Exophobic quasi-realistic heterotic string vacua

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A B S T R A C T

We demonstrate the existence of heterotic string vacua that are free of massless exotic fields. The need to break the non-Abelian GUT symmetries in $k = 1$ heterotic string models by Wilson lines, while preserving the GUT embedding of the weak hypercharge and the GUT prediction $\sin^2 \theta_w(M_{GUT}) = 3/8$, necessarily implies that the models contain states with fractional electric charge. Such states are severely restricted by observations, and must be confined or sufficiently massive and diluted. We construct the first quasi-realistic heterotic string models in which the exotic states do not appear in the massless spectrum, and only exist, as they must, in the massive spectrum. The $SO(10)$ GUT symmetry is broken to the Pati–Salam subgroup. Our PS heterotic string models contain adequate Higgs representations to break the GUT and electroweak symmetry, as well as colour Higgs triplets that can be used for the missing partner mechanism. By statistically sampling the space of Pati–Salam vacua we demonstrate the abundance of quasi-realistic three generation models that are completely free of massless exotics, rendering it plausible that obtaining realistic Yukawa couplings may be possible in this space of models.

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1. Introduction

String theory provides a perturbative calculational framework for quantum gravity, while simultaneously giving rise to the gauge and matter structures that are observed in particle experiments. Furthermore, the gauge and matter sectors are required by its self–consistency constraints. Additionally, by producing spinorial matter representations in the perturbative spectrum, the heterotic string naturally accommodates the $SO(10)$ embedding of the Standard Model, which is well motivated by the experimental data. Absence of higher order Higgs representations in heterotic string models that are based on level one Kac–Moody current algebras necessitates that the $SO(10)$ symmetry is broken directly at the string level by discrete Wilson lines.

A well-known theorem due to Schellekens [1] states that any such string model that preserves the canonical $SU(5)$ or $SO(10)$ GUT embedding of the weak hypercharge, and in which the non-Abelian GUT symmetries are broken by discrete Wilson lines, necessarily contain states that carry charges that do not obey the original GUT quantization rule [1].[2] In terms of the Standard Model charges these exotic states carry fractional electric charge. The existence of such states is severely constrained by experiments. Electric charge conservation implies that the lightest of these states is stable.

Many experimental searches for fractionally charged states have been conducted [3]. However, no reported observation of any such particle has ever been confirmed and there are strong upper bounds on their abundance [3]. This implies that such exotic states in string models should be either confined into integrally charged states [4], or be sufficiently heavy and diluted in the cosmological evolution of the universe [5]. The first of these solutions is, however, problematic, due to the effect of the charged states on the renormalisation group running of the weak hypercharge and gauge coupling unification. The preferred solution is therefore for the fractionally charged states to become sufficiently massive, *i.e.* with a mass which is larger than the GUT scale. In this case the fractionally charged states can be diluted by the inflationary evolution of the universe. Due to their heavy mass they will not be reproduced during re-heating and the experimental constraints can be evaded.

By producing the gauge and matter structures, that are the building blocks of the Standard Particle Model, string theory enables the development of a phenomenological approach. A pivotal task in this regard, is the construction of quasi-realistic three generation models. Such models can in turn be used to explore the properties of string theory and quantum gravity. Quasi-realistic perturbative heterotic string models have been constructed [4,6–11] by using free fermionic [12] and orbifold [13] techniques. The existence of fractionally charged states in string models is endemic. To our knowledge all string models constructed to date contain such states in the massless spectrum. One obvious remedy is to modify the GUT embedding of the weak-hypercharge, that produces integral charges for all states. However, the result is that the GUT prediction of $\sin^2 \theta_W (M_{\mathrm{GUT}})$ is modified, and the GUT embedding of the Standard Model spectrum is lost. Maintaining the GUT embedding and the canonical GUT prediction $\sin^2 \theta_W (M_{\mathrm{GUT}}) = 3/8$ therefore necessitates the existence of fractionally charged states in the physical spectrum.

While in some models the exotic states are chiral and therefore necessarily remain massless, there exist an abundance of models in which they appear in vector-like representations and therefore can gain mass in the effective low energy field theory, by the Vacuum Expectation Value (VEV) of a Standard Model singlet field. Such singlet field VEVs which are of the order of the string scale, are in fact often mandated in the models due to the existence of an anomalous $U(1)$, which is broken by the Green–Schwarz mechanism. The details are model-dependent, but there exist models in which all the exotic states couple to $SO(10)$ singlet fields at the cubic level of the superpotential [6,5]. Assigning VEVs of order that exceeds the GUT scale to these set of fields gives sufficiently heavy mass to the exotic states. Indeed supersymmetric preserving solution with VEVs to the required set of fields were found in Ref. [9]. The caveat is that this demonstration is achieved by an effective field theory analysis. The question therefore remains whether there exist string models that are free of massless fractionally charged states. In such models states with fractional charge necessarily appear in the massive spectrum, but do not appear as massless states.

In this Letter we show that such string models do in fact exist. We use the free fermionic formalism for the analysis. In the orbifold language the free fermionic construction correspond to symmetric, asymmetric or freely acting orbifolds. A subclass of them correspond to symmetric $Z_2 \times Z_2$ orbifold compactifications at enhanced symmetry points in the toroidal moduli space [14,15]. The chiral matter spectrum arises from twisted sectors and thus does not depend on the moduli. This facilitates the complete classification of the topological sectors of the $Z_2 \times Z_2$ symmetric orbifolds. For type II string $N = 2$ supersymmetric vacua the general free fermionic classification techniques were developed in Ref. [16]. The method was extended in Refs. [17,18] for the classification of heterotic $Z_2 \times Z_2$ free fermionic orbifolds, with unbroken $SO(10)$ and E_6 GUT symmetries.

Absence of adjoint Higgs representations in heterotic string models with unbroken GUT symmetries realised as level one Kac–Moody algebras means that the models classified in [17,18] are not realistic. The GUT gauge symmetry must be broken directly at the string level. In the free fermionic models the GUT gauge symmetry generated by untwisted vector bosons is $SO(10)$, and can be enhanced to a larger gauge group by gauge bosons arising from other sectors. Phenomenologically the most appealing case is that of $SO(10)$ by itself, and therefore it is reasonable to demand that gauge bosons which enhance the $SO(10)$ symmetry be projected out by the Generalised GSO (GGSO) projections. The $SO(10)$ symmetry must therefore be broken to one of its subgroups. The cases with $SU(5) \times U(1)$ (flipped $SU(5)$) [4], $SO(6) \times SO(4)$ (Pati–Salam) [7], $SU(3) \times SU(2) \times U(1)^2$ (standard-like) [6,8] and $SU(3) \times SU(2)^2 \times U(1)$ (left–right symmetric) [10] were shown to produce quasi-realistic examples. The Pati–Salam free fermionic heterotic string models utilise only periodic and anti-periodic boundary conditions, whereas the other cases necessarily use also fractional boundary conditions. The Pati–Salam case [19] therefore represents the simplest extension of the classification program of [18] to quasi-realistic models.

2. Pati–Salam heterotic string models

In the free fermionic formulation the 4-dimensional heterotic string, in the light-cone gauge, is described by 20 left-moving and 44 right-moving real fermions. A large number of models can be constructed by choosing different phases picked up by fermions (f_A, $A = 1, \ldots, 44$) when transported along the torus non-contractible loops. Each model corresponds to a particular choice of fermion phases consistent with modular invariance that can be generated by a set of basis vectors v_i, $i = 1, \ldots, n$,

$$v_i = \{\alpha_i(f_1), \alpha_i(f_2), \alpha_i(f_3), \ldots\}$$

describing the transformation properties of each fermion

$$f_A \rightarrow -e^{i\pi \alpha_i(f_A)} f_A, \quad A = 1, \ldots, 44. \tag{2.1}$$

The basis vectors span a space Ξ which consists of 2^N sectors that give rise to the string spectrum. Each sector is given by

$$\xi = \sum N_i v_i, \quad N_i = 0, 1. \tag{2.2}$$

The spectrum is truncated by a generalized GSO projection whose action on a string state $|S\rangle$ is

$$e^{i\pi v_i \cdot F_S} |S\rangle = \delta_S c \begin{bmatrix} S \\ v_i \end{bmatrix} |S\rangle, \tag{2.3}$$

where F_S is the fermion number operator and $\delta_S = \pm 1$ is the space–time spin statistics index. Different sets of projection coefficients $c \begin{bmatrix} S \\ v_i \end{bmatrix} = \pm 1$ consistent with modular invariance give rise to different models. Summarizing: a model can be defined uniquely by a set of basis vectors v_i, $i = 1, \ldots, n$, and a set of $2^{N(N-1)/2}$ independent projections coefficients $c \begin{bmatrix} v_i \\ v_j \end{bmatrix}$, $i > j$.

The free fermions in the light-cone gauge in the usual notation are: ψ^μ, χ^i, y^i, ω^i, $i = 1, \ldots, 6$ (left-movers), and $\bar{y}^i$, $\bar{\omega}^i$, $i = 1, \ldots, 6$, $\bar{\psi}^A$, $A = 1, \ldots, 5$, $\bar{\eta}^B$, $B = 1, 2, 3$, $\bar{\phi}^\alpha$, $\alpha = 1, \ldots, 8$ (right-movers). The class of models we investigate, is generated by a set of thirteen basis vectors $B = \{v_1, v_2, \ldots, v_{13}\}$, where

$$v_1 = 1 = |\{\psi^\mu, \chi^{1,...,6}, y^{1,...,6}, \omega^{1,...,6} \mid \bar{y}^{1,...,6}, \bar{\omega}^{1,...,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,...,5}, \bar{\phi}^{1,...,8}\},$$

$$v_2 = S = \{\psi^\mu, \chi^{1,...,6}\},$$

$$v_{2+i} = e_i = \{y^i, \omega^i \mid \bar{y}^i, \bar{\omega}^i\}, \quad i = 1, \ldots, 6,$$

$$v_9 = b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} \mid \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,...,5}\},$$

$$v_{10} = b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} \mid \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,...,5}\},$$

$$v_{11} = z_1 = \{\bar{\phi}^{1,...,4}\},$$

$$v_{12} = z_2 = \{\bar{\phi}^{5,...,8}\},$$

$$v_{13} = \alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}. \tag{2.4}$$

The first twelve vectors in this set are identical to those used in [17,18]. v_{13} is the additional new vector that breaks the $SO(10)$ GUT symmetry to $SO(6) \times SO(4)$. The second ingredient that is needed to define the string vacuum are the GGSO projection coefficients that appear in the one-loop partition function, $c\begin{bmatrix} v_i \\ v_j \end{bmatrix}$, spanning a 13×13 matrix. Only the elements with $i > j$ are independent, and the others are fixed by modular invariance. A priori there are therefore 78 independent coefficients corresponding to 2^{78} distinct string vacua. Eleven coefficients are fixed by requiring that the models possess $N = 1$ supersymmetry. Additionally, we impose the condition that the only space–time vector bosons that remain in the spectrum are those that arise from the untwisted sector. This restricts further the number of phases, leaving a total of 51 independent GGSO phases. The gauge group in these models is therefore:

observable: $SO(6) \times SO(4) \times U(1)^3$,

hidden: $SO(4)^2 \times SO(8)$.

The untwisted matter is common in these models and is composed of three pairs of vectorial representations of the observable $SO(6)$ symmetry, and $SO(10)$ singlets. The chiral matter spectrum arises from the twisted sectors. The chiral spinorial representations of the observable $SO(6) \times SO(4)$ arise from the sectors:

$$B^1_{\ell^1_3 \ell^1_4 \ell^1_5 \ell^1_6} = S + b_1 + \ell^1_3 e_3 + \ell^1_4 e_4 + \ell^1_5 e_5 + \ell^1_6 e_6, \tag{2.5}$$

$$B^2_{\ell^2_1 \ell^2_2 \ell^2_5 \ell^2_6} = S + b_2 + \ell^2_1 e_1 + \ell^2_2 e_2 + \ell^2_5 e_5 + \ell^2_6 e_6, \tag{2.6}$$

$$B^3_{\ell^3_1 \ell^3_2 \ell^3_3 \ell^3_4} = S + b_3 + \ell^3_1 e_1 + \ell^3_2 e_2 + \ell^3_3 e_3 + \ell^3_4 e_4, \tag{2.7}$$

where $\ell^j_i = 0, 1$; $b_3 = b_1 + b_2 + x = 1 + S + b_1 + b_2 + \sum_{i=1}^6 e_i + \sum_{n=1}^2 z_n$, and x is given by the vector $x = \{\bar{\psi}^{1,...,5}, \bar{\eta}^{1,2,3}\}$. These sectors give rise to 16 and $\overline{16}$ representations of $SO(10)$ decomposed under $SO(6) \times SO(4) \equiv SU(4) \times SU(2)_L \times SU(2)_R$

$$16 = (4, 2, 1) + (\bar{4}, 1, 2),$$

$$\overline{16} = (\bar{4}, 2, 1) + (4, 1, 2).$$

We note that in these models there are three $SO(4)$ group factors, and there a cyclic symmetry among them. We could have therefore defined one of the other two $SO(4)$ group as the observable one, and the other two as the hidden ones. We follow here the convention that keeps the group generated by the world-sheet fermions $\bar{\psi}^{4,5}$ as the observable $SO(4)$ and the ones generated by $\bar{\phi}^{1,2}$ and $\bar{\phi}^{3,4}$ as hidden. The models then give rise to multitude of sectors that produce exotic states with fractional electric charge, given by:

$$Q_{em} = \frac{1}{\sqrt{6}} T_{15} + \frac{1}{2} I_{3_L} + \frac{1}{2} I_{3_R}, \tag{2.8}$$

where T_{15} is the diagonal generator of $SU(4)/SU(3)$ and I_{3_L}, I_{3_R} are the diagonal generators of $SU(2)_L$, $SU(2)_R$, respectively. The models then contain the exotic states in the representations:

$(4, 1, 1) + (\bar{4}, 1, 1)$: $\pm\frac{1}{6}$ exotic coloured particles and SM singlets,

$(1, 2, 1)$: $\pm\frac{1}{2}$ leptons,

$(1, 1, 2)$: $\pm\frac{1}{2}$ SM singlets.

We now enumerate the sectors that give rise to exotic states. The states corresponding to the representations $(4, 2, 1)$, $(4, 1, 2)$, $(\bar{4}, 2, 1)$, $(\bar{4}, 1, 2)$ where 4 and $\bar{4}$ are spinorial (anti-spinorial) representations of the observable $SO(6)$, and the 2 are doublet representations of the hidden $SU(2) \times SU(2) = SO(4)_1$, arise from the following sectors:

$$B^{(4)}_{pqrs} = S + b_1 + b_2 + \beta + pe_1 + qe_2 + re_3 + se_4,$$

$$B^{(5)}_{pqrs} = S + b_1 + b_3 + \beta + pe_1 + qe_2 + re_5 + se_6,$$

$$B^{(6)}_{pqrs} = S + b_2 + b_3 + \beta + pe_3 + qe_4 + re_5 + se_6, \tag{2.9}$$

where $\beta = \alpha + x \equiv \{\bar{\psi}^{1,2,3}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,2}\}$. Similar states arise from the sectors $B_{pqrs}^{(4,5,6)} + z_1$ and correspond to the representations $(4, 2, 1)$, $(4, 1, 2)$, $(\bar{4}, 2, 1)$, $(\bar{4}, 1, 2)$ of $SO(6)_{obs} \times SO(4)_2$.

The states corresponding to the representations $((2, 1), (2, 1))$, $((2, 1), (1, 2))$, $((1, 2), (1, 2))$ and $((1, 2), (2, 1))$ transforming under $SU(2)_L \times SU(2)_R \times SO(4)_1$ arise from the sectors:

$$B_{pqrs}^{(7)} = B_{pqrs}^{(4)} + x = S + b_3 + \beta + pe_1 + qe_2 + re_3 + se_4,$$
$$B_{pqrs}^{(8)} = B_{pqrs}^{(5)} + x = S + b_2 + \beta + pe_1 + qe_2 + re_5 + se_6,$$
$$B_{pqrs}^{(9)} = B_{pqrs}^{(6)} + x = S + b_1 + \beta + pe_3 + qe_4 + re_5 + se_6. \tag{2.10}$$

The remaining sectors give rise to states that transform as representations of the hidden gauge group, and are singlets under the observable $SO(10)$ GUT symmetry. These states are therefore hidden matter states that arise in the string model, but are not exotic with respect to electric charge. The following 48 sectors produce the representations $((2, 1), (2, 1))$ of $SU(2)^4 = SO(4)_1 \times SO(4)_2$:

$$B_{pqrs}^{(10)} = B_{pqrs}^{(1)} + x + z_1 = S + b_2 + b_3 + pe_3 + qe_4 + re_5 + se_6 + z_1,$$
$$B_{pqrs}^{(11)} = B_{pqrs}^{(2)} + x + z_1 = S + b_1 + b_3 + pe_1 + qe_2 + re_5 + se_6 + z_1,$$
$$B_{pqrs}^{(12)} = B_{pqrs}^{(3)} + x + z_1 = S + b_1 + b_2 + pe_1 + qe_2 + re_3 + se_4 + z_1. \tag{2.11}$$

There are 48 sectors producing spinorial 8 and anti-spinorial $\bar{8}$ representations of the hidden $SO(8)$ gauge group:

$$B_{pqrs}^{(13)} = B_{pqrs}^{(1)} + x + z_2 = S + b_2 + b_3 + pe_3 + qe_4 + re_5 + se_6 + z_2,$$
$$B_{pqrs}^{(14)} = B_{pqrs}^{(2)} + x + z_2 = S + b_1 + b_3 + pe_1 + qe_2 + re_5 + se_6 + z_2,$$
$$B_{pqrs}^{(15)} = B_{pqrs}^{(3)} + x + z_2 = S + b_1 + b_2 + pe_1 + qe_2 + re_3 + se_4 + z_2. \tag{2.12}$$

States that transform in vectorial representations are obtained from sectors that contain four periodic world-sheet right-moving complex fermions. Massless states are obtained in such sectors by acting on the vacuum with a Neveu–Schwarz right-moving fermionic oscillator. Vectorial representations arise from the sectors:

$$B_{pqrs}^{(1)} + x = S + b_1 + x + pe_3 + qe_4 + re_5 + se_6,$$
$$B_{pqrs}^{(2)} + x = S + b_2 + x + pe_1 + qe_2 + re_5 + se_6,$$
$$B_{pqrs}^{(3)} + x = S + b_3 + x + pe_1 + qe_2 + re_3 + se_4 \tag{2.13}$$

and produce the following representations:

- $\{\bar{\psi}^{123}\}|R\rangle_{pqrs}^{(i)}$, $i = 1, 2, 3$, where $|R\rangle_{pqrs}^{(i)}$ is the degenerated Ramond vacuum of the $B_{pqrs}^{(i)}$ sector. These states transform as a vectorial representation of $SO(6)$.
- $\{\bar{\psi}^{45}\}|R\rangle_{pqrs}^{(i)}$, $i = 1, 2, 3$, where $|R\rangle_{pqrs}^{(i)}$ is the degenerated Ramond vacuum of the $B_{pqrs}^{(i)}$ sector. These states transform as a vectorial representation of $SO(4)$.
- $\{\bar{\phi}^{12}\}|R\rangle_{pqrs}^{(i)}$, $i = 1, 2, 3$. These states transform as a vectorial representation of $SO(4)$.
- $\{\bar{\phi}^{34}\}|R\rangle_{pqrs}^{(i)}$, $i = 1, 2, 3$. These states transform as a vectorial representation of $SO(4)$.
- $\{\bar{\phi}^{5\ldots8}\}|R\rangle_{pqrs}^{(i)}$, $i = 1, 2, 3$. These states transform as a vectorial representation of $SO(8)$.
- The remaining states in those sectors transform as singlets of the non-Abelian group factors.

3. Exophobic string models

Following the methodology developed in [18] we can write down analytic expressions for the GGSO projections on the states arising from all the sectors listed above. These formulas are inputted into a computer program which is used to scan the space of string vacua generated by random generation of the one-loop GGSO projection coefficients. The number of possible configurations is $2^{51} \sim 10^{15}$, which is too large for a complete classification. For this reason a random generation algorithm is utilised, and a model with the desired phenomenological criteria is fished from the sample generated.

The observable sector of a heterotic string Pati–Salam model is characterized by 9 integers $(n_g, k_L, k_R, n_6, n_h, n_4, n_{\bar{4}}, n_{2L}, n_{2R})$, where

$$n_{4L} - n_{\bar{4}L} = n_{\bar{4}R} - n_{4R} = n_g = \text{\# of generations},$$
$$n_{\bar{4}L} = k_L = \text{\# of non-chiral left pairs},$$
$$n_{4R} = k_R = \text{\# of non-chiral right pairs},$$
$$n_6 = \text{\# of } (6, 1, 1),$$
$$n_h = \text{\# of } (1, 2, 2),$$
$$n_4 = \text{\# of } (4, 1, 1) \text{ (exotic)},$$
$$n_{\bar{4}} = \text{\# of } (\bar{4}, 1, 1) \text{ (exotic)},$$

Table 1
Observable sector spectrum $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^3$ quantum numbers.

Sector	Field	$SU(4) \times SU(2)_L \times SU(2)_R$	$U(1)_1$	$U(1)_2$	$U(1)_3$
S	D_1	$(6, 1, 1)$	$+1$	0	0
	D_2	$(6, 1, 1)$	0	$+1$	0
	D_3	$(6, 1, 1)$	0	0	$+1$
	$\bar{D}_1$	$(6, 1, 1)$	-1	0	0
	$\bar{D}_2$	$(6, 1, 1)$	0	-1	0
	$\bar{D}_3$	$(6, 1, 1)$	0	0	-1
	Φ_{12}	$(1, 1, 1)$	$+1$	$+1$	0
	Φ_{12}^-	$(1, 1, 1)$	$+1$	-1	0
	$\bar{\Phi}_{12}$	$(1, 1, 1)$	-1	-1	0
	$\bar{\Phi}_{12}^-$	$(1, 1, 1)$	-1	$+1$	0
	Φ_{13}	$(1, 1, 1)$	$+1$	0	$+1$
	Φ_{13}^-	$(1, 1, 1)$	$+1$	0	-1
	$\bar{\Phi}_{13}$	$(1, 1, 1)$	-1	0	-1
	$\bar{\Phi}_{13}^-$	$(1, 1, 1)$	-1	0	$+1$
	$\Phi_i, i = 1, \ldots, 6$	$(1, 1, 1)$	0	0	0
	Φ_{23}	$(1, 1, 1)$	0	$+1$	$+1$
	Φ_{23}^-	$(1, 1, 1)$	0	$+1$	-1
	$\bar{\Phi}_{23}$	$(1, 1, 1)$	0	-1	-1
	$\bar{\Phi}_{23}^-$	$(1, 1, 1)$	0	-1	$+1$
$S + b_1 + e_5$	F_{1L}	$(4, 2, 1)$	$\frac{1}{2}$	0	0
$S + b_1 + e_4$	$\bar{F}_{1R}$	$(\bar{4}, 1, 2)$	$-\frac{1}{2}$	0	0
$S + b_1 + e_3$	F_{2L}	$(4, 2, 1)$	$-\frac{1}{2}$	0	0
$S + b_1 + e_3 + e_4 + e_5$	$\bar{F}_{2R}$	$(\bar{4}, 1, 2)$	$\frac{1}{2}$	0	0
$S + b_2 + e_2$	F_{1R}	$(4, 1, 2)$	0	$-\frac{1}{2}$	0
$S + b_2 + e_1 + e_2 + e_6$	$\bar{F}_{3R}$	$(\bar{4}, 1, 2)$	0	$\frac{1}{2}$	0
$S + b_3 + e_4$	F_{3L}	$(4, 2, 1)$	0	0	$-\frac{1}{2}$
$S + b_3 + e_3$	$\bar{F}_{4R}$	$(\bar{4}, 1, 2)$	0	0	$-\frac{1}{2}$
$S + b_1 + b_2 + e_2$	h_1	$(1, 2, 2)$	$-\frac{1}{2}$	$-\frac{1}{2}$	0
$S + b_1 + b_3 + e_1 + e_6$	h_2	$(1, 2, 2)$	$-\frac{1}{2}$	0	$-\frac{1}{2}$
$S + b_1 + b_3$	h_3	$(1, 2, 2)$	$\frac{1}{2}$	0	$\frac{1}{2}$
$S + b_1 + b_2 + e_2 + e_3 + e_4$	D_4	$(6, 1, 1)$	$-\frac{1}{2}$	$-\frac{1}{2}$	0
	$\zeta_a, a = 1, 2$	$(1, 1, 1)$	$\frac{1}{2}$	$-\frac{1}{2}$	0
	$\bar{\zeta}_a, a = 1, 2$	$(1, 1, 1)$	$-\frac{1}{2}$	$\frac{1}{2}$	0
	ξ_1	$(1, 1, 1)$	$\frac{1}{2}$	$\frac{1}{2}$	1
	ξ_2	$(1, 1, 1)$	$\frac{1}{2}$	$\frac{1}{2}$	-1
$S + b_1 + b_2 + e_2 + e_4$	ζ_3	$(1, 1, 1)$	$\frac{1}{2}$	$\frac{1}{2}$	0
	$\bar{\zeta}_3$	$(1, 1, 1)$	$-\frac{1}{2}$	$-\frac{1}{2}$	0

$$n_{2L} = \text{\# of } (1, 2, 1) \text{ (exotic)},$$

$$n_{2R} = \text{\# of } (1, 1, 2) \text{ (exotic)}.$$

Following the methodology developed in [18] we derived analytic formulas for all these quantities, and will reported elsewhere. The spectrum of a viable Pati–Salam heterotic string model should have $n_g = 3$,

$n_g = 3$,   three light chiral of generations,

$k_L \geqslant 0$,   heavy mass can be generated for non-chiral pairs,

$k_R \geqslant 1$,   at least one Higgs pair to break the PS symmetry,

$n_6 \geqslant 1$,   at least one required for missing partner mechanism,

$n_h \geqslant 1$,   at least one light Higgs bi-doublet,

$n_4 = n_{\bar{4}} \geqslant 0$,   heavy mass can be generated for vector-like exotics,

$n_{2L} = 0 \bmod 2$,   heavy mass can be generated for vector-like exotics,

$n_{2R} = 0 \bmod 2$,   heavy mass can be generated for vector-like exotics.

A minimal model which is free of exotics has $k_L = 0$, $k_R = 1$, $n_6 = 1$, $n_h = 3$, $n_4 = n_{\bar{4}} = 0$, $n_{2L} = 0$ and $n_{2R} = 0$. The model given by the following GGSO coefficients matrix:

$$[v_i | v_j] = e^{i\pi (v_i | v_j)}, \tag{3.1}$$

Table 2
Hidden sector spectrum and $SU(2)^4 \times SO(8) \times U(1)^3$ quantum numbers.

Sector	Field	$SU(2)^4 \times SO(8)$	$U(1)_1$	$U(1)_2$	$U(1)_3$
$S+b_1+b_2+e_1+e_4$	H^1_{12}	$(2,2,1,1,1)$	$-\frac{1}{2}$	$-\frac{1}{2}$	0
$S+b_2+b_3+e_3+e_6$	H^2_{12}	$(2,2,1,1,1)$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
$S+b_2+b_3+e_5+e_6$	H^3_{12}	$(2,1,2,1,1)$	0	$-\frac{1}{2}$	$\frac{1}{2}$
$S+b_1+b_2+z_1+e_4$	H^1_{13}	$(2,1,2,1,1)$	$-\frac{1}{2}$	$\frac{1}{2}$	0
$S+b_2+b_3+z_1+e_3+e_5$	H^2_{13}	$(2,1,2,1,1)$	0	$\frac{1}{2}$	$-\frac{1}{2}$
$S+b_2+b_3+z_1$	H^3_{13}	$(2,1,2,1,1)$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
$S+b_1+b_2+z_1+e_1+e_2$	H^1_{14}	$(2,1,1,2,1)$	$\frac{1}{2}$	$\frac{1}{2}$	0
$S+b_1+b_3+z_1+e_1$	H^2_{14}	$(2,1,1,2,1)$	$-\frac{1}{2}$	0	$-\frac{1}{2}$
$S+b_1+b_3+z_1+e_6$	H^3_{14}	$(2,1,1,2,1)$	$\frac{1}{2}$	0	$\frac{1}{2}$
$S+b_1+b_2+z_1+e_1+e_2+e_3+e_4$	H^1_{23}	$(1,2,2,1,1)$	$\frac{1}{2}$	$\frac{1}{2}$	0
$S+b_1+b_2+z_1+e_3$	H^1_{24}	$(1,2,1,2,1)$	$-\frac{1}{2}$	$\frac{1}{2}$	0
$S+b_1+b_3+z_1+e_1+e_2+e_5+e_6$	H^2_{24}	$(1,2,2,1,1)$	$-\frac{1}{2}$	0	$-\frac{1}{2}$
$S+b_1+b_3+z_1+e_2+e_5$	H^3_{24}	$(1,2,2,1,1)$	$\frac{1}{2}$	0	$\frac{1}{2}$
$S+b_2+b_3+z_1+e_3+e_4$	H^4_{24}	$(1,2,1,2,1)$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
$S+b_2+b_3+z_1+e_4+e_5$	H^5_{24}	$(1,2,1,2,1)$	0	$\frac{1}{2}$	$-\frac{1}{2}$
$S+b_1+b_2+e_1+e_3$	H^1_{34}	$(1,1,2,2,1)$	$-\frac{1}{2}$	$\frac{1}{2}$	0
$S+b_1+b_2+e_1+e_2+e_5$	H^2_{34}	$(1,1,2,2,1)$	$\frac{1}{2}$	0	$-\frac{1}{2}$
$S+b_2+b_3+e_2+e_5+e_6$	H^3_{34}	$(1,1,2,2,1)$	$-\frac{1}{2}$	0	$-\frac{1}{2}$
$S+b_2+b_3+e_3+e_4+e_5+e_6$	H^4_{34}	$(1,1,2,2,1)$	0	$-\frac{1}{2}$	$\frac{1}{2}$
$S+b_2+b_3+e_4+e_6$	H^5_{34}	$(1,1,2,2,1)$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
$S+b_1+b_3+e_6$	Z_1	$(1,1,1,1,8_v)$	$\frac{1}{2}$	0	$-\frac{1}{2}$
$S+b_1+b_3+z_2+e_1+e_2+e_5$	Z_2	$(1,1,1,1,8_s)$	$-\frac{1}{2}$	0	$-\frac{1}{2}$
$S+b_1+b_3+z_2+e_1$	Z_3	$(1,1,1,1,8_v)$	$-\frac{1}{2}$	0	$\frac{1}{2}$
$S+b_1+b_3+z_2+e_2+e_5+e_6$	Z_4	$(1,1,1,1,8_s)$	$-\frac{1}{2}$	0	$-\frac{1}{2}$
$S+b_1+b_2+e_1+e_2$	Z_5	$(1,1,1,1,8_s)$	$\frac{1}{2}$	$-\frac{1}{2}$	0

$$(v_i|v_j) = \begin{array}{c} \\ 1 \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ b_1 \\ b_2 \\ z_1 \\ z_2 \\ \alpha \end{array} \begin{array}{c} 1 \ \ S \ \ e_1 \ e_2 \ e_3 \ e_4 \ e_5 \ e_6 \ b_1 \ b_2 \ z_1 \ z_2 \ \alpha \\ \left(\begin{array}{ccccccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{array}\right) \end{array}. \tag{3.2}$$

The twisted massless states generated in the string vacuum of Eq. (3.2) produces the desired spectrum. Namely, it contains three chiral generations; one pair of heavy Higgs states to break the Pati–Salam symmetry along flat direction; one light Higgs bi-doublet to break the electroweak symmetry and generate fermion masses; one vector sextet of $SO(6)$ needed for the missing partner mechanism; it is completely free of massless exotic fractionally charged states. States with fractional electric charge necessarily exist in the massive spectrum of the string model, and it easy to see that indeed they do. All we need is to show that the GSO projection which projects them in a given sector is reversed when additional Neveu–Schwarz oscillators act on the vacuum. Additionally the model contains three pairs of untwisted $SO(6)$ sextets. These can obtain string scale mass along flat directions. The full massless spectrum of the model is shown in Tables 1 and 2.

To explore the abundance of string vacua that do not contain massless exotics we perform a statistical sampling[3] in a space of 5×10^9 models out of the total of 2^{51}–10^{15}. In Fig. 1 we display the total number of exotics that appear in viable three generation models. As can be seen from the figure, in the space of models sampled there are of the order of 10^4 models that are completely free of exotic states. Having established a quasi-realistic spectrum, the next stage is to analyze the Yukawa couplings in the models. The abundance of exotic free three generation models suggests that models with viable Yukawa couplings and fermion mass spectrum do exist in this space of string vacua.

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[3] We note that analysis of large sets of string vacua has also been carried out by other groups [20].
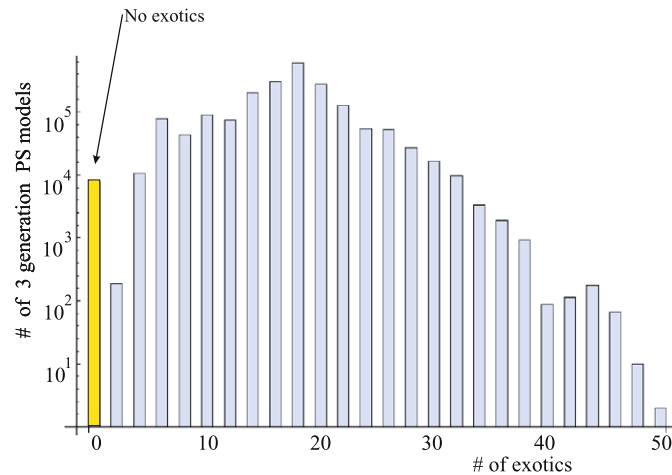
Fig. 1. Number of three generation models versus the total number of exotics.

4. Conclusions

In this Letter we demonstrated the existence of string vacua that are completely free of massless exotic fields. The need to break the non-Abelian GUT symmetries in heterotic string models by Wilson lines, while preserving the GUT embedding of the weak hypercharge and the GUT prediction $\sin^2 \theta_w(M_{GUT}) = 3/8$, necessarily implies that the models contain states with fractional electric charge. Such states are severely restricted by observations, and must be confined or sufficiently massive and diluted. In this Letter we constructed the first quasi-realistic heterotic string models in which the exotic states do not appear in the massless spectrum, and only exist, as they must, in the massive spectrum. The string models that we constructed are three generation models in which the $SO(10)$ GUT symmetry is broken to the Pati–Salam subgroup, and similar analysis can be performed in the case of the other $SO(10)$ subgroups. Our PS heterotic string models contain adequate Higgs representations to break the GUT and electroweak symmetry, as well as colour Higgs triplets that can be used for the missing partner mechanism. By statistically sampling the space of Pati–Salam free fermionic vacua we demonstrated the abundance of three generation models that are completely free of massless exotics, rendering it plausible that obtaining realistic Yukawa couplings may be possible in this space of models.

Acknowledgements

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