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Threshold effects in grand unified string models

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A common feature in most of the string derived models is the appearance of extra matter fields, beyond those needed for the standard model content, which usually become massive at various intermediate scales. In the present work we concentrate on the free fermionic string models based on the intermediate gauge symmetries SU(5)×U(1) and SU(4)×O(4) and investigate the effect on the low energy parameters due to the inclusion of threshold corrections introduced either by string massive states or states becoming massive close to the intermediate symmetry breaking scale.

1. Introduction

Recent calculations have indicated that minimal supersymmetric (SUSY) grand unified theories (GUTs) [1] are in agreement with the precision LEP data, if the unification scale is chosen close to 10^{16} GeV [2]. On the contrary, non-supersymmetric GUTs are not favoured from the recent precision measurements of $\sin^2\theta_w$ and α_3.

String theories on the other hand [3], predict that unification in most of the viable models takes place at a high scale, two orders of magnitude larger than the one predicted in minimal supersymmetric GUTs. This scale is not lowered even if threshold corrections [4–9] from string massive states are included. On the contrary, explicit calculations have shown that, in most of the cases, there is a slight increase of this scale. As a consequence the minimal stringy standard model and certain GUT models with minimal content are possibly ruled out. A viable solution to this problem would be the realization of the string unification scenario in models with intermediate "grand unified" scales having extra representations. In particular, in $k=1$ constructions this scenario may work when the standard model gauge group is obtained through the spontaneous breaking of an intermediate semi-simple group as in the case of the SU(5)×U(1), $SU(4)\times SU(2)_L\times SU(2)_R$ or $SU(3)\times SU(3)_L\times SU(3)_R$ string derived models [10–16]. Certain models of this kind have been investigated the last year in order to determine the necessary ingredients for a consistent unification at the high scale of O(10^{18} GeV). The usual method is to extrapolate the gauge couplings down to low energies and determine the particle content of the model with the requirement that the string predictions coincide with the high precision measurements at LEP.

It has been shown [15] that under certain assumptions, the aforementioned models with the intermediate semi-simple gauge groups may fulfil the above requirements. A reliable calculation however, should necessarily be performed at the two-loop level. In that case, non-logarithmic corrections, arising from states with masses of the order M_X (the scale where the symmetry group breaks down), should be taken into account [17]. In contrast with SUSY GUTs, in string derived models all Yukawa couplings are known; thus one can in principle calculate exactly the effects of the threshold corrections due to these massive states.

Let M_V, M_S and M_F are the masses of the gauge bosons, scalars and fermions that become massive near the scale M_X. Let t_{iV}, t_{iS} and t_{iF} be the generators of the gauge group in the adjoint scalar and fermion representations respectively. Now we may integrate out these superheavy particles and work with an effective gauge theory whose couplings are related with the original unified coupling constant. Thus for the case of a non-SUSY simple gauge group G, which breaks down at a scale M_X, one has

$$\frac{1}{\alpha_i(M_X)}=\frac{1}{\alpha_G(M_X)}-\frac{\lambda_i(M_X)}{12\pi}, \tag{1}$$

where λ_i are the matching functions given by

$$\lambda_i(M_X)=\mathrm{Tr}\{t_{iV}^2\}-21\ \mathrm{Tr}\left\{t_{iV}^2\log\left(\frac{M_V}{M_X}\right)\right\}$$
$$+\mathrm{Tr}\left\{t_{iS}^2\log\left(\frac{M_S}{M_X}\right)\right\}+8\ \mathrm{Tr}\left\{t_{iF}^2\log\left(\frac{M_F}{M_X}\right)\right\}, \tag{2}$$

and $i=1, 2, 3$ stands for the three gauge couplings of the standard model. This formula should be modified properly in the case where the gauge group above the boundary is not simple as well as in the case of SUSY models.

In most of the non-supersymmetric models one may conclude that the predictions for the low energy parameters do not alter significantly, if one ignores threshold corrections (usually arising from heavy scalar particles) at M_X. On the contrary, in SUSY theories since there is an equal number of fermions and bosons the contribution to the matching functions $\lambda_i(M_X)$ becomes larger. In particular, if one considers string derived models, a large number of additional matter and Higgs fields always appears together with the three families and the necessary Higgs; thus the threshold corrections become even more important and should be included in the determination of $\sin^2\theta_w$ and α_3. As a matter of fact, in an ordinary GUT, such calculations would increase the uncertainties of the results since we do not know the precise values of the masses of the superheavy particles we are integrating out.

However, in a more fundamental theory, as in the case of strings, one could in principle evaluate more accurately the masses of the particles and other related parameters, once the specific boundary conditions for the construction of the model are chosen. In conclusion, the calculation of the threshold corrections in string derived models would not lead to uncertainties but to more accurately determined low energy parameters.

In this paper we calculate the threshold corrections at the various mass scales for two of the most successful string models constructed in the free fermionic formulation, namely the string model SU(4)×O(4) [13] based on the gauge group SU(4)× $SU(2)_L\times SU(2)_R$ [10] and the flipped SU(5) [16,18–20] based on the GUT model, proposed a long time ago [11]. In section 2 we start with the computation of string threshold corrections due to massive states at the unification scale M_U for the SU(4)×O(4) case. We subsequently calculate the matching functions at the $SU(4)\times SU(2)_R$ breaking scale M_X and determine the gauge couplings of the effective field theory at M_X. In section 3 we examine the flipped SU(5) model, using the result for the effective string scale derived elsewhere [6], to evolve the gauge couplings from M_U down to low energies and calculate the matching functions at the SU(5) breaking scale as before. Finally in section 4 we present our conclusions.

2. The SU(4)×O(4) model

Before starting our calculation, let us review the basic features of the model. It is constructed within the free fermionic formulation [21] of the heterotic string using nine vectors of b.c. on the world sheet fermions, which lead to the symmetry group of SU(4) × O(4) × $U(1)^4$ × $[SU(8)\times U(1)']_{\mathrm{Hidden}}$ [13]. The spectrum and other properties are analysed elsewhere [9,13,14]. In the most general case one may define the following scales (i) M_U, where $\alpha_4=\alpha_{2L}=\alpha_{2R}\equiv\alpha_U$; (ii) M_A, where the anomalous $U_A(1)$ breaks and a number of fields acquire masses through some singlet fields developing non-zero VEVs. Between M_U and M_A we assume the full string content of the model; (iii) M_X, where the group SU(4) $\times SU(2)_L\times SU(2)_R$ breaks down to SU(3)× $SU(2)_L\times U(1)_Y$ and the relation between the couplings, at that scale, is given by

$$\frac{1}{\alpha_Y}=\frac{1}{\frac{5}{3}\alpha_{2R}}+\frac{1}{\frac{5}{2}\alpha_4}, \tag{3}$$

and finally (iv) M_I, where we assume that below this scale we only have the standard model content. Between M_X and M_I some exotic remnants could survive.

In the one loop approximation the renormalization group equations for the gauge couplings, including string threshold corrections, are written

$$\frac{1}{\alpha_i(\mu)} = \frac{4\pi}{g_i^2(\mu)} = k_i \frac{4\pi}{g^2(M_U)} + \frac{1}{4\pi}\left[b_i \log\left(\frac{M_{\text{string}}^2}{\mu^2}\right) + \Delta_i\right], \quad (4)$$

where b_i are the β-functions coefficients, Δ_i are the string threshold corrections to the inverse gauge couplings and M_{string} is [4,6]

$$M_{\text{string}} = \left(\frac{2\exp(1-\gamma)}{3\sqrt{3}\,\pi\alpha'}\right)^{1/2}, \quad (5)$$

where γ is the Euler–Macheroni constant and $\alpha' = 16\pi^2/g_{\text{string}}^2 M_{\text{Pl}}^2$. Δ_i can be calculated explicitly once the full string spectrum is known. It is possible however to calculate the string threshold corrections at a pretty good level of approximation by examining the dependence of Δ_i on the moduli fields [5]. Using general arguments based on the geometry of the compactified space we are led to write Δ_i in the following form:

$$\Delta_i = -b_i \Delta(T_I) + c_i + Y, \quad (6)$$

where c_i and Y are the gauge dependent and gauge independent pieces respectively while $\Delta(T_I)$ exhibits the non-trivial dependence of Δ_i on the untwisted moduli T_I and arises from the $N=2$ supersymmetry preserving sectors of the model.

Assuming that the moduli T_I take the same value T for the different $N=2$ sectors of the model we may write $\Delta(T_I)$ as follows:

$$\Delta(T) = \log(|\eta(\mathrm{i}T)|^4 \mathrm{Re}\, T), \quad (7)$$

while expanding the η-function we arrive at the approximate formula

$$\Delta(T) \approx -\tfrac{1}{3}\pi\, \mathrm{Re}\, T + \log(\mathrm{Re}\, T), \quad \mathrm{Re}\, T > \tfrac{1}{2}\sqrt{3}, \quad (8)$$

Now, the effective unification scale for two gauge couplings g_i and g_j is determined by the difference

$$\left[b_i \log\left(\frac{M_{\text{string}}^2}{\mu^2}\right) + \Delta_i\right] - \left[b_j \log\left(\frac{M_{\text{string}}^2}{\mu^2}\right) + \Delta_j\right] = 2(b_i - b_j)\log\left(\frac{M_U^{(i,j)}}{\mu}\right), \quad (9)$$

where

$$M_U^{(i,j)} = M_{\text{string}}(\mathrm{Re}\, T)^{-1/2} \exp(\tfrac{1}{6}\pi\, \mathrm{Re}\, T + \tfrac{1}{2}\delta c^{(i,j)}), \quad (10)$$

and $\delta c^{(i,j)} = (c_i - c_j)/(b_i - b_j)$. In our model, i and j indices stand for the $SU(2)_L$, $SU(2)_R$ and $SU(4)$ gauge groups. The quantity $\delta c^{(i,j)}$ is not determined by this method, but explicit calculations in Z_3 orbifold models [4], as well as in the flipped SU(5) [6], have shown that this number is very small ($\delta c^{(i,j)} \approx 0.02$), and therefore for the present level of approximation it can be neglected. Thus the effective unification scale is approximately

$$M_U \approx M_{\text{string}}(\mathrm{Re}\, T)^{(-1/2)} \exp(\tfrac{1}{6}\pi\, \mathrm{Re}\, T). \quad (11)$$

For the free fermionic models we take $\mathrm{Re}\, T = 1$ and we get

$$M_U \approx M_{\text{string}} \exp(\tfrac{1}{6}\pi) \approx 1.7 g_{\text{string}} \times 10^{18}\ \text{GeV}. \quad (12)$$

Once we have estimated the effective unification scale we can make use of the solution of the D-flatness constraints [9] to define the anomalous $U(1)_A$-breaking scale which is found to be

$$M_A \approx 7.8 g_{\text{string}} \times 10^{17}\ \text{GeV}. \quad (13)$$

Now, using the renormalization group analysis and the specific spectrum of the model one may search for the possible $SU(4) \times SU(2)_R$ breaking scales M_X which are consistent with the experimentally determined values of $\sin^2\theta_w$, α_3 and α.

In previous analyses [9,15] it has been shown that the existence of a "grand unified" scale is necessary in order to compromise the high energy unification mass with the experimental value of $\sin^2\theta_w(M_Z)$. This scale was estimated to lie in the range of $M_X \approx (10^{15}-10^{16})$ GeV, provided that some extra matter remains down to some intermediate scale M_I, which varies in the range $M_I \approx (10^8-10^{14})$ GeV.

Here we are going to consider the effect of the threshold corrections to the above scales induced by the superheavy particles. In order to estimate this effect, we need to calculate the matching functions at each particular scale. Let us start with the scale M_X, where the original symmetry breaks down to the standard model gauge group. At this scale the multiplets break in the following way:

$$n_H(\overset{(-)}{4}, 1, 2) \to n'_{31}(\overset{(-)}{3}, 1, \pm\tfrac{2}{3}) + n_3(\overset{(-)}{3}, 1, \pm\tfrac{1}{3})$$
$$+ n_s(1, 1, \pm 1) + n'_s(1, 1, 0)\,,$$
$$n_4(\overset{(-)}{4}, 1, 1,) \to n'_3(\overset{(-)}{3}, 1, \pm\tfrac{1}{6}) + n'(1, 1, \pm\tfrac{1}{2})\,,$$
$$n_{22}(1, 2, 2) \to n_2(1, 2, \pm\tfrac{1}{2})\,,$$
$$n_L(1, 2, 1) \to n'_L(1, 2, 0)\,,$$
$$n_R(1, 1, 2) \to n'(1, 1, \pm\tfrac{1}{2})\,.$$

The matching functions read now

$$\lambda_Y(M_X) = \tfrac{14}{5} - 14 \log\frac{M_V}{M_X}$$
$$+ \tfrac{9}{5}(4n'^{h}_{31} + n^{h}_{3} + 3n^{h}_{s} + \tfrac{1}{4}n'^{h}_{3}$$
$$+ \tfrac{3}{2}n^{h}_{2} + \tfrac{3}{4}n'^{h}) \log\frac{\langle M\rangle}{M_X}\,, \tag{14}$$

$$\lambda_2(M_X) = \tfrac{9}{2}(n^{h}_{2} + n'^{h}_{L}) \log\frac{\langle M\rangle}{M_X}\,, \tag{15}$$

$$\lambda_3(M_X) = 1 - 5 \log\frac{M_V}{M_X}$$
$$+ \tfrac{9}{2}(n'^{h}_{31} + n^{h}_{3} + n'^{h}_{3}) \log\frac{\langle M\rangle}{M_X}\,, \tag{16}$$

where $\langle M\rangle$ stands for an average mass for the matter multiplets. The superscript "h" indicates that the n's give the number of the corresponding heavy multiplets.

Let us examine first the effect of the functions λ_i on the scale M_X at the first loop level. The renormalization group equations give

$$\frac{1}{\alpha_3(M_Z)} = \frac{1}{\alpha_U} + \frac{b'_4}{2\pi}\log\frac{M_U}{M_A} + \frac{b_4}{2\pi}\log\frac{M_A}{M_X}$$
$$+ \frac{b_3}{2\pi}\log\frac{M_X}{M_I} + \frac{b'_3}{2\pi}\log\frac{M_I}{M_Z} - \frac{\lambda_3(M_X)}{12\pi}\,, \tag{17}$$

$$\frac{1}{\alpha_2(M_Z)} = \frac{1}{\alpha_U} + \frac{b'_L}{2\pi}\log\frac{M_U}{M_A} + \frac{b_L}{2\pi}\log\frac{M_A}{M_X}$$
$$+ \frac{b_2}{2\pi}\log\frac{M_X}{M_I} + \frac{b'_2}{2\pi}\log\frac{M_I}{M_Z} - \frac{\lambda_2(M_X)}{12\pi}\,, \tag{18}$$

$$\frac{1}{\alpha_Y(M_Z)} = \frac{1}{\alpha_U} + \frac{1}{2\pi}(\tfrac{3}{5}b'_R + \tfrac{2}{5}b'_4)\log\frac{M_U}{M_A}$$
$$+ \frac{1}{2\pi}(\tfrac{3}{5}b_R + \tfrac{2}{5}b_4)\log\frac{M_A}{M_X} + \frac{b_Y}{2\pi}\log\frac{M_X}{M_I}$$
$$+ \frac{b'_Y}{2\pi}\log\frac{M_I}{M_Z} - \frac{\lambda_Y(M_X)}{12\pi}\,, \tag{19}$$

with an obvious notation for the β-function coefficients corresponding to the different gauge groups and energy regions.

By taking the linear combination

$$\frac{3}{5}\frac{1}{\alpha_Y} + \frac{1}{\alpha_2} - \frac{8}{3}\frac{1}{\alpha_3} = \frac{1}{\alpha} - \frac{8}{3}\frac{1}{\alpha_3}\,, \tag{20}$$

we can define an effective "grand unification" scale $M_X^{\rm eff}$,

$$M_X^{\rm eff} = M_X \exp\left(\frac{\Delta_X}{B_X - B_U}\right), \tag{21}$$

where M_X is now the scale when no threshold corrections are taken into account, while B_U, B_X and Δ_X are combinations of β-functions and matching functions:

$$B_U = b_R + b_L - 2b_4\,, \tag{22}$$

$$B_X = \tfrac{3}{5}b_Y + b_2 - \tfrac{8}{3}b_3\,, \tag{23}$$

$$\Delta_X = \tfrac{1}{6}[\tfrac{3}{5}\lambda_Y(M_X) + \lambda_2(M_X) - \tfrac{8}{3}\lambda_3(M_X)]\,. \tag{24}$$

For an illustration, assume that above M_A we have the usual string content while in the region (M_A, M_X) we have $n_6=4$, $n_H=2$, $n_4=2$, $n_{22}=2$, $n_L=4$ and $n_R=0$. In the range (M_X, M_I) we have $n'_3=n_2=n'=2$ and $n_{31}=n_3=n'_L=0$, while below M_I we have the standard model content (the number of generations is always $n_G=3$). At M_X, the numbers of particles that become heavy are given by

$$n'^{h}_{31} = n_H - 2 - n'_{31}\,, \quad n^{h}_{3} = n_H + 2n_6 - n_3\,,$$
$$n^{h}_{s} = n_H - 2\,, \quad n'^{h}_{3} = n_4 - n'_3\,, \quad n^{h}_{2} = 2n_{22} - n_2\,,$$
$$n'^{h}_{L} = n_L - n'_L\,, \quad n'^{h} = 2n_R + n_4 - n'\,.$$

Let us consider the simplest case where all superheavy particles get masses equal to M_X. In this case one finds that the effective scale $M_X^{\rm eff}$ is higher compared to M_X: $M_X^{\rm eff} \approx 1.32 M_X$. Of course in a more realistic case we should allow the particles to have different masses at M_X. Moreover, since such corrections are of the same order with the two loop ones, we should also make the whole calculation at the two loop level.

We would like to present now a schematic scenario of the effective gauge coupling evolution. In fig. 1a we present the evolution of the gauge couplings, showing expanded the energy region around M_X (where the gauge symmetry $SU(4)\times SU(2)_L\times SU(2)_R$ breaks down to the standard model group), which we assume to be 10^{16} GeV. We have also assumed here that all (matter) multiplets becoming massive at M_X acquire degenerate masses $M_i\approx\langle M\rangle$, where $\langle M\rangle/M_X\approx(g/\sqrt{2}\langle H\rangle)/g\langle H\rangle=1/\sqrt{2}$. For simplicity we ignore corrections from the other

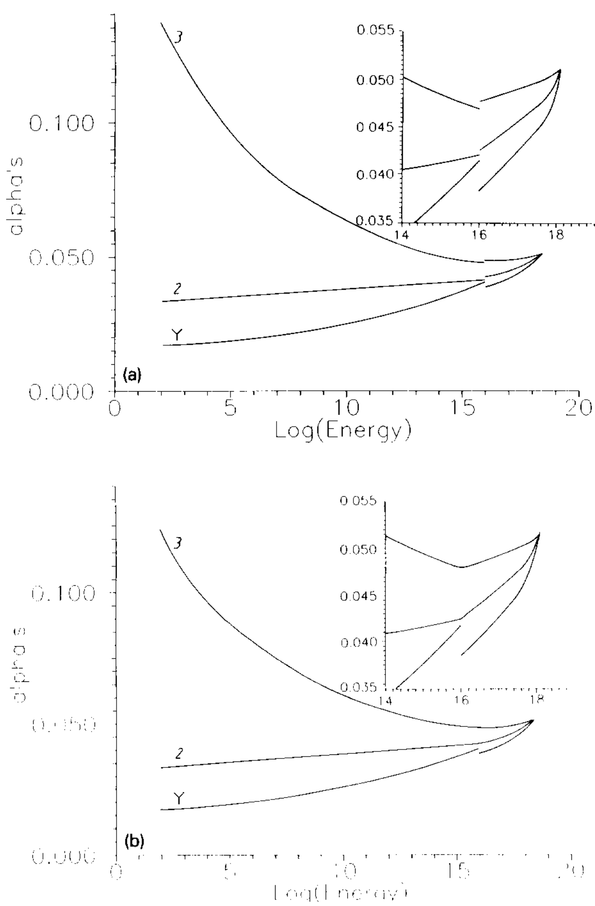

Fig. 1. The evolution of the gauge couplings, for the $SU(4)\times SU(2)_L\times SU(2)_R$ model, from M_U down to M_Z, at the two loop level, (a) including threshold corrections and (b) without threshold corrections.

intermediate states. The discontinuity appearing in the evolution is due to the matching functions λ_i'. For comparison, we draw in fig. 1b, the same graph ignoring the threshold corrections (the appearance of the discontinuity in α_Y is due to eq. (3)).

We turn now to the most interesting part of our analysis. Using eqs. (17)–(19), and the definition of $\sin^2\theta_w(M_Z)$ and $\alpha(M_Z)$, we can write down $\alpha_3(M_Z)$ and $\sin^2\theta_w(M_Z)$ as a function of α_U and M_X. In fig. 2 – using the same particle content used in fig. 1 – we plot curves for constant M_X in the parameter space $(\sin^2\theta_w, \alpha_3)$. For comparison we also plot the corresponding contours when no threshold corrections are included (dashed lines) [9]. Thus when threshold corrections are included we see a general parallel shift of the contours showing that the effective M_X is somewhat higher. Note that these corrections do not alter the qualitative features of previous analyses [9].

Fig. 3 shows the same contours as before but at the two loop level. Again dashed lines represent curves where no threshold corrections are included. We see

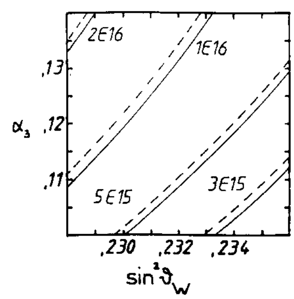

Fig. 2. Contours of constant M_X in the $(\sin^2\theta_w, \alpha_3)$ parameter space for the $SU(4)\times SU(2)_L\times SU(2)_R$ model, at the one loop approximation. Continuous lines include threshold corrections while dashed ones do not. The ratio $r=\langle M\rangle/M_X=1$.

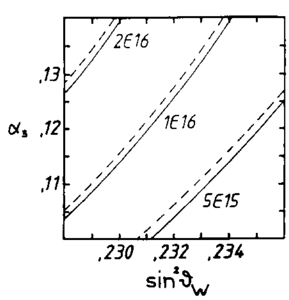

Fig. 3. Same as in fig. 2 but at the two loop approximation.

now a significant shift to lower values of $\sin^2\theta_w$ and α_3, with respect to the one loop order. For both figs. 2 and 3 we have assumed that the superheavy particles acquire mass M_X.

Finally in fig. 4 we present curves of constant M_X for different values of the ratio $r=\langle M\rangle/M_X$, but again assuming degeneracy of the masses of the matter multiplets. In particular the cases $r=1$, 0.71, 0.5 for $(n_L, n_R)=(4, 0)$ and $M_X=2\times10^{16}$, 10^{16} and 5×10^{15} GeV are considered. We conclude that a significant correction arises only if $r<0.5$, i.e. only if $M_X>2\langle M\rangle$. If we further adopt the high precision LEP data [22], where $\sin^2\theta_w=0.2331\pm0.0013$, we conclude that the optimum cases would be $r<0.7$ for the highest acceptable value of $M_X\approx10^{16}$ GeV, or $r>0.5$ for the low value of $M_X\approx5\times10^{15}$ GeV.

3. The SU(5)×U(1) model

Let us turn our discussion to the SU(5)×U(1) model [16]. The effective string unification scale can be defined in terms of the string threshold corrections as [6]

$$M_U=M_{\rm string}\exp\left(\frac{\delta\Delta_{15}}{2\,\delta b_{15}}\right), \tag{25}$$

where $\delta\Delta_{15}=\Delta_1-\Delta_5$ is the difference between the U(1) and SU(5) threshold corrections while $\delta b_{15}=b_1-b_5$ is the difference of the corresponding β-function coefficients. In the "revambed" version of the flipped SU(5), $\delta\Delta_{15}$ has been calculated explicitly $\delta\Delta_{15}=24.13$ (and $\delta b_{15}=\frac{45}{2}$. In that case one finds that

$$M_U\approx1.3g_{\rm string}\times10^{18}\ {\rm GeV}\,. \tag{26}$$

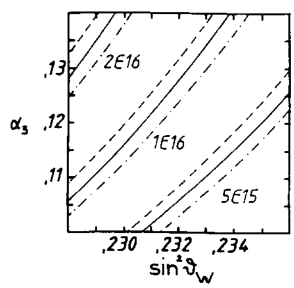

Fig. 4. Same as in fig. 2. The dashed, continuous and dash-dotted lines correspond to $r=\langle M\rangle/M_X=1$, 0.71 and 0.5 respectively.

It can be shown [23], however, that gauge coupling unification at such a high scale needs extra matter representations which should remain massless down to some intermediate scale M_I, much lower than the SU(5) breaking scale. In particular at least one or two extra pairs in the $Q+\bar{Q}=(3,2)+(\bar{3},2)$ vector-like representations, as well as two $D+\bar{D}=(3,1)+(\bar{3},1)$, are necessary in order to obtain the correct $\sin^2\theta_w$ and α_3 at M_Z through the evolution of the gauge couplings down to low energies. Note that $Q+\bar{Q}$ representations arise from the decomposition of $10+\overline{10}$ representations of SU(5), thus the proper number of the latter should be present in the string spectrum of the model. A first attempt to obtain a flipped string SU(5) with extra $10+\overline{10}$ was presented in ref. [18], while a systematic search using computer algorithms was presented in ref. [19].

As in the case of the $SU(4)\times SU(2)_L\times SU(2)_R$ model, which was examined previously, there is a number of extra representations becoming massive at various mass scales. Again one would need the precise number of those multiplets in order to compute the threshold corrections. As an application, we consider a version of the flipped SU(5) string, presented in ref. [19]. The basis of the model consists of eight vectors which break the symmetry to the gauge group $SU(5)\times U(1)_{\tilde{Y}}\times U(1)^5\times[SO(10)\times SU(4)]_{\rm Hidden}$. The Higgs and matter representations of the model are (indicating the transformation properties under $SU(5)\times U(1)_{\tilde{Y}}$):

- Three $F(10,\frac{1}{2})+\bar{f}(\bar{5},-\frac{3}{2})+l^c(1,\frac{5}{2})$ which form the three complete generations with the right-handed neutrinos belonging to the $F(10,\frac{1}{2})$.
- Two $H(10,\frac{1}{2})+\bar{H}(\overline{10},-\frac{1}{2})$ representations which will be used to break the SU(5) symmetry.
- Four $h(5,-1)+h(\bar{5},1)$ pairs of Higgs. One pair is going to provide the necessary Higgs doublet for the standard group breaking, while the remaining two coloured triplets should receive masses at some intermediate scale $M_I\gg M_W$. The rest three pairs of fiveplets receive superheavy masses, as some of the neutral singlets, which are present in the spectrum, develop non-zero VEVs.

The Hidden sector of the model transforms as follows:

- Six pairs of $SU(4)_H\times U(1)_{\tilde{Y}}$ representations $\tilde{F}(4,\frac{5}{4})+\bar{\tilde{F}}(\bar{4},-\frac{5}{4})$, denoted in the following by n_4.
- Seven sextets under $SU(4)_H$, $D_1, D_2, \ldots, D_7$ with zero $\tilde{Y}$-charge, denoted by n_6.

As already mentioned, a number of singlets is also present [19] and some of them will develop non-zero VEVs to cancel the anomalous D-term and provide masses to some of the extra states. In the following we are going to use the RG analysis to determine the range of the various scales that the particles should remain massless (and therefore contribute) in order to obtain the experimental predictions for $\sin^2\theta_w$ and α_3. Since there is a large number of particles becoming massive at each scale, threshold corrections should be taken into account when these states are integrated out at the corresponding scale.

We shall concentrate on the SU(5) breaking scale. At this scale the multiplets break in the following way:

$$n_{10}(\overline{10}, \pm\tfrac{1}{2}) \to n_{32}(3, 2, \pm\tfrac{1}{6}) + n_3(3, 1, \tfrac{1}{3}) + n_1(1, 1, 0)\,,$$

$$n_5(5, \pm 1) \to n_3(3, 1, \tfrac{1}{3}) + n_2(1, 2, \tfrac{1}{2})\,,$$

$$n'_{e^c}(1, \pm\tfrac{5}{2}) \to n_{e^c}(1, 1, \pm 1)\,.$$

The matching functions read

$$\lambda_Y(M_X) = \tfrac{1}{5} - \log\frac{M_V}{M_X} + \tfrac{27}{5}(\tfrac{1}{6}n^h_{32} + n^h_{e^c} + n^h_4 + \tfrac{1}{3}n^h_3 + \tfrac{1}{2}n^h_2)\log\frac{\langle M\rangle}{M_X}\,, \tag{27}$$

$$\lambda_2(M_X) = 3 - 15\log\frac{M_V}{M_X} + 9(\tfrac{3}{2}n^h_{32} + \tfrac{1}{2}n^h_2)\log\frac{\langle M\rangle}{M_X}\,, \tag{28}$$

$$\lambda_3(M_X) = 2 - 10\log\frac{M_V}{M_X} + 9(n^h_{32} + \tfrac{1}{2}n^h_3)\log\frac{\langle M\rangle}{M_X}\,, \tag{29}$$

where, as before, $\langle M\rangle$ stands for an average mass of the heavy matter multiplets. The RGEs give, at the one loop level

$$\frac{1}{\alpha_3} = \frac{1}{\alpha_U} + \frac{b'_5}{2\pi}\log\frac{M_U}{M_A} + \frac{b_5}{2\pi}\log\frac{M_A}{M_X} + \frac{b_3}{2\pi}\log\frac{M_X}{M_I} + \frac{b'_3}{2\pi}\log\frac{M_I}{M_Z}\,, \tag{30}$$

$$\frac{1}{\alpha_2} = \frac{1}{\alpha_U} + \frac{b'_5}{2\pi}\log\frac{M_U}{M_A} + \frac{b_5}{2\pi}\log\frac{M_A}{M_X} + \frac{b_2}{2\pi}\log\frac{M_X}{M_I} + \frac{b'_2}{2\pi}\log\frac{M_I}{M_Z}\,, \tag{31}$$

$$\frac{1}{\alpha_Y} = \frac{1}{\alpha_U} + \frac{1}{2\pi}(\tfrac{1}{25}b'_5 + \tfrac{24}{25}b'_1)\log\frac{M_U}{M_A} + \frac{1}{2\pi}(\tfrac{1}{25}b_5 + \tfrac{24}{25}b_1)\log\frac{M_A}{M_X} + \frac{b_Y}{2\pi}\log\frac{M_X}{M_I} + \frac{b'_Y}{2\pi}\log\frac{M_I}{M_Z}\,. \tag{32}$$

The effective "grand unification" scale M_X^{eff} can be defined, as in the case of the $SU(4)\times SU(2)_L\times SU(2)_R$ model,

$$M_X^{\text{eff}} = M_X \exp\left(\frac{\Delta_X}{B_X - B_U}\right), \tag{33}$$

where

$$B_U = \tfrac{8}{5}(b_1 - b_5)\,, \tag{34}$$

$$B_X = \tfrac{5}{3}b_Y + b_2 - \tfrac{8}{3}b_3\,, \tag{35}$$

$$\Delta_X = \tfrac{1}{6}[\tfrac{5}{3}\lambda_Y(M_X) + \lambda_2(M_X) - \tfrac{8}{3}\lambda_3(M_X)]\,. \tag{36}$$

We express $\sin^2\theta_w(M_Z)$ and $\alpha_3(M_Z)$ as functions of α_U and M_X, and plot contours of constant M_X. At this point we should mention that in plotting the above contours we have used a more complicated (nevertheless necessary) situation, which is not shown in eqs. (30)–(32). Namely, we have introduced the following additional scales:

- For each specific choice of the number of $SU(4)_{\text{Hidden}}$-fourplets n_4 (having charges only under U(1) and $U(1)_Y$) and sextets n_6, we evaluate the scale M_C where the $SU(4)_{\text{Hidden}}$ coupling constant becomes large ($\geqslant 0.2$). At this scale, the fourplets produce bound states and decouple from the spectrum.
- We have also allowed for a scale M_{32} below which no $(3, 2) + (\bar{3}, 2)$ survives. The range of M_{32} is found to be from 10^{12}–10^{13} GeV for the acceptable range of values of $\sin^2\theta_w$ and α_3.

Finally we have included a central value two loop correction to both $\sin^2\theta_w(M_Z)$ and M_X following ref. [18].

In order to achieve unification at the high scale M_U in the $SU(5)\times U(1)$ model, we have used the following content: $n_{10}=6$ and $n_5=2$ between M_U and M_X, while $n_{32}=4$ and $n_3=4$ below M_X. We have also assumed $n_4=6$ and $n_6=3$ above M_X whilst $n_4=4$ and $n_6=1$ in the range $M_X - M_C$. At M_X, the numbers of

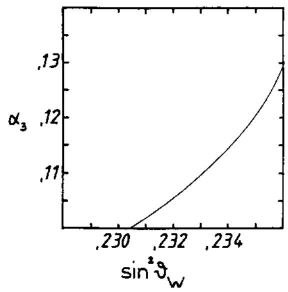

Fig. 5. Contour of constant M_X for the SU(5)×U(1) model. The curve is almost the same for all reasonable changes of the particle content.

representations becoming heavy are given by

$$n^h_{32}=n_{10}-2-n_{32}, \quad n^h_{e^c}=n'_{e^c}-n_{e^c},$$

$$n^h_3=n_{10}+n_5-n_3, \quad n^h_2=n_5-n_2,$$

$$n^h_4=n_4(\text{above } M_X)-n_4(\text{below } M_X).$$

For this particular content $M_X^{\text{eff}}=1.2M_X$.

In fig. 5 we show the contour of constant M_X in the $(\sin^2\theta_w, \alpha_3)$ space. For any reasonable change in the particle content (including n_4 and n_6) as well as in M_X, the deviations from that curve are very small. We have to point out, nevertheless, that M_X should be near M_A and $\alpha_U \sim 0.075$ in order to achieve experimentally accepted values of $\sin^2\theta_w$ and α_3.

4. Conclusions

In the present work we have re-examined the predictions of the fermionic string models based on the intermediate symmetries $SU(4)\times SU(2)_L\times SU(2)_R$ and $SU(5)\times U(1)$, when threshold corrections at the string scale as well as the intermediate gauge group breaking scale M_X are taken into account. In previous analyses it was shown that string threshold corrections increase the unification scale in both models by 70%. As a result extra matter fields should remain massless down to some scale lower than the intermediate scale M_X. Here we find that threshold corrections, due to particles that become massive near the scale M_X, in most realistic cases lead to a similar increase of that scale. The qualitative picture of the above models does not alter. However, it is found that these thresholds are comparable with the two loop corrections and should be included when extrapolates the gauge couplings down to the M_Z scale in order to determine the low energy parameters.

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