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Gauge coupling unification in the SU(5)×U(1) string model

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Recent calculations have shown that the unification scale in the string $SU(5)\times U(1)'\times U(1)^3\times[SU(4)\times SO(10)]_{\text{hidden}}$ model is of the order of 10^{18} GeV. We perform a renormalization group analysis to examine whether it is possible to obtain the experimentally determined values of the low energy parameters $\sin^2\theta_w$ and α_3, including in our calculation the fractionally charges states having non-trivial transformation properties under the hidden SU(4) gauge group. We find that the model – in addition to the three generation multiplets and the standard model Higgses – should contain at least three $(10+\overline{10})$SU(5) representations, while at least two $(3,2)+(\bar{3},2)$ and $(3,1)+(\bar{3},1)$ vector-like SU(3)×SU(2)standard model representations should survive down to an intermediate scale of order 10^8–10^{10} GeV.

Recently it has been shown that threshold corrections [1,2] due to massive string states play a very important role in the determination of the unification scale M_U in effective low energy models derived from superstrings. The scale M_U can be calculated in a large class of string models since in the latter it is possible to know the matter and Higgs spectrum exactly. Such calculations have already appeared for various models derived from the heterotic string [3,4]. In most of the realistic constructions, this scale turns out to be larger than the string scale.

In the $\overline{\text{MS}}$ scheme the string scale is defined by the formula

$$M_{\text{string}}=\left(\frac{2e^{(1-\gamma)}}{3\sqrt{3}}\right)^{1/2} g_{\text{string}}\frac{M_{\text{Pl}}}{\sqrt{8}\,\pi}$$
$$\approx 0.73\, g_{\text{string}}\times 10^{18}\ \text{GeV}\,, \qquad (1)$$

where M_{Pl} is the Planck mass and $g=0.52277$ is the Euler–Mascheroni constant. A scale of this order however, at first sight, is in contradiction with the low energy data. Indeed it is well known that if for example one extrapolates the gauge couplings of the minimal supersymmetric standard model using the renormalization group equations, one finds that the experimentally determined values $\alpha_{\text{em}}(M_Z)=1/128.8$, $\alpha_3(M_Z)\approx 0.11$ and $\sin^2\theta_w(M_Z)\approx 0.233$ are consistent with a unification scale of the order $M_X\approx 10^{16}$ GeV [5]. On the contrary, a scale of order M_U (which is most of the realistic string constructions is found to be of the order of 10^{18} GeV), would lead to $\alpha_3(M_Z)\approx 0.2$ and $\sin^2\theta_w(M_Z)\approx 0.218$, in conflict with the experiment. Thus, the question which naturally arises is whether it is possible to find a viable string model with the proper fermion and Higgs content such as to predict the correct values for the low energy parameters $\alpha_{\text{em}}(M_Z)$, $\alpha_3(M_Z)$ and $\sin^2\theta_w(M_Z)$.

In this letter, we would like to explore in detail the above question in one of the most promising string models which has been derived within the free fermionic formulation of the four dimensional superstring, and is based on the observable SU(5)×U(1) symmetry [6]. A first calculation of the low energy parameters $\alpha_{\text{em}}(M_Z)$, $\alpha_3(M_Z)$ and $\sin^2\theta_w(M_Z)$ with the correct value for M_U has already appeared in ref. [7]. It was shown there, that the low energy values of the latter are consistent with a large scale M_U, only if additional vector-like quark superfields remain in the spectrum of the theory below the SU(5) breaking scale. In the free fermionic construction, the spectrum of the model depends on the choice of a set of basis elements with specific boundary conditions put on the world-sheet fermions. Thus, in order to choose this set it is crucial to know the exact spectrum which is necessary in order to make consistent the large unification scale predicted from the string, with the low energy values of the parameters determined by the experiment. In fact, a more detailed calculation should also include the effects of the fractionally charged states as well as the scale at which the anomalous $U_A(1)$ symmetry of the model breaks down.

It has been pointed out in previous analyses [6,8], that the fractionally charged states (FCPs) which are usually present in this kind of constructions, play a very important role in the evolution of the gauge coupling down to the low energies. In this particular model, these states possess non-trivial transformation properties under the hidden SU(4) gauge group. It is expected that some of them, will earn masses at a high scale, however, some of them will remain unavoidably in the spectrum of the model below the SU(5) breaking scale. It has been argued [9] that they can form bound states at some intermediate scale at which the SU(4) hidden gauge interactions become strong and therefore, in the subsequent, will not affect the renormalization group flow. Thus their effects should be taken into account down to this scale, which has been estimated to be $M_I \approx 10^{10}$–10^{12} GeV. The introduction of the $U_A(1)$ breaking scale is also related to the previous discussion. Indeed, since we must preserve the supersymmetry unbroken, we should let some of the singlet fields Φ_i appearing in the model to develop VEVs in order to cancel the anomalous D-term. The D-flatness condition

$$\sum Q_{iA} |\langle \Phi_i \rangle|^2 + \frac{g^2}{192\pi^2} \operatorname{Tr} U_A = 0 ,$$

$$\operatorname{Tr} U_A = 182 ,$$

predicts an approximate value for the scale $M_A \approx O(10^{-1} M_U)$ at which some of the fractionally charged states as well as other unwanted states receive superheavy masses and decouple from the spectrum.

Let us write down the one loop renormalization group equations for the gauge couplings (we restrict ourselves to models constructed at $k_i = 1$, where k_i is the level of the corresponding Kac–Moody algebra), including the aforementioned threshold corrections:

$$\frac{1}{g_i^2(\mu)} = \frac{1}{g_i^2(M)} + \frac{b_i}{16\pi^2} \ln\left(\frac{M^2}{\mu^2}\right) + \frac{1}{16\pi^2} \Delta_i . \qquad (2)$$

In the above formula, M is the renormalization point below which the effective field theory running of the coupling constants begins, b_i are the one loop beta function coefficients, μ is some field theoretical renormalization scale, while Δ_i are the string threshold corrections to the gauge couplings.

In the flipped SU(5) model, the unification scale has been calculated explicitly in terms of the Planck mass M_{Pl} and the threshold corrections that sum up the effects of all string massive states weighing $M_S > g_{string} M_{Pl}/(8\pi)^{1/2}$. It is determined in terms of the difference between U(1) and SU(5) threshold corrections which are found to be $\delta\Delta = \Delta_1 - \Delta_5 \approx 24.13$ [3], thus in this case one finds that

$$M_U = M_{string} \exp(\delta\Delta / 2\delta b_{15}) \approx 1.3\, g_{string}\, 10^{18}\ \text{GeV} , \qquad (3)$$

where $\delta b_{15} = b_1 - b_5 = \frac{45}{2}$, is the difference between the U(1) and SU(5) one loop beta function coefficients. Thus the only unknown quantity in the above formula is now the value of the gauge coupling g_{string} at the unification scale. This scale however, is by two orders of magnitude larger than the scale at which the couplings in a typical SU(5) supersymmetric scenario meet. In the following we will see that in the case of string SU(5)×U(1) model, one faces the same difficulty.

We are ready now to discuss the implications of the above result on the various low energy parameters in the context of the model under consideration. To this end, let us summarize the various mass scales we are going to use in order to write down the renormalization group equations. In our subsequent analysis, we assume that the mass scales are M_U, M_A, M_X, M_I. M_U, as we already discussed, is the unification scale at which all the gauge couplings of the complete gauge symmetry of the model, namely SU(5)×U(1)×[SU(4)×SO(10)]$_{hidden}$, are equal i.e., $g_5 = g_1 = g_4 = g_{10}$. M_A is the anomalous scale where some of the FCPs and other exotic states may acquire large masses. At the SU(5) breaking scale M_X, which is expected to be close to the scale M_A, we have the following relation between the $U(1)_Y$, U(1) and SU(5) gauge couplings:

$$\frac{25}{\alpha_Y(X)} = \frac{1}{\alpha_5(X)} + \frac{24}{\alpha_1(X)} . \qquad (4)$$

Finally, we have pointed out that the scale M_I is expected to be smaller than M_X. The order of magnitude can be easily calculated if we know the values of the beta function coefficient b_4 between the various mass scales described previously. If we simply assume that some of the SU(4) exotic states (fourplets and sextets) receive masses at the scale M_A, then the scale M_I is found from the formula

$$M_I = M_U \bar{b}_4/b_4 M_A (1-\bar{b}_4/b_4) \exp\left[\frac{2\pi}{b_4}\left(\frac{1}{\alpha_U} - \frac{1}{\alpha_4}\right)\right], \tag{5}$$

where $b_4 = -12 + n_6 + \frac{1}{2}n_4$ is the SU(4) beta function for the range $M_A > \mu > M_I$, while $\bar{b}_4$ is the corresponding one for the range $M_U > \mu > M_A$ (in the case of the revamped version of the model [6], $\bar{b}_4 = -1$). The above formula can be easily generalized in the case where some SU(4) representations earn masses from non-renormalizable terms at some other intermediate scales M_J, $M_U > M_J > M_I$. Now assuming for example an optimistic case where 8 out of the 12 fourplets and 4 out of 5 sextets get masses at M_A, the confinement scale is found to be $M_I \approx 10^{11}$ GeV, in agreement with previous two-loop calculations [8]. From now on, we adopt this value for the scale M_I, while one can easily check that a small change will not have any significance in our calculations.

Let us write down the corresponding equations for the flipped SU(5) model. In the range $M_U > \mu > M_X$ the evolution of the SU(5) and U(1) gauge couplings is given by

$$\frac{1}{\alpha_i(X)} = \frac{1}{\alpha_i(U)} + \frac{\bar{b}_i}{2\pi}\ln\frac{M_U}{M_A} + \frac{b_i}{2\pi}\ln\frac{M_A}{M_X}, \quad i = 1, 5, \tag{6}$$

with an obvious notation for the beta function coefficients which are given by

$$b_5 = -15 + 2n_G + \tfrac{1}{2}n_5 + \tfrac{3}{2}n_{10} + \tfrac{1}{2}n'_5,$$
$$b_1 = 2n_G + \tfrac{1}{2}n_5 + \tfrac{1}{4}n_{10} + \tfrac{5}{8}n_4 + \tfrac{5}{8}n_{ec} + \tfrac{5}{8}n'_5, \tag{7}$$

where n_G is the number of generations, n_{10} is the number of ($`\overline{10}'$, $\frac{1}{2}$) representations, $n_5$ are the representations ($`\bar{5}'$, ± 1) and n'_5 are the ($`\bar{g}'$, $\pm\frac{3}{2}$) pentaplets whilst n_{ec} are the (1, $-\frac{5}{2}$) representations and n_4 are the SU(4) hidden tetraplets. Note that if $n_{10} = n'_5 = n_{ec}$, then these states would form complete generations, but in the string models due to the GSO projection mechanism these numbers are in general different. In the subsequent we will assume that $n'_5 < n_{10}$, n_{ec}, and therefore we absorbe the n'_5 dependence of b_i in the number n_G.

In the range $M_X > \mu > M_Z$, we have the evolution of the standard model gauge couplings which are given by the general formula

$$\frac{1}{\alpha_1(\mu)} = \frac{1}{\alpha_i(X)} + \left(\sum_{J=0}^{N} \frac{b_i^{(J,J+1)}}{2\pi}\ln\frac{M_J}{M_{J+1}}\right), \quad i = Y, 2, 3. \tag{8}$$

The sum runs over all the possible intermediate scales M_J ($M_X = M_0 > M_1 > M_2 > ... > M_N = M_Z$) at which some of the representations might become massive, while $b_i^{(J,J+1)}$ are the corresponding beta function coefficients. The general form of the one-loop beta function coefficients b_i ($i = Y, 2, 3$) is given by

$$b_Y = 2n_G + \tfrac{3}{10}n_2 + \tfrac{1}{5}n_3 + \tfrac{1}{10}n_{32} + \tfrac{3}{5}n_4,$$
$$b_2 = -6 + 2n_G + \tfrac{1}{2}n_2 + \tfrac{3}{2}n_{32},$$
$$b_3 = -9 + 2n_G + \tfrac{1}{2}n_3 + n_{32}, \tag{9}$$

where here n_{32} is the number of $Q = (3, 2, \frac{1}{6})$ supermultiplets, n_3 is the number of $D = (3, 1, \frac{1}{3})$ color triplets and n_2 are the usual Higgs doublets.

Combining the above relations, we can now derive the formulae for the low energy parameters, taking into account the effects of the fractionally charged states of the model. We get

$$\frac{\pi}{6}\left(\frac{3}{5\alpha} - \frac{8}{5\alpha_3}\right) + \tfrac{1}{20}(\boldsymbol{n}_{32} + \boldsymbol{n}_3 - \boldsymbol{n}_4 - \boldsymbol{n}_{ec})\cdot\boldsymbol{Q} - 0.21 = \tfrac{2}{25}(\bar{b}_1 - \bar{b}_5)Q_{U,A} + \tfrac{2}{25}(b_1 - b_5)Q_{A,X} + Q_{X,Z}, \tag{10}$$

$$\sin^2\theta_w = \tfrac{1}{5} + \frac{7}{15}\frac{\alpha}{\alpha_3} - \frac{\bar{b}_1 - \bar{b}_5}{5}\frac{\alpha}{2\pi}Q_{U,A} - \frac{b_1 - b_5}{5}\frac{\alpha}{2\pi}Q_{A,X} + \frac{\alpha}{20\pi}(7\boldsymbol{n}_{32} - 3\boldsymbol{n}_3 - 2\boldsymbol{n}_4 - 2\boldsymbol{n}_{ec})\cdot\boldsymbol{Q} + 0.0029. \tag{11}$$

In the above equations we have denoted $Q_{K,L} = \ln(M_K/M_L)$ and $\boldsymbol{n}_R\cdot\boldsymbol{Q} = \sum_J n_{RJ}Q_{X,J}$ where n_{RJ} is the number of the supermultiplets in the representation R remaining in the massless spectrum from M_U down to the scale M_J. We are not assuming any intermediate scales above M_X since we do not expect nonrenormalizable or any other contributions above this scale. The difference $b_1 - b_5$ is given from (7) (setting $n'_5 = 0$):

$$b_1 - b_5 = 15 + \tfrac{5}{8}(n_4 + n_{ec} - 2n_{10}) . \quad (12)$$

(In the revamped version, in the range (M_U, M_A) we have $\bar{b}_1 - \bar{b}_5 = \frac{45}{2}$.) Finally the above equations contain the two loop corrections for central values of the parameters $\sin^2\theta_w(M_Z)$ and $\alpha_3(M_Z)$. Note that one could also let additional $U^c(\bar{3}, 1, -\frac{2}{3})$ and $L(1, 2, -\frac{1}{2})$ superfields in the light spectrum, however it can be shown that such states in general do not have the desired effect on $\sin^2\theta_w$ and α_3, thus we are assuming that they do not survive below M_X.

Let us now investigate the above equations and see if it is possible to obtain the correct values for $\sin^2\theta_w$ and α_3 parameters at M_Z. First, bearing in mind that such a high unification scale leads to an unacceptably large value for $\alpha_3(M_Z)$ (≈ 0.21), we immediately observe from eqs. (10) and (12) that the value of α_3 at M_Z could be smaller if M_X gets closer to the scale M_A. In fact if we adopt $M_X \approx M_A$ (which seems to be a good choice for other phenomenological reasons to be analyzed elsewhere), we get the maximum benefit for the α_3 value. The same is true for eq. (11). In fact the big positive difference $b_1 - b_5$ will also have a negative effect on the value of $\sin^2\theta_w$ (M_Z), thus again it is desirable to have $M_X \approx M_A$. With these remarks eqs. (10) and (11) are simplified considerably. We may now apply the constraints on the above expressions using the precision LEP data for $\sin^2\theta_w$ and α_3 at M_Z. We get the following constraints on the various mass scales and extra representations:

$$(7\boldsymbol{n}_{32} - 3\boldsymbol{n}_3 - 2\boldsymbol{n}_4 - 2\boldsymbol{n}_{ec}) \cdot \boldsymbol{Q} \approx 8.5 , \quad (13)$$

$$(\boldsymbol{n}_{32} + \boldsymbol{n}_3 - \boldsymbol{n}_4 - \boldsymbol{n}_{ec}) \cdot \boldsymbol{Q} \approx 90 . \quad (14)$$

We may combine the above equations and obtain the constraint

$$(2\boldsymbol{n}_{32} - \boldsymbol{n}_4 - \boldsymbol{n}_{ec}) \cdot \boldsymbol{Q} \approx 56 . \quad (15)$$

For the sake of simplicity, let us assume that all the representations $Q(3, 2, \frac{1}{6})$ get masses at the same scale M_Q while $n_{ec} = 0$. Then the constraint (15) becomes $2n_{32} \ln(M_A/M_Q) - n_4 \ln(M_A/M_I) \approx 56$. If we adopt $n_4 = 4$, $M_A \approx 5 \times 10^{17}$ GeV and $M_I \approx 10^{12}$ GeV, we obtain

$$M_Q \approx M_A \exp(-59/n_{32}) . \quad (16)$$

Thus in order to obtain the correct values for the low energy parameters, we must include additional $Q(3, 2, \frac{1}{6})$ representations in the spectrum of the theory. As it can been seen from (16) their mass depends crucially on their number, while for a reasonable number of them, they become massive at a scale much lower than the grand unification mass. Obviously, a reasonable intermediate scale ($M_I \approx 10^8$–10^{10} GeV), would require at least two additional light $Q + \bar{Q}$ pairs, instead of one if we simply set $M_U \approx M_A$, and ignore the exotic hidden representations [7].

It is interesting to return back now to eqs. (13) and (14), and see what other conditions should be met in order for the flipped scenario to work. Assuming that the only other particles remaining in the light spectrum are the color triplets D, D^c, one finds that they must receive masses at an average scale $M_D \approx M_A \exp(-88/n_3)$. Again two of them would lead to an unacceptably low scale thus there is the need of at least four such representations.

The above results have the following implications in the string spectrum of the model. Since the SU(5) breaking takes place through the VEVs of one pair $(10 + \overline{10})$ of Higgs fields, this means that the $Q + \bar{Q}$ piece of these tenplets is eaten by the Higgs mechanism. Since now we need at least two more such pairs to survive the Higgs mechanism and remain down to a scale of order 10^{10} GeV, the string spectrum of the model in addition to the three $10 + \bar{5} + 1$ complete generations should also include at least $3 \times (10 + \overline{10})$ pairs, as well as those Higgses which are necessary for the electroweak breaking.

A more involved situation arises in the case where some of the hidden gauge group representations develop VEVs. As a matter of fact, a realistic scenario would necessarily include this possibility, since in most of the cases other mechanisms (like non-renormalizable interactions and VEVs of singlet superfields) are not sufficient either to provide with superheavy masses all the unwanted states or to produce the required KM mixing in the quark mass matrix. In such a case the hidden symmetry (SU(4)×SO(10) in this particular model), will break to a smaller one (i.e. SO(5), SU(3), SO(4) etc. for the case of SU(4) and similarly for SO(10)). As a result the confinement mechanism may take place at a different scale or may not take place at al. Then the confinement scale M_C depends crucially on the SU(4) breaking scale M_4 and obviously on the beta function coefficients. Assuming for the sake of simplicity the breaking SU(4)→SU(3), we may obtain M_C, from a formula similar to (3). Indeed if for example we assume that $M_4 \approx 10^{14}$ GeV and $b_4 = -6$, then one finds that $b_3 = 3 + b_4 = -3$ (note that $n_3 = 2n_6 + n_4$ at M_4) and $M_C \approx 10^7$ GeV. However, a small change in the involved parameters will change drastically this scale.

In conclusion, we have discussed the possibility of retaining the successful predictions for $\sin^2\theta_w$ and α_3 parameters in the flipped SU(5) string model, using the renormalization group equations and introducing the unification scale calculated taking into account the string threshold corrections in this particular model [7]. We have included in our calculation the effects of the fractionally charged states and we have assumed that the anomalous U(1) breaking scale is the same with the SU(5) breaking scale. We have found that the unification in this particular model can occur in the case where the string spectrum contains at least two $(10+\overline{10})$ SU(5) representations, in addition to the one $(10+\overline{10})$ such pair needed to break the SU(5) symmetry down to the standard model gauge group. Furthermore, the light spectrum should include at least two vector-like $(3,2)+(\bar{3},2)$ representations until an intermediate scale of the order 10^8–10^{10} GeV, which obviously arise from the decomposition of the two extra $(10+\overline{10})$ SU(5) representations. Moreover there should survive at least two $(3,1)+(\bar{3},1)$ representations at approximately the same scale.

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