FLIPPED SO(10)

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We construct an $N=1$ supersymmetric SO(10) GUT broken down to $SU(3)_C\times SU(2)_L\times U(1)_Y$ with an intermediate flipped $SU(5)\times U(1)_X$ gauge symmetry. A solution to the triplet–doublet mass-splitting problem is proposed in terms of a non-minimal missing-partner mechanism.

Although all contemporary unification efforts are dominated by the superstring philosophy, model building at energies below M_P faces a focusing problem caused by the proliferation of different compactified superstring theories [1,2] or four-dimensional string theories [3]. Despite the fact that uniqueness is lost, the new freedom allows for the construction of problem-free GUTs inspired by the superstring.

In a recent article [4] an $SU(5)\times U(1)_X$ theory was propoesd (termed flipped SU(5)), which later was shown to be obtainable from manifold compactification of the ten-dimensional heterotic string [5]. In the present short letter we construct a conventional SO(10) GUT that breaks down to the standard model through an intermediate flipped $SU(5)\times U(1)_X$ stage. We find that in order to realize the missing-partner mechanism we have to extend the Higgs sector of the model making use of the $126+\overline{126}+210$ representations which break flipped $SU(5)\times U(1)_X$ down to $SU(3)_C\times SU(2)_L\times U(1)_Y$ and naturally split the masses of colour-triplets from Weinberg–Salam isodoublets. Neutrinos come out naturally light through a standard see-saw mechanism.

Among the different breakings of SO(10) down to the standard model, the breaking $SO(10)\to SU(5)\times U(1)_X\to SU(3)_C\times SU(2)_L\times U(1)_Y$ can occur in two distinct ways [6]. One way is to have $SU(3)_C\times SU(2)_L\times U(1)_Y$ contained in SU(5). Then, SU(5) is the standard Georgi–Glashow SU(5). An alternative way is to have SU(5) being a different subgroup of SO(10) than the Georgi–Glashow SU(5). In that case (flipped SU(5)), its decomposition into $SU(3)_C\times SU(2)_L\times U(1)_Z$ is not the standard model but one is led to it after linearly combining Z and X into the weak hypercharge Y. Z is the generator of SU(5) which commutes with $SU(3)_C\times SU(2)_L$. In order to get the standard hypercharge assignments of the Glashow–Weinberg–Salam model, we must have $Y/2=1/5(X-Z)$. Z is normalized for the five-dimensional representation as $\mathrm{diag}(-1/3, -1/3, -1/3, 1/2, 1/2)$. In the flipped model $SU(3)_C\times SU(2)_L$ is embedded in SU(5) but $U(1)_Y$ is embedded in $SU(5)\times U(1)_X$.

Matter fermions, i.e. quarks and leptons, accommodate themselves in chiral superfields in the (10, 1), $(\bar{5}, -3)$ and (1, 5) representations of $SU(5)\times U(1)_X$

$$F(10,1)\equiv\begin{bmatrix} N^c\ Q=\begin{pmatrix}u\\ d\end{pmatrix} \\ d^c\end{bmatrix}\ ,\quad F(\bar{5},-3)\equiv\begin{bmatrix} u^c \\ \ell=\begin{pmatrix}e\\ \nu\end{pmatrix}\end{bmatrix}\ ,\quad F(1,5)\equiv e^c. \tag{1}$$

Notice the flipping $d^c\leftrightarrow u^c$ and $N^c\leftrightarrow e^c$ in comparison to the standard Georgi–Glashow positions. These multiplets fit exacty in the 16 spinorial representation of SO(10)

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$$F(16) \equiv F(10, 1) + F(\bar{5}, -3) + F(1, 5) . \tag{2}$$

Higgs chiral superfields containing the Weinberg–Salam doublets naturally occupy the 10 representation

$$h(10) = h(5,-2) + h^c(\bar{5},2) = \begin{bmatrix} D^c \\ h^c = \begin{pmatrix} h^+ \\ h_0^+ \end{pmatrix} \end{bmatrix} + \begin{bmatrix} D \\ h = \begin{pmatrix} h^- \\ h_0 \end{pmatrix} \end{bmatrix} . \tag{3}$$

The "light" Higgs doublets are accompanied by coloured Higgs triplets D, D^c.

Matter Yukawa interactions are expressible in terms of the superpotential

$$\begin{aligned} W_0 &= F(16)F(16)h(10) \\ &= F(10, 1)F(10, 1)h(5, -1) + F(10, 1)F(\bar{5}, -3)h^c(\bar{5}, 2) + F(\bar{5}, -3)F(1, 5)h(5, -2) \\ &= Qd^c h + Qu^c h^c + \ell e^c h + \ell N^c h^c + QQD + d^c N^c D + e^c u^c D + Q\ell D^c + d^c u^c D^c . \end{aligned} \tag{4}$$

We have suppressed Yukawa coupling constants and family indices. In the final expression of W_0 in terms of standard model fields we recognize the first four terms as the usual set that provides masses for d, u and e as well as a Dirac mass for neutrinos upon $SU(2)_L \times U(1)_Y$ breakdown. The rest of the couplings, involving D and D^c, are baryon-number violating. For example, D–scalar exchange generates a dimension-six operator $ud(u^c)^*(e^c)^*$ that leads to proton decay ($p \to n^0 e^+$). In order to avoid conflict with the observed proton stability D and D^c should be superheavy. We shall return to this problem later and show how to provide masses for the D's.

The first stage of symmetry breaking, namely $SO(10) \to SU(5) \times U(1)_X$, since we are talking about a conventional grand unified theory, can be achieved by a non-vanishing vacuum expectation value of a real antisymmetric tensor representation with an even number of incides such as 45 or 210. The 45 representation decomposes under $SU(5) \times U(1)_X$ as

$$A(45) = A(1, 0) + A(24, 0) + A(10, -4) + A(\overline{10}, 4) . \tag{5}$$

A non-vanishing $\langle A(1, 0) \rangle$ breaks SO(10) to $SU(5) \times U(1)_X$. A drawback of 45 is that its cubic self-coupling is antisymmetric and therefore for a single 45 it vanishes. A quadratic superpotential would maintain 45 at vanishing vacuum expectation value and SO(10) would stay unbroken. This can be avoided by representation doubling or the introduction of other representations. Another useful representation that possesses quadratic and cubic self-couplings and can also couple to 45 is the 210 which decomposes under $SU(5) \times U(1)_X$ as

$$\begin{aligned} B(210) &= B(1, 0) + B(5, 8) + B(\bar{5}, -8) + B(10, -4) + B(\overline{10}, 4) \\ &\quad + B(24, 0) + B(40, 4) + B(\overline{40}, -4) + B(75, 0) . \end{aligned} \tag{6}$$

A non-vanishing $\langle B(1, 0) \rangle$ breaks SO(10) to $SU(5) \times U(1)_X$. We can be agnostic temporarily about the precise form of the superpotential that induces the $SO(10) \to SU(5) \times U(1)_X$ breakdown and assume that the breaking occurs through a VEV $\langle A(45) \rangle = \langle A(1, 0) \rangle \neq 0$. We shall return to the question of how this occurs at the end of the paper.

A beautiful property of the flipped $SU(5) \times U(1)_x$ model [4] is that the minimal Higgs representations that perform the breaking $SU(5) \times U(1)_X \to SU(3)_C \times SU(2)_L \times U(1)_Y$, namely (10, 1) and $(\overline{10}, -1)$, at the same time solve the triplet–doublet mass-splitting problem [7]. This is the simplest known realization of the missing partner mechanism. More specifically, the chiral supermultiplets

$$H(10,1)=\begin{pmatrix} N^c_H & Q_H \\ & d^c_H \end{pmatrix}, \quad \bar{H}(\overline{10},-1)=\begin{pmatrix} \bar{N}^c_H & \bar{Q}_H \\ & \bar{d}^c_H \end{pmatrix} \tag{7}$$

can break $SU(5)\times U(1)_X \to SU(3)_c\times SU(2)_L\times U(1)_Y$ obtaining a non-vanishing vacuum expectation value in the D-flat direction

$$\langle H(10,1)\rangle=\langle \bar{H}(10,-1)\rangle=\langle N^c_H\rangle=\langle \bar{N}^c_H\rangle\neq 0\,. \tag{8}$$

A superheavy value of this scale induces large masses for D and D^c through the couplings

$$H(10,1)H(10,1)h(5,-2)+\bar{H}(10,-1)\bar{H}(10,-1)h^c(\bar{5},2)$$
$$=N^c_H d^c_H D+Q_H d^c_H h+Q_H Q_H D+\bar{N}^c_H \bar{d}^c_H D^c+\bar{Q}_H d^c_H h^c+\bar{Q}_H\bar{Q}_H D^c\,, \tag{9}$$

while the doublets h and h^c remain massless. Thus, nucleon instability is avoided in a natural way.

A naive SO(10) promotion of (10, 1) and $(\overline{10},-1)$ into 16 and $\overline{16}$ is nevertheless disastrous. $H(\bar{5},-3)$ and $\bar{H}(5,3)$ will have couplings

$$H(\bar{5},-3)H(10,1)h^c(\bar{5},2)+\bar{H}(5,3)\bar{H}(\overline{10},-1)h(5,-2)$$
$$=N^c_H \ell_H h^c+Q_H\ell_H D^c+d^c_H u^c_H D^c+Q_H u^c_H h^c+\bar{N}^c_H\bar{\ell}_H h+\bar{Q}_H\bar{\ell}_H D+\bar{d}^c_H\bar{u}^c_H D+\bar{Q}_H\bar{u}^c_H h\,, \tag{10}$$

which will make the Weinberg–Salam isodoublets supermassive as well. These couplings cannot be avoided in SO(10) since they are related to those shown in (9). as a result, the minimal missing-partner mechanism, based on the $16+\overline{16}$, fails in SO(10).

A representation of SO(10) other than 16, which is suitable for breaking $SU(5)\times U(1)_X$, is the 126. It decomposes as

$$C(126)=C(1,10)+C(\bar{5},2)+C(10,6)+C(\overline{15},-6)+C(45,-2)+C(\overline{50},2)\,. \tag{11}$$

$C(\overline{50},2)$ decomposes under $SU(3)_c\times SU(2)_L\times U(1)_Y$ as

$$C(\overline{50},2)=C(1,1,0)+C(\bar{3},1,2/3)+C(3,2,1/3)+(6,3,2/3)+C(\bar{6},1,4/3)+C(8,2,1)\,. \tag{12}$$

A vacuum expectation value $\langle C(\overline{50},2)\rangle=\langle C(1,1,0)\rangle\neq 0$ breaks $SU(5)\times U(1)_X$ down to $SU(3)\times SU(2)_L\times U(1)_Y$. Let us assume that we have $C(126)+\bar{C}(\overline{126})$ and that a D-flat VEV $\langle C(1,1,0)\rangle=\langle \bar{C}(1,1,0)\rangle\neq 0$ is generated at some superheavy scale. In order to couple $126+\overline{126}$ to the other Higgses present, namely the 10, we need other representations. Since $126\times 10=210+1050$, let us introduce 210 which, as it was mentioned before, could also be useful for the first stage of SO(10) breakdown. The superpotential needed reads

$$W_1=C(126)B(210)h(10)+\bar{C}(\overline{126})B(210)h(10)\,. \tag{13}$$

Assuming that B(210) does not have an expectation value, we obtain

$$C(\overline{50},2)B(75,0)h(5,-2)+\bar{C}(50,-2)B(75,0)h^c(\bar{5},2)+C(1,10)B(\bar{5},-8)h(5,-2)$$
$$+C(\bar{5},2)B(1,0)h(5,-2)+C(\bar{5},2)B(24,0)h(5,-2)+C(10,6)B(\bar{5},-8)h^c(\bar{5},2)$$
$$+C(10,6)B(10,-4)h(5,-2)+C(15,-6)B(5,8)\,h(5,-2)$$
$$+C(45,-2)B(24,0)h^c(\bar{5},2)+C(45,-2)B(75,0)h^c(\bar{5},2)+(\text{analogous terms with }\bar{C})\,. \tag{14}$$

The contributions to masses come from the first two terms. The decomposition of B(75, 0) under $SU(3)_C\times SU(2)_L\times U(1)_Y$ is

$$B(75)=B(1,1,0)+B(3,1,-2/3)+B(3,2,1/3)+B(\bar{3},1,2/3)+B(\bar{3},2,-1/3)$$
$$+B(\bar{6},2,1/3)+B(6,2,-1/3)+B(8,1,0)+B(8,3,0)\,. \tag{15}$$

The first two terms in (14) give ($D=(3,1,-2/3)$, $D^c=(\bar{3},1,2/3)$)

$$\langle C(1,1,0)\rangle DB(\bar{3},1,2/3)+\langle \bar{C}(1,1,0)\rangle D^c B(3,1,-2/3)+\ldots\,, \tag{16}$$

while no corresponding terms for h and h^c exist since there are no colourless isodoublets in B(75).

Therefore, we conclude that (13) can lead to superheavy masses for D and D^c in combination with triplets in $B(75,0)\in B(210)$ when $\langle C(126)\rangle=\langle C(\overline{50},2)\rangle\neq 0$ breaks $SU(5)\times U(1)_X$ down to the standard model. The Weinberg–Salam doublets are kept massless.

B(210) can be given a superheavy mass through a quadratic superpotential and its components pose no threat to mess up the light spectrum. C(126) however, apart from those components of the $C(\overline{50},2)$ that are eaten, would leave us with a huge number of massless stuff unless we include a suitable mass term for it in the superpotential. Since we assumed the B(210) has no expectation value, our only reasonable choice for the first stage of SO(10) breaking is to assume that $\langle A(45)\rangle=\langle A(1,0)\rangle\neq 0$ and give B(126) a mass through its coupling to the A(45) VEV. This is possible only with a 120 representation. The required term reads

$$W_2=C(126)A(45)G(120)+\bar{C}(\overline{126})A(45)G(120)\,. \tag{17}$$

Decomposing G(120) under $SU(5)\times U(1)_X$, we get

$$G(120)=G(5,-2)+G(\bar{5},2)+G(10,6)+G(\overline{10},-6)+G(45,-2)+G(\overline{45},2)\,. \tag{18}$$

We see that, except $C(\overline{15},-6)$, $\bar{C}(15,6)$ and the singlets $C(1,10)$, $\bar{C}(1,-10)$, the components of $C(126)+\bar{C}(\overline{126})$ match exactly with the components of G(120). Therefore, they can obtain large masses from the singlet expectation value $\langle A(1,0)\rangle$. The mass terms induced by (17) are

$$W_2=\langle A(1,0)\rangle[G(5,-2)C(\bar{5},2)+G(\bar{5},2)\bar{C}(5,-2)+G(10,6)\bar{C}(\overline{10},-6)+G(\overline{10},-6)C(10,6)$$
$$+G(45,-2)\bar{C}(\overline{45},2)+G(\overline{45},2)C(45,-2)+\ldots]\,. \tag{19}$$

The Yukawa couplings in W_0 generate quark and lepton masses but also neutrino Dirac masses which are naturally as large as m_d, m_u or m_e. It is however possible to realize the see-saw mechanism if we couple C(126) to quarks and leptons. The required superpotential term is

$$W_3=F(16)F(16)\bar{C}(\overline{126})$$
$$=F(10,1)F(10,1)\bar{C}(50,-2)+F(\bar{5},-3)F(\bar{5},-3)\bar{C}(15,6)+F(1,5)F(1,5)\bar{C}(1,-10)$$
$$+F(\bar{5},-3)F(1,5)\bar{C}(5,-2)+F(10,1)F(10,1)\bar{C}(5,-2)+F(10,1)F(10,1)\bar{C}(5,-2)$$
$$+F(10,1)F(\bar{5},-3)\bar{C}(45,2)\,. \tag{20}$$

The last three couplings do not play any role since $\bar{C}(5,-2)$ and $\bar{C}(45,2)$ are supermassive. The $SU(3)_c\times SU(2)_L\times U(1)_Y$ content of $\bar{C}(15,6)$ is

$$\bar{C}(15,6)=\bar{C}(1,3,2)+\bar{C}(3,2,7/3)+\bar{C}(6,1,8/3)\,, \tag{21}$$

while $\bar{C}(1,-10)=\bar{C}(1,1,-4)$. The second and third terms in (20), in terms of standard model fields, are

$$e^c e^c \bar{C}(1,1,-4)+u^c u^c \bar{C}(6,1,8/3)+\ell\ell\bar{C}(1,3,2)+\ell u^c\bar{C}(3,2,7/3)\,. \tag{22}$$

The first term gives

$$\begin{pmatrix} N^c_{(1,1,0)} & Q_{(3,2,1/3)} \\ & d^c_{(\bar{3},1,2/3)}\end{pmatrix}\begin{pmatrix} N^c & Q \\ & d^c\end{pmatrix}$$
$$\times[\bar{C}(1,1,0)+\bar{C}(3,1,-2/3)+\bar{C}(\bar{3},2,-1/3)+\bar{C}(\bar{6},3,-2/3)+\bar{C}(6,1,-4/3)+\bar{C}(8,2,-1)]$$
$$=N^cN^c\langle\bar{C}(1,1,0)\rangle+N^cd^c\bar{C}(3,1,-2/3)+QQ\bar{C}(3,1,-2/3)+QQ\bar{C}(\bar{6},3,-2/3)$$
$$+d^cd^c\bar{C}(6,1,-4/3)+Qd^c\bar{C}(8,2,-1)\,. \tag{23}$$

Apart from $\bar{C}(\bar{3}, 2, -1/3)$ which together with C(3, 2, 2/3) gets eaten, the rest of the $\bar{C}$-fields appearing in (22) and (23) stay light. Since they participate in no other couplings to light fields they do not induce baryon-number violation apart from higher-dimensional baryon-violating operators in which at least one superheavy field participates.

These fields however can lead to very interesting exotic processes such as $\mu \to e e \bar{e}$ since their couplings need not be flavor diagonal [#1].

The first term in (32) is a Majorana mass for the right-handed neutrino which allows for the standard see-saw mechanism through

$$\begin{pmatrix} \langle \bar{C}(1,1,0) \rangle & \langle h^c \rangle \\ \langle h^c \rangle & 0 \end{pmatrix}, \tag{24}$$

with $m_N \sim \langle \bar{C} \rangle$ and $m_\nu \sim \langle h^c \rangle^2 / \langle \bar{C} \rangle$.

We postponed talking about the superpotential sector that is responsible for the $SO(10) \to SU(5) \times U(1)_X$ symmetry breaking. We have just assumed that A(45) at least is present and that it receives an expectation value $\langle A(1,1,0) \rangle \neq 0$. The simplest possible superpotential that has this property is

$$W_3 = \tfrac{1}{2}\lambda_1 \phi(1) A^2(45) + \tfrac{1}{3}\lambda_2 \phi^3(1) - \mu^2 \phi(1) , \tag{25}$$

where $\phi(1)$ is an SO(10) singlet chiral superfield. A(45) analyzed under $SU(5) \times U(1)_X$ contains A(1, 0), A(10, 4), A($\overline{10}$, −4) and A(24, 0). As long as the singlet acquires a non-vanishing VEV the 10's get eaten and we are left with a singlet and the A(24, 0). The supersymmetric minimum is determined from (15) by

$$\partial W_3 / \partial A = \lambda_1 \phi A = 0 , \quad \partial W_3 / \partial \phi = \tfrac{1}{2}\lambda_1 A^2 + \lambda_2 \phi^2 - \mu^2 = 0 , \tag{26}$$

which get solved for $\phi = 0$ and $\langle A(1,0) \rangle = \sqrt{2}\mu^2/\lambda_1$.
The resulting scalar potential

$$V_F = 2\lambda_1 \mu^2 |\phi(1)|^2 + (\lambda_1 \mu^2 / 2) |A(1,0) + A^*(1,0)|^2 + (\text{cubic and quartic terms}) \tag{27}$$

leaves A(24, 0) massless. This cannot be altered by couplings to B(210). A(24, 0) together with the left-overs of C(126) and $\bar{C}(\overline{126})$ survive massless down to low energies and presumably need a more complicated superpotential structure in order to avoid influencing renormalization group predictions. Nevertheless, the questions of $SO(10) \to SU(5) \times U(1)$ breaking and the realization of the see-saw mechanism are not addressed to the particular model we have proposed but are general questions adressed to any supersymmetric SO(10) GUT.

Summarizing, we have proposed a conventional SO(10) GUT based on the superpotential

$$W = F(16)F(16)h(10) + F(16)F(16)\bar{C}(\overline{126}) + [C(126) + \bar{C}(\overline{126})]B(210)h(10)$$
$$+ [C(126) + \bar{C}(126)]A(45)G(120) + M[B(210)]^2 + W(B(210), A(45), \phi(1)) , \tag{28}$$

with a flipped intermediate $SU(5) \times U(1)_X$ symmetry which exhibits a missing-partner mechanism among 210, 10 and 126. It is an open question whether such a theory can emerge from a manifold compactification of a superstring theory or from a four-dimensional superstring theory with suitable boundary conditions.

[#1] The couplings $N^c d^c$ and QQ to $\bar{C}(3, 1, -2/3)$ are harmless since N^c is supermassive. The operator $QQ(d^c \nu)^*$ is suppressed by the neutrino mass.

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