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Family symmetries in F-theory GUTs

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Abstract

We discuss F-theory $SU(5)$ GUTs in which some or all of the quark and lepton families are assigned to different curves and family symmetry enforces a leading order rank one structure of the Yukawa matrices. We consider two possibilities for the suppression of baryon and lepton number violation. The first is based on Flipped $SU(5)$ with gauge group $SU(5) \times U(1)_\chi \times SU(4)_\perp$ in which $U(1)_\chi$ plays the role of a generalised matter parity. We present an example which, after imposing a Z_2 monodromy, has a $U(1)_\perp^2$ family symmetry. Even in the absence of flux, spontaneous breaking of the family symmetry leads to viable quark, charged lepton and neutrino masses and mixing. The second possibility has an R-parity associated with the symmetry of the underlying compactification manifold and the flux. We construct an example of a model with viable masses and mixing angles based on the gauge group $SU(5) \times SU(5)_\perp$ with a $U(1)_\perp^3$ family symmetry after imposing a Z_2 monodromy.

1. Introduction

The origin of quark and lepton masses and mixing remains one of the key unanswered questions in the Standard Model. Recently there has been much interest in the possibility that the fermion mass structure might emerge from F-theory [1–6]. Most of the analyses to date have focused on the possibility that the families belong to a single matter curve and the fermion mass

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hierarchy results from the case that the mass matrices have rank one in the absence of fluxes [7–19]. While this provides a promising structure it requires that there is only a single intersection of the matter and Higgs curves in the up down and charged lepton sectors. However explicit calculations [20] for simple geometries suggest that the number of intersections must be even. Although there are ways to recover the rank one starting point, for example imposing factorization of the matter curves into irreducible pieces [11,12,21–23], it does not seem to be the norm with the generic case having, a large number of intersections.

An alternative possibility that can lead to fermion mass hierarchy even for the case of multiple intersections has been explored by Dudas and Palti [24]. Starting with the group $SU(5) \times SU(5)_\perp$ they explored the possibility that the family fields belong to *different* matter curves. As the fields carry different charges under the $U(1)$ factors of $SU(5)_\perp$ (after identifying the monodromy group) the latter act as family symmetries. Allowing for spontaneous breaking of these symmetries can lead to an hierarchical structure for the fermion masses. As we shall discuss in this case multiple intersections do not disturb the hierarchy. Note that, unlike Dudas and Palti, we will also consider cases with more than one state on a matter curve.

The survey of all possible monodromies presented in [24] gave rise to models with promising mass structure but they all suffered from the problem that some R-parity violating term(s) was not forbidden by the family symmetries and thus the models had unacceptable levels of baryon and/or lepton number violating processes. In this paper we shall discuss how this conclusion can be avoided and illustrate the possibilities by constructing two models with viable fermion mass matrix structure. The first model is based on the 'flipped' $SU(5)$ group, $SU(5) \times U_\chi(1)$, in which the $SU(2)$ singlet, charge conjugate down and up quarks belong to the 10 and $\bar{5}$ representations respectively, the opposite assignment to the case of conventional $SU(5)$. In this case the $U(1)_\chi$ acts as a generalised matter parity and eliminates the leading unwanted baryon and lepton number violating terms. The second model invokes the R-parity that the authors of [20] argue can arise in F-theory models through a symmetry of the underlying Calabi–Yau manifold and the flux. In this case one can build viable models based on the normal $SU(5)$ multiplet assignments.

Of course the ultimate aim is to obtain phenomenologically acceptable quark and lepton mass matrices. The structure of the quark mass matrices is not completely determined by the measured quark masses and mixing angles. To a good approximation for the hierarchical structure that follows from spontaneously broken family symmetries it is the terms on the diagonal and above the diagonal (assuming left–right convention) in the current quark basis that are fixed by the quark masses and the Cabibbo–Kobayashi–Maskawa (CKM) matrix. The terms below the diagonal (again assuming left–right convention) depend on the rotation of the right-handed (RH) quark components needed to diagonalise the mass matrix and, due to the absence of charged gauge bosons coupling to the RH quark sector, we have no constraint on it. Assuming a symmetric structure a fit to the available data [25] has the form[1]

$$M^d = \begin{pmatrix} 0 & -1.9i\epsilon^3 & 2.3\epsilon^3 e^{-i\pi/3} \\ -1.9i\epsilon^3 & \epsilon^2 & 2.1\epsilon^2 \\ 2.3\epsilon^3 e^{-i\pi/3} & 2.1\epsilon^2 & 1 \end{pmatrix} m_{b_0} \tag{1}$$

$$M^u = \begin{pmatrix} 0 & 0.4\epsilon^4 & 0 \\ 0.4\epsilon^4 & 0.8\epsilon^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_{t_0} \tag{2}$$

[1] Ref. [25] also discusses further ambiguities associated with the phases and threshold effects.

where $\epsilon = 0.15$. Note that CKM mixing matrix is unchanged if M^d and M^u are rotated by the same amount (of course the eigenvalues are unchanged by rotations). This will be important when we discuss the form of the mass matrices in the flipped $SU(5)$ case. The structure of Eqs. (1), (2) has a texture zero in the $(1,1)$ position that leads to the prediction [26]

$$V_{us}(M_X) \approx \left|\sqrt{\frac{m_d}{m_s}} + i\sqrt{\frac{m_u}{m_c}}\right|$$

that gives an excellent fit to V_{us}.

Note also that the magnitude of the $(2,3)$ element of M^d is comparable to the $(2,2)$ element; this is potentially a problem for mass matrices ordered by $U(1)$ symmetries that typically give $O(\epsilon)$. A non-zero entry in the $(1,3)$ position of $O(\epsilon^3)$ is necessary to avoid the relation $V_{ub}/V_{cb} = \sqrt{m_u/m_c}$.[2]

As discussed above the data does not strongly constrain the elements of $M^{u,d}$ below the diagonal and they are limited only by the constraint that the eigenvalues should approximately remain the same. The same is true of the $(1,3)$ and $(2,3)$ elements of M^u.

With this brief summary of the desired form of the quark mass matrices we turn to the structure that can come from F-theory in the case that the mass hierarchy is controlled by the Abelian symmetries.

2. Flipped $SU(5)$

In flipped $SU(5)$ [28,29] the chiral matter fields of a single generation, as in ordinary $SU(5)$, constitute the three components of the $16 \in SO(10)$, ($16 = 10_{-1} + \bar{5}_3 + 1_{-5}$ under the $SU(5) \times U(1)_\chi$ decomposition). However, the definition of the hypercharge includes a component of the external $U(1)_\chi$ in such a way that flips the positions of u^c, d^c and e^c, ν^c within these representations, while leaves the remaining unaltered. Indeed, employing the hypercharge definition

$$Y = \frac{1}{5}\left(x + \frac{1}{6}y\right)$$

where, x is the charge under the $U(1)_\chi$ and y the 'non-flipped' $SU(5)$ hypercharge generator, we obtain the following 'flipped' embedding of the SM representations

$$F_i = 10_{-1} = \left(Q_i, d_i^c, \nu_i^c\right) \tag{3}$$

$$\bar{f}_i = \bar{5}_{+3} = \left(u_i^c, \ell_i\right) \tag{4}$$

$$\ell_i^c = 1_{-5} = e_i^c \tag{5}$$

In the field theory model the Higgs fields are found in

$$H \equiv 10_{-1} = \left(Q_H, D_H^c, \nu_H^c\right), \qquad \overline{H} \equiv \overline{10}_{+1} = \left(\bar{Q}_H, \bar{d}_H^c, \bar{\nu}_H^c\right) \tag{6}$$

$$h \equiv 5_{+2} = (D_h, h_d), \qquad \bar{h} \equiv \bar{5}_{-2} = (\bar{D}_h, h_u) \tag{7}$$

When $H, \overline{H}$ acquire non-zero vacuum expectation values (vevs) along their neutral components $\langle \nu_H^c \rangle = \langle \bar{\nu}_H^c \rangle = M_{GUT}$, they break the $SU(5) \times U(1)_\chi$ symmetry down to the Standard Model

[2] However an alternative symmetric fit (not considered here) is possible with $(1,3)$ elements of both M^u and M^d being zero providing one allows for a non-zero $(1,1)$ element in M^u (maintaining a zero $(1,1)$ element of M^d) [27]. Such a fit allows a simple explanation of the right unitarity triangle via a phase sum rule.

(SM) one. The breaking of the SM gauge symmetry occurs via vev's of the two fiveplets $h, \bar{h}$ of (7) while the coloured triplets become heavy via the superpotential terms $HHh + \overline{H}\overline{H}\bar{h} \to \langle \nu^c_H \rangle D^c_H D_h + \langle \bar{\nu}^c_H \rangle \bar{D}^c_H \bar{D}_h$. In F-theory the breaking of the GUT may be due to the fluxes rather than fundamental Higgs fields.

Note that matter antifiveplets (4) are completely distinguished from the Higgs antifiveplets (7), since they carry different $U(1)_\chi$ charges and they do not contain exactly the same components. As a result $U(1)_\chi$ or a discrete factor of it can be used to forbid the R-parity violating terms. This will be crucial in the F-theory version of the model that we turn to now.

2.1. *Flipped $SU(5)$ in F-theory*

Our starting point is the sequence

$$E_8 \supset E_5\{= SO(10)\} \times SU(4) \to [SU(5) \times U(1)_\chi] \times SU(4) \\ \to [SU(5) \times U(1)_\chi] \times U(1)^3 \tag{8}$$

The adjoint representation of E_8 then has the $SO(10) \times SU(4)$, $SU(5) \times U(1)_\chi \times SU(4)$ decomposition given by

$$\begin{aligned} 248 &\to (45,1) + (16,4) + (\overline{16},\bar{4}) + (10,6) + (1,15) \\ &\to (24,1)_0 + (1,15)_0 + (1,1)_0 + (1,4)_{-5} + (1,\bar{4})_5 + (10,4)_{-1} + (10,1)_4 \\ &\quad + (10,\bar{4})_1 + (10,1)_{-4} + (\bar{5},4)_3 + (\bar{5},6)_{-2} + (5,\bar{4})_{-3} + (5,6)_2 \end{aligned} \tag{9}$$

respectively. We further assume that appropriate fluxes exist to induce the required chirality for the matter fields. At the $SO(10)$ level in particular, this means that $\#16\text{'s} - \#\overline{16}\text{'s} = 3$.

To accommodate the $U(1)_\chi$ we see that the monodromies must lie in the $U(1)^3 \subset SU(4)$. There are three possible choices for the monodromy group, namely S_3, $\mathcal{Z}_2 \times \mathcal{Z}_2$ and $\mathcal{Z}_2$. The first two cases reduce the number of the available matter curves to two. The $\mathcal{Z}_2$ case gives three matter curves and only it has the possibility of distinct localization of the three families. Although the first two cases are not a priori excluded, in this paper we will consider in detail only the $\mathcal{Z}_2$ monodromy.

We label the weights of the $SU(4)$ factor in Eq. (9) by t_i, $i = 1, \ldots, 4$, with $\sum_{i=1}^4 t_i = 0$. The $\mathcal{Z}_2$ monodromy acts on $\{t_1, t_2\}$. The $SU(5)$ matter representations $F_{1,2,3} \in 10$ belong to $(10,4)_{-1}$. There are three matter curves and we assign one family to each:

$$10^{(3)}_{-1} : \{t_1, t_2\},\ 10^{(2)}_{-1} : \{t_3\},\ 10^{(1)}_{-1} : \{t_4\} \tag{10}$$

The fiveplets, h, $\bar{h}$, $\bar{f}_i$, accommodating the Higgs and matter fields must lie on a subset of the following curves: The Higgs fiveplet responsible for up quark masses is in $\bar{h} \in (\bar{5},\bar{6})_{-2}$ so there are four possible Higgs curves

$$\begin{aligned} \bar{h} \in\ & \bar{5}^{(h_1)}_{-2} : \{-t_1 - t_2\},\ \bar{5}^{(h_2)}_{-2} : \{-t_3 - t_4\},\ \bar{5}^{(h_3)}_{-2} : \{-t_1 - t_3, -t_2 - t_3\}, \\ & \bar{5}^{(h_4)}_{-2} : \{-t_1 - t_4, -t_2 - t_4\} \end{aligned} \tag{11}$$

The down quark Higgs is in $h \in (5,6)_2$ and lies on one of the curves[3]

[3] Since $\sum_{i=1}^4 t_i = 0$, we could also label h-curves as $5^{(h_1)}_2 : \{t_3 + t_4\}$, $5^{(h_2)}_2 : \{t_1 + t_2\}$ and so on.

Table 1
Field representation content under $SU(5) \times U(1)_\chi \times SU(4)_\perp$.

Field	Representation	$SU(4)$ component
Q_3, D_3^c, ν_3^c	10_{-1}^3	$\{t_1, t_2\}$
Q_2, D_2^c, ν_2^c	10_{-1}^2	t_3
Q_1, D_1^c, ν_1^c	10_{-1}^1	t_4
U_3^c, L_3	$\bar{5}_3^3$	$\{t_1, t_2\}$
U_2^c, L_2	$\bar{5}_3^1$	t_4
U_1^c, L_1	$\bar{5}_3^1$	t_4
l_3^c	$1_{-5}^{c_3}$	$\{t_1, t_2\}$
l_2^c	$1_{-5}^{c_2}$	t_3
l_1^c	$1_{-5}^{c_1}$	t_4
h_u	$\bar{5}_{-2}^{h_1}$	$-t_1 - t_2$
h_d	$5_2^{h_1}$	$-t_1 - t_2$
θ_{ij}	1_0^{ij}	$t_i - t_j$
10_H^3	10_{-1}^3	$\{t_1, t_2\}$
$\overline{10}_H^3$	$\overline{10}_1^3$	$-\{t_1, t_2\}$

$$h \in 5_2^{(h_1)} : \{-t_1 - t_2\}, 5_2^{(h_2)} : \{-t_3 - t_4\}, 5_2^{(h_3)} : \{-t_1 - t_3, -t_2 - t_3\},$$
$$5_2^{(h_4)} : \{-t_1 - t_4, -t_2 - t_4\} \tag{12}$$

The fiveplets accommodating the matter fields belong to $(\bar{5}, 4)_3$ so there are three possibilities

$$\bar{f}_i \in \bar{5}_3^{(3)} : \{t_1, t_2\}, \bar{5}_3^{(2)} : \{t_3\}, \bar{5}_3^{(1)} : \{t_4\} \tag{13}$$

Charged singlet fields accommodating the right handed electrons belong to $(1, 4)_{-5}$ curves

$$\ell_i^c \in 1_{-5}^{c(3)} : \{t_1, t_2\}, 1_{-5}^{c(2)} : \{t_3\}, 1_{-5}^{c(1)} : \{t_4\} \tag{14}$$

The neutral singlets descending from the decomposition of $(1, 15)$ lie on the curves $t_i - t_j$ and designated as θ_{ij} and

$$\theta_{ij} = 1_0^{(ij)} : \{t_i - t_j\}, \quad i \neq j,\ i, j = 1, 2, 3, 4 \tag{15}$$

2.2. *Fermion masses*

2.2.1. *Rank-1 structure for the quarks and charged leptons*

As discussed above the $U(1)_\chi$ plays the role of an R-symmetry. As we shall see the Abelian symmetries in the $SU(4)$ factor play the role of family symmetries. We want to have rank one mass matrices in the absence of family symmetry breaking so it immediately follows for the down quarks that the down quark Higgs should lie in $5^{(h_1)}$ giving mass to the third generation through the superpotential coupling $W_{down} = 10^{(3)} \cdot 10^{(3)} \cdot 5^{(h_1)}$.

Similarly for the up quarks, assigning $\bar{f}_3$ to $\bar{5}^{(3)}$ we must choose the up quark Higgs to lie on $\bar{5}^{(h_1)}$ and the third generation up quark gets mass from the coupling $10^{(3)} \cdot \bar{5}^{(3)} \cdot \bar{5}^{(h_1)}$. Turning to the charged lepton mass matrix we must assign the RH τ-lepton to the $1^{c(3)}$ matter field and it gets mass from the coupling $1^{(c_3)} \cdot \bar{5}^{(3)} \cdot 5^{(h_1)}$. The assignment of the fields is summarised in Table 1.

Note that the rank one structure of these mass matrices follows from the $U(1)$ symmetries and does not require a single intersection of the matter curves with the Higgs curve.

2.2.2. The light quark masses

In order to generate masses for the first two generations of quarks and charged leptons it is necessary to break the family symmetries. This will happen if some of the singlet (familon) fields θ_{ij} develop non-vanishing vevs. In fact, as discussed in Appendix A, two fields, θ_{13} and θ_{14}, do acquire vevs due to the Fayet–Iliopoulos (FI) terms [30] associated with the family $U(1)$. Allowing for these vevs the down quark mass matrix, which is symmetric as it comes from the $10 \cdot 10 \cdot 5$ coupling, has the form ($\mathcal{O}(1)$ couplings are suppressed)

$$M^d = \begin{pmatrix} \theta_{14}^2 & \theta_{13}\theta_{14} & \theta_{14} \\ \theta_{13}\theta_{14} & \theta_{13}^2 & \theta_{13} \\ \theta_{14} & \theta_{13} & 1 \end{pmatrix} m_{b_0} \tag{16}$$

Here vevs are understood for the familon fields and we have suppressed the messenger mass scale, M, associated with the higher dimension operators, *i.e.* $\theta_{13} \equiv \langle\theta_{13}\rangle/M$, etc. Comparing this with Eq. (1) one sees that the down quark eigenvalues are reproduced with the choice $\theta_{13} = \epsilon$ and $\theta_{14} = \epsilon^2$.

At this stage we cannot yet determine the CKM matrix as it involves the up quark mass matrix. The form of the latter requires assignment of the two light generations of $SU(2)$ singlet up quarks to matter curves. If, as for the $SU(2)$ doublet assignment, we assign them to different matter curves they have the same weight structure as the doublets and the form of the up quark mass matrix is the same as for the down quarks. Unless there are unnatural cancelations involving the $\mathcal{O}(1)$ couplings this means the up quark eigenvalues hierarchy will be similar to that of the down quarks and hence unacceptable. To avoid this we assign both light generations of $SU(2)$ singlet up quarks to the *same* matter curve $\bar{5}^{(1)}$. Then we have

$$M^u = \begin{pmatrix} \lambda_1\theta_{14}^2 & \theta_{14}^2 & \theta_{14} \\ \lambda_2\theta_{13}\theta_{14} & \theta_{13}\theta_{14} & \theta_{13} \\ \lambda_3\theta_{14} & \theta_{14} & 1 \end{pmatrix} m_{t_0} \tag{17}$$

In this matrix we have explicitly included the factors λ_i that determine the ratios of the $(i,1)$ to $(i,2)$ elements because they play an important role in generating an acceptable up quark mass matrix. Since we have assigned two families to a single matter curve, if there is only a single intersection of the matter and Higgs curves generating each of the entries in the first two columns of the mass matrix, then the λ_is are equal and, by a rotation acting on the first two families of $SU(2)$ singlet up quarks, we can make $\lambda_i = 0$. However, as discussed above, we expect multiple intersections and in this case the λ_is need not be equal and the rotation can only change them by a common constant λ. Thus the mass matrix can have rank three. However, for a large number of intersections or if the intersections are very close together, we expect $(\lambda_i - \lambda) \ll \lambda$ and so in the rotated basis we arrive at the form of Eq. (17) but with small λ_is.

With this preamble we can now ask whether the form of Eq. (17) gives an acceptable mass matrix. The eigenvalues are in the ratio $1 : \theta_{13}\theta_{14} : \lambda_i\theta_{14}^2 = 1 : \epsilon^3 : \lambda_i\epsilon^4$. Comparing this with Eq. (2) we see an acceptable pattern of mass eigenvalues is possible if $\lambda_i = \mathcal{O}(\epsilon^2)$.

2.2.3. The CKM matrix

Finally what about the CKM matrix? Clearly the up and down quark mass matrices are not of the form given in Eqs. (1) and (2). However a simultaneous rotation of the up and down quark mass matrices (which leaves the CKM matrix unchanged) can make the $(1,3)$ and $(2,3)$ elements of M^u and M^d vanish provided the $\mathcal{O}(1)$ coefficients of these elements in the up and down sectors are equal. The latter is expected to be the case if the symmetry at the intersection point of the quark and Higgs curves is enhanced to $SO(10)$ as is possible since the weight structure of the matter curves in the up and the down sector involved in the $(i,3)$ Yukawa couplings are the same. In this case the CKM elements V_{13} and V_{23} (approximately) vanish. However we know $SO(10)$ must be broken by fluxes so the equality of the $(1,3)$ and $(2,3)$ elements of M^u and M^d can only be approximate. Taking this into account and performing a common rotation of the up and down quark mass matrices we obtain the form

$$M^d = \begin{pmatrix} \theta_{14}^2 & \theta_{13}\theta_{14} & \delta_1\theta_{14} \\ \theta_{13}\theta_{14} & \theta_{13}^2 & \delta_2\theta_{13} \\ \delta_1\theta_{14} & \delta_2\theta_{13} & 1 \end{pmatrix} m_{b_0} \tag{18}$$

$$M^u = \begin{pmatrix} \lambda_1\theta_{14}^2 & \theta_{14}^2 & 0 \\ \lambda_2\theta_{13}\theta_{14} & \theta_{13}\theta_{14} & 0 \\ \lambda_3\theta_{14} & \theta_{14} & 1 \end{pmatrix} m_{t_0} \tag{19}$$

where $\delta_1 \approx \delta_2$ takes account of the flux breaking effects. Choosing $\delta_1 \approx \delta_2 = \mathcal{O}(\epsilon)$ we obtain the same form as is given in Eqs. (1) and (2) and hence an acceptable CKM matrix.

2.3. The lepton sector

In flipped $SU(5)$ leptons and down quarks receive masses from couplings not related by $SU(5)$. Geometrically, RH electrons and down quarks reside on different matter curves. Thus, in contrast to $SU(5)$, in flipped $SU(5)$ there is no GUT relation between the Yukawa couplings of the down quarks and the leptons. However, if we distribute lepton doublets to distinct curves as we did for the down quarks, the structure of M^d and M^ℓ will be the same. In this case the situation is similar to that in normal $SU(5)$ and one expects the magnitude of the coefficients to be similar if the geometrical structure of the relevant intersections giving rise to the Yukawa couplings in the down quark and charged lepton sectors are the same. Since the situation is the same as for ordinary $SU(5)$ we postpone a discussion of how this can lead to an acceptable charged lepton mass matrix to Section 3.2.

Turning to neutrino masses, note that the Dirac neutrino mass matrix originates from the coupling $10 \cdot \bar{5} \cdot \bar{5}$ and therefore is related to the up quarks. Since the latter is related to the CKM mixing and has small mixing angles, the large neutrino angles must be attributed to the see-saw mechanism [31] and the specific form of the RH Majorana mass matrix. Doing this is a non-trivial task but may be possible [32]. Starting from a near diagonal Dirac neutrino mass matrix $M^\nu_{Dirac} \approx \mathrm{diag}(m_u, m_c, m_t)$ the condition on the heavy RH Majorana mass matrix M_R in order to yield bi-large neutrino mixing is obtained from the following generalization of the string instanton results in [33] to the case of right-handed neutrinos and arbitrary lepton mixing:

$$M_R = \frac{AA^T}{m_1} + \frac{BB^T}{m_2} + \frac{CC^T}{m_3} \tag{20}$$

where $A = M^\nu_{Dirac}\Phi_1$, $B = M^\nu_{Dirac}\Phi_2$, $C = M^\nu_{Dirac}\Phi_3$, with Φ_i being the three columns of the lepton mixing matrix $U = (\Phi_1\ \Phi_2\ \Phi_3)$, while m_i are the physical neutrino masses.

We now turn to the question whether it is possible to achieve such right-handed neutrino masses in flipped $SU(5)$. For this purpose we introduce $\overline{10}^3_H$ additional heavy fields, part of additional vectorlike pairs, $\overline{10}^3_H$, 10^3_H living on the matter curves. The relevant superpotential couplings needed to obtaining right-handed neutrino masses are given by (suppressing dimensionless order one coefficients),

$$\overline{10}^3_H\big(10^3+\theta_{13}10^2+\theta_{14}10^1\big)S_{1,2,3} \tag{21}$$

where $S_{1,2,3}$ are singlet fields, part of the massive string sector with masses M_S. After integrating out these fields we find effective operators of the form,

$$M_R \sim \begin{pmatrix} \theta_{14}^2 & \theta_{14}\theta_{13} & \theta_{14} \\ \theta_{14}\theta_{13} & \theta_{13}^2 & \theta_{13} \\ \theta_{14} & \theta_{13} & 1 \end{pmatrix} \langle \overline{10}^3_H \overline{10}^3_H \rangle \tag{22}$$

where we have suppressed not only the dimensionless order one coefficients but also all the dimensional mass scales of order M_S in the denominators which if reinserted would lead to a rank 3 right-handed neutrino mass matrix after the $\overline{10}^3_H$ acquires a vacuum expectation value $\langle \overline{10}^3_H \rangle = \langle \bar{\nu}^c_H \rangle$. Its magnitude fixes the magnitude of the right-handed neutrino masses, the heaviest of which should have an approximate mass $\langle \overline{10}^3_H \overline{10}^3_H \rangle / M_S \sim \mathcal{O}(10^{14\text{–}15})$ GeV in order to get light neutrino masses in the observed range, and this is readily achieved.

Comparing Eq. (22) here to the desired form (20) we see that each of the column vectors A, B, C has the general form $(\theta_{14}\ \theta_{13}\ 1)^T \sim (\epsilon^2\ \epsilon\ 1)^T$ to be compared to the desired general form $(m_u\ m_c\ m_t)^T \sim (\epsilon^6\ \epsilon^3\ 1)^T$. This demonstrates the underlying difficulty in obtaining bi-large mixing in flipped $SU(5)$. It is insensitive to the precise details of the see-saw, following simply from the observation that the field combinations 10^3, $\theta_{13}10^2$ and $\theta_{14}10^1$ have the same $U(1)^3_\perp$ charges and thus are always generated with the same coefficients. The only way we can see to get bi-large mixing without fine tuning combinations of $\mathcal{O}(1)$ coefficients is to have strong $SU(5)$ breaking so that the messenger mass, M_{ν^c}, in the ν^c sector is much greater than the messenger mass M in the quark and charged lepton sector. Then terms proportional to θ_{13}/M_{ν^c} can be of order ϵ^3 as required for bi-large mixing provided $M/M_{\nu^c} = \epsilon^2$. Terms involving θ_{14} require a further suppression and this will be the case if we replace θ_{14}/M in the quark and charged lepton sector by $\theta_{13}\theta_{34}/M^2$ where $\theta_{34}/M = \epsilon$. Then the term $\theta_{13}\theta_{34}/M^2_{\nu^c} = \epsilon^6$ as required for bi-large mixing (up to the $\mathcal{O}(1)$ coefficients). While this may be a possible solution to get a viable neutrino mixing pattern it is certainly not very convincing. The price one pays for a viable mass matrix is a complicated choice of vevs and messenger masses; essentially one exchanges the parameters in the neutrino mass matrix for another set of parameters, the vevs, and the problem of understanding the neutrino mass matrix structure is replaced by the problem of determining the vacuum structure of the multi-field familon potential. As we shall discuss the situation is better in the normal $SU(5)$ case where the Dirac neutrino mass matrix is not related to the quark mass matrices.

2.4. Nucleon decay

A big advantage of flipped $SU(5)$ is that the $U(1)_\chi$ factor eliminates the unacceptable dimension 4 baryon- and lepton-number violating operators of the form $10^i_M \bar{5}^j_M \bar{5}^k_M$. The symmetry does however allow baryon and lepton number operators of dimension five that mediate nucleon decay. They have the form $10^i_M 10^j_M 10^k_M \bar{5}^l_M$ and their family structure is given by

$$\mathcal{W}_5 \supset 10^3 10^3 10^2 \bar{5}^1 + 10^3 10^3 10^1 \bar{5}^2 + 10^3 10^2 10^1 \bar{5}^3$$

Table 2
Field representation content under $SU(5) \times SU(5)_\perp$.

Field	Representation	$SU(5)_\perp$ component	R-parity
Q_3, U_3^c, l_3^c	$(10, 5)$	$t_{1,2}$	$-$
Q_2, U_2^c, l_2^c	$(10, 5)$	t_3	$-$
Q_1, U_1^c, l_1^c	$(10, 5)$	t_4	$-$
D_3^c, L_3	$(\bar{5}, 10)$	$t_3 + t_5$	$-$
D_2^c, L_2	$(\bar{5}, 10)$	$t_1 + t_3$	$-$
D_1^c, L_1	$(\bar{5}, 10)$	$t_1 + t_4$	$-$
H_u	$(5, \overline{10})$	$-t_1 - t_2$	$+$
H_d	$(\bar{5}, 10)$	$t_1 + t_4$	$+$
θ_{ij}	$(1, 24)$	$t_i - t_j$	$+$
θ'_{ij}	$(1, 24)$	$t_i - t_j$	$-$
S'	$(1, 1)$	$-$	$-$

Note that since we have not assigned matter to the $\bar{5}^2$ curve the second operator is absent. The remaining operators are generated via heavy triplet mediated graphs and are expected to be suppressed by the string scale. By itself this is not sufficient suppression but note that each of the allowed operators involves two matter fields belonging to the third family of current quarks. This means that the proton decay operators involving light quarks are further suppressed by small mixing angles and this can provide the additional suppression needed to bring nucleon decay within experimental limits.

3. An $SU(5)$ model

As pointed out by Hayashi et al. [20] it is possible that the F-theory has an R-symmetry that descends from a symmetry of the underlying Calabi–Yau manifold and the flux. In this case it was shown that there may be both R-parity odd and even zero modes on a given curve. Assigning the quarks and leptons to odd R-parity states and the Higgs to even R-parity states, the leading baryon and lepton number violating interactions are forbidden even though the $U(1)$s may allow them. This opens up the possibilities for constructing realistic models based on $SU(5)$ so one must reconsider the models first analyzed by Dudas and Palti [24]. Here we present a model that can closely duplicate the phenomenologically viable mass matrices of Eqs. (1) and (2).

3.1. Quark masses

The starting point is the $SU(5) \times SU(5)_\perp$ group. The weights of $SU(5)_\perp$ are labeled by t_i, $i = 1, \ldots, 5$. We will analyse the model with monodromy group Z_2 relating $t_1 \leftrightarrow t_2$. We assign the quarks and Higgs fields to the curves as shown in Table 2. In addition there are familon fields θ_{ij} belonging to the $(1, 24)$ representation. With these assignments the up quark matrix mass matrix has the form:

$$M^u/m_t = \begin{pmatrix} \theta_{14}^2 & \theta_{13}\theta_{14} & \theta_{14} \\ \theta_{13}\theta_{14} & \theta_{13}^2 & \theta_{13} \\ \theta_{14} & \theta_{13} & 1 \end{pmatrix} \tag{23}$$

where we have written $\theta_{(1,2)j} = \theta_{1j}$ and, for the moment, we allow for all possible vevs of the familon fields.

The down quark mass matrix has the form:

$$M^d/m_b = \begin{pmatrix} \theta_{54}\theta_{34} & \theta_{54} & \theta_{14} \\ \theta_{54} & \theta_{53} & \theta_{13} \\ \theta_{31}\theta_{54} + \theta_{34}\theta_{51} & \theta_{51} & 1 \end{pmatrix} \tag{24}$$

For $\theta_{34} = 0$ there is a $(1,1)$ texture zero in the down quark mass matrix. The choice $\theta_{51} = 0$ gives further zeros in the $(3,1)$ and $(3,2)$ positions, consistent with the data since the elements below the diagonal are poorly determined. To determine the non-zero familon vevs consider the magnitudes of the quark masses. We assume that there are no (unnatural) cancelations involving the unknown $\mathcal{O}(1)$ coefficients in determining the eigenvalues. Then $m_c/m_t = \theta_{13}^2$, $m_u/m_t = \theta_{14}^2$, $m_s/m_b = \theta_{53}$ and $m_d/m_b = \theta_{54}^2/\theta_{53}$. The choice $\theta_{53} = \epsilon^2$, $\theta_{54} = \epsilon^3$, $\theta_{13} = 3\epsilon^2$, $\theta_{14} = \epsilon^3$ and $\theta_{31} = 0$ gives a good description of these mass ratios (up to $\mathcal{O}(1)$ coefficients) and has the mass matrices

$$M^u/m_t = \begin{pmatrix} \epsilon^6 & 3\epsilon^5 & \epsilon^3 \\ 3\epsilon^5 & 9\epsilon^4 & 3\epsilon^2 \\ \epsilon^3 & 3\epsilon^2 & 1 \end{pmatrix} M^d/m_b = \begin{pmatrix} 0 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & 3\epsilon^2 \\ 0 & 0 & 1 \end{pmatrix} \tag{25}$$

again up to $\mathcal{O}(1)$ coefficients.

Of course one must check that this choice is consistent with the familon potential and this is discussed in Appendix A. Since the theory has three anomalous $U(1)$s we expect at least three familon fields should acquire vevs. As discussed in Appendix A, because the soft SUSY breaking parameters are scale dependent, it may readily happen that additional familon fields acquire vevs. The important thing to check is that the theory is F-flat with this choice of vevs and this is demonstrated in Appendix A.

Turning to the mixing angles one may see that the contribution to V_{cb} from the up and the down matrices is of the same order and, as discussed above for the case of flipped $SU(5)$, allowing for some cancelation between them one may readily obtain the measured value. The same is true for V_{ub}. Finally consider the effect of the texture zero in the $(1,1)$ position of M^d. If the symmetry at the intersection points of the quark and Higgs curves that generate the Yukawa couplings in the $(1,2)$ block is enhanced to $SO(10)$ the $(1,2)$ couplings will be symmetric as they correspond to the $SO(10)$ coupling $16 \cdot 16 \cdot 10$. This with the texture zero gives a down quark contribution to $V_{us} = \sqrt{m_d/m_s}$. Including the contribution from the up quark sector gives $V_{us} = \sqrt{m_d/m_s} + \mathcal{O}(\sqrt{m_u/m_c})$, again in good agreement with the measured value. It is interesting to note that geometry could ensure a further texture zero in the $(1,1))$ of the up quark mass matrix so that one obtains the full Gatto–Sartori–Tonin relation [26]. This happens if there is no intersection of the up quark and Higgs curves corresponding to the Yukawa coupling in the $(1,1)$ position.

3.2. Charged lepton masses

There are hints at a stage of Grand Unification coming from the structure of the charged lepton masses. In particular, after including radiative corrections corresponding to threshold corrections and the running to low scales, they can be consistent with the mass relations $m_b = m_\tau$ and $\mathrm{Det}(M^d) = \mathrm{Det}(M^\ell)$ at the GUT scale [25,34]. In F-theory it is possible to explain the origin of such relations provided we assign the LH and charged conjugate RH charged leptons to the same $SU(5)$ representations as the charge conjugate RH down quarks and LH quark doublets respectively as given in Table 2. Then the structure of the charged lepton mass matrix will be the same as that of the down quarks, Eq. (26), although the $\mathcal{O}(1)$ coefficients may differ. However, provided the symmetry at the intersection points of the lepton, Higgs and familon curves that generate the Yukawa couplings in the $(1,2)$, $(2,1)$ and $(3,3)$ positions is enhanced to $SU(5)$, the $\mathcal{O}(1)$ coefficients in the down quark mass matrix will be the same as that for the charged leptons, giving the mass relations $m_b = m_\tau$ and $\mathrm{Det}(M^d) = \mathrm{Det}(M^l)$. Of course these relations will have corrections due to flux breaking but this may be small. However the big problem is to explain why there is no equivalent relation for the second generation, namely $m_\mu = m_s$. Taking account of the radiative corrections, the measured values of the masses are in gross disagreement with this relation and favour instead $m_\mu \approx 3m_s$.[4] In an $SU(5)$ GUT one may explain the factor of 3 by arranging through additional symmetries that the $(2,2)$ element involves a coupling to the vacuum expectation value of a 45-dimensional representation which is proportional to $B-L$ [35]. As required this gives a relative enhancement by a factor 3 for the muon compared to the strange quark. In the case of F-theory this option is not available as, *cf.* Eq. (9), the 45 representation of $SU(5)$ are not present. If the $SU(5)$ were enhanced to $SO(10)$ then the 45 representation of $SO(10)$ could in principle be used in a similar way but since, *cf.* Eq. (9), it is a family singlet it cannot selectively couple to the $(2,2)$ element. However in F-theory a geometrical explanation is possible because the intersection points of the lepton, Higgs and familon curves that generate the Yukawa couplings in the $(2,2)$ element need not be at an $SO(10)$ enhanced symmetry point relating the strange quark and muon couplings. In particular if there happens to be a single intersection for the strange quark and a triple intersection for the muon one expects there to be the required factor of 3 enhancement for the muon mass.

3.3. Neutrino masses

Finally we consider the neutrino masses. The R-parity allows operators quadratic in the matter fields and so we can construct operators that violate lepton number by 2 units provided they are invariant under the gauge symmetries. We note that the combinations $L_1 h^u \theta_{14}$ and $L_2 h^u \theta_{13}$ are invariant under the gauge symmetries and so any combination of two of these operators will be allowed. These give rise to a Majorana mass matrix for neutrinos given by

$$M^{\nu}_{Majorana} = \begin{pmatrix} 9\epsilon^4 & 3\epsilon^5 & 0 \\ 3\epsilon^5 & \epsilon^6 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{(h^u)^2}{M} \tag{26}$$

For the messenger scale M at the string scale $M \gg 10^{10}$ GeV and these masses are negligible. This means there should be light messengers and the obvious possibility is that there are light right-handed neutrinos. The R-parity odd $SU(5)$ singlet fields θ'_{ij} and S' are candidate right-handed neutrinos.

A choice that can accommodate the observed neutrino masses starts with the odd R-parity zero modes θ'_{15} and S'. Through the superpotential coupling $\lambda S S'^2$ the field S' acquires a Majorana mass, $M'_S = \lambda S$, if the R-parity even field S acquires a vev. As shown in Appendix A F-flatness requires that θ_{51} also acquires a vev of $\mathcal{O}(S\frac{\theta_{53}}{\theta_{13}})$ and this in turn generates a Majorana mass, M_{15} for θ'_{15}, $M_{15} = \mathcal{O}(\lambda'^2 \theta_{51}^2 / M_S)$ through the coupling $\lambda' S' \theta'_{15} \theta_{51}$, assuming a hierarchy

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4 But see [34] for more general possibilities.

$M_{15} \ll M_{S'}$. With such a hierarchy the right-handed neutrinos θ'_{15} and S' have suppressed mixing and we may apply the conditions of sequential dominance [36,37] to achieve a neutrino mass hierarchy with large atmospheric and solar mixing as discussed below.

Now the coupling of the LH-neutrino states to θ'_{15} and S will generate Majorana masses for two combinations of the LH neutrino states. The dominant term generating the heaviest (atmospheric) neutrino mass involves the lightest RH neutrino state, θ'_{15}. Its coupling to the light neutrinos is through the term (suppressing the $\mathcal{O}(1)$ coefficients) $(L_3\theta_{13} + L_2\theta_{53} + L_1\theta_{54})\theta'_{15}h^u$ and, through the see-saw mechanism generates the neutrino mass term

$$(L_3\theta_{13} + L_2\theta_{53} + L_1\theta_{54})^2\langle h^u\rangle^2/M_{15} \tag{27}$$

In the fit to the quark masses quoted above we had $\theta_{13} = 3\epsilon^2$, $\theta_{53} = \epsilon^2$, $\theta_{14} = \epsilon^3$ and $\theta_{54} = \epsilon^3$. This does not give the observed atmospheric neutrino mixing angles unless the $O(1)$ coefficients play a role. As a simple example of this we suppose that the coefficient of the (2, 3) entry of M^u has a relative factor of 3 in its coupling (as mentioned above this could readily happen if there are three intersections generating the coupling). Then the fit to M^u gives $\theta_{13} = \epsilon^2$, $\theta_{53} = \epsilon^2$, $\theta_{54} = \epsilon^3$ and $\theta_{14} = 3\epsilon^3$. In this case, up to $\mathcal{O}(1)$ coefficients, we have the atmospheric neutrino mass term given by

$$m_{@}(\nu_\tau + \nu_\mu + \epsilon\nu_e) \tag{28}$$

where $m_{@} = \epsilon^4\langle H^u\rangle^2/M_{15}$. To $\mathcal{O}(\epsilon)$ one obtains near-maximal atmospheric mixing in agreement with the observed value.

A second Majorana mass is generated through the see-saw mechanism via the coupling $(L_2\theta_{13} + L_1\theta_{14})S'h^u \approx ((-\nu_\tau + \nu_\mu)\theta_{13}/2 + \nu_e\theta_{14})S'h^u$ where we have kept only the components left light by the dominant first Majorana mass term. This gives the second neutrino mass term

$$m_\odot(-\nu_\tau + \nu_\mu + 6\epsilon\nu_e)^2 \tag{29}$$

where $m_\odot = \epsilon^4\langle h^u\rangle^2/(4m_S)$. Since $6\epsilon \approx 0.9$ this gives large solar mixing. The absolute value of the neutrino masses requires that $S = \mathcal{O}(\epsilon^9)$ corresponding to Majorana masses for the RH-neutrinos of $\mathcal{O}(10^{10}$ GeV). The ratio of the solar to atmospheric masses is of $\mathcal{O}(1/4)$ up to the $\mathcal{O}(1)$ factors. Our analysis assumes $M_{15} < M'_S$ and this can be justified with a reasonable choice of the $\mathcal{O}(1)$ factors since several of these factors are involved. The estimates above of the bi-large mixing pattern are only valid up to $\mathcal{O}(\epsilon)$ corrections and further (small) corrections from the charged lepton sector.

A final comment is in order. The assumption that there are light singlet fields S and S' can be questioned as they do not couple to fluxes and so fluxes cannot ensure their chirality. An alternative is to replace S and S' by θ_{31} and θ'_{31}. Then with $\theta_{31} = \epsilon^7$ one generates a singlet vev for $\theta_{31}\theta_{13}$ of the required order. Similarly we can replace S' by $\theta_{13}\theta'_{31}$. One may readily check that the structure of the light neutrino masses and mixing remains the same.

4. Doublet–triplet splitting, the μ term and FCNC

So far, we have discussed how the above GUT models are capable of reproducing the fermion mass hierarchy and the CKM mixing. However it is also necessary to inhibit nucleon decay by making the colour triplets of the fiveplet Higgs fields h, $\bar{h}$ heavy. In the Flipped $SU(5)$ model we have already argued that in the presence of Higgs tenplets $H, \overline{H}$, there is a doublet–triplet splitting mechanism and triplets acquire a mass due to the missing partner mechanism. In the normal $SU(5)$ case this solution is not possible. It has been suggested that the splitting can be achieved by putting the up and down Higgs on different matter curves. As a result there is no direct mass term inducing a dimension-five proton decay operator, whilst heavy mass terms for the triplets are generated when combined with the heavy KK-modes [4]. However it was shown in [24] that this solution is not available in the case that the matter fields reside on different matter curves. Given this we must assume that the geometry accommodates Wilson line breaking in which case it is possible to project out the light triplet states.

It is also necessary to have a mechanism to generate the μ-term. For the case that the up and down Higgs curves intersect each other, a μ term can be naturally generated through their interaction with a chiral superfield localised on a curve normal to the GUT surface [4].

Finally we consider the bounds on family symmetries imposed by requiring consistency with the measurements sensitive to flavour changing neutral currents (FCNC). In supersymmetric models the limits on FCNC give rise to stringent bounds on dimension 2 and 3 soft supersymmetry breaking terms [38]. The latter are very dependent on the precise origin of supersymmetry breaking and can be suppressed in specific schemes so we concentrate here on the former. Of these the strongest bound in the squark sector is on the left-handed $\Delta_{ds}^{LL}\phi_{dL}^{\dagger}\phi_{sL}$ and right-handed $\Delta_{ds}^{RR}\phi_{dR}^{\dagger}\phi_{sR}$ soft mass terms mixing the first two generations.[5] For gaugino and squarks of comparable order and allowing for the running between the mediator scale and the SUSY breaking scale [40] the most stringent experimental bounds are $\Delta_{ds}^{LL}/\tilde{m}^2 < \mathcal{O}(\epsilon)$ and $\sqrt{\Delta_{ds}^{LL}\Delta_{ds}^{RR}}/\tilde{m}^2 < \mathcal{O}(\epsilon^3)$, where $\tilde{m}^2$ is the mean squark mass squared taken here to be $(350\ \text{GeV})^2$. Both the models discussed here $\phi_{dL,R}^{\dagger}\phi_{sL,R}$ have weight structure $t_4 - t_3$ and the associated mass terms will arise at $\mathcal{O}(\theta_{31}\theta_{14}^{\dagger})$. In the flipped $SU(5)$ case these terms are of $\mathcal{O}(\epsilon^3)$ while in the normal $SU(5)$ case it is of $\mathcal{O}(3\epsilon^5)$, both consistent with the bounds. In gauge family symmetry models there is a second source of these terms coming from the D-terms of the family symmetry. On rotating to the down quark mass eigenstate basis these induce the off-diagonal d–s mixing terms. The D-terms are proportional to the familon soft mass squared masses [41,40] and if these are of the same order as the mean squark mass the contribution is of $\mathcal{O}(\epsilon)$, violating the bounds. Allowing for mean squark masses to be of $\mathcal{O}(1)$ TeV only reduces the experimental bound by a factor ϵ so it is necessary that the familon soft masses should be somewhat smaller than the squark masses, a factor of ϵ being consistent with a $(350\ \text{GeV})^2$ mean squark mass. This may readily happen if the SUSY breaking messenger fields are more weakly coupled to the familons than the squarks.

These estimates readily extend to the slepton sector. In this case the predicted value of the μ–e mixing terms at the messenger mass scale is reduced by approximately $1/3$ because $m_d/m_\mu \approx 1/3$ at that scale giving a reduction in the mixing angle needed to diagonalise the lepton mass matrix. The experimental bounds on $\Delta_{e\mu}$ and Δ_{ds} are comparable and so the overall bound on the familon soft mass coming from the slepton sector is somewhat weaker than that coming from the squark sector.

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[5] For an updated summary of results and extensive references see [39].

5. Summary and conclusions

In this work we have presented two examples of viable fermion mass textures of quarks charged leptons and neutrinos in the context of local F-theory GUTs. In these models the fermion mass hierarchy is ensured by family symmetries and spontaneous breaking of these symmetries can give viable masses and mixings even in the absence of flux corrections.

The first example is based on the Flipped $SU(5) \times U(1)_\chi$ gauge symmetry in which the fermion generations carry charges under the two Abelian factors of the enhanced (family) gauge symmetry $U(1)_\perp^2$, left after imposing a $\mathcal{Z}_2$ monodromy relating two Abelian factors of $SU(4)_\perp$. A fermion mass pattern consistent with the low energy data arises when matter assigned in 10's resides on different matter curves and matter transforming under $\bar{5}$ is accommodated only in two matter curves. Furthermore, it is shown that $U(1)_\chi$ acts as a generalised matter parity, preventing all dangerous R parity breaking (dimension-four) operators. While it may be possible to accommodate a viable pattern of neutrino masses and mixings it must be admitted the resulting structure looks very contrived.

The second example is based on the $SU(5)$ GUT gauge symmetry with matter transforming under the family symmetry $U(1)_\perp^3 \subset SU(5)_\perp$, while again a Z_2 monodromy is imposed among two $U(1)_\perp$ factors of $SU(5)_\perp$. Invoking an R-parity that can arise in certain Calabi–Yau compactifications with appropriate fluxes, we construct an R-parity conserving model capable of generating the observed quark and lepton masses and mixing angles. In contrast to the previous example, each fermion family is localised on a different matter curve. Giving vevs to only a few familon fields we break the $U(1)_\perp$ family symmetries and generate charged fermion mass matrices with the required hierarchy of masses and mixing angles. In addition, using parity-odd singlet fields for right-handed neutrinos, and mildly extending the singlet (familon) field content that acquire vevs along F- and D-flat directions, we demonstrate how to construct an effective light neutrino Majorana mass matrix with bi-large mixing and mass squared differences in the experimentally required region.

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Appendix A

A.1. The familon potential in flipped $SU(5)$

The superpotential terms involving the familon fields θ_{ij} is

$$\begin{aligned}\mathcal{W}_\theta &= \lambda_{ijk}\theta_{ij}\theta_{jk}\theta_{ki} \\ &= \lambda_1'\theta_{13}\theta_{34}\theta_{41} + \lambda_2'\theta_{31}\theta_{14}\theta_{43}\end{aligned} \tag{30}$$

If only θ_{ij} acquire vevs at a high scale, the flatness conditions read

$$\frac{\partial \mathcal{W}_\theta}{\partial \theta_{ij}} = \lambda_{ijk}\theta_{jk}\theta_{ki} = 0 \tag{31}$$

For our choice of non-zero vevs ($\langle\theta_{13}\rangle \neq 0$, $\langle\theta_{14}\rangle \neq 0$) conditions (31) are automatically satisfied.

To write down the corresponding D-flatness conditions, we must take into account the monodromies. For the $\mathcal{Z}_2$ monodromy, $t_1 \leftrightarrow t_2$ the D-flatness conditions can be written in compact form

$$\sum_{j=3,4} \left|\langle\theta_{nj}\rangle\right|^2 - \left|\langle\theta_{jn}\rangle\right|^2 + \xi_n = 0, \quad n = 3,4 \tag{32}$$

where ξ_n are – moduli dependent – FI terms. For the specific choice of vevs these read,

$$-\left|\langle\theta_{13}\rangle\right|^2 + \xi_3 = 0$$
$$-\left|\langle\theta_{14}\rangle\right|^2 + \xi_4 = 0$$

Note that these equations require two familon fields acquire vevs and these must be θ_{13} and θ_{14} if ξ_3 and ξ_4 are positive.

In the presence of large vevs for possible $H_i = 10_i$, $\overline{H}_i = \overline{10}_i$ Higgs fields, the D-flatness conditions are modified as follows

$$10\left(\left|\langle H_n\rangle\right|^2 - \left|\langle\overline{H}_n\rangle\right|^2\right) + \sum_{j=1,3,4} \left|\langle\theta_{nj}\rangle\right|^2 - \left|\langle\theta_{jn}\rangle\right|^2 + \xi_n = 0, \quad n = 3,4 \tag{33}$$

and an analogous solution can be worked out.

The familon potential in $SU(5)$

In this case there are twelve familon fields of the form θ_{ij} $i, j = 1,3,4,5$ and three $U(1)$s. This means we expect at least three vevs for the familon fields to be required by the D-flatness condition. To generate the quark and charged lepton masses we require vevs for four fields θ_{53}, θ_{54}, θ_{13} and θ_{14} and so we must check that it is possible for more than three familons to get vevs. From Eq. (31) we see that the choice of vevs is F-flat. The D-flatness conditions are

$$\begin{aligned}
&-\left|\langle\theta_{13}\rangle\right|^2 - \left|\langle\theta_{53}\rangle\right|^2 + \xi_3 = 0\\
&-\left|\langle\theta_{14}\rangle\right|^2 - \left|\langle\theta_{54}\rangle\right|^2 + \xi_4 = 0\\
&\left|\langle\theta_{53}\rangle\right|^2 + \left|\langle\theta_{54}\rangle\right|^2 + \xi_5 = 0
\end{aligned} \tag{34}$$

and clearly can be satisfied for ξ_5 negative and ξ_3, ξ_4 positive. However these equations have a flat direction corresponding to the fact that we require four familon vevs but there are only three D-terms. The familon potential also has soft SUSY breaking mass terms. If these are constant then only the three familon fields with the smallest (positive) mass squared will acquire vevs. However the mass squared terms are scale dependent due to the Yukawa couplings that increase the soft mass squared as the scale is increased. Thus the contribution to the potential of the soft mass squared terms has the form

$$\begin{aligned}
V(\theta_{ij}) = {}& m_{13}^2(\phi_{13})\left|\langle\theta_{13}\rangle\right|^2 + m_{14}^2(\phi_{14})\left|\langle\theta_{14}\rangle\right|^2 + m_{53}^2(\phi_{53})\left|\langle\theta_{53}\rangle\right|^2\\
&+ m_{54}^2(\phi_{54})\left|\langle\theta_{54}\rangle\right|^2
\end{aligned} \tag{35}$$

Minimising Eqs. (34) and (35) can readily require all four vevs to be non-zero.

The discussion has so far dealt with the vevs required to give the quarks and charged leptons a mass. However in order to generate a mass for the neutrinos further (much smaller) vevs were needed. Consider the case where the additional vevs are for the fields S and θ_{51}. In this case the F-term conditions may change due to the additional couplings of the form $S\theta_{ij}\theta_{ji}$. If only the fields acquiring vevs are light no additional F-terms appear. If however the field θ_{35} is also light we have non-trivial term given by

$$\left|\langle F_{35}\rangle\right|^2 = \left|\langle\theta_{13}\theta_{51} + S\theta_{53}\rangle\right|^2$$

This requires $S = O(\theta_{51}\frac{\theta_{13}}{\theta_{53}})$. The D-term conditions can be satisfied with only very small changes in the dominant vevs because they are quadratic in the fields. This changes the F-terms (linear in the fields) by small corrections and they can be compensated by small corrections to the S and θ_{51}. Repeating the procedure one obtains a rapidly convergent perturbative solution to the D- and F-flatness conditions. No additional non-trivial F-terms are generated in the case that S is replaced by the field θ_{31} that acquires a vev.

References

[1] R. Donagi, M. Wijnholt, Model building with F-theory, arXiv:0802.2969 [hep-th].

[2] C. Beasley, J.J. Heckman, C. Vafa, GUTs and exceptional branes in F-theory. I, JHEP 0901 (2009) 058, arXiv:0802.3391 [hep-th].

[3] R. Donagi, M. Wijnholt, Breaking GUT groups in F-theory, arXiv:0808.2223 [hep-th].

[4] C. Beasley, J.J. Heckman, C. Vafa, GUTs and exceptional branes in F-theory. II: Experimental predictions, JHEP 0901 (2009) 059, arXiv:0806.0102 [hep-th].

[5] R. Blumenhagen, T.W. Grimm, B. Jurke, T. Weigand, Global F-theory GUTs, Nucl. Phys. B 829 (2010) 325, arXiv:0908.1784 [hep-th].

[6] R. Blumenhagen, T.W. Grimm, B. Jurke, T. Weigand, F-theory uplifts and GUTs, JHEP 0909 (2009) 053, arXiv:0906.0013 [hep-th].

[7] J.J. Heckman, C. Vafa, Flavor hierarchy from F-theory, arXiv:0811.2417 [hep-th].

[8] J.L. Bourjaily, Local models in F-theory and M-theory with three generations, arXiv:0901.3785 [hep-th].

[9] J. Marsano, N. Saulina, S. Schafer-Nameki, Monodromies, fluxes, and compact three-generation F-theory GUTs, JHEP 0908 (2009) 046, arXiv:0906.4672 [hep-th].

[10] A. Font, L.E. Ibanez, Yukawa structure from U(1) fluxes in F-theory grand unification, JHEP 0902 (2009) 016, arXiv:0811.2157 [hep-th].

[11] R. Tatar, Y. Tsuchiya, T. Watari, Right-handed neutrinos in F-theory compactifications, Nucl. Phys. B 823 (2009) 1, arXiv:0905.2289 [hep-th].

[12] J.J. Heckman, A. Tavanfar, C. Vafa, The point of E_8 in F-theory GUTs, arXiv:0906.0581 [hep-th].

[13] H. Hayashi, T. Kawano, R. Tatar, T. Watari, Codimension-3 singularities and Yukawa couplings in F-theory, Nucl. Phys. B 823 (2009) 47, arXiv:0901.4941 [hep-th].

[14] L. Randall, D. Simmons-Duffin, Quark and lepton flavor physics from F-theory, arXiv:0904.1584 [hep-ph].

[15] V. Bouchard, J.J. Heckman, J. Seo, C. Vafa, F-theory and neutrinos: Kaluza–Klein dilution of flavor hierarchy, JHEP 1001 (2010) 061, arXiv:0904.1419 [hep-ph].

[16] J. Jiang, T. Li, D.V. Nanopoulos, D. Xie, Flipped $SU(5) \times U(1)_X$ models from F-theory, Nucl. Phys. B 830 (2010) 195, arXiv:0905.3394 [hep-th].

[17] J.P. Conlon, E. Palti, Aspects of flavour and supersymmetry in F-theory GUTs, JHEP 1001 (2010) 029, arXiv:0910.2413.

[18] S. Cecotti, M.C.N. Cheng, J.J. Heckman, C. Vafa, Yukawa couplings in F-theory and non-commutative geometry, arXiv:0910.0477 [hep-th].

[19] F. Marchesano, L. Martucci, Non-perturbative effects on seven-brane Yukawa couplings, arXiv:0910.5496.

[20] H. Hayashi, T. Kawano, Y. Tsuchiya, T. Watari, Flavor structure in F-theory compactifications, arXiv:0910.2762.

[21] J. Marsano, N. Saulina, S. Schafer-Nameki, Compact F-theory GUTs with $U(1)_{PQ}$, JHEP 1004 (2010) 095, arXiv:0912.0272 [hep-th].

[22] J. Marsano, N. Saulina, S. Schafer-Nameki, F-theory compactifications for supersymmetric GUTs, JHEP 0908 (2009) 030, arXiv:0904.3932 [hep-th].

[23] H. Hayashi, T. Kawano, Y. Tsuchiya, T. Watari, More on dimension-4 proton decay problem in F-theory – spectral surface, discriminant locus and monodromy, arXiv:1004.3870.
[24] E. Dudas, E. Palti, Frogatt–Nielsen models from E8 in F-theory GUTs, JHEP 1001 (2010) 127, arXiv:0912.0853.
[25] G. Ross, M. Serna, Unification and fermion mass structure, Phys. Lett. B 664 (2008) 97, arXiv:0704.1248 [hep-ph].
[26] R. Gatto, G. Sartori, M. Tonin, Weak selfmasses, Cabibbo angle, and broken $SU(2) \times SU(2)$, Phys. Lett. B 28 (1968) 128.
[27] S. Antusch, S.F. King, M. Malinsky, M. Spinrath, Quark mixing sum rules and the right unitarity triangle, Phys. Rev. D 81 (2010) 033008, arXiv:0910.5127.
[28] S.M. Barr, A new symmetry breaking pattern for $SO(10)$ and proton decay, Phys. Lett. B 112 (1982) 219.
[29] I. Antoniadis, J.R. Ellis, J.S. Hagelin, D.V. Nanopoulos, Supersymmetric flipped $SU(5)$ revitalized, Phys. Lett. B 194 (1987) 231.
[30] P. Fayet, J. Iliopoulos, Spontaneously broken supergauge symmetries and Goldstone spinors, Phys. Lett. B 51 (1974) 461;
M. Dine, N. Seiberg, E. Witten, Fayet–Iliopoulos terms in string theory, Nucl. Phys. B 289 (1987) 589.
[31] P. Minkowski, $\mu \to e\gamma$ at a rate of one out of 10^9 muon decays?, Phys. Lett. B 67 (1977) 421;
M. Gell-Mann, P. Ramond, R. Slansky, in: Sanibel Talk, CALT-68-709, February 1979, and in: Supergravity, North-Holland, Amsterdam, 1979;
T. Yanagida, in: Proc. of the Workshop on Unified Theory and Baryon Number of the Universe, KEK, Japan, 1979;
S.L. Glashow, Cargese Lectures, 1979;
R.N. Mohapatra, G. Senjanovic, Neutrino mass and spontaneous parity nonconservation, Phys. Rev. Lett. 44 (1980) 912.
[32] A.Y. Smirnov, Seesaw enhancement of lepton mixing, Phys. Rev. D 48 (1993) 3264, arXiv:hep-ph/9304205.
[33] S. Antusch, L.E. Ibanez, T. Macri, Neutrino masses and mixings from string theory instantons, JHEP 0709 (2007) 087, arXiv:0706.2132 [hep-ph].
[34] S. Antusch, M. Spinrath, GUT predictions for quark and lepton mass ratios, AIP Conf. Proc. 1200 (2010) 928, arXiv:0908.1520 [hep-ph].
[35] H. Georgi, C. Jarlskog, A new lepton–quark mass relation in a unified theory, Phys. Lett. B 86 (1979) 297, arXiv:0808.1571 [hep-th].
[36] S.F. King, Large mixing angle MSW and atmospheric neutrinos from single right-handed neutrino dominance and U(1) family symmetry, Nucl. Phys. B 576 (2000) 85, arXiv:hep-ph/9912492.
[37] S. Antusch, S. Boudjemaa, S.F. King, Neutrino mixing angles in sequential dominance to NLO and NNLO, arXiv:1003.5498 [hep-ph];
S. Antusch, S.F. King, M. Malinsky, G.G. Ross, Solving the SUSY flavour and CP problems with non-Abelian family symmetry and supergravity, Phys. Lett. B 670 (2009) 383, arXiv:0807.5047 [hep-ph].
[38] F. Gabbiani, A. Masiero, FCNC in generalized supersymmetric theories, Nucl. Phys. B 322 (1989) 235;
F. Gabbiani, E. Gabrielli, A. Masiero, L. Silvestrini, A complete analysis of FCNC and CP constraints in general SUSY extensions of the standard model, Nucl. Phys. B 477 (1996) 321, arXiv:hep-ph/9604387;
W. Altmannshofer, A.J. Buras, S. Gori, P. Paradisi, D.M. Straub, Anatomy and phenomenology of FCNC and CPV effects in SUSY theories, Nucl. Phys. B 830 (2010) 17, arXiv:0909.1333 [hep-ph].
[39] G. Isidori, B physics in the LHC era, arXiv:1001.3431 [hep-ph];
G. Isidori, Y. Nir, G. Perez, Flavor physics constraints for physics beyond the Standard Model, arXiv:1002.0900.
[40] I. de Medeiros Varzielas, G.G. Ross, Family symmetries and the SUSY flavour problem, arXiv:hep-ph/0612220.
[41] Y. Kawamura, H. Murayama, M. Yamaguchi, Low-energy effective Lagrangian in unified theories with nonuniversal supersymmetry breaking terms, Phys. Rev. D 51 (1995) 1337, arXiv:hep-ph/9406245.