minimum. When gravitation is included, the time-dependent expectation value of Im z can lead to time-dependence of the symmetry breaking scales arising from the minimization of the potential. This can give rise to a hierarchy of mass scales.

Chiral superfields in $N = 1$ supergravity can be considered as the coordinates of a Kähler manifold. The lagrangian is then expressible in terms of the Kähler potential $G(z^A, \bar{z}^A)$, a real function of chiral superfields. This serves to define the Kähler metric

$$G_{A\bar{B}} = \partial^2 G/\partial z^A \partial \bar{z}^{\bar{B}} ,$$

and the auxiliary fields

$$G_A = \partial G/\partial z^A , \quad G_{\bar{B}} = \partial G/\partial \bar{z}^{\bar{B}} .$$

The spin-zero part of the action can be written as

$$S = \int d^4x \, [-G_{A\bar{B}} \partial^\mu z^A \partial_\mu \bar{z}^{\bar{B}} + e^G (G^{A\bar{B}} G_A G_{\bar{B}} - 3)] , \tag{1}$$

where we have neglected gauge interactions. The classical vacua are the solutions of the equations of motion with least energy. Assuming a trivial gravitational background, the equations of motion take the form

$$\partial_\mu \partial^\mu z^A + \Gamma^A_{BC} \partial_\mu z^B \partial^\mu z^C + V^A = 0 . \tag{2}$$

The Kähler connection is just $\Gamma^A_{BC} = G^{A\bar{D}} \partial_B G_{C\bar{D}}$, and $V^A = G^{A\bar{B}} \partial_{\bar{B}} V$. It is perhaps suggestive to call these equations geodesic equations on the Kähler manifold.

Let us start with the simple case of an SU(1, 1)/U(1) Kähler manifold [1,4] spanned by one complex field with

$$G = -3 \ln(z + \bar{z}) + \text{const.} . \tag{3}$$

In this case the potential is exactly zero. Concentrating on homogeneous, i.e., spatially constant, fields, we get

$$\ddot{z} - 2\dot{z}^2/(z + \bar{z}) = 0 . \tag{4}$$

The solutions of (4) expressed in terms of the real and imaginary parts of $z = u + iv$, are

$$u_Q(t) = [(12CE)^{1/2}/Q] \times [\exp(-2\sqrt{\tfrac{1}{3}E}t) + C\exp(2\sqrt{\tfrac{1}{3}E}t)]^{-1} ,$$

$$v_Q(t) = V(0) - (4\sqrt{3}E/Q) \times \{[1 + C\exp(4\sqrt{\tfrac{1}{3}E}t)]^{-1} - (1 + C)^{-1}\} . \tag{5}$$

We have inserted into (5) the two constants of motion

$$E = 3|\dot{z}|^2/(z + \bar{z})^2 ,$$

and

$$Q = -3i(\dot{z} - \dot{\bar{z}})/(z + \bar{z})^2 = \tfrac{3}{2}\dot{v}/u^2 .$$

The constant of integration C is defined as

$$C = \{1 - [1 - Q^2 u(0)^2/E]^{1/2}\} \times \{1 + [1 + Q^2 u(0)^2/E]^{1/2}\}^{-1} .$$

Note the qualitative difference between the solutions (5) for $Q \neq 0$ and the $Q = 0$ ($\dot{v} = 0$) solution, which for a given E is

$$u_0(t) = u(0)\exp(2\sqrt{E}t/3) . \tag{6}$$

In the $Q \neq 0$ case, $\lim_{L\to\infty} u_Q(t) = 0$, while in the $Q = 0$ case, $\lim_{t\to\infty} u_0(t) = \infty$. Small fluctuations in Q for a given energy will drive u to exponentially small values, no matter what $u(0)$ is. In terms of the properly normalized field

$$\phi \equiv \sqrt{\tfrac{3}{2}} \ln u \quad (u > 0) , \tag{7}$$

this runaway behaviour translates into $\lim_{t\to\infty} \phi_Q(t) = -\infty$, while $\lim_{t\to\infty} \phi_0(t) = +\infty$. The form of the energy in terms of ϕ suggests that the Q-term acts like a potential:

$$E = \tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{3}Q^2 \exp(2\sqrt{\tfrac{2}{3}}\phi) .$$

The situation should be contrasted with a minimal example. For instance, a massless complex scalar $\phi = \rho e^{i\theta}$ exhibits a time evolution that is not qualitatively altered by a non-zero $\tilde{Q} = 2\rho^2\dot{\theta}$. We are led to conclude that the observed runaway behaviour should be attributed to the curved structure of the SU(1,1)/U(1) Kähler manifold.

In general, however, a potential is present. Such a potential can be induced radiatively due to the presence of other fields. As happens in no-scale supergravity models, the z-field of our SU(1,1)/U(1) example does not participate in gauge interactions directly, but any loops generated by other fields will give contributions always weighted by the "gravitino mass", $m^2_{3/2} = \exp[G(z, \bar{z}, ...)]$. Since $m_{3/2}$ depends only on the real part of z, and since the z-dependence of the potential is only through $m_{3/2}$, any induced potential will be only a function of u = Re z and not of v. In terms of the normalized field ϕ, the lagrangian can be written as

$$\mathcal{L} = \tfrac{1}{2}\dot{\phi}^2 + \tfrac{3}{4}\exp(-2\sqrt{\tfrac{2}{3}}\,\phi)\dot{v}^2 - V(\phi)\,. \tag{8}$$

Using the equation of motion, $\dot{v}$ can be eliminated to yield an effective Q-dependent potential

$$V_Q = \tfrac{1}{3}Q^2\exp(2\sqrt{\tfrac{2}{3}}\,\phi) + V(\phi)\,. \tag{9}$$

To illustrate this phenomenon, let us consider the radiatively-induced potential of no-scale models. The general form of such a potential is

$$V(\phi) = m_{3/2}^2(\phi)H_1(\ln m_{3/2}^2) + m_{3/2}^4 H_2(\ln m_{3/2}^2) + V_0$$
$$= \exp(-\sqrt{6}\phi)H_1(\phi) + \exp(-2\sqrt{6}\phi)H_2(\phi) + V_0\,, \tag{10}$$

where H_1 and H_2 are polynomials and V_0 is a constant. As an example, we will disregard the logarithms and take $H_1 = -\Lambda^2$ and $H_2 = C^2$, where Λ^2 and C^2 are positive constants. The potential (8) is then well-behaved and has a minimum at $\phi_0 = (1/\sqrt{6})\ln(2C^2/\Lambda^2)$. The energy is given by

$$E = \tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{3}Q^2\exp(\sqrt{\tfrac{8}{3}}\phi) - \Lambda^2\exp(-\sqrt{6}\phi)$$
$$+ C^2\exp(-2\sqrt{6}\phi) + V_0\,, \tag{11}$$

so that ϕ_0 will be the vacuum when $Q = 0$. However, in the case $Q \neq 0$, one should minimize the full potential V_Q of eq. (9). If $V_Q(\phi)$ is of the form that has a minimum, the general solution to the field equations will show oscillations about the minimum of $V_Q(\phi)$. The situation that we encountered in the free case was simply a manifestation of the fact that in the absence of $V(\phi)$ the minimum of $V_Q(\phi)$ is at $\phi = -\infty$.

The field oscillations are damped by particle production which dissipates the energy of the oscillations. In principle, this dissipation can be evaluated by computing quantum corrections to the field equations. In the absence of such a computation for Kähler models, we can only argue that it is unlikely that the initial value of Q, certainly present because of field fluctuations, will not be eroded through dissipation, since $v = \mathrm{Im}\,z$ is massless. Therefore, the theory will not settle to a vacuum with $Q = 0$. Indeed, one can verify that this is true for small Q and for small oscillations about the minimum of any potential, where the effects of damping [6] can be expected to be the same as in the case of ordinary field theories with minimal kinetic energies. Such a situation may be a source of difficulty for SU(1,1)/U(1) models, because conceivably the initial value of Q should vary randomly at a length scale of the horizon distance.

A very interesting situation arises when $V(\phi)$ does not possess any minimum. Then it is nevertheless possible that the $Q \neq 0$ theory has a vacuum. Consider for example

$$V(\phi) = \Lambda^2\exp(\alpha G) = \Lambda^2\exp(-\alpha\sqrt{b}\phi)\,, \tag{12}$$

which does not have a minimum. α is a parameter. In contrast, the full effective potential V_Q [see eq. (9)] has a minimum at

$$\phi_0 = (\sqrt{\tfrac{8}{3}} + \sqrt{6}\alpha)^{-1}\ln[(9\Lambda^2/2Q^2)^\alpha]\,.$$

The gravitino mass term at this minimum reads (here $\alpha = 1$)

$$m_{3/2}^2 = \tfrac{1}{8}(2Q^2/9\Lambda^2)^{9/11}\,.$$

It is therefore possible that a small but non-zero value of Q, due to quantum fluctuations, could easily induce a hierarchy of mass scales. In the case where the vacuum has $Q \neq 0$, there might even exist stable space-dependent configurations [7].

Let us also point out that in SU(1,1)/U(1), $Q \neq 0$ will always induce a positive contribution to the vacuum energy, which may counterbalance the usually negative contribution from the electroweak radiative corrections. The appearance of the Q-term is a consequence of the direct coupling of $\mathrm{Im}\,\dot{z}$ to $\mathrm{Re}\,z$ in the kinetic energy.

This is a general feature of any model with a flat direction. Consider a single complex field z and let the Kähler metric and potential depend on $z + z^*$ only. Then the equations of motion yield

$$\dot{v} = QG_{z\bar{z}}^{-1}\,,$$
$$\ddot{u} + \tfrac{1}{2}(\dot{u}^2 - Q^2G_{z\bar{z}}^{-1})\,\partial\ln G_{z\bar{z}}/\partial u + \tfrac{1}{2}G_{z\bar{z}}^{-1}\partial V/\partial u = 0\,. \tag{13}$$

Defining $\dot{\phi} = \dot{u}G_{z\bar{z}}^{1/2}$, the second field equation above becomes

$$\ddot{\phi} + \partial V_{\mathrm{eff}}/\partial\phi = 0\,,$$

where

$$V_{\mathrm{eff}} = \tfrac{1}{2}Q^2G_{z\bar{z}}^{-1} + \tfrac{1}{2}V(\phi)\,. \tag{14}$$

Therefore, at the classical level, the field theory of the lagrangian

$$\mathcal{L} = G_{z\bar{z}}(z+\bar{z})|\dot{z}|^2 - V(z+\bar{z}),$$

is equivalent to the field theory of $\mathcal{L}' = \frac{1}{2}\dot{\phi}^2 - V_{\rm eff}$ with $V_{\rm eff}$ given in eq. (14). Of course, at the quantum level, differences may arise.

It is important to investigate to what extent the somewhat unusual properties of the model in question are modified due to a non-trivial gravitational background induced by the stress energy of the field z. Starting with the Einstein action

$$S = \int d^4x\,\sqrt{-g}\left(-\tfrac{1}{2}\mathcal{R} - G_{A\bar{B}}g^{\mu\nu}\partial_\mu z^A\partial_\nu \bar{z}^{\bar{B}} + V\right), \tag{15}$$

one obtains, upon variation of the metric $g_{\mu\nu}$,

$$\mathcal{R}_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}\mathcal{R} = -2G_{A\bar{B}}\partial_\mu z^A\partial_\nu\bar{z}^{\bar{B}} - g_{\mu\nu}\mathcal{L}. \tag{16}$$

For the Robertson–Walker metric

$$ds^2 = dt^2 - [R^2(t)/(1+\tfrac{1}{2}kr^2)](dr^2 + r^2\,d\Omega^2), \tag{17}$$

one obtains from (12)

$$(3/R^2)(k+\dot{R}^2) = G_{A\bar{B}}z^A\dot{\bar{z}}^{\bar{B}} + V,$$

$$2\ddot{R}/R + (\dot{R}/R)^2 + k/R^2 = -G_{A\bar{B}}\dot{z}^A\dot{\bar{z}}^{\bar{B}} + V. \tag{18}$$

The field equation (2) is modified to

$$\ddot{z}^A + 3(\dot{R}/R)\dot{z}^A + \Gamma^A_{BC}\dot{z}^B\dot{z}^C + V^A = 0. \tag{19}$$

Let us first examine the case of vanishing potential. It is obvious from eqs. (18) that the curvature of the Kähler manifold will not affect gravity, and one can solve $R(t)$ implicitly by

$$t = \int_{R(0)^3}^{R(t)^3} dR^3(A - 9kR^4)^{1/2}, \tag{20}$$

where $A = 9R^4(0)[\dot{R}^2(0)+k] > 0$. For $k = 0$, the solution is

$$R(t) = R(0)\{1 + 3[\dot{R}(0)/R(0)]t\}^{1/3}, \tag{21}$$

whereas for $k > 0$ one obtains oscillating solutions (with singularities) and for $k < 0$ one has asymptotically $R \sim t$.

Gravity, however, will affect the evolution of the scalar field z. In the SU(1,1)/U(1) model with vanishing potential, the equations of motion can now be solved to yield

$$u(t) = \{2\Lambda\dot{R} \pm [4\Lambda^2\dot{R}^2 + 4k(\Lambda^2 + \tfrac{4}{9}Q^2R^{-4})]^{1/2}\} \times(\Lambda^2 + \tfrac{4}{9}Q^2R^{-4})^{-1}, \tag{22}$$

where

$$\Lambda = -[R(0)/u(0)]\,[\dot{u}(0)/u(0) - 2\dot{R}(0)/R(0)],$$

is an integration constant. If $k \leqslant 0$, the boundary conditions must satisfy the condition $\Lambda^2 C < 2|k|Q^2$. Asymptotically

$$u_Q(t) \to (2/\Lambda)|k|, \tag{23}$$

which is independent of Q. We therefore conclude that gravity affects strongly the evolution of u. Of course, the presence of other matter may modify considerably the result (23).

In order to examine what happens in the presence of potential energy, it is more convenient to use the normalized field ϕ defined in eq. (7). Let us assume that $V_Q(\phi)$ possesses a minimum that depends on Q, as in the case of the potential (14). Now the crucial parameter is the effective time-dependent charge $Q(t) \equiv Q/R^3(t)$. A time-dependent "gravitino mass" would then be

$$m^2_{3/2}(t) = \tfrac{1}{8}[\tfrac{2}{9}Q^2/\Lambda^2R^6(t)]^{9/11}. \tag{24}$$

Let us now assume that the vacuum energy is zero at this time-dependent minimum $\phi_0(R(t))$. From the equations of motion (18), we then find that

$$d\ln(\dot{\phi}R^3)/dt = 0, \quad \dot{\phi}^2 - 6(\dot{R}/R)^2 = 0, \tag{25}$$

which yield

$$R = R(0)\{1 + 3[\dot{R}(0)/R(0)]t\}^{1/3},$$

$$\phi = \phi(0) + \sqrt{\tfrac{2}{3}}\ln[R^3(t)/R^3(0)]. \tag{26}$$

Is the solution for ϕ compatible with being the minimum of any reasonable potential? The answer is yes, as we will demonstrate below by considering the example (12). The minimum of $V_Q(\phi)$ occurs at

$$\phi_0(t) = (\alpha\sqrt{6} + \sqrt{\tfrac{8}{3}})^{-1}\ln[9\alpha\Lambda^2R^6(t)/2Q^2]$$
$$= \text{const.} + 2(\alpha\sqrt{6} + \sqrt{\tfrac{8}{3}})^{-1}\ln[R(t)/R(0)]^3. \tag{27}$$

This is compatible with (26) provided that $\alpha = \frac{1}{3}$. Therefore (27) serves as a proof of existence for potentials in supergravity which may have time-dependent minima. These co-evolve with the expansion of the universe. Therefore, one could expect hierarchically separated scales arising from the evolution of R. The above example also illustrates that although gravity couples directly only to the kinetic terms, it can influence indirectly the mass scales appearing at the minimum of the potential.

In conclusion, in this paper we have considered the vacua and field equations of the simplest SU(1, 1)/U(1) no-scale model with and without gravity. We found that the coupling of $\mathrm{Im}\,z$ to $\mathrm{Re}\,z$ can greatly affect the evolution of the latter. The runaway behaviour encountered in the absence of a potential is, however, modified due to a non-trivial gravitational background. In the presence of a potential, we found that a time-dependent expectation value of $\mathrm{Im}\,z$ has a decisive influence on the stability properties of the classical vacua. In particular, the coupling of $\mathrm{Im}\,z$ may render the settling of $\mathrm{Re}\,z$ to a minimum of a potential not a straightforward matter. However, in the absence of quantum corrections to the field equations one cannot make very precise claims, and clearly these questions merit further investigation. In the presence of gravity, there are cases where the time-dependent universal scale will determine the symmetry breaking scales of the induced potential.

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