# HIGGS MIXING IN SUPERGRAVITY

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We show that the mass mixing term between Higgs doublets in a phenomenological $N = 1$ supergravity theory is naturally of the same order of magnitude as the supersymmetry breaking gaugino mass term.

The presence of global supersymmetry (SUSY) broken at low energies of O(100) GeV is an aid to assaults on the naturalness and hierarchy problems in GUTs. SUSY stabilizes [1] the masses of the doublets of Higgs scalars responsible for gauge symmetry breaking, which must be comparable with the scale of global SUSY breaking. Also, in some models the scales of weak symmetry breaking [2,3] and global SUSY [3] are determined dynamically to be $\ll m_P$. Phenomenologically realistic models also require [4] mixing between the two light Higgs doublets $H_1H_2$, which is specified by an additional mass parameter $\mu$ in the superpotential. This mass parameter $\mu$ must also be $O(m_W)$, and therefore poses its own special version of the naturalness problem. Is $\mu$ naturally of $O(m_W)$? and can $\mu$ be determined dynamically along with the scales of SUSY and weak symmetry breaking? The appropriate theoretical framework for constructing phenomenological supersymmetric theories is $N = 1$ supergravity, used to provide an effective low-energy theory with global SUSY broken by terms such as scalar and gaugino masses $m_0$, $m_{1/2}$ [‡1]. These are of the order of the gravitino mass $m_{3/2}$ in minimal supergravity theories, but could be quite different in non-minimal theories, such as the no-scale models [3,6] in which $m_W$, $m_0$ and $m_{1/2}$ are determined dynamically. $N = 1$ supergravity theories are specified [7] by a Kahler potential $G$ which is a real function of the chiral fields $\phi$, and characterizes their kinetic terms and superpotential interactions $g(\phi)$, and a chiral function $f_{\alpha\beta}(\phi)$ which transforms as the symmetric product of two adjoint representations of the gauge group and gives kinetic terms to the gauge superfields $W_\alpha$:

$$S = \int d^4x \,\mathrm{Re} \int d^2\theta \,[f_{\alpha\beta}(\phi) W^\alpha W^\beta + g(\phi)] + \int d^4x \, d^4\theta \, \Phi(\phi^+ e^{2V}, \phi) , \quad G = -3 \ln(-\tfrac{1}{3}\phi) + \ln |g|^2 . \tag{1}$$

We do not [8] interpret such a theory as fundamental, but extract from it an effective theory believed to describe physics at energies $\ll m_P$. To be natural, this theory should contain all possible terms consistent with desired symmetries, with coefficients at least as large as would be generated by radiative corrections. Srednicki and Theisen [9] have recently made an interesting calculation to check this philosophy, showing that one-loop corrections to the

‡1 For reviews, see ref. [5].

effective potential in minimal $N = 1$ supergravity do not generate corrections to the scalar and fermion masses which are larger than $O(m_{3/2})$. Their result is true if the Higgs fields $H_1$ and $H_2$ only enter linearly into the superpotential, as in the "missing partner" mechanism [10]. This is a rather dramatic result because in this case there is no way of creating radiatively a $\mu$ term by integrating out the GUT fields [11]. Such radiative creations are in any case absent in no-scale GUT models [12]. So we face another troublesome hierarchy problem, which we now proceed to solve.

Here we analyze the naturalness of small $H_1H_2$ mixing: the superpotential parameter $\mu = O(m_W)$ in the effective lagrangian philosophy. It is clear that small $\mu$ is natural as far as radiative corrections in the low-energy renormalizable field theory are concerned, but what is the order of magnitude of radiative corrections due to non-renormalizable terms in the lagrangian? We allow the maximum ‡2 reasonable $H_1H_2$ mixing term in the gauge kinetic function

$$f_{\alpha\beta}(\phi) = f_\alpha \delta_{\alpha\beta} = \delta_{\alpha\beta}(f_\alpha^0 + f_\alpha^1 H_1 H_2) + \dots , \tag{2}$$

where $f_\alpha^1/f_\alpha^0 = O(1)$, and show that the quadratically divergent loops generated by (2) contribute to the superpotential mixing an amount

$$\delta\mu = O(m_{1/2}) , \tag{3}$$

if the loop cut-off $\Lambda = O(m_P)$, where $m_{1/2}$ is the gaugino mass. Thus not only is $\mu = O(m_W)$ technically natural, but $\mu$ is dynamically determined to be of this order in no-scale models [3,6] where the scale of global symmetry breaking is fixed by radiative corrections.

It is well known [7] that non-trivial chiral functions $f_{\alpha\beta}$ such as in eq. (2) can generate inter alia gaugino masses:

$$\mathcal{L}_{\lambda\lambda} \ni \tfrac{1}{4} e^{G/2} G_i (G^{-1})^i_j \frac{\partial f^*_{\alpha\beta}}{\partial \phi^*_j} (\bar\lambda^\alpha_R \lambda^\beta_R) - \tfrac{1}{32} (G^{-1})^i_j \frac{\partial f_{\alpha\beta}}{\partial \phi^i} \frac{\partial f^*_{\gamma\delta}}{\partial \phi^*_j} (\bar\lambda^\alpha_L \lambda^\beta_L)(\bar\lambda^\gamma_R \lambda^\delta_R)$$
$$+ \tfrac{1}{16} \left( -4 G^k_{ij} (G^{-1})^l_k \frac{\partial f_{\gamma\delta}}{\partial \phi^l} + 4 \frac{\partial^2 f_{\gamma\delta}}{\partial \phi^i \partial \phi^j} - \mathrm{Re}\, f^{-1}_{\alpha\beta} \frac{\partial f_{\alpha\gamma}}{\partial \phi^i} \frac{\partial f_{\beta\delta}}{\partial \phi^j} \right) (\bar\chi^i_L \chi^j_L)(\bar\lambda^\gamma_L \lambda^\delta_L) + \text{h.c.} , \tag{4}$$

where we follow the notation of Cremmer et al.:

$$G_i \equiv \partial G/\partial \phi^i , \quad G^j_i \equiv \partial^2 G/\partial \phi^i \partial \phi^*_j , \quad (G^{-1})^j_i G^i_k = \delta^j_k , \ \text{etc.} \tag{5}$$

We take $f_{\alpha\beta}$ to be real and diagonal in the ground state, as in eq. (2), and to get the correct normalization of the gauge kinetic terms we rescale the gauge fields according to

$$\hat A^\alpha_\mu = A^\alpha_\mu f^{1/2}_\alpha , \quad \hat\lambda^\alpha = f^{1/2}_\alpha \lambda^\alpha . \tag{6}$$

Simultaneously, we must redefine the gauge couplings by

$$g^2 = g^2_\alpha f_\alpha . \tag{7}$$

In terms of the rescaled gaugino fields, eq. (5) becomes

$$\mathcal{L}_{\lambda\lambda} \ni \mathcal{M}_{\alpha\beta} \bar{\hat\lambda}^\alpha \hat\lambda^\beta - \tfrac{1}{32} (G^{-1})^j_i \frac{\partial f_{\alpha\beta}}{\partial \phi^j} \frac{\partial f^*_{\gamma\delta}}{\partial \phi^*_i} (f_\alpha f_\beta f_\gamma f_\delta)^{-1/2} (\bar{\hat\lambda}^\alpha \hat\lambda^\beta)(\bar{\hat\lambda}_\gamma \hat\lambda^\beta_\delta)^*$$
$$+ \tfrac{1}{16} (f_\gamma f_\delta)^{-1/2} \left( -4 G^k_{ij} (G^{-1})^l_k \frac{\partial f_{\gamma\delta}}{\partial \phi^l} + 4 \frac{\partial^2 f_{\gamma\delta}}{\partial \phi^i \partial \phi^j} - \mathrm{Re}\, f^{-1}_{\alpha\beta} \frac{\partial f_{\alpha\gamma}}{\partial \phi^i} \frac{\partial f_{\beta\delta}}{\partial \phi^j} \right) (\bar\chi^i \chi^j)(\bar{\hat\lambda}_\gamma \hat\lambda_\delta) + \text{h.c.} , \tag{8}$$

where we have defined

$$\mathcal{M}_{\alpha\beta}(\hat\lambda) \equiv \tfrac{1}{4} (f_\alpha f_\beta)^{-1/2} e^{G/2} G_i (G^{-1})^i_j \frac{\partial f^*_{\alpha\beta}}{\partial \phi^*_j} . \tag{9}$$

‡2 We use natural units: $M \equiv m_P/\sqrt{8\pi} = 1$.

We are interested in no-scale models [3] based on an SU($n$, 1)/SU($n$) × U(1) Kahler manifold [10] for which

$$G = -3\ln(z + z^* - \tfrac{1}{3}\phi^i\phi_i^*) + \dots . \qquad (10)$$

In this case, the Kahler metric

$$G_i^j = e^{G/3}\delta_i^j + O(H^2) , \qquad (11)$$

and eq. (10) becomes

$$\mathcal{L}_{\hat\lambda\hat\lambda} \ni \mathcal{M}_{\alpha\beta}\bar{\hat\lambda}^\alpha\hat\lambda^\beta - \tfrac{1}{32}e^{-G/3}\, 2\left(\frac{\partial f_{\alpha\beta}}{\partial H_1}\frac{\partial f^*_{\gamma\delta}}{\partial H_1^*} + \frac{\partial f_{\alpha\beta}}{\partial H_2}\frac{\partial f^*_{\gamma\delta}}{\partial H_2^*}\right)(f_\alpha f_\beta f_\gamma f_\delta)^{-1/2}(\bar{\hat\lambda}^\alpha\hat\lambda^\beta)(\bar{\hat\lambda}^\gamma\hat\lambda^\delta)^*$$

$$+ \tfrac{1}{4}(\bar{\tilde H}_{1L}\tilde H_{2L})(\bar{\hat\lambda}^\gamma\hat\lambda^\delta)(f_\gamma f_\delta)^{-1/2}\, 2\,\frac{\partial f_{\gamma\delta}}{\partial H_1\,\partial H_2} + \text{h.c.} , \qquad (12)$$

plus terms which are higher order in the scalar fields $H_1$ and $H_2$. In terms of the parameters $f_\alpha^0, f_\alpha^1$ introduced in eq. (2), we finally obtain

$$\mathcal{L}_{\hat\lambda\hat\lambda} \ni \mathcal{M}_{\alpha\beta}\bar{\hat\lambda}^\alpha\hat\lambda^\beta - \tfrac{1}{16}e^{-G/3}(|H_1|^2 + |H_2|^2)\left(\frac{f_\alpha^1}{f_\alpha^0}\frac{f_\gamma^1}{f_\gamma^0}\right)(\bar{\hat\lambda}^\alpha\hat\lambda^\alpha)(\bar{\hat\lambda}^\gamma\hat\lambda^\gamma)^* + \tfrac{1}{2}(\bar{\tilde H}_{1L}\tilde H_{2L})(\bar{\hat\lambda}^\alpha\hat\lambda^\alpha)\frac{f_\alpha^1}{f_\alpha} + \text{h.c.} , \qquad (13)$$

plus irrelevant higher order terms.

Suppose we now close the gaugino loops spanned by eq. (13). Each one will be quadratically divergent, but we should cut them all off in the same way:

$$\int^\Lambda \frac{d^4p}{(2\pi)^4}\frac{1}{p^2} \equiv \xi\left(\frac{m_P}{\sqrt{8\pi}}\right)^2 = \xi . \qquad (14)$$

Then we get the Higgs/higgsino mass terms

$$\mathcal{L}_{H_1H_2} \ni -\tfrac{1}{4}\xi^2 e^{-G/3}|f_\alpha^1\mathcal{M}_{\alpha\alpha}(f_\alpha^0)^{-1}|^2(|H_1|^2 + |H_2|^2) - \tfrac{1}{2}\xi f_\alpha^1\mathcal{M}_{\alpha\alpha}(f_\alpha^0)^{-1}(\bar{\tilde H}_1\tilde H_2) + \text{h.c.} . \qquad (15)$$

Rescaling [10] the Higgs field according to eq. (11), we write

$$\hat H_i \equiv e^{G/6}H_i , \quad \hat{\tilde H}_i \equiv e^{G/6}\tilde H_i , \qquad (16a)$$

and thus, through (2),

$$\hat f_\alpha^1 \equiv e^{G/3}f_\alpha^1 , \qquad (16b)$$

which transforms eq. (15) to

$$\mathcal{L}_{H_1H_2} \ni -\mu^2(|\hat H_1|^2 + |\hat H_2|^2) - \mu(\bar{\hat{\tilde H}}_1\hat{\tilde H}_2 + \text{h.c.}) , \qquad (17)$$

where

$$\mu \equiv \tfrac{1}{2}\xi\left(\frac{\hat f_\alpha^1}{f_\alpha^0}\mathcal{M}_{\alpha\alpha}\right) = \tfrac{1}{8}\xi\frac{f_\alpha^1}{(f_\alpha^0)^2}e^{G/2}G^i(G^{-1})_i^j\frac{\partial f^*_{\alpha\alpha}}{\partial\phi_j^*} . \qquad (18)$$

We note two important features of this result: (i) it is supersymmetric as would come from a superpotential term

$$g(\phi) \ni \mu H_1H_2 , \qquad (19)$$

and (ii) assuming $f_\alpha^0, \hat f_\alpha^1 = O(1)$, $\mu$ has the same order of magnitude as the gaugino mass $m_{1/2}$ in SU($n$, 1)/SU($n$) × U(1) models [10,6].

Although derived using component language, the fact that the result (17) looks globally supersymmetric suggests that its derivation could be couched in supergraph language. Gauge superfield loops will always generate a $D$-term: symbolically

$$\delta I = \int d^4x \int d^4x' \int d^2\theta \int d^2\bar{\theta} \, \frac{\Phi_1(x,\theta)\Phi_2(x,\theta)}{M^2} \, \Delta(\theta,\bar{\theta};x,x')\Delta(\theta,\bar{\theta};x',x) f^{0\dagger}(x',\bar{\theta}) \,, \tag{20}$$

where $\Delta$ is the gauge superfield propagator and we retain for future reference factors of $1/M$. We can rewrite (20) as

$$\delta I = \int d^4x \int d^2\theta \, \Phi_1(x,\theta)\Phi_2(x,\theta)\mu(x,\theta) \,, \tag{21}$$

where

$$\mu(x,\theta) \equiv \int \frac{d^4x'}{M^2} \int d^2\bar{\theta} \, \Delta^2(\theta,\bar{\theta};x,x') f^{0\dagger}(x',\bar{\theta}) \,. \tag{22}$$

A non-zero limit for $\mu(x,\theta)$ when $\theta \to 0$ would enable the expression (21) to mimic an $F$-term. Such a non-zero limit is impossible in a renormalizable SUSY field theory with only logarithmic divergences and no mass factors $\propto 1/M$ in the denominators of the couplings. However, with non-renormalizable couplings, power-counting enables us to obtain a cut-off dependent $\theta \to 0$ limit of $\mu(x,0) \propto \Lambda^2/M^2$, and hence generate an effective $F$-term for the low-energy phenomenological lagrangian. We also note that loop diagrams based on the last term in eq. (4) can easily provide a supersymmetry-breaking Higgs boson mixing term

$$m_3^2 = \tfrac{1}{16} \sum_\alpha f_\alpha^{-1} (f_\alpha^1)^2 m_\alpha(\lambda) m(\mathrm{H}) \tag{23}$$

in the effective low-energy potential, which is very valuable phenomenologically [13]. It corresponds to a non-zero value of $B$ in the conventional notation [5].

From this analysis, we see that the natural order of magnitude of a Higgs mixing term in the low-energy superpotential suggested by radiative corrections due to non-renormalizable interactions is $\mu = O(m_{1/2}) = O(m_\mathrm{W})$. Moreover, if we play the game of using renormalizable radiative corrections in the low-energy sector of a no-scale model to generate $m_{1/2}$ and hence $m_\mathrm{W}$ dynamically, it is consistent to keep [2,6,10]

$$\mu/m_{1/2} = O(1) \,, \tag{24}$$

fixed during the calculation of the minimum of the potential. Phenomenology [14] indeed requires this behaviour. From this point of view, the introduction of Higgs mixing with a magnitude $O(m_\mathrm{W})$ presents none of the technical or conceptual problems normally associated with the introduction of a new small mass parameter.

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