GRAND UNIFICATION IN SIMPLE SUPERGRAVITY

John ELLIS [1], D.V. NANOPOULOS and K. TAMVAKIS
CERN, Geneva, Switzerland

Received 13 October 1982

We discuss aspects of grand unified theories (GUTs) combined with $N = 1$ local supersymmetry, interpreting the general form of simple supergravity theory as a phenomenological effective lagrangian applicable at energies less than the Planck mass. We consider light fermion masses, baryon decay and a possible resolution of the QCD vacuum angle θ problem in this general framework. We propose two mechanisms for weak symmetry breaking based on radiative corrections originating from supergravity effects, notably by a direct gaugino mass in theories with non-minimal coupling to $N = 1$ supergravity. We give a specific example of a GUT employing this mechanism.

Grand Unified Theories (GUTs) of the strong, weak and electromagnetic interactions [1] are incomplete and unsatisfactory because they do not include gravity. This is all the more regrettable because a naïve logarithmic extrapolation of the known strong and weak coupling constants leads one to expect that grand unification occurs at a mass-scale m_X within a few orders of magnitude of the Planck mass m_P at which quantum gravity effects become O(1). Nevertheless, one might hope that one could neglect gravitational effects in GUTs as a first approximation, and perhaps include them as small perturbations of order $(m_X/m_P)^n$ to the results of conventional GUTs. Baryon decay [2] and the light fermion masses [3,4] have been singled out as cases where gravitational effects might be important. The combination of GUTs and gravity has recently become very topical for two reasons. One is the current interest in cosmological applications of GUTs in the very early Universe, particularly during a possible de Sitter phase where quantum gravitational temperature and curvature effects may be crucial [5]. The other reason is the current interest [6] in global supersymmetry as a possible framework for understanding the hierarchy of different mass scales ($m_W \ll m_X < m_P$) in GUTs. By now we are used to believing that Nature abhors global symmetries and prefers them to be local, and making supersymmetry local gets us into supergravity. It may well be that the ultimate and fundamental truly unified theory is an extended supergravity theory, but only simple supersymmetry can easily accommodate the chiral fermion representations needed in GUTs. Therefore we restrict our attention to a simple $N = 1$ supergravity theory [7] as a phenomenlogical lagrangian suitable for describing physics at energy scales $E < m_P$. In contrast to conventional gravity which does not require any other interactions scaled by inverse powers of the Planck mass, $N = 1$ supergravity generates [8] many new interactions of order $(\phi/m_P)^n$ (where ϕ is a generic particle field) relative to conventional renormalizable interactions. It has already been observed [9] that these interactions generate masses of the order of the gravitino mass for all spin-zero fields, and make it difficult to maintain scenarios with supersymmetry breaking at a scale $\geqslant O(10^{10})$ GeV [10,11].

In this paper we give a general discussion of supergravity effects on GUTs. We consider their possible impact on light fermion masses (the ratio m_d/m_e [3]), neutrino masses [4], baryon decay and a possible resolution of the QCD vacuum angle θ problem. We also show how the spontaneous symmetry breaking of the weak interactions may be due to radiative corrections triggered by direct supergravity contributions either to the gaugino masses, or else to spin-zero boson masses. These mechanisms work for top quark masses greater

[1] Address after Oct. 10th 1982: Theory Group, SLAC, P.O. Box 4349, Stanford, CA 94305, USA.

than 1.4 and 2.5 m_W respectively. We give an explicit example of a supersymmetric GUT employing these mechanisms.

We start with a reminder of the structure of $N = 1$ supergravity actions [8] containing gauge and matter fields (if not explicitly stated, we use natural units $\mathcal{K}^2 \equiv 8\pi G_N = 8\pi/m_P^2 = 1$)

$$\mathcal{A} = \int d^4x\, d^4\theta E\{\Phi(\phi, \bar{\phi} e^{2V}) + \mathrm{Re}[R^{-1} g(\phi)] + \mathrm{Re}[R^{-1} f_{\alpha\beta}(\phi) W_a^\alpha \epsilon^{ab} W_b^\beta]\}\,, \tag{1}$$

where E is the superspace determinant, Φ is an arbitrary real function of the chiral superfields ϕ and their complex conjugates $\bar{\phi}$, V is the gauge vector supermultiplet, R is the chiral scalar curvature superfield, g is the chiral superpotential, $f_{\alpha\beta}$ is another chiral function of the chiral superfields ϕ, and W_α^a is a gauge-covariant chiral superfield containing the gauge field strength. In addition to all the obvious general coordinate transformations, local supersymmetry and gauge invariance, the action (1) is also invariant [8] under the transformations

$$J \equiv 3 \ln(-\tfrac{1}{3}\Phi) \to J + f(\phi) + f^*(\bar{\phi})\,,$$
$$g(\phi) \to e^{f(\phi)} g(\phi)\,. \tag{2}$$

These transformations can be related to a description of the chiral superfields ϕ as coordinates on a Kähler manifold with Kähler potential Φ, and the transformations (2) are known as Kähler gauge transformations [12]. One particular manifestation of this Kähler gauge symmetry is in the effective scalar potential

$$V = -\exp(-G)(3 + G_i' G_j''^{\,i\ -1} G'^j) + (\text{gauge terms})\,, \tag{3}$$

where

$$G \equiv J - \ln(\tfrac{1}{4}|g|^2)\,, \tag{4}$$

which is clearly invariant under the transformations (2). In general, the action depends on a real function

$$\hat{\Phi} \equiv \Phi/|g(\phi)|^{2/3} \tag{5}$$

and on the chiral function $f_{\alpha\beta}(\phi)$. The most familiar forms of these functions are $J = -\frac{1}{2}\phi\bar{\phi}$ giving canonical kinetic energy terms for the chiral superfields, $g(\phi)$ a cubic polynomial giving renormalizable matter interactions of dimension $\leqslant 4$, and $f_{\alpha\beta} = \delta_{\alpha\beta}$. We expect that more complicated functions will contain terms $O(\phi/m_P)^n$ relative to these canonical leading terms.

We will interpret equation (1) as an effective action suitable for describing particle interactions at energies $\ll m_P$, just as chiral SU(N) × SU(N) lagrangians were suitable for describing hadronic interactions at energies $\ll 1$ GeV. In much the same way as we know that physics gets complicated at $E = 1$ GeV with many new hadronic degrees of freedom having masses of this order, we also expect many new "elementary particles" to exist with masses $O(m_P)$. It may well be that all the known light "elementary particles" as well as these heavy ones are actually composite, and that at energies $\gg m_P$ a simple preonic picture will emerge, analogously to the economical description of high-energy hadronic interactions in terms of quarks and gluons. It may even be that these preonic constituents are themselves ingredients in an extended supergravity theory [13]. But let us ignore these speculations for the moment and return to our pedestrian phenomenological interpretation of the action (1).

The well-known rules of phenomenological lagrangians [14] are that one should write down all possible interactions consistent with the conjectured symmetries [e.g. chiral SU(2) × SU(2)] and only place absolute belief in predictions which are independent of the general form of the lagrangian (e.g. $\pi\pi$ scattering lengths). These are the reliable results which could also be obtained using current algebra arguments. It does not make sense to calculate strong interaction radiative corrections (read: supergravity loop corrections) to these unimpeachable predictions: these are ambiguous until we know what happens at the 1 GeV scale (read: m_P) and our ignorance can be subsumed in the general form of the phenomenological lagrangian in which any and all possible terms are present a priori (read: non-trivial J, non-polynomial g and $f_{\alpha\beta}$). On the other hand, non-strong interaction radiative corrections can often be computed meaningfully (e.g. the $\pi^+ - \pi^0$ mass difference, large numbers of pseudo-Goldstone boson masses in extended technicolour theories). Similarly, it makes sense to compute matter interaction (gauge, Yukawa, Higgs) corrections to the tree-level predictions of the effective action (1).

As an example of this philosophy for using (1), let us consider the cosmological constant C. The canonical procedure [8] is to look for a local minimum of the effective potential (3) at which $V = 0$ and hence also $C = 0$. If this is done, one can derive a formula for the supertrace of the (mass)^2 matrix. If $J = -\frac{1}{2}\phi\bar{\phi}$ and $f_{\alpha\beta} = \delta_{\alpha\beta}$, this is [8]

$$\mathrm{Str}\,\mathcal{M}^2 = 2(N-1)m_G^2\,, \tag{6}$$

where N is the total number of chiral superfields. The sum rule (6) apparently contains the seeds of its own destruction, since the radiative corrections to the cosmological constant are [15]

$$\delta C = \sum_s (-1)^{2s+1}(8\pi^2)^{-1}(2s+1)\Lambda^4 - (16\pi^2)^{-1}(\mathrm{Str}\,\mathcal{M}^2)\Lambda^2 + (64\pi^2)^{-1}(\mathrm{Str}\,\mathcal{M}^4)\ln\Lambda^2\,, \tag{7}$$

where Λ is a momentum cut-off which is probably $O(m_P)$. The first term in eq. (7) vanishes in any supersymmetric theory and so does not concern us. The second term is guaranteed by eq. (6) not to vanish except in the physically uninteresting case $N = 1$. But if $C \neq 0$, then the sum rule (6) is no longer valid: apparently [16]

$$\mathrm{Str}\,\mathcal{M}^2 = 4m_G^2[\tfrac{1}{2}(N-1) + \tfrac{1}{2}(N-1)(C/m_G^2) - \tfrac{2}{9}(C/m_G^2)^2]\,, \tag{8}$$

although this statement begs the question of the obscure interpretation of mass terms in de Sitter space, which is where we start from if $C \neq 0$. A possible way out of the conundrum might be to solve the conditions (7), (8) self-consistently by requiring $C + \delta C = 0$ [16]. But then what about higher order contributions to δC? And if one could sum all these, what about radiative corrections to other parameters in the effective action (1) which should presumably also be included in the self-consistent calculation of $\mathrm{Str}\,\mathcal{M}^2$ and C? We believe that the correct procedure is to keep $C = 0$ and use the most general forms [17] of J and $f_{\alpha\beta}$, and only believe results obtained with non-gravitational loop corrections.

Our basic reason for this belief is the phenomenological lagrangian philosophy outlined above. This is buttressed by two other arguments. One is the automatic appearance of non-trivial J and $f_{\alpha\beta}$ functions in extended supergravity theories [8]. The other is the appearance of gaugino masses at the one-loop level as soon as local supersymmetry is broken [18]. We would not have had a non-zero gaugino mass at the tree level if $f_{\alpha\beta} = \delta_{\alpha\beta}$, but one is generated by loop diagrams [18] and this can only be mimicked in the phenomenological lagrangian by choosing non-trivial $f_{\alpha\beta}$, which one should introduce ab initio into the effective action as a possible renormalization counterterm. Since the supergravity action is non-renormalizable, and since both the Φ and $f_{\alpha\beta}$ terms in the action (1) have a $\int d^4\theta$ form, we expect general variants of them to be generated by loop corrections. Presumably radiative corrections maintain the essential geometry of the Kähler manifold [12]. Therefore we expect loop corrections to fall into the class of Kähler gauge transformations (2). The only analogous transformation allowed in a conventional renormalizable theory is f = constant, corresponding to a wave function renormalization. In our case more general gauge functions $f(\phi)$ might appear.

What might be some of the phenomenological manifestations of these non-minimal couplings of matter to simple supergravity?

Non-minimal superpotential. There could be a quartic term in $g(\phi)$ such as

$$g_4 \ni (\lambda/m_P)\bar{F}\bar{H}\Sigma T\,, \tag{9}$$

where $\bar{F}$ is a $\bar{5}$ of matter (quark + lepton) chiral superfields in SU(5), T is a **10** of matter superfields, Σ is an adjoint **24** of Higgs superfields, $\bar{H}$ is a $\bar{5}$ of Higgs superfields, and λ is some generalized Yukawa coupling. When SU(5) is broken down to SU(3) × SU(2) × U(1) at a scale of 10^{16} GeV, and SU(2) × U(1) is broken down to $U(1)_{em}$ at $O(10^2)$ GeV, the Higgs superfields in the expression (9) acquire vacuum expectation values which give contributions to the charge-$\frac{1}{3}$ quark and charge -1 lepton masses which could easily be O(1) MeV and remedy the old disaster $m_d = m_e$ of minimal SU(5) [‡1]. Likewise a quartic term such as

$$g_4 \ni (\lambda/m_P)\bar{F}H\bar{F}H\,, \tag{10}$$

could give rise to neutrino masses $O(10^{-5}$ eV which could show up in solar neutrino experiments. Both of the interactions (9), (10) are counterparts of analogous terms in conventional GUTs [3,4].

Turning now to baryon decay, an interaction of the form

‡1 This possibility is particularly important in view of the difficulties with ideas for rectifying m_d/m_e by using radiative corrections recently pointed out by Ibáñez [19].

$$g_4 \ni (\lambda/m_P)\bar{F}TTT\,, \qquad (11)$$

could replace the Higgs exchange in the Weinberg–Sakai–Yanagida loop diagram [20] for baryon decay. The magnitude of the diagram with (11) relative to the conventional Higgs diagram is

$$(\lambda/m_P)/(\lambda^2/m_H) \approx (m_H/\lambda m_P)\,, \qquad (12)$$

where λ is a generic Yukawa coupling. The ratio (12) could easily be >1, making a non-renormalizable superpotential interaction the dominant contribution to baryon decay. There is no particular reason that it should respect the selection rules worked out for Higgs exchange. A possible signature would be if baryon decay modes turn out different from all the suggestions [21–23] made so far. This mechanism could give observable baryon decay even if the grand unification mass $m_X \approx m_P$.

Non-minimal $f_{\alpha\beta}$. It has already been observed [8] that the gauge kinetic term in (1), as well as giving rise to the canonical

$$-\tfrac{1}{4}F^{\alpha}_{\mu\nu}F^{\beta}_{\mu\nu}(\mathrm{Re}\, f_{\alpha\beta})\,, \qquad (13)$$

could also yield the *CP*-violating θ vacuum term

$$\epsilon^{\mu\nu\rho\sigma}F^{\alpha}_{\mu\nu}F^{\beta}_{\rho\sigma}(\mathrm{Im}\, f_{\alpha\beta})\,. \qquad (14)$$

We know that θ_{QCD} is $<O(10^{-9})$ experimentally, and that θ is not renormalized in a supersymmetric theory [24]. It is finitely renormalized when supersymmetry is broken, but [24] this is plausibly only by an amount $\delta\theta = O(10^{-16})$ in the popular Kobayashi–Maskawa model [25]. Thus we see that θ should be less than $O(10^{-9})$ in a supersymmetric GUT and may be very small. An attractive hypothesis is that $f_{\alpha\beta}$ is a function with only real coefficients as found in extended supergravities [17]. In this case $\mathrm{Im}\, f_{\alpha\beta} = 0$ when $\langle 0|\phi|0\rangle = 0$, and the theory is *CP*-invariant in the gauge sector. If some of the ϕ then acquire complex vacuum expectation values they will induce a non-zero value of $\mathrm{Im}\, f_{\alpha\beta}$ and hence violate *CP* spontaneously in the gauge sector, which is a new twist on an old proposal [26]. If the moduli of some of these complex $\langle 0|\phi|0\rangle$ were $O(m_P)$, then the effective θ parameter would be $O(1)$ which is phenomenologically unacceptable. However, it is easy to imagine scenarios where θ is much smaller. For example if $f_{\alpha\beta} = \delta_{\alpha\beta} + O(\phi^2/m_P^2)$ and the culprit $\langle 0|\phi|0\rangle = O(m_X)$ then

$$\theta = O((m_X^2/m_P^2) \times \text{some small (?) angle}) \lesssim O(10^{-7}) \qquad (15)$$

and the phenomenological constraint on θ_{QCD} could easily be respected. If the only complex $\langle 0|\phi|0\rangle$ were $O(m_W)$, or if all the $\langle 0|\phi|0\rangle$ were real as in all supersymmetric GUTs proposed to date, then the bare $\theta = 0$, and the low-energy θ_{QCD} would just be given by $\delta\theta = O(10^{-16})$ [24,25]. Hence supergravity offers the other half of an answer to the θ vacuum problem. This is fortunate since the rival axion solution now seems to have serious cosmological problems [27].

Even if J and $f_{\alpha\beta}$ are canonical, the effective potential (3) contains many additional terms besides the usual ones derivable from a canonical superpotential $g(\phi)$:

$$V = \exp\left(+\tfrac{1}{2}\sum_{\phi}|\phi|^2\right) \times \left(2\sum_{\phi}|\partial g/\partial\phi + \tfrac{1}{2}\phi^* g(\phi)|^2 - 3|g(\phi)|^2\right)\,. \qquad (16)$$

In a previous paper [9], two of us (J.E. and D.V.N.) have pointed out the striking implications of eq. (16) for spin-zero boson masses M_0. For states which have associated vacuum expectation values $\langle 0|\phi|0\rangle \ll m_P$ and do not mix with the spin-zero partners of the goldstino (e.g. squarks, sleptons) the average of the spin-zero masses in each chiral supermultiplet obeys

$$M_0^2 \gtrsim \exp\left(+\tfrac{1}{2}\sum_{\phi}|\phi|^2\right)|g|^2 = m_G^2\,. \qquad (17)$$

This immediately implies that

$$O(10^2\ \mathrm{GeV}) \gtrsim \exp\left(+\tfrac{1}{4}\sum_{\phi}|\phi|^2\right)|g| = m_G\,, \qquad (18)$$

which is a stringent upper limit on the scale of local supersymmetry breaking [8].

It has been suggested [28] that this theorem could be evaded by having a global symmetry broken at the same time as local supersymmetry, in which case one should have many massless Goldstone bosons. However, these cannot include both the spin-zero particles in all the "light" chiral supermultiplets. The great majority of these would presumably actually be pseudo-Goldstones corresponding to symmetries which are not respected by the non-gravitational interactions, and which have masses calculable using standard techniques. However, even in this case loop diagrams generate (mass)^2 for gauge non-singlet spin-zero fields in O(α), and the previous upper bound on the gravitino mass and the scale of local supersymmetry breaking cannot be greatly relaxed [28]. Another possible way of evading the theorem might be to abandon the constraint that the cosmological constant vanish [16]. Indeed, the dangerous contributions to the spin-zero boson masses vanish if

$$\sum_i |G'^i|^2 = 1 , \quad \text{or} \quad \sum_\phi |\partial g/\partial\phi + \tfrac{1}{2}\phi^* g(\phi)|^2 = |g(\phi)|^2, \tag{19}$$

instead of the canonical conditions with 3/2 on the right-hand side of eq. (19). However, as we said above, in the absence of a complete self-consistent calculation of all the loop corrections to particle masses as well as to the cosmological constant, we believe the correct phenomenological procedure is to work with the most general effective action (1), set $C = 0$ at the tree level, and try to live with the consequences.

It has also been observed recently [18] that loop corrections give quadratically divergent gaugino and scalar masses

$$\delta m_{\tilde{g}}^2, \delta m_0^2 = \mathrm{O}(m_G^2)(\Lambda^2/m_P^2)(8\pi^2)^{-1} , \tag{20}$$

which are probably O(m_G^2) when one takes a plausible cut-off $\Lambda = \mathrm{O}(m_P)$. We presume that these effects can be absorbed into non-canonical forms of the functions g, J and $f_{\alpha\beta}$. Indeed the gaugino mass term

$$(\tfrac{1}{4} f'^k_{\alpha\beta} e^{-G/2} G'_l G''^{-1}{}^l_k)\bar{\lambda}^\alpha_L \lambda^\beta_L , \tag{21}$$

is clearly also O(m_G) if $f_{\alpha\beta}$ is non-trivial and contains terms $\mathrm{O}(\phi/m_P)^n$ where $\langle 0|\phi|0\rangle = \mathrm{O}(m_P)$. We therefore think that the most natural possibility is that gaugino masses are O(m_G) as well as spin-zero boson masses.

This observation suggests two novel scenarios for weak interaction symmetry breaking that we would like to discuss briefly. It has been shown previously [29,11,30] that radiative corrections can generate an interesting pattern of symmetry breaking starting off from an input gaugino mass M: including radiative corrections at low energies

$$m_{\tilde{g}} = \alpha_3 M/\alpha_G , \quad m_{\tilde{W}} = \alpha_2 M/\alpha_G ,$$

$$m_{\tilde{B}} = \tfrac{5}{3}\alpha_1 M/\alpha_G , \tag{22}$$

where we will here take M to be O(m_G) generated by supergravity effects. Gaugino loop diagrams [11] generate

$$\delta m_{\tilde{q}}^2 = (4\alpha_3/3\pi)L_3 m_{\tilde{g}}^2 + \ldots ,$$

$$\delta m_H^2, \delta m_{\tilde{l}}^2 = (3\alpha_2/4\pi)L_2 m_{\tilde{W}}^2 + \ldots , \tag{23}$$

where $L_n = 6\pi/(11n - 12)\alpha_n$ and $\alpha_{n=3,2}$ are evaluated at momenta O(m_W). If there are also bare scalar boson masses μ whose renormalization from m_X down to low energies are easily calculated:

$$m_{\tilde{q}} = (\alpha_3/\alpha_{GUT})^{8/9}\mu + \ldots ,$$

$$m_H, m_{\tilde{l}} = (\alpha_2/\alpha_{GUT})^{-3/2}\mu + \ldots , \tag{24}$$

then one has the following general form for the total squark and slepton masses:

$$M_{\tilde{q}}^2 = \alpha_3^3(4L_3/3\pi\alpha_{GUT}^2)M^2 + \alpha_3^{16/9}(\alpha_{GUT}^{16/9})^{-1}\mu^2 + \ldots , \tag{25a}$$

$$M_{\tilde{l}}^2 = \alpha_2^3(3L_2/4\pi\alpha_{GUT}^2)M^2 + \alpha_2^{-3}(\alpha_{GUT}^3)\mu^2 + \ldots . \tag{25b}$$

For the Higgs fields there are in addition interesting contributions coming from the quark Yukawa couplings h_q:

$$\delta m_H^2 = -(36/1225\pi)(h_t^2/\alpha_{GUT}^2)\alpha_3 M^2 + (-27/322\pi)(h_t^2/\alpha_3)(\alpha_3/\alpha_{GUT})^{16/9}\mu^2 + \ldots, \tag{26}$$

where

$$h_t v_1 = m_t , \quad h_b v_2 = m_b , \tag{27a}$$

for the top and bottom quarks, and similarly for the lighter quarks, with $v_{1,2}$ the vacuum expectation values of the doublets of Weinberg–Salam Higgs fields:

$$m_W^2 = \tfrac{1}{2} g_2^2(v_1^2 + v_2^2) . \tag{27b}$$

Clearly

$$M_H^2 = M_{\tilde{l}}^2 + \delta m_H^2 , \tag{28}$$

is less than zero for sufficiently large h_t (heavy t quark) and spontaneous gauge symmetry breaking takes place [29,11,30]. If we neglect the μ terms in the expressions (29) then spontaneous symmetry breaking occurs for

$$m_t^2 + m_b^2 > (\tfrac{1225}{160}\alpha_2/\alpha_3)m_W^2 , \tag{29}$$

which means numerically

$$m_t > 1.4\, m_W . \tag{30}$$

Thus it is possible to imagine that weak gauge symmetry breaking is triggered by a gaugino mass generated by supergravity effects. In this case one expects $m_{\tilde{g}}, m_{\tilde{W}}, m_G = O(m_W)$.

Another scenario is suggested by eqs. (25), namely that weak symmetry breaking occurs even if the direct gaugino mass M is negligible. Retaining only the μ terms in eqs. (25) and using eq. (26) as before, we find that M_H^2 (28) is negative if

$$m_t^2 + m_b^2 > (\tfrac{161}{27}\,\alpha_{GUT}^{43/9}/\alpha_3^{7/9}\alpha_2^4)m_W^2\,, \qquad (31)$$

corresponding to

$$m_t > 2.5\,m_W\,. \qquad (32)$$

This is more difficult to realize than was the gaugino scenario, but by no means impossible. In this case one expects to have $m_G = O(m_W)$ but even lighter gauginos.

It may be worth commenting that in both these scenarios the dominant contributions to the squark and slepton masses are universal and generation-independent, so that difficulties [31] with flavour-changing neutral interactions do not arise from violations of super-GIM cancellations. It should also be noted that in both of these scenarios for weak symmetry breaking there will be radiative corrections to the vacuum energy, and hence to the cosmological constant, which are $O(\alpha^n m_W^4)$. We do not mind starting with a non-zero cosmological constant of this order of magnitude and then renormalizing, since it is a non-gravitational radiative correction. Previously we have argued only that all gravitational loop corrections should be lumped together into the initial value of C.

A model that embodies the features described above can easily be constructed. The usual SU(5) superfields [6] should be supplemented by a singlet superfield à la Polonyi [32] that appears linearly in the superpotential so that the cosmological constant can be set to zero. The superspace potential is

$$g = \mu m_P z + B + \tfrac{1}{2}m\,\mathrm{Tr}(\Sigma^2) + \tfrac{1}{3}\lambda\,\mathrm{Tr}(\Sigma^3) + \mathrm{h}\bar{\mathrm{H}}\Sigma\mathrm{H}$$
$$+ m'\bar{\mathrm{H}}\mathrm{H} + \bar{\mathrm{F}}\mathrm{T}\bar{\mathrm{H}} + \mathrm{TTH}\,. \qquad (33)$$

The different mass scales appearing in (33) are the Planck mass m_P, m, m' of $O(10^{16})$ the grand unification scale, and $\mu \sim O(m_W)$. The effective scalar potential (3) can be written in terms of the generalized g-terms

$$g_z = \mu m_P + \tfrac{1}{2}z^* g\,,$$

$$(g_\Sigma)_i^j = m\Sigma_i^j + \lambda(\Sigma^2)_i^j + \mathrm{h}\mathrm{H}_i\bar{\mathrm{H}}^j + \tfrac{1}{2}\Sigma_i^{+j}g\,,$$

$$g^i_{\mathrm{H},\bar{\mathrm{H}}} = h(\mathrm{H},\bar{\mathrm{H}})^j\Sigma_j^i + m'(\mathrm{H},\bar{\mathrm{H}})^i + \tfrac{1}{2}(\bar{\mathrm{H}}^+,\mathrm{H}^+)^i g, \quad \text{etc.} \qquad (34)$$

We find

$$V = \exp[\tfrac{1}{2}(|z|^2 + \mathrm{Tr}|\Sigma|^2 + |\mathrm{H}|^2 + |\bar{\mathrm{H}}|^2)]$$
$$\times \{2[|g_z|^2 + |g_{\mathrm{H},\bar{\mathrm{H}}}|^2 + |g^j_{\Sigma_i} - \tfrac{1}{5}\delta_i^j\,\mathrm{Tr}\,g_\Sigma|^2]$$
$$- 3|g|^2\} + \text{etc.} \qquad (35)$$

[The quark and lepton fields are irrelevant in the determination of the vacuum and hence omitted in (35).]

The minimization equations reduce to $\langle\mathrm{H}\rangle = \langle\bar{\mathrm{H}}\rangle = 0$, and

$$\tfrac{1}{2}z^+ V + \exp(...)$$
$$\times(\mu z^+ g_z^+ + g^+ g_z^+ + \mu\phi^+ g_\phi^+ - 3\mu g^+) = 0\,, \qquad (36)$$

$$\tfrac{1}{2}\phi^+ V + \exp(...)$$
$$\times\{z^+ g_z^+(m - \lambda\phi/\sqrt{30})\phi$$
$$+ 2g_\phi^+[m - 2\lambda\phi/\sqrt{30} + \tfrac{1}{2}|\phi|^2(m - \lambda\phi/\sqrt{30})]$$
$$+ g_\phi g - 3g^+\phi(m - \lambda\phi/\sqrt{30})\} = 0\,, \qquad (37)$$

where Σ has been replaced by

$$\Sigma = (\phi/\sqrt{30})\,\mathrm{diag}(2, 2, 2, -3, -3)\,,$$

$$g_\phi = \phi(m - \lambda\phi/\sqrt{30}) + \tfrac{1}{2}\phi^+ g\,.$$

It is not very easy to determine exactly which ϕ and z satisfy (36) and (37), but a few simple conclusions can be drawn. For $\phi = 0$ [SU(5)] we find (for a vanishing potential $V = 0$) that (36) cannot be satisfied unless B is fine tuned to $\mu(2 - \sqrt{3})\sqrt{2}$. For any other value of B, SU(5) is not a minimum with $\Lambda = 0$.

Looking now for a solution to (37) with $\langle\phi\rangle \sim O(m)$ we can discard the first term in the parenthesis, as it is of the order of at most $\mu^2 m$, while the rest of the terms are of the order of μm^2. Then, for a vanishing cosmological constant [i.e. to $O(\mu^4)$ as a first approximation] we obtain

$$\lambda\phi/\sqrt{30} \approx m + \mu/\sqrt{2} - (\sqrt{3}/2)\mu^2/m\,. \qquad (38)$$

Turning now to eq. (36), we obtain

$$(\mu z + g)(\mu + \tfrac{1}{2}zg) - 3\mu g + (\sqrt{3}\cdot 15/\lambda^2)(\mu^3 m) = O(\mu^4). \qquad (39)$$

For a vanishing cosmological constant, (39) is supplemented by

$$2[\mu + \tfrac{1}{2}zg]^2 - 3g^2 + (45/\lambda^2)(\mu^4) = O(\mu^5/m) . \quad (40)$$

It is not difficult to check that for $g = \mu\sqrt{2}$ and $z = \sqrt{2}(\sqrt{3} - 1)$ (39) and (40) are satisfied up to terms of $O(\mu^2)$. Of course in order to have $g = \mu\sqrt{2}$ we must fine tune the parameter B to the value

$$B = \sqrt{2}(2 - \sqrt{3})\mu - (5/\lambda^2)(m^3) + (15/2\lambda^2)(m\mu^2) + O(\mu^3) . \quad (41)$$

When we do this the SU(5) minimum is excluded.

The model (33) is very crude and not to be taken seriously, employing as it does repugnant fine-tuning. However, it does serve as an example of how the two scenarios of SU(2) gauge symmetry breaking triggered by supergravity effects can be realized. It may well turn out that supergravity effects play a crucial role in the construction of supersymmetric GUTs.

We thank participants in the CERN "Not the standard model" workshop, especially H.P. Nilles and M. Srednicki, for their valuable comments on this work.

[1] For reviews see:
D.V. Nanopoulos, Ecole d/Eté de Physique des Particules, Gif-sur-Yvette, 1980, (IN2P3, Paris, 1980) p. 1;
J. Ellis, Gauge theories and experiments at high energies, eds. K.C. Bowler and D.G. Sutherland (Scottish Universities Summer School in Physics, Edinburgh, 1981) p.201; CERN preprint TH.3174 (1981), to be published in Proc. 1981 Les Houches Summer School;
P. Langacker, Phys. Rep. 72C (1981) 185; Proc. 1981 Intern. Symp. on Lepton and photon interactions at high energies, ed. W. Pfeil (University Bonn, 1981) p. 823.
[2] Ya.B. Zel'dovich, Phys. Lett. 59A (1976) 254.
[3] J. Ellis and M.K. Gaillard, Phys. Lett. 88B (1979) 315.
[4] R. Barbieri, J. Ellis and M.K. Gaillard, Phys. Lett. 90B (1980) 249.
[5] S.W. Hawking and I.G. Moss, Phys. Lett. 110B (1982) 35.
[6] S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150;
N. Sakai, Z. Phys. C11 (1982) 153;
E. Witten, Nucl. Phys. B188 (1981) 513.
[7] A.H. Chamseddine, R. Arnowitt and P. Nath, Northeastern University preprints NUB-2559, 2565 (1982);
L.E. Ibáñez, CERN preprint TH-3374 (revised) (1982).
[8] E. Cremmer et al., Phys. Lett. 79B (1978) 231; Nucl. Phys. B147 (1979) 105;
E. Cremmer, S. Ferrara, L. Girardello and A. van Proeyen, CERN preprints TH-3319 and TH-3348 (1982).
[9] J. Ellis and D.V. Nanopoulos, Phys. Lett. 116B (1982) 133.
[10] R. Barbieri, S. Ferrara and D.V. Nanopoulos, Z. Phys. C13 (1982) 276; Phys. Lett. 116B (1982) 16.
[11] J. Ellis, L.E. Ibáñez and G.G. Ross, Phys. Lett. 113B (1982) 283.
[12] B. Zumino, Phys. Lett. 87B (1979) 203.
[13] J. Ellis, M.K. Gaillard, L. Maiani and B. Zumino, Unification of the fundamental particle interactions, eds. S. Ferrara, J. Ellis and P. van Nieuwenhuizen (Plenum, New York, 1980) p. 69;
J. Ellis, M.K. Gaillard and B. Zumino, Phys. Lett. 94B (1980) 343.
[14] S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1968) 2239;
C. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1968) 2247;
S. Weinberg, Phys. Rev. 166 (1968) 1568.
[15] B.S. De Witt, Dynamical theory of groups and fields (Gordon and Breach, New York, 1965) pp. 231, 233;
B. Zumino, Nucl. Phys. B89 (1975) 535.
[16] See H.P. Nilles, CERN preprint TH-3398 (1982).
[17] M.T. Grisaru, M. Roček and A. Karlhede, Caltech preprint CALT-68-949 (1982).
[18] R. Barbieri, S. Ferrara, D.V. Nanopoulos and K. Stelle, Phys. Lett. 113B (1982) 219.
[19] L.E. Ibáñez, CERN preprint TH-3352 (1982).
[20] S. Weinberg, Phys. Rev. D26 (1982) 287;
N. Sakai and T. Yanagida, Nucl. Phys. B197 (1982) 533.
[21] J. Ellis, D.V. Nanopoulos and S. Rudaz, Nucl. Phys. B209 (1982) 43;
S. Dimopoulos, S. Raby and F. Wilczek, Phys. Lett. 112B (1982) 113.
[22] D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 110B (1982) 449; 113B (1982) 151; 114B (1982) 235.
[23] A. Masiero, D.V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. 115B (1982) 298.
[24] J. Ellis, S. Ferrara and D.V. Nanopoulos, Phys. Lett. 114B (1982) 231.
[25] J. Ellis and M.K. Gaillard, Nucl. Phys. B150 (1979) 141.
[26] T.D. Lee, Phys. Rep. 9 (1973) 143.
[27] P. Sikivie, Phys. Rev. Lett. 48 (1982) 1156;
L.F. Abbott and P. Sikivie, Brandeis University preprint (1982);
M. Dine and W. Fischler, Pennsylvania University preprint UPR-0201T (1982);
J. Preskill, M.B. Wise and F. Wilczek, Harvard University preprint HUTP-82/A048 (1982).
[28] S. Weinberg, M.K. Gaillard, L. Hall and G.G. Ross, private communications.
[29] L.E. Ibáñez and G.G. Ross, Phys. Lett. 110B (1982) 215.
[30] J. Ellis, L.E. Ibáñez and G.G. Ross, CERN preprint TH-3382 (1982).
[31] J. Ellis and D.V. Nanopoulos, Phys. Lett. 110B (1982) 44.
[32] J. Polonyi, Budapest preprint KFKI-1977-93 (1977).