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FRIEDMANN-LIKE COLLAPSING MODEL OF A RADIATING SPHERE WITH HEAT FLOW

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ABSTRACT

We consider a spherical body consisting of a fluid with heat flow which radiates in its exterior a null fluid described by the outgoing Vaidya's metric. We give a Friedmann-like exact solution of the interior Einstein field equations. We prove that this solution matched with the outgoing Vaidya metric represents a physically reasonable collapsing model which, when the heat flow is switched off, reduces to the well-known collapsing model with dust. Our model has the remarkable property that even if the heat flow is small, the horizon will never be formed because before this happens the collapsing body will be destroyed by opposite gradients of pressure.

Subject headings: hydrodynamics — relativity

I. INTRODUCTION

In two recent papers by de Oliveira, Santos, and Kolassis (1985) and de Oliveira, Pacheco, and Santos (1986), the model of a collapsing sphere consisting of a fluid with heat flow and emitting radiation in the form of a null fluid[2] is investigated. One essential, although not so realistic, property of this model is that the collapse is due only to the heat flow in the sense that when the heat flow is switched off the model becomes static. In this paper we propose a similar collapsing model which does not have this disadvantage. In contrast to the preceding model and to the other known perfect fluid collapsing models, we find that the heat flow has here the remarkable property of preventing the horizon formation. This is due to a reverse pressure gradient which develops before the instant of the horizon formation and which must literally destroy the collapsing sphere even if the heat flow is arbitrarily small (but not null).

Recently the generalization of the nonadiabatic collapse to a magnetohydrodynamic fluid has been given by de Oliveira and Santos (1987).

In § I of this paper we give the field equations which in a general relativistic context govern the gravitational collapse of the sphere. We also obtain in this section the formulae for the total luminosity perceived at infinity and for the redshift of the emitted radiation, which are of particular interest since they are observable quantities. In § II, by making the simplifying assumption that the fluid trajectories are geodesics, we obtain an exact solution of the Einstein field equations for the interior of the sphere which, when the heat flow is switched off, reduces to the Friedmann solution with dust. In § III of this paper the physical conditions are studied, i.e., the behavior of the mass, energy density, pressure, and heat flow of the sphere, to show that the model is physically reasonable.

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[2] The null fluid is also frequently mentioned in the literature as a pure radiation field.

The unique spherically symmetric solution of the Einstein's field equations with a null fluid is known to be that of Vaidya (1953):

$$ds_+^2 = -\left[1 - \frac{2m(v)}{\imath}\right]dv^2 - 2\,dv\,d\imath + \imath^2(d\theta^2 + \sin^2\theta\,d\varphi^2)\,, \tag{1}$$

where the radial coordinate $\imath$ is an affine parameter along the outgoing rays of the null fluid and the quantity $m(v)$ is interpreted as the Newtonian mass of the gravitating body measured by an observer at infinity. The line element (1) describes the exterior part of the spacetime. The interior part is described by a spherically symmetric shear-free line element which in isotropic and comoving coordinates reads

$$ds_-^2 = -A^2(r, t)dt^2 + B^2(r, t)[dr^2 + r^2(d\phi^2 + \sin^2\theta\,d\varphi^2)]\,. \tag{2}$$

The energy momentum tensor which via the Einstein field equations $R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = 8\pi T_{\alpha\beta}$ generates this metric is assumed to be that of a fluid with heat flow

$$T_{\alpha\beta} = (\mu + p)w_\alpha w_\beta + pg_{\alpha\beta} + q_\alpha w_\beta + q_\beta w_\alpha\,, \tag{3}$$

where μ is the energy density in the rest frame of the fluid, p is the isotropic pressure, w^α the four-velocity of the fluid, and q^α is the heat flux vector which has to satisfy $q_\alpha w^\alpha = 0$. Since we use comoving coordinates, the four-velocity of the fluid can be written

$$w^\alpha = A^{-1}\delta_0^\alpha\,. \tag{4}$$

Taking into account the spherical symmetry, the heat flow must be radial, and because of equation (4) and the fact that q^α is orthogonal to w^α, we shall have

$$q^\alpha = q\,\delta_1^\alpha\,. \tag{5}$$

Einstein's field equations for the metric (2) and for the energy momentum tensor (3) with equations (4) and (5) yield (de Oli-