GAUGINO MASSES AND GRAND UNIFICATION

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Neither the SU(3), SU(2) and U(1) gauge couplings nor the gaugino masses need be universal at the grand unification scale M_X in supergravity theories. The experimental value of $\sin^2\theta_W$ is naturally reproduced only if M_X is somewhat less than the Planck mass M_P. In this case with SU(5) broken by an adjoint 24 of Higgs there is a simple sum rule relating the SU(3), SU(2) and U(1) gaugino masses $\mathcal{M}_i$. Requiring that the supersymmetry breaking leptoquark mass splitting vanish as in no-scale models would impose the specific non-universal ratios $\mathcal{M}_3:\mathcal{M}_2:\mathcal{M}_1 = 5:-5:-1$ at M_X. In this case $m_{\tilde{\gamma}}/m_{\tilde{g}}$ is half the usual value derived from universality, while m_b/m_τ takes its usual value.

With the recent discovery [1] of the intermediate gauge bosons at the CERN $\bar{p}p$-collider, one of the central predictions of the SU(3) × SU(2) × U(1) standard model has been confirmed. Although the standard model is not completely tested, there is a growing belief among physicists that it describes physics correctly at least up to energies of the order of the electroweak scale. A theoretically unsatisfying feature of the model however, is that it contains a number of free parameters whose determination requires one to go beyond SU(3) × SU(2) × U(1). This has led to the enterprise of grand unification in which the standard gauge group is unified in a larger group such as SU(5) at an exponentially distant mass scale. Parameters such as the electroweak mixing angle or quark to lepton mass ratios, which were free in the framework of SU(3) × SU(2) × U(1), can now be determined in remarkable agreement with measured values. Supersymmetry has been introduced in the unification programme as an improvement over conventional perturbatively renormalizable gauge field theories since it automatically solves the technical aspects of the gauge hierarchy problem associated with fundamental scalars [‡1]. More specifically, unification in the framework of $N = 1$ local supersymmetry (supergravity) offers unexpected fruits such as a possible understanding of the origin of the electroweak scale itself and thus completely solving the gauge hierarchy problem of conventional GUTs, as well as improving enormously our understanding of the absence of any observable cosmological constant [3]. Unification in the framework of $N = 1$ supergravity should of course reproduce the standard predictions of $\sin^2\theta_W$ and m_b/m_τ as well as a set of relations among the parameters associated with the new particles of the theory. In particular, one would naturally expect to have mass relations among gauge fermions.

In this paper, under general assumptions we derive and analyze the gauge fermion mass relations implied by unification. Contrary to the chiral sector of the theory whose spectrum is determined in a model-dependent way, gauge fermion masses are strongly con-

[‡1] For reviews see ref. [2].

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strained by the unifying group and at the same time related in a model-independent way.

Let us consider a supersymmetric SU(5) gauge theory with an arbitrary chiral matter superfield content coupled to $N = 1$ supergravity. All gauge and matter terms in the component lagrangian are expressible in terms of two fundamental functions of chiral superfields [4]. These are a chiral function $f_{\alpha\beta}(\phi)$ which transforms as the symmetric product of two adjoint representations [in the case of SU(5) $\alpha, \beta = 1, ..., 24$] and is an analytic function of the left-handed chiral superfields ϕ_i, and the Kähler potential $G(\phi_i, \phi_i^*)$ which is a singlet under the gauge group and is a real function of chiral superfields. The structure of the kinetic terms of gauge superfields is determined by $f_{\alpha\beta}$. The same term in the superspace lagrangian that gives rise to gauge supermultiplet kinetic terms also generates bilinear gauge fermion terms that lead to gaugino masses when supersymmetry is broken. In component field form the lagrangian reads (we use natural units in which $M_P/\sqrt{8\pi} = 1$)

$$
\begin{aligned}
e^{-1}\mathcal{L} &= \tfrac{1}{2}\,\mathrm{Re} f_{\alpha\beta}(-\tfrac{1}{2}\bar{\lambda}^{\alpha}\not{D}\lambda^{\beta}) \\
&- \tfrac{1}{8}\mathrm{i}\,\mathrm{Im} f_{\alpha\beta}\, e^{-1} \mathrm{D}_{\mu}(e\bar{\lambda}^{\alpha}\gamma^{\mu}\gamma_5\lambda^{\beta}) \\
&- \tfrac{1}{4}\,\mathrm{Re} f_{\alpha\beta} F^{\alpha}_{\mu\nu}F^{\beta\mu\nu} + \tfrac{1}{4}\mathrm{i}\,\mathrm{Im} f_{\alpha\beta}F^{\alpha}_{\mu\nu}\tilde{F}^{\beta\mu\nu} \\
&+ \tfrac{1}{4}\,\mathrm{e}^{-G/2}G^{i}(G^{-1})_i^j(\partial f^*_{\alpha\beta}/\partial\phi^{*j})\lambda^{\alpha}\lambda^{\beta} + \mathrm{h.c.} + ... \,, \quad (1)
\end{aligned}
$$

in the notation of Cremmer et al. [4]: $G^i \equiv \partial G/\partial\phi_i$ and $(G^{-1})_i^j$ is the inverse matrix of $G_i^j \equiv \partial G/\partial\phi^{*i}\partial\phi_j$.

In the minimal case ($f_{\alpha\beta} = \delta_{\alpha\beta}$), the gauge kinetic terms acquire their familiar form and the gaugino mass term vanishes. A non-trivial $f_{\alpha\beta}$ will in principle be a function of all chiral superfields, but we need only consider those that get large expectation values. Such are gauge-singlet fields z_i that make up the "hidden" sector, and "observable" fields associated with the SU(5) breakdown to SU(3) × SU(2) × U(1), such as a chiral superfield in the adjoint (**24**) representation $\Sigma_j^i = \Sigma^{\alpha}(\lambda_{\alpha})_j^i/\sqrt{2}$. The SU(5) breaking can also be achieved with a field in the **75** representation [5] but for the moment let us restrict ourselves to the simplest case of the **24** and come back later to the **75**. Keeping in mind that $f_{\alpha\beta}$ transforms as the symmetric product of two adjoint representations, the most general form it could have is

$$f_{\alpha\beta} = A(z, ...)\delta_{\alpha\beta} + B(z, ...)d_{\alpha\beta\gamma}\Sigma^{\gamma} \,, \quad (2)$$

plus higher order terms in Σ like $C(z, ...)\Sigma_{\alpha}\Sigma_{\beta}$ [‡2], etc. As we will see later, to explain in a natural way the nearness of $\sin^2\theta_W$ to conventional GUT predictions, we will need to assume $\langle 0|\Sigma|0\rangle \ll 1$, in which case higher orders may be neglected. In eq. (2), A and B are gauge invariant functions of the chiral superfields. Therefore, if we ignore all other fields except z and Σ for the reasons stated, they should be analytic functions of z, $\mathrm{Tr}(\Sigma^2) = \Sigma^{\alpha}\Sigma^{\alpha}$ and $d_{\alpha\beta\gamma}\Sigma^{\alpha}\Sigma^{\beta}\Sigma^{\gamma}$, although the latter two are negligible if $\langle 0|\Sigma|0\rangle \ll 1$, as we suppose. The group-theoretic symmetric coefficient $d_{\alpha\beta\gamma}$ is defined as $d_{\alpha\beta\gamma} = 2\mathrm{Tr}(\{\lambda_{\alpha}/2, \lambda_{\beta}/2\}\lambda_{\gamma}/2)$ with $\lambda_{\alpha}/2$ the generators of SU(5) in the adjoint representation.

Returning to the lagrangian (1), we see that in order to regain the familiar minimal form of the kinetic terms

$$\tfrac{1}{2}(-\tfrac{1}{2}\bar{\hat{\lambda}}^{\alpha}\not{D}\hat{\lambda}^{\alpha}) - \tfrac{1}{4}\hat{F}^{\alpha}_{\mu\nu}\hat{F}^{\alpha\mu\nu} \,,$$

the gauge fields have to be rescaled according to

$$F^{\alpha}_{\mu\nu}(f^{\alpha})^{1/2} = \hat{F}^{\alpha}_{\mu\nu}, \quad A^{\alpha}_{\mu}(f^{\alpha})^{1/2} = \hat{A}^{\alpha}_{\mu}, \quad \lambda^{\alpha}(f^{\alpha})^{1/2} = \hat{\lambda}^{\alpha}, \quad (3)$$

where the coefficients f_{α} are taken from

$$
\begin{aligned}
f_{\alpha\beta}(z_0, \Sigma_0) &= f_{\alpha}\delta_{\alpha\beta} = A(z_0, ...)\delta_{\alpha\beta} + d_{\alpha\beta\gamma}\Sigma_0^{\gamma}B(z_0, ...) \\
&= [A(z_0, ...) + \Sigma_0 B(z_0, ...)c_{\alpha}]\delta_{\alpha\beta} \,, \quad (4)
\end{aligned}
$$

with z_0 and $\Sigma_0^{\gamma} = \delta^{\gamma,24}\Sigma_0$ the values of the scalar fields at the minimum. The representation invariants $d_{\alpha\beta 24} \equiv c_{\alpha}\delta_{\alpha\beta}$ are

$$
\begin{aligned}
c_{\alpha} &= 2/\sqrt{15} \quad (\alpha = 1, ..., 8) \,, \\
&= -1/2\sqrt{15} \quad (\alpha = 9, ..., 20) \,, \\
&= -3/\sqrt{15} \quad (\alpha = 21, 22, 23) \,, \\
&= -1/\sqrt{15} \quad (\alpha = 24) \,, \quad (5)
\end{aligned}
$$

where we have assumed for simplicity that $f_{\alpha\beta}$ is real, but our results are true more generally. The rescaling conditions (3) go together with the definition of rescaled gauge couplings g_{α} according to

$$g_{\alpha}f_{\alpha}^{1/2} = g \,. \quad (6)$$

[‡2] Such terms give contributions to both $f_{\alpha\beta}$ and its derivatives suppressed by extra powers of Σ. We shall see later that it is reasonable to ignore them.

Thus, the first striking consequence of a non-trivial $f_{\alpha\beta}$ is the modification of the standard unification condition at the unification scale M_X. Now, this condition reads

$$\alpha_G = \alpha_3(M_X)f_3 = \alpha_2(M_X)f_2 = \tfrac{5}{3}\alpha'(M_X)f_1 . \quad (7)$$

The gauge couplings are no longer equal at M_X. The classic renormalization group analysis that led to the predictions of $\sin^2\theta_W$ and M_X has to be re-examined and we intend to do so later.

Let us now analyze the gaugino mass matrix

$$\mathcal{M}_{\alpha\beta} = \tfrac{1}{4}e^{-G/2}G^i(G^{-1})^j_i(\partial f^*_{\alpha\gamma}/\partial\phi^{*j})f^{-1}_{\gamma\beta} . \quad (8)$$

Using SU(5) covariance, the Kähler potential dependent factor can always be written as

$$G^i(G^{-1})^j_i = P_1(z_0, \Sigma_0)\delta^{jz} + P_2(z_0, \Sigma_0)\delta^{j\gamma}\Sigma_0^{*\gamma} \quad (9)$$

(in no-scale models [3] $P_2 = 0$). Finally, we get

$$\mathcal{M}_{\alpha\beta} = \delta_{\alpha\beta}m_{3/2}(\Gamma + c_\alpha\Delta\Sigma_0)/(A + c_\alpha\Sigma_0 B) , \quad (10)$$

where as a shorthand notation we have introduced

$$\Gamma \equiv \tfrac{1}{4}[P_1(\partial A/\partial z)_0 + 2P_2\Sigma_0^2(\partial A/\partial\Sigma^2)_0] ,$$

$$\Delta \equiv \tfrac{1}{4}[P_1(\partial B/\partial z)_0 + P_2 B + 2\Sigma_0^2(\partial B/\partial\Sigma^2)_0] . \quad (11)$$

Thus, grand unification in general allows different gaugino masses at M_X, contrary to the commonly assumed equal values. Eq. (10) can be put in a more transparent form using eqs. (4) and (6). Then,

$$\mathcal{M}_\alpha = m_{3/2}(\Gamma + c_\alpha\Delta\Sigma_0)f_\alpha^{-1}$$
$$= m_{3/2}(g_\alpha^2/g^2)(\Gamma + c_\alpha\Delta\Sigma_0),$$

or

$$\mathcal{M}_\alpha/g_\alpha^2 = (m_{3/2}/g^2)(\Gamma + c_\alpha\Delta\Sigma_0) \quad (\text{at } M_X) . \quad (12)$$

It is already evident that since $\mathcal{M}_\alpha$ depends only on two arbitrary functions Γ and Δ, we must find relations among the different gaugino masses. Substituting the group factor c_α, we get

$$\mathcal{M}_3/\alpha_3 = (m_{3/2}/\alpha_G)(\Gamma + 2\Delta\Sigma_0/\sqrt{15}) ,$$
$$\mathcal{M}_2/\alpha_2 = (m_{3/2}/\alpha_G)(\Gamma - 3\Sigma_0\Delta/\sqrt{15}) ,$$
$$\mathcal{M}_1/\tfrac{5}{3}\alpha' = (m_{3/2}/\alpha_G)(\Gamma - \Sigma_0\Delta/\sqrt{15}) , \quad (13)$$

which implies

$$\mathcal{M}_3/\alpha_3 = -\tfrac{3}{2}\mathcal{M}_2/\alpha_2 + \tfrac{5}{2}\mathcal{M}_1/\tfrac{5}{3}\alpha' . \quad (14)$$

Since the ratios $\mathcal{M}_i/\alpha_i$ are renormalization group invariant, the same relation holds for all energy scales $\mu < M_X$. Relation (14) depends only on the assumption that SU(5) is broken to SU(3) × SU(2) × U(1) with a non-vanishing expectation value of the adjoint Σ which is <1. No assumption was made on the form of the Kähler potential (i.e., P_1 and P_2) or on the form of A and B. If we were to include in $f_{\alpha\beta}$ a term of the form $C(z, \Sigma^2, ...)\Sigma_\alpha\Sigma_\beta$, the expression for $\mathcal{M}_1$ would change to

$$\mathcal{M}_1/\tfrac{5}{3}\alpha' = (m_{3/2}/\alpha_G)(\Gamma - \Sigma_0\Delta/\sqrt{15} + \Sigma_0^2 E) ,$$

where

$$E \equiv P_1\partial C/\partial z + 2P_2(C + \Sigma_0^2\partial C/\partial\Sigma^2) .$$

This extra term, however, is suppressed even if Σ_0 is just an order of magnitude below M_P.

In the case that we set the heavy leptoquark gaugino Majorana masses equal to zero, as required if their loops are not to make large contributions to the vacuum energy, which would be disastrous in many models [6], we obtain an additional constraint. Indeed, for

$$\mathcal{M}_{\tilde{X},\tilde{Y}} = m_{3/2}(\Gamma - \Sigma_0\Delta/2\sqrt{15})/(A - \Sigma_0 B/2\sqrt{15}) = 0 ,$$

we get

$$\mathcal{M}_3/\alpha_3 = 5(m_{3/2}\Sigma_0\Delta/2\sqrt{15}\alpha_G) ,$$
$$\mathcal{M}_2/\alpha_2 = -5(m_{3/2}\Sigma_0\Delta/2\sqrt{15}\alpha_G) , \quad (15)$$
$$\mathcal{M}_1/\tfrac{5}{3}\alpha' = -(m_{3/2}\Sigma_0\Delta/2\sqrt{15}\alpha_G) ,$$

Therefore, in this case the gaugino masses scale as

$$(\mathcal{M}_3/\alpha_3) : (\mathcal{M}_2/\alpha_2) : (\mathcal{M}_1/\tfrac{5}{3}\alpha') = 5 : -5 : -1. \quad (16)$$

This is in sharp contrast to the usually assumed equality $\mathcal{M}_3(M_X) = \mathcal{M}_2(M_X) = \mathcal{M}_1(M_X)$ which together with $\alpha_3(M_X) = \alpha_2(M_X) = \tfrac{5}{3}\alpha'(M_X) = \alpha_G$ leads to

$$\mathcal{M}_3/\alpha_3 = \mathcal{M}_2/\alpha_2 = \mathcal{M}_1/\tfrac{5}{3}\alpha' .$$

It should be pointed out also that if we neglect neutral Higgs fermion mixings, as is appropriate for $\mathcal{M}_{1,2} \ll M_W$, the photino now has a mass

$$\mathcal{M}_{\tilde{\gamma}} = \tfrac{4}{3}\sin^2\theta_W\,\mathcal{M}_2 ,$$

in contrast to the usual expression [7] which is twice as large.

As we have already stated, our mass relation (14) can change if SU(5) is broken down [5] to SU(3) ×

SU(2) × U(1) with the help of the **75** representation whose expectation value is

$$\Sigma_{jl}^{[ik]} = \Sigma_0\{\Delta_{cj}^{[i}\Delta_{cl}^{k]} + 2\Delta_{wj}^{[i}\Delta_{wl}^{k]} - \tfrac{1}{2}\delta_j^{[i}\delta_l^{k]}\}\,, \tag{17}$$

with

$$\Delta_c = \mathrm{diag}(1,1,1,0,0),\quad \Delta_w = \mathrm{diag}(0,0,0,1,1)\,.$$

The corresponding choice for $f_{\alpha\beta}$ is then

$$f^{\alpha\beta} = A(z,\mathrm{Tr}\,\Sigma^2)\delta^{\alpha\beta} + B(z,\mathrm{Tr}\,\Sigma^2)\lambda_{[i}^{\alpha j}\lambda_{k]}^{\beta l}\Sigma_{jl}^{[ik]}$$
$$\equiv (A + B\Sigma_0 D^\alpha)\delta^{\alpha\beta}\,. \tag{18}$$

The representation coefficients D^α are calculated to be

$$D^\alpha = -1 \quad (\alpha = 1, ..., 8)\,,$$
$$= \ \ 1 \quad (\alpha = 9, ..., 20)\,,$$
$$= -3 \quad (\alpha = 21, 22, 23)\,,$$
$$= \ \ 5 \quad (\alpha = 24)\,. \tag{19}$$

Finally, the gaugino masses are

$$\mathcal{M}_3/\alpha_3 = (m_{3/2}/\alpha_G)(\Gamma' - \Delta')\,,$$
$$\mathcal{M}_2/\alpha_2 = (m_{3/2}/\alpha_G)(\Gamma' - 3\Delta')\,,$$
$$\mathcal{M}_1/\tfrac{5}{3}\alpha' = (m_{3/2}/\alpha_G)(\Gamma' + 5\Delta')\,, \tag{20}$$

and the mass relation that follows is quite different from the one previously obtained,

$$\mathcal{M}_3/\alpha_3 = \tfrac{3}{4}\,\mathcal{M}_2/\alpha_2 + \tfrac{1}{4}\,\mathcal{M}_1/\tfrac{5}{3}\alpha'\,. \tag{21}$$

Again, in the case of vanishing leptoquark gaugino Majorana masses we get

$$(\mathcal{M}_3/\alpha_3) : (\mathcal{M}_2/\alpha_2) : (\mathcal{M}_1/\tfrac{5}{3}\alpha') = -1 : -2 : 2\,, \tag{22}$$

which is also different from (16).

Let us now go back to the modified boundary condition for the gauge couplings in the case with an adjoint **24** of Higgses Σ, and examine its consequences for the standard predictions of $\sin^2\theta_W$ and M_X. The unification condition now reads

$$\alpha_3(M_X) = \alpha_2(M_X)f_2/f_3 = \tfrac{5}{3}(f_1/f_3)\alpha'(M_X) = f_3^{-1}\alpha_G\,. \tag{23}$$

Notice that (23) requires $f_3, f_2, f_1 > 0$. Independently of any constraint, the f's satisfy an identity imposed by the fact that they all depend on only two arbitrary functions A and B,

$$f_3 = -\tfrac{3}{2}f_2 + \tfrac{5}{2}f_1\,. \tag{24}$$

Since they must all be positive, we must in addition have

$$f_2 < \tfrac{5}{3}f_1 \quad \text{or} \quad f_3 < \tfrac{5}{2}f_1\,. \tag{25}$$

The f's do not appear in any expressions for the gaugino masses at any energy, since

$$\mathcal{M}_\alpha/g_\alpha^2 = m_{3/2}(\Gamma + c_\alpha\Sigma_0\Delta)/g^2\,,$$

is a renormalization group invariant quantity. However, the renormalization group equations for the three gauge couplings certainly depend on the f's explicitly.

We find it convenient for later use to define

$$m_0 \equiv m_{3/2}\Gamma\,, \tag{26}$$

and

$$\eta \equiv 5/f_3\,,\quad \xi \equiv 1/f_1\,. \tag{27}$$

Then automatically $f_2 = \frac{5}{3}(1/\xi - 2/\eta)$, and the boundary condition at M_X can be written

$$\alpha_G = 5\alpha_3(M_X)/\eta = \tfrac{5}{3}(1/\xi - 2/\eta)\alpha_2(M_X)$$
$$= 5\alpha'(M_X)/3\xi \qquad (\eta > 2\xi > 0)\,. \tag{28}$$

The one-loop renormalization group equations for the three gauge couplings are

$$\alpha_3^{-1}(\mu) = \alpha_3^{-1}(M_X) - (b_3/2\pi)\ln(M_X/\mu)$$
$$= 5/\eta\alpha_G - (b_3/2\pi)\ln(M_X/\mu)\,,$$
$$\alpha^{-1}(\mu)\sin^2\theta_W(\mu) = \alpha_2^{-1}(M_X) - (b_2/2\pi)\ln(M_X/\mu)$$
$$= (5/3\alpha_G)(1/\xi - 2/\eta) - (b_2/2\pi)\ln(M_X/\mu)\,,$$
$$\tfrac{3}{5}\alpha^{-1}(\mu)\cos^2\theta_W(\mu) = \tfrac{3}{5}\alpha'^{-1}(M_X)$$
$$- (b_1/2\pi)\ln(M_X/\mu) = 1/\xi\alpha_G - (b_1/2\pi)\ln(M_X/\mu)\,.$$

Defining

$$D \equiv \tfrac{2}{3}b_3(\eta/\xi - 1) - \tfrac{5}{3}b_1 - b_2\,,$$

we obtain

$$\ln(M_X/\mu) = (2\pi/D)[\alpha^{-1}(\mu) - \tfrac{8}{3}(\eta/\xi - 1)\alpha_3^{-1}(\mu)]\,,$$
$$\sin^2\theta_W(\mu) = \tfrac{1}{3}(\eta/\xi - 2)\alpha(\mu)\alpha_3^{-1}(\mu)$$
$$+ [1 - \tfrac{2}{3}\alpha(\mu)\alpha_3^{-1}(\mu)(\eta/\xi - 1)]$$
$$\times [\tfrac{1}{3}b_3(\eta/\xi - 2) - b_2]D^{-1}\,, \tag{29}$$
$$\alpha_G^{-1} = \tfrac{1}{5}\eta\{\alpha_3^{-1}(\mu) + b_3[\alpha^{-1}(\mu) - \tfrac{2}{3}(\eta/\xi - 1)\alpha_3^{-1}(\mu)]D^{-1}\}$$

In the minimal case of two Higgs doublets the renormalization group coefficients are $b_3 = 3, b_2 = -1$ and $b_1 = -33/5$.

If we now demand that the electroweak mixing angle lies in the experimentally observed [8] range

$$0.21 \lesssim \sin^2\theta_W(M_W) \lesssim 0.23 ,$$

we find that the ratio η/ξ is forced to take values in the rather narrow range [with $\alpha_3(M_W) = 0.122$]

$$4.5 \lesssim \eta/\xi \lesssim 5 . \quad (30)$$

This result holds independently of any possible vanishing of the supersymmetry breaking contribution to the masses of the $\tilde{X}$, $\tilde{Y}$ gauginos in the coset SU(5)/SU(3) × SU(2) × U(1). The behaviour of $\sin^2\theta_W(M_W)$ as a function of η/ξ is shown in fig. 1, where we also display the unification mass M_X as a function of η/ξ. As can be seen from there, the observed range of $\sin^2\theta_W$ indicates that

$$2.4 \times 10^{16}\ \mathrm{GeV} \lesssim M_X \lesssim 3.5 \times 10^{17}\ \mathrm{GeV} . \quad (31)$$

In fig. 2 we show the behaviour of the combination $\eta\alpha_G$ as a function of η/ξ. Since in the experimentally favoured range (30) we find $M_X < M_P$, it presumably

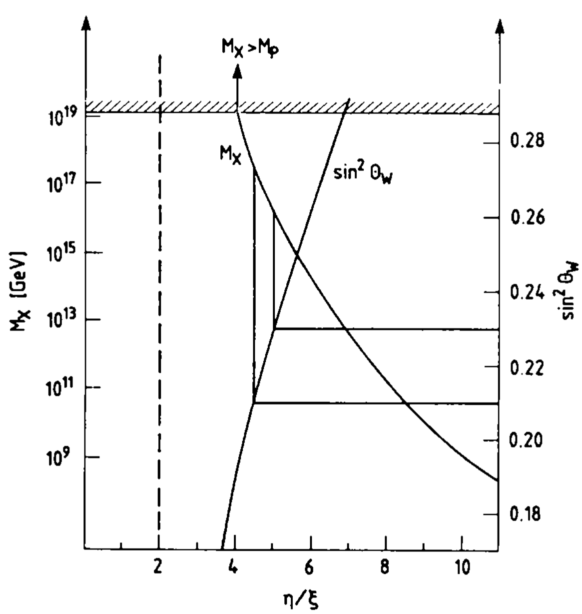

Fig. 1. The unification mass M_X and $\sin^2\theta_W$ as functions of η/ξ [see (29)]. Shown also is the experimentally allowed range of $\sin^2\theta_W$.

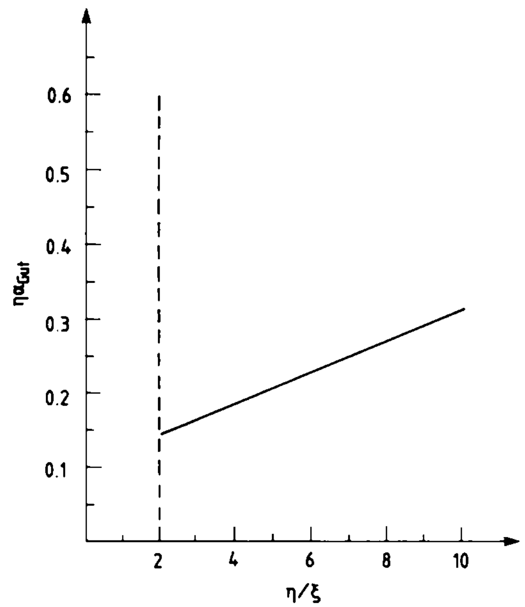

Fig. 2. The combination $\eta\alpha_{GUT} = 5\alpha_3(M_X)$ as a function of η/ξ [see (29)].

makes sense to demand that $\alpha_G \lesssim 1$, which implies that $\eta \gtrsim 0.2$. It is also interesting that in the range (30) the successful prediction for the ratio m_b/m_τ remains practically unchanged. In general we would obtain [9] (at one-loop)

$$m_b/m_\tau = [5\alpha_3(\mu)/\eta\alpha_G]^{\Gamma_3} [\alpha_1(\mu)/\xi\alpha_G]^{\Gamma_1} , \quad (32)$$

where $\Gamma_i = (\gamma_\tau^{(i)} - \gamma_b^{(i)})/2b_i$, where $\gamma^{(i)}$ are the anomalous dimensions. With $\eta\alpha_G \sim 0.2$, the ratio (32) is practically equal to the "naive" SUSY value.

It is remarkable that the low-energy predictions (30)–(32) show such a consistency. In particular, the result (31) justifies *a posteriori* the neglect of terms of the type $\Sigma_\alpha \Sigma_\beta$ in $f_{\alpha\beta}$, which are suppressed by an extra power of M_X/M_P.

The spectrum of sparticles can now be expected to differ considerably from previous analyses [10,11]. If we parametrize the bare gaugino masses as

$$\mathcal{M}_i(M_X) = \xi_i m_0 \quad (i = 1, 2, 3) ,$$

with m_0 as defined in (26), the isodoublet squarks and right-handed sleptons in radiative models will acquire masses

$$m_{\tilde{q}}^2 = m^2 + C_q(\mu) m_0^2 , \quad m_{\tilde{\ell}_R}^2 = m^2 + C_R(\mu) m_0^2 , \quad (33)$$

where

$$C_q(\mu) = \tfrac{16}{3}\,\xi_3^2 J_3(\mu) + 3\xi_2^2 J_2(\mu) + \tfrac{1}{15}\,\xi_1^2 J_1(\mu)\,,$$

$$C_R(\mu) = \tfrac{12}{5}\,\xi_1^2 J_1(\mu)\,,$$

$$J_i(\mu) \equiv (1/2b_i)\{[\alpha_i(\mu)/\alpha_i(M_X)]^2 - 1\}\,. \qquad (34)$$

Setting $\sin^2\theta_W(M_W) \simeq 0.21$ determines η/ξ and $\alpha_G\eta$ (see figs. 1 and 2). Let us now concentrate on the interesting case of vanishing supersymmetry breaking mass splitting in the leptoquark vector supermultiplets as expressed by (16). At $\mu = M_W$ we find that

$$m_{\tilde{q}}^2/m_{\tilde{g}}^2 = 0.882 + 0.104(m/\eta m_0)^2\,,$$

$$m_{\tilde{\ell}_R}^2/m_{\tilde{g}}^2 = 0.0008 + 0.104(m/\eta m_0)^2\,. \qquad (35)$$

For $\eta m_0 = m$ we obtain $m_{\tilde{q}} \simeq 0.99 m_{\tilde{g}}$ and $m_{\tilde{\ell}_R} \simeq 0.32 m_{\tilde{g}}$. Using the PETRA/PEP bound $m_{\tilde{e}_R} \gtrsim 18$ GeV we find that

$$m_{\tilde{g}} \gtrsim 55.6\ \text{GeV}\,, \quad m_{\tilde{q}} \gtrsim 55.2\ \text{GeV}\,. \qquad (36)$$

Therefore, the simplest possibility favours a two-squark or -gluino explanation [12] of the UA1 monojet events. Another explanation [13] of the monojet events has a light [~O(5) GeV] gluino and O(100) GeV squarks. This spectrum can be realized if $m/\eta m_0 \simeq 62$: a possibility we cannot exclude although it appears surprising.

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