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Theta angle versus CP violation in the leptonic sector

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Assuming that the axion mechanism of solving the strong CP problem does not exist and the vanishing of θ at the tree level is achieved by some model-building means, we study the naturalness of having large CP-violating sources in the leptonic sector. We consider the radiative mechanisms which transfer a possibly large CP-violating phase in the leptonic sector to the θ parameter. It is found that a large θ cannot be induced in the models with one Higgs doublet as at least three loops are required in this case. In the models with two or more Higgs doublets the dominant source of θ is the phases in the scalar potential, induced by CP violation in the leptonic sector. Thus, in the minimal supersymmetric standard model framework the imaginary part of the trilinear soft-breaking parameter A_l generates the corrections to the theta angle already at one loop. These corrections are large, excluding the possibility of large phases, unless the universality in the slepton sector is strongly violated.

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I. INTRODUCTION AND MOTIVATION

The strong CP problem whose existence was realized over twenty years ago [1] remains a complete mystery. The theta term of the QCD Lagrangian breaks P and CP invariance, and thus induces a variety of P-, T-odd observable effects, among which the electric dipole moments (EDMs) of the neutron and heavy atoms play a prominent role [2]. The conflict between strong limits on θ resulting from experimental searches of EDMs and natural expectations of $\theta\sim 1$ presents a severe fine-tuning problem, usually referred to as the strong CP problem. Using the experimental limits on the EDM of the neutron [3] together with the result of a recent QCD sum rule calculation of $d_n(\theta)$ [4] one can place a very stringent limit on the theta term:

$$\theta < 6\times 10^{-10}. \tag{1}$$

A common and universal solution to the strong CP problem may come through a dynamical relaxation mechanism [5] which requires the existence of a light pseudoscalar (axion [6]) in the particle spectrum. Negative results from experimental searches [7] of an axion together with very restrictive astrophysical [8] and cosmological bounds [9] on its coupling constant stimulate searches for alternative solutions.

Another possibility is a model-building construction where θ can be naturally chosen to be zero at some high-energy scale due to exact parity or CP symmetry [10,11]. In this case however, θ is not protected against radiative corrections at lower scales where parity and/or CP symmetry are spontaneously broken. Thus the theta term is extremely sensitive to the presence of additional, other than Kobayashi-Maskawa (KM), CP-violating sources in the hadronic sector. This sensitivity is unique: θ can receive contributions from the CP-violating phases in the "heavy" sector of the theory without powerlike suppression, in contrast with other CP-violating operators. Thus, in the SUSY variants of these models large soft-breaking phases in the squark and gluino sectors are excluded, as they penetrate into the low-energy effective expression for θ already at one loop level. Therefore, a necessary consequence of these constructions seems to be a strong restriction on CP violation, i.e., no CP violation other than the KM phase. Is this also true for CP violation which resides solely in the leptonic sector? In other words, how susceptible is θ to the CP violation in the leptonic sector?

If the axion mechanism does not exist, the theta term is expected to be a dominant source of CP violation at low energy as it is the CP-odd operator of lowest dimension. What would be a signal of the "θ dominance" among CP-violating observables? Both neutron and mercury EDMs produce similar bounds on θ and one should naturally expect that

$$\begin{aligned} d_n &\simeq 10^{-26}\ e\,\text{cm}\ \frac{\theta}{10^{-10}} \quad (\text{Ref. [4]}) \\ d_{Hg} &\simeq 10^{-28}\ e\,\text{cm}\ \frac{\theta}{10^{-10}}, \quad (\text{Ref. [4]}) \\ d_{Tl} &\sim 2\times 10^{-29}\ e\,\text{cm}\ \frac{\theta}{10^{-10}}. \quad (\text{Ref. [12]}) \end{aligned} \tag{2}$$

Comparing the predictions of θ-dominated EDMs with current experimental limits [3,13,14], one can easily see that for $\theta = 10^{-10}$, d_n and d_{Hg} are within a factor of 2–3 from the current experimental figures, whereas d_{Tl} is smaller by five orders of magnitude than its present limit. In other words, $\theta = 10^{-10}$ will produce thallium EDM at the level equivalent to d_{Tl}, induced by the electron EDM $d_e \sim 4\times 10^{-32} e$ cm. Thus, it appears that the signal of θ-dominance could be easily distinguished from the case of the minimal supersymmetric standard model (MSSM) with large CP SUSY phases and axion-type solution to the strong CP problem. In the latter case d_{Tl} is expected to be much more important than in Eq. (2) and competitive with d_n and d_{Hg}.

However, if CP violation is initially concentrated in the leptonic sector, the ''θ signal'' (2) could be different. In this case the EDM of the electron and d_{Tl} could be enhanced relative to (2) and, at the same time, the θ term, induced by a lepton CP phase via radiative corrections would still dominate d_n and d_{Hg}.

The purpose of this article is to study the mechanisms of transferring CP violation from the leptonic sector to the theta term in the context of different models without an axion. Assuming no fine tuning which would compensate an induced value of θ, we find a ''maximal'' amount of CP violation in the lepton sector, which can be consistent with the bound (1). At the same time, we study possible enhancement of d_e and d_{Tl} due to the same sources of CP violation and the departure from the θ-dominance signal, Eq. (2).

II. NON-SUSY MODELS

We begin with some remarks about the way the low energy value of θ should be calculated in a generic theory with CP violation. Besides the initial value of θ_{QCD}, the relevant low energy parameter $\bar{\theta}$ receives tree level contributions from the phases of the quark masses and other $SU(3)_c$-charged fermions.

$$\bar{\theta}=\theta_{QCD}+\arg\det(M_uM_d)+\cdots. \tag{3}$$

It is often assumed in the literature that the radiative corrections to $\bar{\theta}$ are simply contained in the imaginary parts of the quark and gluino masses. This is certainly true at the tree level, but at the loop level the structure of radiative corrections is more complicated. To give a simplest example, one can consider an effective Lagrangian for gluons and quark field q which arises after integrating out some unknown CP-violating physics at the scale Λ:

$$\mathcal{L}_{eff}=\theta(\Lambda)\frac{g_3^2}{16\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}+\bar{q}(i\partial_\mu\gamma^\mu-m-im'\gamma_5)q$$
$$-\frac{im''}{2\Lambda^2}\bar{q}G^a_{\mu\nu}t^a\sigma^{\mu\nu}\gamma_5 q+\cdots. \tag{4}$$

Here $\theta(\Lambda)$ denotes the theta term, coming from the scale Λ. Let us take for simplicity $m\gg\Lambda_{QCD}$ and $m'\ll m$. Then the field q can be also integrated out and the theta parameter below the scale m reads as

$$\bar{\theta}=\theta(\Lambda)+\frac{m'}{m}+\frac{mm''}{\Lambda^2}\log(\Lambda^2/m^2). \tag{5}$$

The second term in this expression is the ''usual'' correction due to the phase of the mass term, whereas the third term is generated by the ''chromoelectric dipole'' in Eq. (4). It is usually smaller than the second term due to $\Lambda\gg m$, although not necessarily negligible. For example, the scale of new physics Λ could be comparable to the mass of heaviest fermions (top quark) so that the ratio mm''/Λ^2 is not small, or m' can be simply zero from additional symmetry arguments and then the third term dominates the expression for $\bar{\theta}$. The latter is exactly the case in the minimal SM, where $\bar{\theta}$ receives corrections from ''dipole'' contributions as it was first shown by Khriplovich [15]. Technically, the corrections to $\bar{\theta}$ can be easily calculated within the external field formalism which will automatically account for all contributions.

In what follows we determine possible mechanisms of transmitting CP violation from the leptonic sector into the theta term in various possible models [16]. As representative examples we take the standard model extended by right-handed neutrino fields, the dilepton Zee model [17], multi-Higgs models, and the MSSM in particular.

It turns out that the main criterion which governs the efficiency of transmitting the CP violation from the leptonic sector into the theta term is the number of weak doublets which give masses to the quark fields. The contribution of the quark masses into $\bar{\theta}$ can be separated into the contributions of Yukawa couplings and Higgs vacuum expectation values (VEVS):

$$\arg\det(M_uM_d)=\arg\det(Y_u)+\arg\det Y_d$$
$$+3(\arg v_u+\arg v_d). \tag{6}$$

We take the vanishing of this expression due to some symmetry arguments (for example, hermiticity of Y_i, reality of v_i's) as the starting point for our analysis.

In the SM and in other models where $v_u\equiv v_d^*$, the contribution from the second line in Eq. (6) is identically zero, irrespective of the presence of CP violation. Therefore the only way to insert CP violation into the theta term is to ''complexify'' quark Yukawa couplings and/or create quark chromoelectric dipoles.

Nontrivial corrections to quark Yukawa couplings sensitive to a CP phase in the leptonic sector must be induced via Yukawa and $SU(2)\times U(1)$ gauge interactions. Furthermore, it is clear that in the presence of only one Higgs doublet Yukawa interactions alone are not sufficient to achieve this. In any possible graph, involving a quark line and leptons in the loop, it is convenient to separate the loop part where actual CP violation takes place. Let us suppose now that the particles circulating in the loop are heavy (Majorana neutrinos, for example) and the lines connecting the leptonic loop to a quark line are ''soft.'' Then it is possible to classify the effects of CP violation in the leptonic sector in terms of effective CP odd operators with dimension 6 and larger: $H^\dagger H(B^{\mu\nu}\tilde{B}_{\mu\nu})$; $H^\dagger H(W^{a\mu\nu}\tilde{W}^a_{\mu\nu})$; $\tilde{W}^a_{\mu\nu}W^{b\nu\alpha}W^{c\mu}_{\alpha}\epsilon_{abc}$, etc. One needs at least two loops to attach these operators to a quark line with no external $SU(2)$ or $U(1)$ fields allowed. Together with at least one (leptonic) loop needed to generate these operators, three loops is the *minimal* order in which CP violation from the leptonic sector penetrates into $\bar{\theta}$!

In practice, the loop level is often higher. In the SM with heavy Majorana neutrinos, singlets of the SM gauge group, one should have a minimum of four flavor-changing vertices on the lepton line. In the weak basis, which is more convenient because the momenta flowing in the loop are large, of the order of the heavy Majorana masses, these can only come from interactions with the Higgs doublet. This adds another

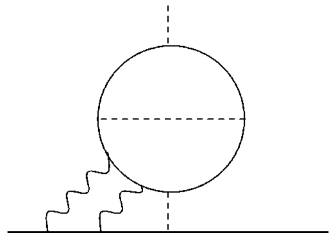

FIG. 1. A typical diagram, which gives a phase to the quark Yukawa coupling. The circle is ordinary leptons and Majorana neutrinos, wavy lines are gauge bosons of the electroweak group, and dashed lines the Higgs field. Similar diagrams work for the Zee model with the dashed line inside the circle being a dilepton.

loop and indicates that the effect may first appear at the four-loop order and a typical diagram is shown in Fig. 1.

This diagram will be further suppressed by at least the square of the charged lepton mass as CP violation disappears if all charged leptons are massless. A more detailed calculation may reveal further suppression factors. For our purposes it is sufficient to acknowledge that the suppression factor is at least

$$\bar{\theta} < \left(\frac{\alpha}{4\pi}\right)^2 \left(\frac{1}{16\pi^2}\right)^2 \frac{m_\tau^2}{M^2} J_{CP}^L, \tag{7}$$

where M is the relevant high energy scale, at least as heavy as M_W. No matter how large the CP-violating combination of mixing angles J_{CP}^L in the leptonic sector is, the result for $\bar{\theta}$ is well within the experimental bound. Therefore all CP violating phenomena discussed in the literature, such as CP violation in neutrino oscillations, CP violation in the heavy Majorana neutrino decay, needed for leptogenesis and others, are entirely possible without causing problems for θ.

Precisely the same estimates (in this crude approach) can be applied to the Zee model to produce similar conclusions, i.e., θ generated from the CP violation in the leptonic sector is small. However, unlike the SM with Majorana neutrinos, where the possible electron EDM is likely to be very small [18], the Zee model can have d_e at a measurable level [16].

Another group of models has two or more Higgs doublets which give masses to quarks via two or more *different* VEVs. In this case one should look for the effects which introduce phases into the scalar potential. Thus the operators $H_u^i H_d^j \epsilon_{ij}$, $(H_u^\dagger H_u) H_u^i H_d^j \epsilon_{ij}$, etc. may enter with complex coefficients which then can lead to $(\arg v_u + \arg v_d) \neq 0$.

In the nonsupersymmetric framework a consideration of the radiative corrections to the scalar potential are somewhat flawed. Indeed, the dimension 2 $H_u^i H_d^j \epsilon_{ij}$-proportional term enters in the Lagrangian multiplied by some mass squared parameter. In the non-SUSY framework, at the radiative level, this parameter will be sensitive to the square of the cutoff Λ^2 which by itself requires fine tuning to ensure the stability of the electroweak scale. Thus, we believe that the question of induced phases in the soft-breaking sector and θ cannot be solved without a specified framework which ensures the stability of the scalar potential. Thus we abandon non-SUSY two Higgs doublet models and take the case of MSSM where we study in detail the value of θ versus complex soft-breaking terms in the leptonic sector.

III. MSSM WITH COMPLEX SOFT-BREAKING PARAMETERS IN THE LEPTON SECTOR

We concentrate only on the leptonic sector of the MSSM superpotential, i.e.,

$$\mathcal{W} \supset \epsilon_{ab} (\mathbf{Y}_e)_{ij} L_i^a H_1^b \bar{E}_j, \tag{8}$$

and the soft breaking terms,

$$\mathcal{L}_{soft} \supset -\epsilon_{ab}[(\mathbf{A}_e \mathbf{Y}_e)_{ij} \tilde{l}_{Li}^a H_1^b \tilde{e}_{Rj}^* + \text{H.c.}] - [\mu B \epsilon_{ab} H_1^a H_2^b + \text{H.c}], \tag{9}$$

where as usual $\tilde{e}, \tilde{l}$ are the corresponding scalar components of the chiral superfields $L, \bar{E}$ appearing in Eq. (8). Let us assume universality of the soft trilinear couplings at the grand unified theory (GUT) scale, $\mathbf{A}_e = \mathbf{A}_\mu = \mathbf{A}_\tau$, and one common phase ϕ_A associated with them. We consider the third generation of leptons, i.e., A_τ where the Yukawa couplings are large as compared to those of the first and the second generation. Then the renormalization group running of the imaginary part of the parameter A_τ, denoted as $\overline{A_\tau}$, induces an imaginary part of the parameter B, denoted as $\bar{B}$, at a scale below the GUT scale and their renormalization group equations (RGEs) are given by [19],

$$\frac{d\overline{A}_\tau}{dt} = \frac{8|Y_\tau|^2}{16\pi^2} \overline{A}_\tau, \tag{10}$$

$$\frac{d\bar{B}}{dt} = \frac{2|Y_\tau|^2}{16\pi^2} \overline{A}_\tau. \tag{11}$$

All the other parameters of the SUSY or the soft SUSY breaking sector remain real. The tau lepton Yukawa coupling has a weak running (especially for small values of $\tan\beta$). Thus the system of differential equations of Eq. (11) can be solved trivially and gives

$$\overline{A}_\tau(Q) = \overline{A}_\tau(M_G) \left(\frac{Q}{M_G}\right)^{\frac{|Y_\tau|^2}{2\pi^2}}, \tag{12}$$

$$\bar{B}(Q) = -\frac{\overline{A}_\tau(M_G)}{4} \left[1 - \left(\frac{Q}{M_G}\right)^{\frac{|Y_\tau|^2}{2\pi^2}}\right]. \tag{13}$$

So even if all the parameters at the GUT scale are real apart from the leptonic trilinear coupling i.e., $\overline{A}_\tau$, then this parameter affects the running of the $\bar{B}$ parameter and generates a nonzero $\bar{B}$. We can easily see from Eq. (13) that the running of the phase of the parameter B, i.e., ϕ_B at a scale Q is given by

$$\sin\phi_B(Q) = -\frac{1}{4}\frac{|A_\tau(M_G)|}{|B(Q)|}\sin\phi_A(M_G)\left[1-\left(\frac{Q}{M_G}\right)^{\frac{|Y_\tau|^2}{2\pi^2}}\right]. \tag{14}$$

Let us now see what happens at the electroweak scale, i.e., $Q=M_Z$. It is reasonable to take $|A_\tau(M_G)|\simeq|B(M_Z)|$ in the MSSM with radiative electroweak breaking. This assumption of course depends on the choice of the other MSSM parameters, M_0, $M_{1/2}$, A_0 and $\tan\beta$. We display the numerical solutions below. For $|Y_\tau|^2/4\pi\simeq 4\times10^{-5}$ and $\tan\beta=2$ with $M_{GUT}=3\times10^{16}$ GeV we get, from Eq. (14),

$$\sin\phi_B(M_Z)\simeq -2\times10^{-4}\sin\phi_A. \tag{15}$$

Now from the minimization conditions of the scalar Higgs potential we have

$$v_1 v_2 = \frac{\mu^* B^* |v|^2}{m_1^2+m_2^2}, \tag{16}$$

where $m_{1,2}^2 = m_{H_{1,2}}^2+\mu^2$ and $|v|^2=|v_1|^2+|v_2|^2$. Note also that the parameter μ remains real (if it originally is real) at every scale because its renormalization is multiplicative.

The θ angle is generated if B is complex and given by eq. (6). Putting the experimental bound (1) of θ parameter into Eq. (15) we get

$$\phi_A(M_G) < 10^{-6}, \tag{17}$$

an unnaturally small number at the GUT scale. We conclude that the phase in the leptonic sector produces large additive renormalization of the θ-QCD parameter which constitutes a fine tuning problem unless this phase is tiny, of the order of 10^{-6} or smaller.

Three remarks are in order: (i) Even if we assume an appropriate phase for the parameter μ at a scale Q which cancels the contribution of the ϕ_B, i.e., $\phi_\mu(Q)=-\phi_B(Q)$, then eventually this pattern will be destroyed by the running of ϕ_B of Eq. (13) since ϕ_μ does not run, (ii) the constraint (17) on $\phi_A(M_G)$ is relaxed if we consider nonuniversality of the soft SUSY breaking trilinear couplings at the GUT scale in the case of the electron and muon Yukawa couplings, (iii) if we start with the (trivial) case $A=0$ GeV at the GUT scale then there is no contribution to the θ term and no CP violation in the leptonic sector.

We perform a numerical analysis of the RGEs by also taking into account low energy threshold effects [20]. We present our results in Fig. 2. We see that $\phi_A \lesssim 10^{-6}$ unless A_0 is exactly zero. Small departures from zero (see the line with $|A_0|=1$ GeV in Fig. 2 for instance) put a strong bound on the phase ϕ_A. As $|A_0|$ increases the bound becomes stronger, as strong as $\phi_A\lesssim10^{-8}$ for $|A_0|\gtrsim 300$ GeV. This happens because $|A_0|=|A_\tau(M_G)|$ gets much larger than $|B(M_Z)|$ which further enhances the value of theta, as seen from Eq. (14).

Therefore we face two possible choices if we still want to keep large CP violation in the leptonic sector: (i) relax the universality pattern of the phases at the GUT scale (however,

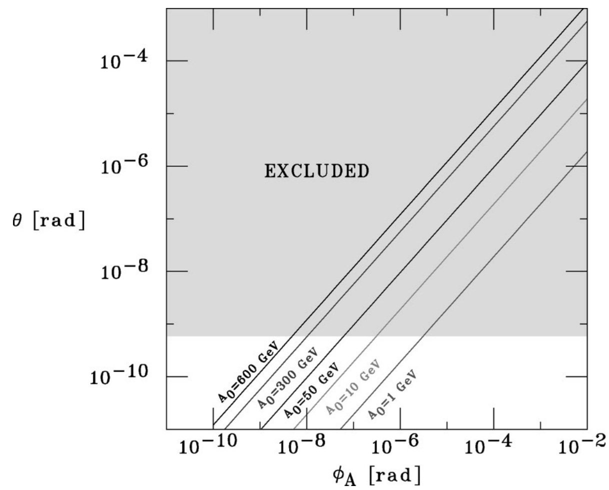

FIG. 2. The extracted value of the θ term as a function of the common phase ϕ_A at the GUT scale of the lepton trilinear soft SUSY breaking couplings. The other SUSY breaking parameters have been fixed $M_0=M_{1/2}=200$ GeV and $\tan\beta=10$. The shaded region is excluded by the experiment; see Eq. (1). Results on θ from different values of the modulo $|A_0|=1,10,50,300,600$ GeV are also indicated.

even in that case the phases of the τ trilinear soft breaking coupling must be unnaturally small as we prove above); (ii) introduce PQ symmetry and the axion solution to the strong CP problem.

IV. CONCLUSIONS

We have studied the question of naturalness for CP violation in the leptonic sector to be large without inducing large corrections to $\bar{\theta}$. This is an important question in the context of nonaxionic solutions to the strong CP problem. We find that the main criterion dividing models into two classes is the number of Higgs doublets giving masses to quarks. In the case of one doublet the contribution of CP phases from the leptonic sector to $\bar{\theta}$ is always small, being suppressed by at least a three-loop factor so that a ''maximal'' CP violation in the leptonic sector is allowed. In some of these models (dilepton model, for example), d_e can be quite large, enhancing d_{Tl} with respect to the ''θ-dominance'' signal, Eq. (2), usually expected when the axion mechanism is absent.

In the models with several doublets there is an efficient way of transmitting CP violation from the leptonic sector into θ via complex parameters in the scalar potential. In MSSM without an axion, a large phase of the leptonic A_l parameter is excluded on the grounds of naturalness, unless the lepton universality is broken in a peculiar way that only A_e (or A_μ) has the phase.

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[1] C. Callan, R. Dashen, and D. Gross, Phys. Lett. **63B**, 334 (1976); R. Jackiw and C. Rebbi, Phys. Rev. Lett. **37**, 172 (1976).

[2] I.B. Khriplovich and S.K. Lamoreaux, *CP Violation Without Strangeness* (Springer, Berlin, 1997).

[3] K.F. Smith *et al.*, Phys. Lett. B **234**, 191 (1990); I.S. Altarev *et al.*, *ibid.* **276**, 242 (1992); P.G. Harris *et al.*, Phys. Rev. Lett. **82**, 904 (1999).

[4] M. Pospelov and A. Ritz, Phys. Rev. Lett. **83**, 2526 (1999); M. Pospelov and A. Ritz, hep-ph/9908508.

[5] R.D. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); , Phys. Rev. D **16**, 1791 (1977); J.E. Kim, Phys. Rev. Lett. **43**, 103 (1979); M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. **B166**, 493 (1980); A.R. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 260 (1980) [Yad. Fiz. **31**, 497 (1980)]; M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981).

[6] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, *ibid.* **40**, 279 (1978).

[7] C. Hagmann *et al.*, in Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).

[8] For reviews, see M.S. Turner, Phys. Rep. **197**, 67 (1990); G.G. Raffelt, *ibid.* **198**, 1 (1990).

[9] J. Preskill, M.B. Wise, and F. Wilczek, Phys. Lett. **120B**, 127 (1983); L.F. Abbott and P. Sikivie, *ibid.* **120B**, 133 (1983); M. Dine and W. Fischler, *ibid.* **120B**, 137 (1983). For reviews, see J.E. Kim, Phys. Rep. **150**, 1 (1987); M.S. Turner, *ibid.* **197**, 68 (1990).

[10] M.A.B. Beg and H.S. Tsao, Phys. Rev. Lett. **41**, 278 (1978); R.N. Mohapatra and G. Senjanovic, Phys. Lett. **79B**, 278 (1978).

[11] A. Nelson, Phys. Lett. **136B**, 387 (1984); S. Barr, Phys. Rev. Lett. **53**, 329 (1984); P.H. Frampton, *ibid.* **68**, 2129 (1992); D. Bowser-Chao, D. Chang, and Wai-Yee Keung, Phys. Rev. Lett. **81**, 2028 (1998); H. Georgi and S.L. Glashow, Phys. Lett. B **451**, 372 (1999). The renormalization of θ due to the color EDM was discussed also in D. Chang, Kiwoon Choi, and W.-Y. Keung, Phys. Rev. D **44**, 2196 (1991).

[12] We are not aware of any work which calculates an EDM of a paramagnetic atom, d_{Tl} or d_{Cs}, induced by the theta term. One way to estimate the effect [M. Pospelov, 1992 (unpublished)] is to determine a size of the effective *T*-odd nucleon-electron interaction $\overline{NN}\overline{e}\gamma_5 e$, induced by θ via a π_0 exchange. In this case *CP* violation resides in the $\pi\overline{N}N$ vertex, R. Crewther *et al.*, Phys. Lett. **88B**, 123 (1979), and $\overline{e}\gamma_5 e\pi_0$ interaction can be extracted from the $\pi_0\to e^+e^-$ branching ratio. The final estimate is given in the third line of Eq. (2).

[13] J.P. Jacobs *et al.*, Phys. Rev. Lett. **71**, 3782 (1993).

[14] E.D. Commins *et al.*, Phys. Rev. A **50**, 2960 (1994).

[15] I.B. Khriplovich, Yad. Fiz. **44** 1019 (1986) [Sov. J. Nucl. Phys. **44**, 659 (1986)]. The value of θ, induced by the KM phase δ at the electroweak threshold is much larger than the renormalization group mixing of θ and δ considered in J. Ellis and M. Gaillard, Nucl. Phys. **150B**, 141 (1979).

[16] W. Bernreuther and M. Suzuki, Rev. Mod. Phys. **63**, 313 (1991).

[17] A. Zee, Phys. Rev. Lett. **55**, 2382 (1985).

[18] D. Ng and J. Ng, Mod. Phys. Lett. A **11**, 211 (1996). The two-loop diagrams considered in this paper have additional suppression factors driving d_e below the level of $10^{-32}e$ cm.

[19] S.P. Martin and M.T. Vaughn, Phys. Rev. D **50**, 2282 (1994); Y. Yamada, *ibid.* **50**, 3537 (1994); I. Jack and D.R. Jones, Phys. Lett. B **333**, 372 (1994).

[20] A. Dedes, A.B. Lahanas, and K. Tamvakis, Phys. Rev. D **53**, 3793 (1996).