FURTHER ASPECTS OF SUPERCOSMOLOGY

D.V. NANOPOULOS, K.A. OLIVE and K. TAMVAKIS
CERN, Geneva, Switzerland

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Further aspects of the cosmological consequences of supersymmetric GUTs are studied. Special emphasis is placed on the nature of the phase transition and the dynamical creation of matter–antimatter asymmetry. We envisage that the inclusion of an SU(5) singlet(s) can in principle lead to paths with small $[O(10^{10}\ \mathrm{GeV})^4]$ barrier heights. SU(5) singlets will also lead invariably to the desired baryon excess even at temperatures $\lesssim 10^9$ GeV.

It has been known for quite some time that supersymmetric field theories possess remarkable naturalness properties not shared by general renormalizable theories [1]. In particular, quadratic divergences are absent in supersymmetric theories. This property, together with powerful non-renormalization theorems [2], solves, at least technically, the problem posed by scalars in unified gauge models. Scalar fields, a necessary ingredient of spontaneously broken gauge theories, have quadratically divergent masses and thus the only "natural" mass scale for a fundamental scalar would be the cut-off (M_X in GUTs). Scalars can, however, be protected by supersymmetry. Thus, the gauge hierarchy problem ($M_W/M_X \ll 1$), although still put in by hand, is stable in perturbation theory [2]. Nevertheless, no fully satisfactory supersymmetric GUT has yet been constructed, basically because of the absence of a satisfactory supersymmetry breaking scheme.

Supersymmetry, in sharp contrast with gauge symmetries, is manifestly violated at finite temperatures [3] due to the different statistics of bosons and fermions. Supersymmetric GUTs describe the evolution of the early universe, in the framework of the standard Big-Bang cosmological model, in terms of successive phase transitions corresponding to the breakdowns suffered by the gauge symmetries. However, due to the non-trivial constraints imposed by supersymmetry, one is led to cosmological scenarios [4–6] quite different from those encountered in ordinary GUTs.

Recently two of us [5] have examined the cosmological implications of supersymmetry in the framework of an SU(5) GUT and concluded that the universe had to pass through a strongly coupled SU(5) symmetric phase before the phase transition to the SU(3) × SU(2) × U(1) phase becomes possible roughly at temperatures 10^9–10^{10} GeV. As a consequence, monopoles were found to be suppressed and the baryon asymmetry of the universe could be generated. In this paper we return to our scenario in order to clarify the nature of the phase transition on the one hand and various aspects of the dynamics of baryon generation on the other. We find that

(1) the phase transition from SU(5) to SU(3) × SU(2) × U(1) can take place provided the barrier separating the two phases is kept at $O(10^{10}\ \mathrm{GeV})^4$ and

(2) the desired baryon asymmetry can be generated with relatively light Higgs bosons ($m_H \sim 10^{10}$ GeV), which during the intermediate confining phase form SU(5) singlet bound states that subsequently decay.

Let us consider a supersymmetric SU(5) gauge theory with the standard superfield content (V_α, Σ_{24}, H_5, $H_{\bar{5}}$, Q_{10}, $Q_{\bar{5}}$). The Higgs sector of the superpotential is [7]

$$W = \tfrac{1}{2}\lambda M\ \mathrm{tr}(\Sigma^2) + \tfrac{1}{3}\lambda\ \mathrm{tr}(\Sigma^3) + f\bar{H}\Sigma H + h\chi\bar{H}H + \dots \quad (1)$$

where M is the unification mass ($M \sim 10^{16}$ GeV) and χ is an SU(5) singlet superfield introduced to make the triplet–doublet separation in mass natural. The adjoint Higgs contribution to the potential is, at zero temperature,

$$V = \lambda^2 \operatorname{tr}|M\Sigma + \Sigma^2 - \tfrac{1}{5}\operatorname{tr}(\Sigma^2)|^2 + \tfrac{1}{2}g^2 \operatorname{tr}([\Sigma, \Sigma^\dagger])^2. \tag{2}$$

At least three degenerate supersymmetric minima of zero energy occur at

$\langle\Sigma\rangle = 0$ [SU(5) symmetric]

$\langle\Sigma\rangle = \frac{1}{3}M \operatorname{diag}(1, 1, 1, 1, -4)$

[SU(4) × U(1) symmetric]

$\langle\Sigma\rangle = M \operatorname{diag}(2, 2, 2, -3, -3)$

[SU(3) × SU(2) × U(1) symmetric].

At temperatures $T \gg M$ one loop corrections due to the adjoint Higgs superfield Σ yield a temperature dependent term

$$\Delta V \approx \tfrac{1}{20}(21\lambda^2 + 25g^2)T^2 \operatorname{tr}|\Sigma|^2. \tag{3}$$

At such high temperatures, as expected only the SU(5) symmetric phase corresponds to a minimum. As the universe cools down, the broken minima SU(3) × SU(2) × U(1) and SU(4) × U(1) appear. The global minimum, however, is still SU(5) and the unifying gauge symmetry remains unbroken.

Since the natural scale of the unbroken phase is the temperature itself, the gauge coupling runs with temperature. SU(5) is asymptotically free, and therefore the coupling becomes increasingly strong as the temperature falls, finally reaching values of O(1) around [5] $T_{c_1} \approx 10^9$–10^{10} GeV. What happens in the strong coupling region is hard to estimate quantitatively. Nevertheless, one could try to draw conclusions based on qualitative arguments. In the low temperature region, the potential could be approximated by

$$V(T) \approx V(0) - \tfrac{1}{90}\pi^2(N_B + \tfrac{7}{8}N_F)T^4, \tag{4}$$

where $N_B(N_F)$ is the number of bosonic (fermionic) light degrees of freedom. Provided supersymmetry is not broken, one would guess that the dominant effect of "confinement" is to reduce the number of light degrees of freedom and thus shift the symmetric minimum upwards (see fig. 1). As soon as the broken minima lie below the SU(5) minimum, the transition could in principle take place. Two independent questions then arise. First, towards which phase [SU(4) × U(1) or SU(3) × SU(2) × U(1)] does the transition occur, and second, how fast does it occur.

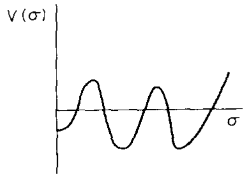

Fig. 1. Effective Higgs potential at $T \sim \Lambda_{SU(5)}$. SU(5) confinement raises the free energy of the SU(5) minimum relative to the SU(3) × SU(2) × U(1) and SU(4) × U(1) minima.

The answer to the first question can be found by counting the number of light degrees of freedom in the broken phases since the zero temperature potential is zero for all three minima (we assume that supersymmetry remains unbroken). The SU(4) × U(1) phase at $T \sim 10^9$–10^{10} GeV is still weakly coupled and contains 16 gauge bosons, three generations of ordinary fermions as well as their superpartners. Thus [‡1]:

$$N = N_B = N_F = 16 \times 2 + 3 \times 15 \times 2 = 122. \tag{5a}$$

Similarly, for the SU(3) × SU(2) × U(1) phase,

$$N = 114 + 4N_H, \tag{5b}$$

where N_H is the number of light complex Higgs doublets. For $N_H = 2$, the free energies are the same,

$$V(4, 1) = V(3, 2, 1) \approx (-61\pi^2/24)T^4, \tag{6}$$

and we can draw no conclusion as to which phase is preferred. However, in order to generate a non-vanishing baryon asymmetry in our scenario, we are forced to consider at least four Higgs doublets [8]. For $N_H = 4$,

$$V(3, 2, 1) \approx (-65\pi^2/24)T^4 < V(4, 1) \approx (-61\pi^2/24)T^4. \tag{7}$$

Thus, for four Higgs doublets the SU(3) × SU(2)

‡1 Notice that in the SU(4) × U(1) phase, the entire Higgs pentuplet is heavy, since the superpotential was chosen to lead to massless doublets in the SU(3) × SU(2) × U(1) phase.

$\times$ U(1) phase is preferred. It is remarkable that the existence of a baryon asymmetry has led to the choice of the (3, 2, 1) minimum. It should also be emphasized that the corresponding value of $\sin^2\theta_W$ is compatible [7] with experiment, in contrast with other varieties of supersymmetric SU(5) [9], mainly due to the fact that we keep the associated Higgs triplets "light", i.e., $m_H \sim 10^{10}$ GeV.

Let us proceed next to examine the second question of how fast the transition occurs. Recently, Srednicki [10] pointed out that the barrier separating the unbroken and broken phases is always of order $(10^{16})^4$ GeV^4 and hence even if the broken minimum were much deeper than the symmetric one, the transition might, in practice, never occur due to the incredibly slow rate of bubble nucleation [11]. He bases his use of the tunnelling picture on the observation that the strong coupling phenomena are generally of order 10^{10} GeV and since the barrier (in his model) in all directions is of order $M^4 \sim (10^{16})^4$ GeV^4, only a small region near the origin is going to be influenced by them. The conclusion of this reasoning is that the transition is never completed and such a universe gets stuck in a donfined SU(5) phase down to temperatures of the order of supersymmetry breaking. This looks like a very sad picture and it certainly does not have to be true. It appears to us that the effective potential is not a very well-defined quantity in a discussion involving confinement. Nevertheless, even in terms of the effective potential, we can argue that the above conclusions are premature.

One should remember that along directions

$$\Sigma = (\sigma/\sqrt{30})\,\mathrm{diag}(2, 2, 2, -3, -3) \qquad (8)$$

we have no contribution from the D term and the height of the barrier can be *arbitrarily low* [$\leqslant$ O(10^{10} GeV)^4] by choosing the parameter λ *small*. The gauge boson masses after the breaking, as well as the D term contribution to the adjoint Higgs masses

$$\mathcal{M}^2_{ab} \sim g^2\,\mathrm{tr}([\langle\Sigma\rangle, T_a]\,[\langle\Sigma\rangle, T_b]) \qquad (9)$$

will be large [$gM \sim$ O(10^{16} GeV)]. Nevertheless, thanks to supersymmetry the height O(10^{10} GeV)^4 of the barrier will be altered only by temperature-dependent corrections which are also O(10^{10} GeV)^4. Consequently at low temperatures ($T \ll gM$), strong coupling phenomena would influence a large part of the barrier and the tunnelling picture should not be

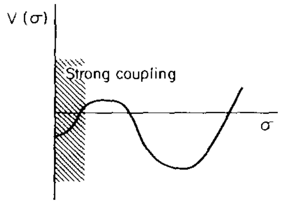

Fig. 2. The potential at $T \sim \Lambda_{SU(5)}$ with a barrier height of the order $\sim(10^{10}$ GeV$)^4$. Strong coupling phenomena affect a region (shaded comparable to the dimensions of the barrier.

applicable. On the contrary, one should expect that at temperatures of O($\Lambda_{SU(5)}$) thermal fluctuations would be of the order of the height of the barrier and the transition would proceed at a reasonable pace (see fig. 2).

The alert reader might object that keeping the barrier low would result in the adjoint Higgs being essentially massless before the transition and thus having to include it in the renormalization group computation of $\Lambda_{SU(5)}$. This would give too low a value for $\Lambda_{SU(5)}$ and might invalidate the scenario at least in its original form. Nevertheless, this appears to be a superficial defect of this particular model and can be avoided by introducing SU(5) singlet superfields in the superpotential in such a way that the adjoint Higgs in the SU(5) phase is massive, but at the same time there are paths in field space along which the barrier will be negligible. In any case one should not take seriously any calculation of the barrier penetration rate since the height is comparable to the range of strong coupling phenomena. Furthermore, we feel that a perturbative expression for $V(\sigma)$ is meaningless in the region of strong couplings. In conclusion, we believe that it is possible to have a fast transition provided we keep the barrier low enough in the SU(3) $\times$ SU(2) $\times$ U(1) direction.

A very important consequence of the fact that the phase transition can only take place below 10^{10} GeV is the suppression of superheavy magnetic monopoles [12,5,6]. The argument goes as follows. Since monopoles are a measure of the lack of correlation between Higgs fields, there should be about one monopole per horizon volume at the time of the phase transition to SU(3) $\times$ SU(2) $\times$ U(1) (remember that the horizon defines the maximum region of causal correlation), i.e.,

$$n_m \sim (2t)^{-3}. \qquad (10)$$

The time temperature relation on the other hand is given by

$$t \sim 10^{-2} M_P/T^2 \tag{11}$$

where M_P is the Planck mass. Hence the number of monopoles per photon is roughly

$$n_m/n_\gamma \sim 10^3 (T/M_P)^3. \tag{12}$$

Since the transition takes place at $T \sim 10^9$ GeV, $n_m/n_\gamma \lesssim 10^{-27}$ and is within the observational limit. Note that any delay in completion of the phase transition results in an even lower monopole abundance.

Let us now examine in some detail the creation of the universal baryon asymmetry in our scheme. As is well known [13] by now, there are three necessary conditions to be satisfied in order to create a baryon asymmetry:

(1) baryon number violating interactions;

(2) C and CP non-conservation and

(3) a departure from thermal equilibrium.

In the previous paper [5] on cosmological implications of supersymmetric GUTs, we had proposed a scenario for the creation of the baryon asymmetry through the decays of relatively light Higgs bosons of mass $m_H \approx 10^{10}$ GeV. In the case of SU(5) these would be 5 and $\bar{5}$ supermultiplets. As the temperature falls down to $T \approx m_H$ any previous asymmetry generated by gauge bosons or more massive Higgs bosons will be washed away [14]. Of course, an initial asymmetry with a nonzero value of some global quantum number such as $B - L$ or 5-ness will not be washed away [15]. The baryon excess that would result from such a light Higgs would be rather small. In standard SU(5) we would have [16]

$$kn_B/s \approx 2 \times 10^{-3} \epsilon [(3K)^{-1.2}/(1 + (3K)^{-1.2})] \tag{13}$$

where s/k is the specific entropy, ϵ is the net baryon number produced by the decay of an H, $\bar{\mathrm{H}}$ pair and $K = 2.9 \times 10^{17}\, \alpha_H/m_H$ is the ratio of the decay rate to the expansion rate at $T = m_H$. If we take the Yukawa coupling $\alpha_H \sim 10^{-4}$ ‡2 and $m_H \sim 10^{10}$ GeV we get

$$kn_B/s \approx 4 \times 10^{-8}\, \epsilon, \tag{14}$$

which seems to be too small to agree with the present cosmological bound $kn_B/s \approx (3.7) \times 10^{-11}$ [17]. In addition to that, in a supersymmetric model the number of annihilation channels for H and $\bar{\mathrm{H}}$ is much greater than in a standard GUT. Therefore, annihilations around $T \simeq m_H$ would dominate and hence greatly reduce the value of kn_B/s given above [18].

Although at first glance baryon generation in supersymmetric SU(5) seems in trouble, the situation is saved by the confining nature of SU(5) near temperatures $T \sim m_H \sim \Lambda_{SU(5)}$ ‡3. As we have seen, the gauge coupling becomes of order one near temperatures $T_{c_1} \approx 10^9$ GeV and SU(5) confinement sets in. In particular, all particles with SU(5) charge such as the Higgs 5 and $\bar{5}$ become confined. If the universe were to remain in the SU(5) phase for temperatures $T < T_{c_1}$, the only allowable degrees of freedom would be SU(5) singlets, i.e., bound states of 5 and $\bar{5}$, etc. What are the properties of such an SU(5) singlet superfield χ? Its coupling to H and $\bar{\mathrm{H}}$ would be effectively as in the expression given by (1). Its total decay rate (both for χ_F and χ_B) would be

$$\Gamma = \tfrac{1}{2}\alpha_\chi (m_\chi^2 - 4m_H^2)^{1/2}, \tag{15}$$

where $\alpha_\chi = h^2/4\pi$. In order to have Higgs triplets of mass $\sim 10^{10}$ GeV, we must choose $f \sim 10^{-6} \sim m_H/M_X$. It is then natural to choose $h \sim f$. Such a small coupling is in itself enough to guarantee that these singlets will live long enough so that when they decay to their constituents, all baryon violating scatterings and H, $\bar{\mathrm{H}}$ annihilations will be ineffective. H and $\bar{\mathrm{H}}$ pairs will subsequently decay and produce the desired baryon asymmetry.

To check this, let us consider the expansion rate of the universe

$$\dot{R}/R \equiv H \approx 10^{-18}\, T^2. \tag{16}$$

The SU(5) singlets χ will begin to decay when $\Gamma \approx H$ or

$$T^2 \approx 5 \times 10^{17}\, \alpha_\chi (m_\chi^2 - 4m_H^2)^{1/2}. \tag{17}$$

If we assume $(m_\chi^2 - 4m_H^2)^{1/2} \sim 10^{10}$ GeV (i.e., we

‡2 It is amusing to note [5] that if we were to couple a pair of Higgs fields to all three generations with couplings $10^{-8} - 10^{-9}$, i.e., identical to the coupling to the lightest generation, the value of kn_B/s would be adequate.

‡3 Incidentally, this value of Λ coincides roughly with the lower mass limit imposed by the stability of the proton [19]. Such "light" Higgs triplets lead [7] to a proton decay rate of $\sim 10^{-31}$ yr^{-1} dominated by the decay modes $p \to \mu^+ K^0$ and $\bar{\nu}_\mu K^+$.

are not requiring any fine tuning between m_χ and m_H) and set $h \sim m_H/M_X$ we find that

$$T/m_H \sim O(10^{13})/M_X \sim O(10^{-3}). \tag{18}$$

Certainly at these temperatures H, $\bar{H}$ annihilations and H mediated scatterings are frozen out.

To be certain that the number of χ's does not diminish through annihilations, we must also compare the expansion rate H with the annihilation rate Γ_α. Without performing a complete numerical integration of these rates, we have estimated the upper limit on the low temperature behaviour of the rate for $\chi_f + \chi_f^* \to H_f + H_f^*$ (this is the dominant reaction). The upper limit for Γ_α is

$$\Gamma_\alpha < [O(10)\alpha_\chi^2/m_\chi^4]\, T^5 \tag{19}$$

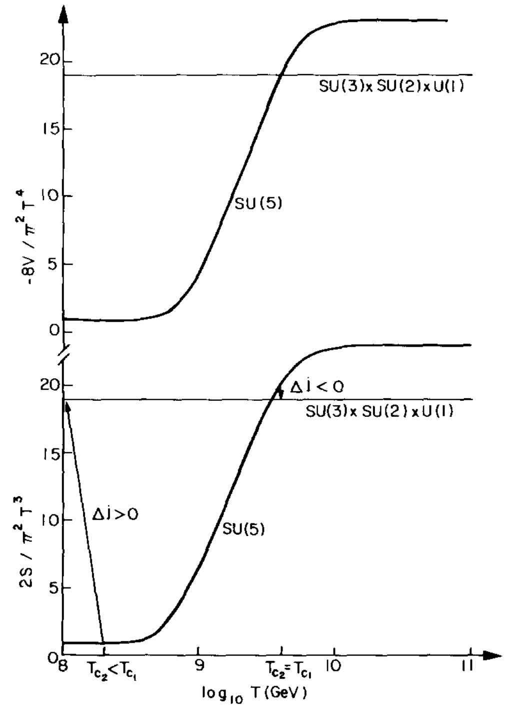

Fig. 3. The free energy density and entropy density (in dimensionless units) are plotted schematically as a function of temperature for the SU(5) and SU(3) × SU(2) × U(1) phases. For definiteness we have chosen $T_{c_1} = 4 \times 10^9$ GeV showing the onset of confinement. If the transition to SU(3) × SU(2) × U(1) takes place at $T_{c_2} = T_{c_1}$ (case 1) the latent heat $l = (\Delta j)T^4$ is negative and the transition occurs at constant temperature (see fig. 4) if $T_{c_2} < T_{c_1}$ (case 2), the latent heat is positive representing the barrier between the two phases.

and at $T \sim 10^{10}$ GeV we see that

$$\Gamma_\alpha/H \sim 10^{48}\alpha_\chi^2/m_\chi^4 \ll 1. \tag{20}$$

Thus annihilations are also not occurring. All that remains to be seen is what is the ratio n_χ/n_γ at the time of the phase transition.

In figs. 3 and 4 we show schematically the free energy entropy and time-temperature relation for the SU(5) and SU(3) × SU(2) × U(1) phases. The quantities for SU(5) have been chosen to show confinement in analogy to the quark–hadron transition [20]. In the absence of supercooling, one would expect the transition to the SU(3) × SU(2) × U(1) phase to occur at a temperature $T_{c_2} \simeq T_{c_1}$. If supercooling does occur $T_{c_2} < T_{c_1}$. Let us consider each of these two possibilities to compute n_χ/n_γ.

(1) *No supercooling.* At the temperature $T = T_{c_1} = T_{c_2}$ the number of χ's formed will be their equilibrium number density at T_{c_1}

$$n_\chi = \frac{T^3}{2\pi^2} \int \frac{\chi^2 d\chi}{\exp[(\chi^2 + z^2)^{1/2} \pm 1} = \frac{T^3}{2\pi^2} I(z), \tag{21}$$

where $z = m_\chi/T$ (the ± refers to χ_F or χ_B). We then have

$$n_\chi/n_{tot} = 4I(z)/7I(0)N \tag{22}$$

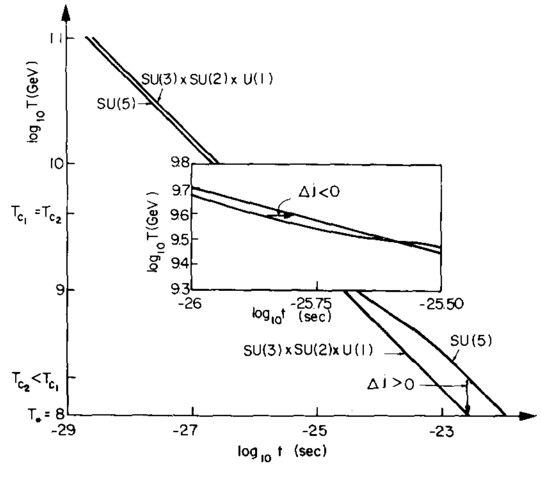

Fig. 4. The time–temperature relation for the SU(5) and SU(3) × SU(2) × U(1) phase, derived from fig. 3. For $T_{c_1} = T_{c_2}$ ($\Delta j < 0$), the transition takes place at constant temperature and takes $\sim 10^{-27}$ s until completion. For $T_{c_2} < T_{c_1}$ ($\Delta j > 0$) there is a decrease in temperature $(T_{c_2}/T_*)^3 \gtrsim 10$ where T_* is the final temperature after the phase transition.

where $I(0) = 2.404$. The number of Higgs produced by the decay of the singlets is just double this. Today, this translates to a net baryon to photon ratio

$$n_B/n_\gamma \approx 0.5\, I(z)\epsilon/N. \qquad (23)$$

If we take as an example $m_\chi = 4 \times 10^{10}$ GeV, $T_{c_1} = 4 \times 10^9$ GeV and $N = 130$ we have

$$n_B/n_\gamma \approx 8 \times 10^{-6}\epsilon \qquad (24)$$

thus in order to have n_B/n_γ between $2-5 \times 10^{-10}$ [17] we need ϵ in the range $2-6 \times 10^{-5}$, a perfectly acceptable value in GUTs [13].

(2) *Supercooling.* In this case, $T_{c_2} < T_{c_1}$. However, the number of χ's formed is the same (corresponding to the temperature T_{c_1}). Whereas in case 1, the latent heat of the transformation (shown in fig. 3 as Δj) is negative, supplying the energy to keep the temperature constant during the transition, in case 2 it may be positive and large. Since the χ's are effectively decoupled (i.e., they behave as a non-interacting gas) there will be a sudden drop in the temperature to compensate for the large Δj. This behaviour is shown schematically in fig. 4. Again since the χ's are unaffected by the transition, this results in "photon" temperature which is different from the temperature of the singlets, we find then that the resulting baryon to photon ratio is increased relative to case 1 by the factor $(T_{c_2}/T_*)^3$ where T_{c_2} is the initial transition temperature and T_* is the final transition temperature. From fig. 4 we can estimate that $(T_{c_2}/T_*)^3 \gtrsim 10$. Thus supercooling enhances the production of the baryon asymmetry in this model. Of course, the supercooling should not be associated with any exponential expansion or de Sitter period which certainly does not occur in this model.

In conclusion we would like to restate our results.

(a) We find that the transition to the broken phase can take place at temperatures 10^9-10^{10} GeV provided that the height of the barrier in the SU(3) $\times$ SU(2) $\times$ U(1) direction is kept small [$\sim(10^{10})^4$ GeV^4].

(b) A baryon asymmetry of the right magnitude can arise due to relatively light Higgs bosons ($m_H \sim 10^{10}$ GeV) which form SU(5) singlet bound states during the confining phase that subsequently decay.

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