Physics Letters B 278 (1992) 257–265
North-Holland

PHYSICS LETTERS B

Gauge symmetry breaking in the hidden sector of the flipped SU(5)×U(1) superstring model

I. Antoniadis, J. Rizos [1]
Centre de Physique Théorique, École Polytechnique, F-91128 Palaiseau Cedex, France

and

K. Tamvakis
Theoretical Physics Division, University of Ioannina, GR-451 10 Ioannina, Greece

Received 20 December 1991

We analyze the $SU(5)\times U(1)'\times U(1)^4\times SO(10)\times SU(4)$ superstring model with a spontaneously broken hidden sector down to $SO(7)\times SO(5)$ taking into account non-renormalizable superpotential terms up to eight order. As a result of the hidden sector breaking the "exotic" states get a mass and the "observable" spectrum is composed of the standard three families. In addition, Cabibbo mixing arises at sixth order and an improved fermion mass hierarchy emerges.

One of the very few realistic superstring models [1,2] that are phenomenologically promissing is the "flipped" [3][#1] $SU(5)\times U(1)'\times U(1)^4\times SU(10)\times SU(4)$ superstring model. In recent articles [5–9] the model has been studied thoroughly taking into account non-renormalizable corrections [6] to the chiral superpotential up to fifth order. The result of this analysis was to determine a VEV pattern that satisfies *F*- and *D*-flatness and predicts an almost realistic hierarchical mass structure for the matter fermions. The gauge symmetry breaking pattern $SU(5)\times U(1)'\times U(1)^4\to SU(5)\times U(1)'\to SU(3)_c\times SU(2)_L\times U(1)_Y$ leaves the "hidden sector" gauge group $SO(10)\times SU(4)$ unbroken. A problematic leftover of the analysis was the presence of the massless exotic states f_4, $\bar{l}_4^c$ (see table 1) together with a combination of surplus superfields in standard matter representations ($\bar{f}_i$, l_i^c, $i=1, ..., 3$). Another potential problem of the model is the existence of baryon number violating terms appearing as effective dimension 5 operators [10,11]. This problem, as it has been argued [11] is related to the previous one since these terms involve the surplus matter-like fields.

The model is derived in the free fermionic formulation of four dimensional strings and is defined by eight vectors of boundary conditions for all world sheet fermions. The massless spectrum generated by this basis is composed of the graviton supermultiplet, the gauge supermultiplets, and the seventy chiral superfields listed in tables 1 and 2. In a previous article [7] the quartic and quintic corrections to the cubic superpotential were computed and it was shown that the *F*- and *D*-flatness conditions were satisfied with the following basic choice of vanishing VEVs:

$$\Phi_{12}=\bar{\Phi}_{12}=\Phi_3=\Phi_4=\phi_3=F_4=T_i=\Delta_i=0\,, \qquad (1)$$

together with eight solvable constraint equations. This solution predicts two pairs of Higgs isodoublets and one extra pair of triplets. In fact, it can been shown [12,13] that, ignoring the $SU(5)\times U(1)$ breaking VEVs (i.e. taking $F_i=\bar{F}_5=0$), this basic choice satisfies *F*-flatness to all orders. Nevertheless, an unpleasant feature of this solution is that the exotic states f_4, $\bar{l}_4^c$ that cannot be part of the low energy phenomenology stay massless to the computed order of non-

[1] Research supported by an EEC contract Ref. B/SC1*-915053.
[#1] See also ref. [4].

Table 1
Chiral superfields in terms of their SU(5)×U(1)′×U(1)^4 quantum numbers.

$F_1(10, \frac{1}{2}; -\frac{1}{2}, 0, 0, 0)$	$\bar{f}_1(\bar{5}, -\frac{3}{2}; -\frac{1}{2}, 0, 0, 0)$	$l_1^c(1, \frac{5}{2}; -\frac{1}{2}, 0, 0, 0)$
$F_2(10, \frac{1}{2}; 0, -\frac{1}{2}, 0, 0)$	$\bar{f}_2(\bar{5}, -\frac{3}{2}; 0, -\frac{1}{2}, 0, 0)$	$l_2^c(1, \frac{5}{2}; 0, -\frac{1}{2}, 0, 0)$
$F_3(10, \frac{1}{2}; 0, 0, \frac{1}{2}, -\frac{1}{2})$	$\bar{f}_3(\bar{5}, \frac{3}{2}; 0, 0, \frac{1}{2}, \frac{1}{2})$	$l_3^c(1, \frac{5}{2}; 0, 0, \frac{1}{2}, \frac{1}{2})$
$F_4(10, \frac{1}{2}; -\frac{1}{2}, 0, 0, 0)$	$f_4(5, \frac{3}{2}; \frac{1}{2}, 0, 0, 0)$	$\bar{l}_4^c(1, -\frac{5}{2}; \frac{1}{2}, 0, 0, 0)$
$F_5(\overline{10}, -\frac{1}{2}; 0, \frac{1}{2}, 0, 0)$	$\bar{f}_5(\bar{5}, -\frac{3}{2}; 0, -\frac{1}{2}, 0, 0)$	$l_5^c(1, \frac{5}{2}; 0, -\frac{1}{2}, 0, 0)$
$h_1(5, -1; 1, 0, 0, 0)$	$h_2(5, -1; 0, 1, 0, 0)$	$h_3(5, -1; 0, 0, 1, 0)$
$\bar{h}_1(\bar{5}, 1; -1, 0, 0, 0)$	$\bar{h}_2(\bar{5}, 1; 0, -1, 0, 0)$	$\bar{h}_3(\bar{5}, 1; 0, 0, -1, 0)$
$h_{45}(5, -1; -\frac{1}{2}, -\frac{1}{2}, 0, 0)$	$\bar{h}_{45}(\bar{5}, 1; \frac{1}{2}, \frac{1}{2}, 0, 0)$	
$\phi_{45}(1, 0; \frac{1}{2}, \frac{1}{2}, 1, 0)$	$\phi_+(1, 0; \frac{1}{2}, -\frac{1}{2}, 0, 1)$	$\phi_-(1, 0; \frac{1}{2}, -\frac{1}{2}, 0, -1)$
$\bar{\phi}_{45}(1, 0; -\frac{1}{2}, -\frac{1}{2}, -1, 0)$	$\bar{\phi}_+(1, 0; -\frac{1}{2}, \frac{1}{2}, 0, -1)$	$\bar{\phi}_-(1, 0; -\frac{1}{2}, \frac{1}{2}, 0, 1)$
$\Phi_{23}(1, 0; 0, -1, 1, 0)$	$\Phi_{31}(1, 0; 1, 0, -1, 0)$	$\Phi_{12}(1, 0; -1, 1, 0, 0)$
$\bar{\Phi}_{23}(1, 0; 0, 1, -1, 0)$	$\bar{\Phi}_{31}(1, 0; -1, 0, 1, 0)$	$\bar{\Phi}_{12}(1, 0; 1, -1, 0, 0)$
$\phi_i(1, 0; \frac{1}{2}, -\frac{1}{2}, 0, 0)$, $i=1, ..., 4$	$\bar{\phi}_i(1, 0; -\frac{1}{2}, \frac{1}{2}, 0, 0)$, $i=1, ..., 4$	$\Phi_I(1, 0; 0, 0, 0, 0)$, $I=1, ..., 5$

Table 2
Chiral superfields in terms of their U(1)′×SO(10)×SO(6) ×U(1)^4 quantum numbers.

$\Delta_1(0; 1, 6; 0, -\frac{1}{2}, \frac{1}{2}, 0)$	$T_1(0; 10, 1; 0, -\frac{1}{2}, \frac{1}{2}, 0)$
$\Delta_2(0; 1, 6; -\frac{1}{2}, 0, \frac{1}{2}, 0)$	$T_2(0; 10, 1; -\frac{1}{2}, 0, \frac{1}{2}, 0)$
$\Delta_3(0; 1, 6; -\frac{1}{2}, -\frac{1}{2}, 0, \frac{1}{2})$	$T_3(0; 10, 1; -\frac{1}{2}, -\frac{1}{2}, 0, -\frac{1}{2})$
$\Delta_4(0; 1, 6; 0, -\frac{1}{2}, \frac{1}{2}, 0)$	$T_4(0; 10, 1; 0, \frac{1}{2}, -\frac{1}{2}, 0)$
$\Delta_5(0; 1, 6; \frac{1}{2}, 0, -\frac{1}{2}, 0)$	$T_5(0; 10, 1; -\frac{1}{2}, 0, \frac{1}{2}, 0)$
$\bar{X}_1(-\frac{5}{4}; 1, \bar{4}; -\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{2})$	$\bar{X}_2(-\frac{5}{4}; 1, \bar{4}; -\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$
$Y_1(\frac{5}{4}; 1, 4; -\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, \frac{1}{2})$	$Y_2(\frac{5}{4}; 1, 4; -\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, -\frac{1}{2})$
$Z_1(-\frac{5}{4}; 1, 4; \frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, \frac{1}{2})$	$\bar{Z}_1(\frac{5}{4}; 1, \bar{4}; -\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$
$Q_1(\frac{5}{4}; 1, 4; -\frac{1}{4}, \frac{3}{4}, +\frac{1}{4}, 0)$	$\bar{Q}_1(-\frac{5}{4}; 1, \bar{4}; -\frac{3}{4}, \frac{1}{4}, -\frac{1}{4}, 0)$
$Y'_2(\frac{5}{4}; 1, 4; -\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, -\frac{1}{2})$	$\bar{Y}_1(-\frac{5}{4}; 1, \bar{4}; \frac{1}{4}, -\frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$
$X_1(\frac{5}{4}; 1, 4, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{2})$	$\bar{X}'_2(-\frac{5}{4}; 1, \bar{4}; -\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$

renormalizable interactions. On the other hand, as it has been pointed out [11], at fifth and sixth order there exist terms that involve $\bar{f}_3$, and l_3^c which generate $d=5$ baryon number violating operators and would lead to proton decay if these states were to stay massless and to be interpreted as part of a matter generation. It is therefore highly desirable to render the pairs $\bar{f}_3, f_4$ and l_3^c, $\bar{l}_4^c$ superheavy.

As it can be seen in tables 1 and 2, the quantum number signature of the products $f_4\bar{f}_3$ and $\bar{l}_4^c l_3^c$ is (**1**, 0; $\frac{1}{2}$, 0, $\frac{1}{2}$, $\frac{1}{2}$) under SU(5)×U(1)′×U(1)^4. All chiral fields have an integer charge for the last U(1) gauge group except $F_3(-\frac{1}{2})$, $\Delta_3(\frac{1}{2})$ and $T_3(-\frac{1}{2})$. Thus, a mass term for the unwanted exotic states must necessarily involve at least one of these fields an odd number of times. This holds to all orders of non-renormalizable couplings. Already at fourth order such a term is present in the form

$$(f_4\bar{f}_3+\bar{l}_4^c l_3^c)(T_3\cdot T_4)/M\,,$$

where $M\sim 10^{18}$ GeV. At sixth order also we have the similar terms

$$(f_4\bar{f}_3+\bar{l}_4^c l_3^c)\left[\frac{T_3\cdot T_4}{M^3}\left(\sum_{i=1}^{4}\bar{\phi}_i\phi_i+\bar{\phi}_+\phi_+ +\bar{\phi}_-\phi_- +\bar{\phi}_{45}\phi_{45}+\bar{\Phi}_{31}\Phi_{31}\right) + \frac{T_3\cdot T_5}{M^3}\,(\phi_2\bar{\Phi}_{23}+\bar{\phi}_2\Phi_{31})\right].$$

This suggests to investigate the (partial) breakdown of the hidden SO(10) group. Note that although in principle the desired mass terms could arise also as $\bar{f}_3 f_4\phi^n(F_3\cdot\bar{F}_5)^m/M^{1-n-2m}$, in which case there would be no necessity to break the hidden sector, no such terms were found up to eight order.

Similar considerations apply to our search for a solution to another drawback of the model, namely, the absence of Cabibbo mixing. The only field that contains charge $\frac{1}{3}$ quarks that stay naturally massless up to fifth order is F_3. It seems reasonable to adopt the interpretation that d-quarks are dominantly in F_3. The Cabibbo angle would arise from the appearance of terms of the type $F_i\cdot F_3$ with $i=1, 2, 4$. The quantum number structure of these is either ($\bar{\mathbf{5}}$, 1; $-\frac{1}{2}$, 0, $\frac{1}{2}$, $-\frac{1}{2}$) or ($\bar{\mathbf{5}}$, 1; 0, $-\frac{1}{2}$, $\frac{1}{2}$, $-\frac{1}{2}$). Again, all chiral fields have an integer charge under the fourth U(1) except F_3, Δ_3 and T_3. In fact, at sixth order a term of this kind appears. This term is

$$F_2\cdot F_3\, h_1\,\frac{(\Delta_2\cdot\Delta_3)\Phi_{23}}{M^3}$$

and requires a broken SU(4) via $\Delta_2, \Delta_3\neq 0$.

Before we proceed to investigate in detail the development of non-vanishing VEVs by hidden sector fields, we should mention that there exists an alternative approach that appears to provide us with similar results based on the existence of non-vanishing bilinear hidden sector condensates [14]. Nevertheless, compatibility with the F-flatness constraints is not easy to achieve for the desired pattern of condensates. Contrary to the case of individual field VEVs that are only constrained by F- and D-flatness, the bilinear condensates are in addition related to the gaugino condensate in the limit of global supersymmetry. It seems that condensation phenomena are unavoidably entangled with the question of the supersymmetry breaking mechanism which has not yet been understood. A point that needs to be mentioned is that in the "partially broken hidden sector" approach the strength of condensation phenomena is significantly decreased in proportionality to the scale of the surviving hidden gauge groups. Thus, they become much less important and it is justifiable to ignore them. On the other hand, the scale of the surviving hidden gauge group that results from SU(4), i.e. either SO(5) or SU(2)×SU(2), should not be too low so that fractionally charged states are permanently confined [15].

Let us now write down the F-flatness equations up to sixth order adopting the following initial vanishing VEV choice:

$$\begin{aligned}&\Phi_{12}=\bar\Phi_{12}=\Phi_{I=1,\ldots,5}=\Delta_1=\Delta_4=\Delta_5=T_1=F_2=F_4\\&\quad=\phi_1=\bar\phi_2=\phi_3=0\,.\end{aligned}\tag{2}$$

According to ref. [7] the choice $\Phi_{12}=\bar\Phi_{12}=\Phi_I=F_4=\phi_3=0$ is required by the tree level F- and D-flatness constraints, $\bar\phi_2=\phi_1=0$ preserves the tree level top mass coupling and $F_2=0$ is necessary to avoid $m_b=m_s$. Among the other hidden sector VEVs we shall insist that at least Δ_2, Δ_3, T_3, $T_4\neq 0$ for the reasons we have explained before and we shall also allow T_2, $T_5\neq 0$ since the F-flatness constraints assocciate these VEVs with Δ_2 and T_4 respectively. The non-trivial F-flatness constraints in this case are

$$\frac{\delta W}{\delta F_2}=\bar F_5\phi_2(T_2\cdot T_4)/M^2\approx 0\,,\tag{3a}$$

$$\frac{\delta W}{\delta F_3}=F_3\bar F_5^2\bar\phi_{45}\phi_+\approx 0\,,\tag{3b}$$

$$\frac{\delta W}{\delta \bar F_5}=F_3^2\bar F_5\bar\phi_{45}\phi_+\approx 0\,,\tag{3c}$$

$$\begin{aligned}\frac{\delta W}{\delta\phi_2}&=(T_4\cdot T_5)/\sqrt{2}\\&\quad+[\phi_2(T_2^2+\Delta_2^2+T_5^2)\bar\Phi_{23}+T_4^2\bar\Phi_{31}]/M^2\approx 0\,,\end{aligned}\tag{3d}$$

$$\frac{\delta W}{\delta\phi_4}=\phi_4[\bar\Phi_{23}(T_5^2+T_2^2+\Delta_2^2)+T_4^2\bar\Phi_{31}]/M^2\approx 0\,,\tag{3e}$$

$$\begin{aligned}\frac{\delta W}{\delta\phi_+}&=\phi_-[\bar\Phi_{23}(T_5^2+T_2^2+\Delta_2^2)+\bar\Phi_{31}T_4^2]/M^2\\&\quad+\{(\bar F_5\cdot F_3)^2\bar\phi_{45}+[(T_3^2T_4^2)+(T_3\cdot T_4)^2]\phi_{45}\}/M^3\\&\quad\approx 0\,,\end{aligned}\tag{3f}$$

$$\begin{aligned}\frac{\delta W}{\delta\phi_-}&=\phi_+[\bar\Phi_{23}(T_5^2+T_2^2+\Delta_2^2)+\bar\Phi_{31}T_4^2]/M^2\\&\quad+T_4^2\Delta_3^2\phi_{45}/M^3\approx 0\,,\end{aligned}\tag{3g}$$

$$\begin{aligned}\frac{\delta W}{\delta\phi_{45}}&=\{T_4^2\Delta_3^2\phi_-+[(T_3^2T_4^2)+(T_3\cdot T_4)^2]\phi_+\}/M^3\\&\quad\approx 0\,,\end{aligned}\tag{3h}$$

$$\frac{\delta W}{\delta\bar\phi_{45}}=F_3^2\bar F_5^2\phi_+\approx 0\,,\tag{3i}$$

$$\frac{\delta W}{\delta\Phi_{12}}=\Phi_{23}\Phi_{31}+\tfrac12(\phi_2^2+\phi_4^2)^+\phi_-\phi_+=0\,,\tag{3j}$$

$$\begin{aligned}\frac{\delta W}{\delta\bar\Phi_{12}}&=\bar\Phi_{23}\bar\Phi_{31}+\bar\phi_+\bar\phi_-+\tfrac12(\bar\phi_1^2+\bar\phi_3^2+\bar\phi_4^2)\\&\quad+[(F_1\cdot\bar F_5)^2+(T_4\cdot T_5)^2+(T_4\cdot T_2)^2\\&\quad+T_4^2(T_5^2+T_2^2+\Delta_2^2)]/M^2\approx 0\,,\end{aligned}\tag{3k}$$

$$\frac{\delta W}{\delta\Phi_{23}}=\tfrac12 T_4^2\approx 0\,,\tag{3ℓ}$$

$$\frac{\delta W}{\delta\bar\Phi_{23}}=(T_2^2+\Delta_2^2+T_5^2)(\phi_-\phi_++\phi_2^2+\phi_4^2)/M^2\approx 0\,,\tag{3m}$$

$$\frac{\delta W}{\delta \Phi_{31}} = \tfrac{1}{2}(T_2^2 + \Delta_2^2 + T_5^2) = 0\,, \tag{3n}$$

$$\frac{\delta W}{\delta \bar{\Phi}_{31}} = T_4^2(\phi_-\phi_+ + \phi_2^2 + \phi_4^2)/M^2 \approx 0\,, \tag{3o}$$

$$\frac{\delta W}{\delta \Phi_1} = (\Delta_2^2 + T_2^2)\bar{\Phi}_{23}(\phi_-\phi_+ + \phi_2^2 + \phi_4^2)/M^3 \approx 0\,, \tag{3p}$$

$$\frac{\delta W}{\delta \Phi_3} = \tfrac{1}{2}(\phi_4\bar{\phi}_4 + \phi_{45}\bar{\phi}_{45} + \phi_-\bar{\phi}_- + \phi_+\bar{\phi}_+) = 0, \tag{3q}$$

$$\frac{\delta W}{\delta \Phi_4} = \bar{\phi}_1\phi_2 = 0\,, \tag{3r}$$

$$\frac{\delta W}{\delta \Phi_5} = \bar{\phi}_3\phi_4 = 0\,, \tag{3s}$$

$$\frac{\delta W}{\delta \Delta_2} = \Delta_2[\Phi_{31} + \bar{\Phi}_{23}(\phi_-\phi_+ + \phi_2^2 + \phi_4^2)/M^2] \approx 0\,, \tag{3t}$$

$$\frac{\delta W}{\delta \Delta_3} = T_4^2\Delta_3\phi_-\phi_{45}/M^3 \approx 0\,, \tag{3u}$$

$$\frac{\delta W}{\delta \Delta_4} = (\bar{F}_5\cdot F_3)\Delta_3\bar{\phi}_3\Phi_{31}/M^3 \approx 0\,, \tag{3v}$$

$$\begin{aligned}\frac{\delta W}{\delta \Delta_5} &= (\bar{F}_5\cdot F_3)\Delta_3[1 + (\phi_4\bar{\phi}_4 + \phi_{45}\bar{\phi}_{45} \\ &\quad + \bar{\phi}_+\phi_+ + \phi_-\bar{\phi}_- + \bar{\Phi}_{23}\Phi_{23})/M^2]/M \\ &\quad + [\Delta_2(T_2\cdot T_4)\phi_2]/M^2 \approx 0\,,\end{aligned} \tag{3w}$$

$$\frac{\delta W}{\delta T_2} = T_2[\Phi_{31} + \bar{\Phi}_{23}(\phi_-\phi_+ + \phi_2^2 + \phi_4^2)/M^2] \approx 0\,, \tag{3x}$$

$$\frac{\delta W}{\delta T_3} = [T_4^2 T_3 + (T_4\cdot T_3)T_4]\phi_+\phi_{45}/M^3 \approx 0\,, \tag{3y}$$

$$\begin{aligned}\frac{\delta W}{\delta T_4} &= \phi_2 T_5/\sqrt{2} \\ &\quad + T_4[\Phi_{23} + \bar{\Phi}_{31}(\phi_-\phi_+ + \phi_2^2 + \phi_4^2)/M^2 \\ &\quad + (\Delta_3^2\phi_-\phi_{45} + T_3^2\phi_+\phi_{45})/M^3 \\ &\quad + (T_4\cdot T_3)T_3\phi_+\phi_{45}/M^3] \approx 0\,,\end{aligned} \tag{3z}$$

$$\begin{aligned}\frac{\delta W}{\delta T_5} &= \phi_2 T_4/\sqrt{2} \\ &\quad + T_5[\Phi_{31} + \bar{\Phi}_{23}(\phi_-\phi_+ + \phi_2^2 + \phi_4^2)/M^2] \approx 0\,,\end{aligned} \tag{3aa}$$

where W denotes the superpotential. The wiggled equality symbol indicates that in front of all non-renormalizable interactions we do not display the calculable numerical coefficient. Note also that all couplings involving Δ_2^2 and T_2^2 are identical up to a fixed phase.

In addition to F-flatness we also have the D-flatness conditions. Starting with the observable sector $SU(5)\times U(1)'\times U(1)^4$ we have the following five conditions:

$$|F_1|^2 + |F_3|^2 = |\bar{F}_5|^2\,, \tag{4a}$$

$$\begin{aligned}|\phi_{45}|^2 - |\bar{\phi}_{45}|^2 &= \tfrac{1}{15}\xi + \tfrac{1}{4}(|F_1|^2 - |F_3|^2 - |\bar{F}_5|^2) \\ &\quad + \tfrac{1}{2}(|\Delta_3|^2 + |T_3|^2)\,,\end{aligned} \tag{4b}$$

$$\begin{aligned}|\Phi_{23}|^2 - |\bar{\Phi}_{23}|^2 &= |\bar{\phi}_+|^2 - |\phi_+|^2 - \tfrac{1}{2}(|\phi_2|^2 + |\phi_4|^2) \\ &\quad + \tfrac{1}{2}(|\bar{\phi}_1|^2 + |\bar{\phi}_3|^2 + |\bar{\phi}_4|^2) \\ &\quad + \tfrac{1}{8}(|F_1|^2 + |F_3|^2 + 3|\bar{F}_5|^2) \\ &\quad + \tfrac{1}{2}(|T_4|^2 - |\Delta_3|^2)\,,\end{aligned} \tag{4c}$$

$$\begin{aligned}&|\Phi_{23}|^2 - |\bar{\Phi}_{23}|^2 - |\Phi_{31}|^2 + |\bar{\Phi}_{31}|^2 \\ &\quad = -\tfrac{1}{5}\xi - \tfrac{1}{2}(|\Delta_2|^2 + |\Delta_3|^2) \\ &\quad - \tfrac{1}{2}(|T_2|^2 + |T_3|^2 + |T_5|^2 - |T_4|^2)\,,\end{aligned} \tag{4d}$$

$$\begin{aligned}&|\bar{\phi}_-|^2 - |\bar{\phi}_+|^2 + |\phi_+|^2 - |\phi_-|^2 \\ &\quad = \tfrac{1}{15}\xi + \tfrac{1}{2}(|F_3|^2 + |T_3|^2 - |\Delta_3|^2)\,,\end{aligned} \tag{4e}$$

with $\xi = 90g^2 e^{\Phi}/96\pi^2$, where g is the gauge coupling constant and Φ is the dilaton field. Note that the conditions (4) naturally imply VEVs of order $\frac{1}{10}M$ for the various fields justifying a perturbative treatment. The $SO(10)\times SU(4)$ D-flatness constraints are generally expressed as

$$\sum_{I=2,3} \Delta_I^* \tau^\alpha \Delta_I = 0\,, \tag{4f}$$

$$\sum_{I=2,3,4,5} T_I^* \lambda^A T_I = 0\,, \tag{4g}$$

where τ^α, with $\alpha = 1, ..., 15$, are the $SU(4)$ generators and λ^A, with $A = 1, ..., 45$, are the $SO(10)$ ones.

The most important prerequisite of the flatness solution that we are after is that the Higgs mass matrix should allow for at least one pair of massless Higgs isodoublets which should include $\bar{h}_{45}$ and some linear combination of h_1, h_2 in order to preserve the tree level up–down quark masses. We have computed the non-renormalizable corrections to the Higgs pentaplet mass matrix up to eighth order. There is a multitude of contributing terms that, even within the special vacuum choice (2), leave no doublets massless in general. A necessary requirement in order to protect the above doublets, is to enlarge the vanishing VEV choice further by taking [#2]

$$\phi_+ = \phi_- = \bar{\phi}_+ = \bar{\phi}_- = 0 , \tag{5a}$$

and also imposing the additional constraints

$$\Delta_2^2 + T_2^2 = T_2 \cdot T_3 = 0 . \tag{5b}$$

This choice makes all corrections vanish and preserves the standard form of the Higgs pentaplet mass matrix that is derived at the cubic level [7].

Let us proceed now to solve the flatness conditions keeping in mind the conditions that control the corrections to the Higgs pentaplet mass matrix as expressed by eqs. (5). We start our analysis considering eqs. (3z), (3aa). It is clear that there exist two distinct cases depending on wheather $\phi_2 = 0$ or not. The case $\phi_2 = 0$, requires $\phi_4 \neq 0$ (otherwise eqs. (3z), (3x) imply $\Phi_{31} = \Phi_{23} = 0$) and then (3aa), (3x), (3s) lead to $\bar{\phi}_1 = T_5 = 0$. Then eqs. (3x), (3t), (3r), (3j), (3f), (3w) are impossible to solve unless if we assign VEVs of order M to a large set of fields, namely Φ_{31}, $\bar{\Phi}_{31}$, Φ_{23}, $\bar{\Phi}_{23}$, ϕ_4, $\bar{\phi}_4$, ϕ_{45}, $\bar{\phi}_{45}$. Thus this case is obviously rejected.

We can now continue our analysis by looking at the second case, namely $\phi_2 \neq 0$. This must be accompanied with $T_5 \neq 0$, $\bar{\phi}_1 = 0$, according to eqs. (3aa), (3r). Eqs. (3a), (3d), (3ℓ), (3aa) now require

$$T_2 \cdot T_4 = T_4 \cdot T_5 = T_4^2 = T_5^2 = 0 \tag{6}$$

in addition to (5a), (5b), while eqs. (3z), (3aa) lead to the constraint

$$\tfrac{1}{2}\phi_2^2 \approx [\Phi_{23} + \bar{\Phi}_{31}(\phi_2^2 + \phi_4^2)/M^2] \times [\Phi_{31} + \bar{\Phi}_{23}(\phi_2^2 + \phi_4^2)/M^2] . \tag{7}$$

Further, we notice that the choice $\phi_4 = 0$ is not allowed since it leads to incompatibility of eqs. (3t), (3x) and (7). Thus we have to impose $\bar{\phi}_3 = 0$ in order to satisfy (3s). The F-flatness equations (3a)–(3aa) are then reduced to

$$[(\bar{F}_5 \cdot F_3)^2 \bar{\phi}_{45} + (T_3 \cdot T_4)^2 \phi_{45}]/M^3 \approx 0 , \tag{8a}$$

$$\Phi_{23}\Phi_{31} + \tfrac{1}{2}(\phi_2^2 + \phi_4^2) = 0 , \tag{8b}$$

$$\bar{\Phi}_{23}\bar{\Phi}_{31} + \tfrac{1}{2}\bar{\phi}_4^2 + (F_1 \cdot \bar{F}_5)^2/M^2 \approx 0 , \tag{8c}$$

$$\phi_4\bar{\phi}_4 + \phi_{45}\bar{\phi}_{45} = 0 , \tag{8d}$$

$$\Phi_{31} + \bar{\Phi}_{23}(\phi_2^2 + \phi_4^2)/M^2 \approx 0 , \tag{8e}$$

$$(\bar{F}_5 \cdot F_3)\Delta_3 [1/M + (\phi_4\bar{\phi}_4 + \phi_{45}\bar{\phi}_{45} + \bar{\Phi}_{23}\Phi_{23})/M^3] \approx 0 , \tag{8f}$$

$$\tfrac{1}{2}\phi_2^2 \approx [\Phi_{23} + \bar{\Phi}_{31}(\phi_2^2 + \phi_4^2)/M^2] \times [\Phi_{31} + \bar{\Phi}_{23}(\phi_2^2 + \phi_4^2)/M^2] , \tag{8g}$$

$$T_4 \approx (T_5/\phi_2)\sqrt{2}\,[\Phi_{31} + \bar{\Phi}_{23}(\phi_2^2 + \phi_4^2)/M^2] \approx 0 . \tag{8h}$$

An immediate implication of eqs. (8a), (8g) is

$$\Phi_{23}\bar{\Phi}_{23} \approx M^2 , \tag{9}$$

which combined with eqs. (4) implies large VEVs of order M for both untwisted moduli fields Φ_{23}, $\bar{\Phi}_{23}$. This choise does not spoil the perturbative treatment of non-renormalizable terms, because any non-vanishing superpotential term (of order $n > 3$) involving these fields should also include at least four twisted fields [13] and is therefore sufficiently suppressed. Furthermore, in this case the associated Higgs phenomenon can also be realized directly at the string level [16].

For the choice (9), eq. (8f) requires $F_3 = 0$ in order to avoid also $\phi_4\bar{\phi}_4 \approx \phi_{45}\bar{\phi}_{45} \approx M^2$. Then (8a) suggests $\phi_{45} = 0$ but unfortunately this leads to $\bar{\phi}_4 = 0$ through eq. (4d). This is impossible since we must have $\bar{\phi}_4 \neq 0$ in order to guarantee the charm quark mass term [7]. The only way out is to "satisfy"the constraint (8a) approximately, in the sence that this F-term is smaller than the expected value of the supersymmetry breaking. Guessing [#3]

[#2] Note that the choice $\phi_+ = \bar{\phi}_- = 0$ also gets rid of the sixth order terms $(\phi_+ \Phi_{23} + \bar{\phi}_- \Phi_{31}) F_4 F_3 \cdot F_3 \bar{f}_5$ that cause proton decay, without using any antisymmetry assumptions as in ref. [10].

[#3] The radiative corrections to scalar masses are necessarily of the form

$$(\delta m^2)\phi^*\phi = \int \frac{d^2\theta\, d^2\bar{\theta}}{M^2} (\phi + ...)(\phi^* + ...)(\theta^2 F + ...)(\bar{\theta}^2 F^* + ...) = \phi\phi^*(FF^*)/M^2 .$$

Thus, $F \leqslant M\,\delta m = O(10^{10}\,\text{GeV})$.

$$(T_3 \cdot T_4)^2 \phi_{45}/M^3 \leqslant (10^{10}\,\text{GeV})^2$$

leads us to take [#4]

$$T_3, T_4 \sim 10^{14-15}\,\text{GeV}\,. \tag{10}$$

Note that the generated $\bar{f}_3 f_4$ mass in this case of "small" $T_3 \cdot T_4$ is of the order of 10^{14} GeV which is very acceptable and safe.

The $SU(4) \times SO(10)$ symmetry breaking is subject to the constraints of D-flatness (4f), (4g), as well as to the constraints (5b), (6). The $SU(4)$ D-flatness can be easily satisfied by taking Δ_2 and Δ_3 to be real. Choosing $\Delta_2 = (\alpha, 0, ..., 0) \| \Delta_3 = (\beta, 0, ..., 0)$ with $\alpha^* = \alpha$, $\beta^* = \beta$ statisfies the $SU(4)$ D-flatness (4f) automatically and breaks $SU(4)$ just to $SO(5)$. This is desirable since we do not want to lower too much the confinement scale of the fractionally charged states. A general VEV for Δ_2 and Δ_3 breaks $SU(4)$ down to $SO(4) = SU(2) \times SU(2)$ and this is actually the case $T_2 = 0$. For the $SO(10)$ D-flatness we first note that (8h) can be written as $T_5 = \delta T_4$ with δ depending on singlet VEVs. The choice

$$\begin{aligned}
&T_4 = (\gamma, \mathrm{i}\gamma, 0, ..., 0)\,,\\
&T_5 = \delta(\gamma, \mathrm{i}\gamma, 0, ..., 0)\,,\\
&T_3 = (1+\delta^2)^{1/2}(\gamma, -\mathrm{i}\gamma, 0, ..., 0)\,,\\
&T_2 = (0, 0, \mathrm{i}\alpha, 0, ..., 0)\,,
\end{aligned} \tag{11}$$

with $\gamma^* = \gamma$ and $\delta^* = \delta$, breaks $SO(10)$ to $SO(7)$, is $SO(10)$ D-flat and satisfies all the constraints (5), (6).

Let us now return to our F- and D-flatness constraints (8), (4), (9). Eqs. (8b)–(8h), (9) yield

$$\phi_2 \approx \phi_4 \approx \sqrt{\Phi_{23}\Phi_{31}}\,, \tag{12a}$$

$$\bar{\phi}_4 \approx \sqrt{\bar{\Phi}_{23}\bar{\Phi}_{31}}\,, \tag{12b}$$

$$\phi_{45}\bar{\phi}_{45} \approx M\sqrt{\Phi_{31}\bar{\Phi}_{31}}\,, \tag{12c}$$

$$\delta \approx \sqrt{\Phi_{31}/\Phi_{23}}\,, \tag{12d}$$

in addition to (9). Assuming real VEVs for the singlet fields we can now proceed to the solution of the D-flatness equations (8). According to eqs. (10)–(12) we can ignore T_3, T_4, T_5 terms and obtain

$$\beta^2 = \tfrac{1}{15}\xi\,, \tag{13a}$$

$$\phi_{45}^2 - \bar{\phi}_{45}^2 = \tfrac{1}{10}\xi\,, \tag{13b}$$

$$\begin{aligned}
V^2 = |F_1|^2 = |\bar{F}_5|^2 = \tfrac{1}{15}\xi + 2(|\Phi_{23}|^2 - |\bar{\Phi}_{23}|^2)&\\
+ (|\phi_2|^2 + |\phi_4|^2 - |\bar{\phi}_4|^2)\,,&
\end{aligned} \tag{13c}$$

$$\alpha^2 = -\tfrac{1}{15}\xi - (|\Phi_{23}|^2 - |\bar{\Phi}_{23}|^2 - |\Phi_{31}|^2 + |\bar{\Phi}_{31}|^2)\,. \tag{13d}$$

Eqs. (12), (13) together with (9) determine all the non-vanishing VEVs of our solution, except γ, in terms of the three NS fields, namely Φ_{31}, $\bar{\Phi}_{31}$ and one of Φ_{23}, $\bar{\Phi}_{23}$. Thus we are left with only four free parameters.

Summarizing our results, up to this point, we have found a solution that violates sixth order flatness by at most $(10^{10}\,\text{GeV})^2$, an amount expected to be tolerated by supersymmetry breaking. The non-vanishing VEVs are the singlets ϕ_2, ϕ_4, $\bar{\phi}_4$, Φ_{31}, $\bar{\Phi}_{31}$, Φ_{23}, $\bar{\Phi}_{23}$ and the hidden fields Δ_2, Δ_3, T_2, T_3, T_4, T_5. This solution has $\Phi_{23}, \bar{\Phi}_{23} \sim O(M)$, $T_3, T_4, T_5 \sim 10^{14}$ GeV and all non-vanishing VEVs can be determined in terms of four parameters as given in eqs. (9), (11), (12) and (13).

Let us now come to the doublet and triplet mass matrices. For our flatness solution all non-renormalizable contributions to the doublet mass matrix (M_2), up to the eighth order, vanish. M_2 has therefore the following tree level form:

$$M_2 = \begin{pmatrix} 0 & 0 & \bar{\Phi}_{31} & 0 \\ 0 & 0 & \Phi_{23} & 0 \\ \Phi_{31} & \bar{\Phi}_{23} & 0 & \bar{\phi}_{45} \\ 0 & 0 & \phi_{45} & 0 \end{pmatrix}, \tag{14}$$

which is characterized by two pairs of massless doublets [7]. The triplet mass matrix takes the form

$$M_3 = \begin{pmatrix} & & & & V \\ & M_2 & & & aV \\ & & & & 0 \\ & & & & bV \\ c\bar{V} & \bar{V} & 0 & d\bar{V} & 0 \end{pmatrix}, \tag{15}$$

in a D_1, D_2, D_3, D_{45}, $\bar{d}_5^c$ and $\bar{D}_1$, $\bar{D}_2$, $\bar{D}_3$, $\bar{D}_{45}$, d_1^c, basis, where M_2 is given in (14).

The entries in (15) stand for

$$V \equiv \langle F_1 \rangle (1 + ...), \tag{16}$$

[#4] Note that in this case the confinement scale (Λ_{10}) of $SO(10)$ is $\Lambda_{10} \leqslant 10^4$ GeV.

$$aV \equiv \langle F_1 \rangle (\phi_4^2/M^2 + \ldots),$$
$$bV \equiv \langle F_1 \rangle [\phi_{45}(\Phi_{31} + \phi_4^2 \bar{\Phi}_{23}/M^2)/M^2 + \ldots],$$
$$\bar{V} \equiv \langle \bar{F}_5 \rangle (1 + \ldots),$$
$$c\bar{V} \equiv \langle \bar{F}_5 \rangle [\bar{\phi}_4^2(1 + \Phi_{23}\bar{\Phi}_{23}/M^2)/M^2 + \ldots],$$
$$d\bar{V} \equiv \langle \bar{F}_5 \rangle [\bar{\phi}_{45}\Phi_{23}(1 + \Phi_{23}\bar{\Phi}_{23}/M^2)/M^2 + \ldots], \tag{16 cont'd}$$

where dots stand for higher order corrections. Two pairs of triplets acquire mass of order M together with the corresponding doublets in M_2. Two more pairs receive masses through the triplet doublet splitting mechanism [17,3,4] of order V which is the $SU(5) \times U(1)'$ breaking scale. Finally the remaining pair of triplets stays massless; these are approximately

$$D_0 \sim \frac{\phi_{45}}{\Phi_{23}} D_2 + D_{45},$$

$$\bar{D}_0 \sim \bar{D}_1 + \frac{(\Phi_{31} - c\bar{\Phi}_{23})^2}{\Phi_{31}\bar{\phi}_{45} + cd\bar{\Phi}_{23}^2 - d\Phi_{31}\bar{\Phi}_{23} - c\bar{\Phi}_{23}\bar{\phi}_{45}} \bar{D}_{45}. \tag{17}$$

Since the lightest generation up-quarks are still without a mass term to sixth order, we cannot declare this triplet harmless or dangerous and the issue is postponed for a higher order of computation. It is possible that the massless triplet pair will become massive at some higher order. For instance, if at some order a pentaplet mass term $h_{45}\bar{h}_2$ appears, all triplets will become heavy and one pair of doublets will remain massless.

The terms relevant to the matter–fermion mass matrix, including non-renormalizable corrections only where it is necessary, are [7]

$$F_4 \cdot F_4\, h_1 \to g\sqrt{2}\, bb^c H_1, \tag{18a}$$
$$F_2 \cdot F_2\, h_2 \to g\sqrt{2}\, ss^c H_2, \tag{18b}$$
$$F_4 \bar{f}_5 \bar{h}_{45} \to g\sqrt{2}\, tt^c \bar{H}_{45}, \tag{18c}$$
$$F_2 \bar{f}_2 \bar{h}_{45}(\bar{\phi}_4/M) \to g\sqrt{2}\, c_1 cc^c \bar{H}_{45}(\bar{\phi}_4/M), \tag{18d}$$
$$\bar{f}_1 l_2^c h_1 \to g\sqrt{2}\, \tau\tau^c H_1, \tag{18e}$$
$$(\bar{f}_2 l_2^c + \bar{f}_5 l_5^c) h_2 \to g\sqrt{2}\, (ee^{c'} + \mu\mu^c) H_2, \tag{18f}$$
$$F_2 \cdot F_3\, h_1(\Delta_2 \cdot \Delta_3\, \bar{\Phi}_{23}/M^3) \to g\sqrt{2}\, c_2(\alpha\beta\bar{\Phi}_{23}/M^3)(sd^c + ds^c)H_1, \tag{18g}$$

where c_1 and c_2 denote the numerical values of the corresponding non-renormalizable coefficients which are of order O(1) [6]. The fields H_1, H_2 and $\bar{H}_{45}$ denote the doublet components of the pentaplet Higgses in self-explanatory notation. They are expressed in terms of the massless isodoublets (H_0, H_0', $\bar{H}_0$, $\bar{H}_0'$) [7]:

$$H_1 = -\cos\theta'\, H_0 + N \tan\theta \sin\theta'\, H_0' + \ldots,$$
$$H_2 = \sin\theta'\, H_0 - N \tan\theta \cos\theta'\, H_0' + \ldots,$$
$$\bar{H}_{45} = -(\bar{N}/\cos\bar{\theta}')\, \bar{H}_0' + \ldots, \tag{19}$$

where the dots stand for the contribution of the massive isodoublets and $\theta \equiv \tan^{-1}(-\phi_{45}/\Phi_{23})$, $\theta' \equiv \tan^{-1}(\bar{\Phi}_{31}/\Phi_{23})$, $\bar{\theta} \equiv \tan^{-1}(-\bar{\phi}_{45}/\bar{\Phi}_{23})$, $\bar{\theta}' \equiv \tan^{-1}(\Phi_{31}/\bar{\Phi}_{23})$, $N \equiv (1 + \tan^2\theta + \tan^2\theta')^{-1/2}$ and $\bar{N} \equiv (1 + \tan^2\bar{\theta} + \tan^2\bar{\theta}')^{-1/2}$.

The expressions (18) are based on the assignments

$$F_4 \equiv ((t, b); b^c; \nu_4^c), \quad F_2 \equiv ((c, s); s^c; \nu_2^c),$$
$$F_3 \equiv ((u, d); d^c; \nu_3^c),$$
$$\bar{f}_5 \equiv ((\mu, \nu_{\mu'}); t^c), \quad \bar{f}_2 \equiv ((e, \nu_3'); c^c),$$
$$\bar{f}_1 \equiv ((\tau, \nu_\mu'); u^c),$$
$$l_5^c \equiv \mu^c, \quad l_2^c \equiv e^{c'}, \quad l_1^c \equiv \tau^c.$$

For the proposed flatness solution $\phi \in \{\phi_{45}, \bar{\phi}_{45}, \Phi_{31}, \bar{\Phi}_{31}\} \ll \Phi \in \{\Phi_{23}, \bar{\Phi}_{23}\}$ according to (9), (12) and thus expanding we get the fermion masses

$$m_t = g\sqrt{2}\, \langle \bar{H}_0' \rangle + O(\phi/\Phi),$$
$$m_b = m_\tau = g\sqrt{2}\, \langle H_0 \rangle + O(\phi/\Phi),$$
$$m_c = m_t c_1(\langle \bar{\phi}_4 \rangle / M),$$
$$m_s = m_e = m_\mu = g\sqrt{2}\, [(\bar{\Phi}_{31}/\Phi_{23})\langle H_0 \rangle + (\phi_{45}/\Phi_{23})\langle H_0' \rangle] + O(\phi/\Phi)^2,$$
$$m_u = 0,$$
$$m_d = 2(gc_2\alpha\beta\bar{\Phi}_{23}/M^3)^2 \langle H_0 \rangle^2 / m_s + O(\phi/\Phi). \tag{20}$$

A nice mass relation is $m_b/m_\tau = 1$ while a problematic one is $m_s/m_\mu = 1$. We also see that the m_s and

m_μ are suppressed by ϕ/Φ relatively to m_b, m_τ, for natural isodoublet VEVs ($\langle H_0\rangle \sim \langle H'_0\rangle$), although all these masses arise from the tree level superpotential. This hierarchy persists even in the case where one pair of isodoublets ($H'_0, \bar{H}_0$) become heavy. Finally the lepton (e, $e^{c\prime}$) is not really the electron yet since in the cubic superpotential there exists a term $\bar{X}'_2 Z_1 l^c_2$. In fact, this term, as well as higher order corrections to it, is unique in the sense that it mixes directly through chiral interactions the ordinary matter to "hidden" fractionally charged states. Nevertheless, since these states are strongly coupled at high energies it is certain that the bound state $L \sim \bar{X}'_2 Z_1$, with quantum numbers conjugate to $l^c_2 = e^{c\prime}$, will form. Then, the above chiral term would be interpreted as a mass term of order Λ for the normalized chiral composite field L and $e^{c\prime}$, where Λ is the scale of the relevant gauge group SO(5) or SU(4). If only that term were present, then $e^{c\prime}$ would have become supermassive and the model would lack a right-handed electron. However, one should expect the formation of the conjugate bound states as well ($\bar{L} = \bar{Z}_1 X_1, \bar{Z}_1 Y'_2, \ldots$). Although it is not theoretically impossible that these bound states are massless, it seems much more plausible that they acquire masses smaller by a few orders of magnitude than the scale Λ. For a mass term $L\bar{L}$ of order $\Lambda' \approx 10^{-2}\Lambda$ the electron problem would be solved through the mass matrix

$$(e \quad \bar{L})\begin{pmatrix} \Lambda & m_\mu \\ \Lambda' & 0 \end{pmatrix}\begin{pmatrix} L \\ e \end{pmatrix}, \tag{21}$$

which gives a large mass for one combination and a small mass of the correct order $m_e \approx m_\mu(\Lambda'/\Lambda)$ for the other combination. In contrast to quarks and charged leptons the determination of neutrino masses is more complicated because of the mixing with the various singlet fields. This is not a trivial task and will be reported elsewhere.

Coming back to our flatness solution (9), (12) we see that only three parameters are involved in the fermion masses. If we had one pair of Higgs doublets only one additional parameter would be introduced (the VEV ratio) and the relations (20) would lead to definite predictions for the fermion masses. Although this could be probably the case when higher order terms are taken into account, as we have already argued, unfortunately this predictability is lost when two pairs of doublets survive, since three additional parameters are introduced.

Our conclusions can be summarized as follows. We found a D- and F-flat solution up to sixth non-renormalizable order that correspond to a spontaneously broken hidden sector $SO(10)\times SU(4) \to SO(7)\times SO(5)$ and is characterized by four free parameters. The surplus states $\bar{f}_3$, l^c_3, f_4, $\bar{l}^c_4$ become massive and decouple from low energy phenomenology due to the development of non-vanishing VEVs by hidden sector fields. There are two pairs of massless Higgs isodoublets and one pair of massless coloured triplets up to eighth order. Higher order terms could in principle give masses to these triplets as well as to one of the doublet pairs. All quarks and charged leptons, except the up-quark which remains massless, obtain masses that have the correct hierarchical structure. In particular Cabibbo mixing arises through VEVs of the hidden sector fields that break the SU(4) symmetry.

One of us (K.T.) wishes to thank the Ministry of Research and Technology, the CERN Theory Division and the EEC (Science grant SC1-0221-CCTT) for travelling support.

References

[1] I. Antoniadis, C. Bachas, C. Kounnas and P. Windey, Phys. Lett. B 171 (1986) 51;
H. Kawai, D.C. Lewellen and S.H. Tye, Phys. Rev. Lett. 57 (1986) 1832; Nucl. Phys. B 288 (1987) 1;
I. Antoniadis, C. Bachas and C. Kounnas, Nucl. Phys. B 289 (1987) 87;
I. Antoniadis and C. Bachas, Nucl. Phys. B 298 (1988) 586.

[2] K.S. Narain, Phys. Lett. B 169 (1986) 41;
W. Lerche, D. Lüst and A.N. Schellekens, Nucl. Phys. B 287 (1987) 477.

[3] I. Antoniadis, J. Ellis, J.S. Hagelin and D.V. Nanopoulos, Phys. Lett. B 194 (1987) 231; B 231 (1989) 65.

[4] I. Antoniadis, J. Ellis, J.S. Hagelin and D.V. Nanopoulos, Phys. Lett. B 205 (1988) 459; B 208 (1988) 209.

[5] J. Lopez and D.V. Nanopoulos, Nucl. Phys. B 338 (1990) 73.

[6] S. Kalara, J. Lopez and D.V. Nanopoulos, Phys. Lett. B 245 (1990) 421; Nucl. Phys. B 353 (1991) 650

[7] J. Rizos and K. Tamvakis, Phys. Lett. B 251 (1990) 369;
K. Tamvakis, Nucl. Phys. B (Proc. Suppl.) 22 A (1991) 84.

[8] G. Leontaris, J. Rizos and K. Tamvakis, Phys. Lett. B 251 (1990) 83.

[9] J. Lopez and D.V. Nanopoulos, preprint CERN-TH-6097/91.

[10] J. Ellis, J. Lopez and D.V. Nanopoulos, Phys. Lett. B 252 (1990) 53.
[11] G. Leontaris and K. Tamvakis, Phys. Lett. B 260 (1991) 333.
[12] S. Kalara, J. Lopez and D.V. Nanopoulos, Phys. Lett. B 256 (1991) 150.
[13] J. Rizos and K. Tamvakis, Phys. Lett. B 262 (1991) 227;
[14] J. Lopez and D.V. Nanopoulos, Phys. Lett. B 251 (1990) 73;
S. Kalara, J. Lopez and D.V. Nanopoulos, preprint CTP-TAMU 96/91.
[15] G. Leontaris, J. Rizos and K. Tamvakis, Phys. Lett. B 243 (1990) 220;
J. Ellis, J. Lopez and D.V. Nanopoulos, Phys. Lett. B 247 (1990) 257.
[16] I. Antoniadis, C. Bachas and C. Kounnas, Phys. Lett. B 200 (1988) 297;
I. Antoniadis, Nucl. Phys. B (Proc. Suppl.) 5B (1988) 1.
[17] A. Masiero, D.V. Nanopoulos and K. Tamvakis and T. Yanashida, Phys. Lett. B 115 (1982) 298.