HORIZONTAL INTERACTIONS AND PROTON DECAY

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We point out that if all flavour mixing is due to horizontal interactions, proton decay via gauge boson exchange could be rotated away, when horizontal interactions are switched off.

A fundamental question of elementary particle physics concerns the existence of generations of quarks and leptons in nature. As a possible way to understand their structure in the framework of gauge theories, one introduces [1–17] horizontal gauge interactions. These interactions by construction break the flavour conservation in neutral currents which is guaranteed in a natural way by the GIM mechanism [18] and its generalizations [19]. Thus, if horizontal interactions exist they should respect the known bounds on flavour changing neutral currents [20], which in turn require that their scale [1–17] should be at least 10^{3-4} times the scale of the standard Weinberg–Salam–Glashow–Iliopoulos–Maiani model [21,18].

A standard role that horizontal interactions could play in electroweak theories is that they are the source of family mixing. Switching off the horizontal interactions could render the weak fermion eigenstates identical to those of the fermion mass matrix in which case family mixing disappear. Turning them on could produce such corrections to the fermion mass matrix in higher order in perturbation theory that family mixing could emerge. A model was proposed [7] along these lines which in the quark sector resulted to Cabbibo mixing with $\tan^2\theta_c = m_d/m_s$. Demanding, in addition, the possible Majorana neutrino mass terms to be at the scale of horizontal interactions one obtains in the leptonic sector [12] an effective lepton mixing with $\tan^2\theta = m_e/m_\mu$. One might further associate the existence and feebleness of the measured *CP*-violation with horizontal interactions [13–15,17] providing an attractive alternative to the KM matrix [22] and to the *CP*-violation induced by Higgs exchanges [23]. Finally in the same framework one can obtain [16] realistic Cabibbo-like angles in the quark sector and Pontecorvo angles in the lepton sector.

Here we point out, in a variant of the above scenario, that horizontal interactions render the proton unstable, while switching them off (no flavour mixing) makes the proton stable.

We use an $SU(5) \times SU(3)_H$ gauge group, where SU(5) is the usual Georgi–Glashow [24,25] grand unified model and $SU(3)_H$ is a horizontal gauge group. We leave open the option that $SU(5) \times SU(3)_H$ can be embedded in a larger group like SU(8) which might be related to more general physics which is not well understood till now [26]. We first recall the conditions under which the proton is stable in the SU(5) model materializing Jarlskog's idea [27–29].

Let us first consider the gauge sector of the SU(5) model. The effective four-Fermi interaction for

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baryon-number violating processes in the SU(5) model is:

$$(4G/\sqrt{2})\epsilon_{\alpha\beta\gamma}\{(\bar{U}^c_{0\alpha}\gamma^\mu U_{0\beta})(\bar{E}^c_0\gamma_\mu D_{0\gamma}) - (\bar{U}^c_{0\alpha}\gamma^\mu D_{0\beta})(\bar{E}^c_0\gamma_\mu U_{0\gamma}) - (\bar{U}^c_{0\alpha}\gamma^\mu U_{0\beta})(\bar{D}^c_{0\gamma}\gamma_\mu E_0) + (\bar{U}^c_{0\alpha}\gamma^\mu D_{0\beta})(\bar{D}^c_{0\gamma}\gamma_\mu N_0)\}, \quad (1)$$

where

$$U_{0\alpha} \equiv \begin{pmatrix} u_\alpha \\ c_\alpha \\ t_\alpha \end{pmatrix}_0, \quad D_{0\alpha} \equiv \begin{pmatrix} d_\alpha \\ s_\alpha \\ b_\alpha \end{pmatrix}_0, \quad E_0 \equiv \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_0, \quad N_0 \equiv \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_0 \quad (2)$$

and α, β, γ are colour indices. G is the effective four fermion coupling of leptoquark interactions. All the fields appearing in eq. (1) are left-handed. The subscript zero means that the fields in eq. (1) are current eigenstates. In eq. (1) and in the rest of our discussion we assume that baryon decay proceeds via gauge interactions.

When the $SU(2)_L \times U(1)$ gauge group is broken and the fermions acquire masses, the current eigenstates of eq. (1) must be replaced by the corresponding mass eigenstates. The fermion masses can be written as

$$-\mathcal{L}_{\mathcal{M}} = \bar{U}_{0L}\mathcal{M}^u D_{0R} + \bar{D}_{0L}\mathcal{M}^d D_{0R} + \bar{E}_{0L}\mathcal{M}^e E_{0R} + \text{h.c}, \quad (3)$$

The above mass matrices will in general be complex and can be diagonalized by the following biunitary transformations

$$U^u_L\mathcal{M}^u U^{u+}_R = \mathcal{M}^u_{diag}, \quad U^d_L\mathcal{M}^d U^{d+}_R = \mathcal{M}^d_{diag},$$
$$U^e_L\mathcal{M}^e U^{e+}_R = \mathcal{M}^e_{diag}, \quad U^\nu_L\mathcal{M}^\nu U^{\nu+}_R = \mathcal{M}^\nu_{diag}. \quad (4)$$

The physical fermion states are then given by

$$U_L = U^u_L U_{0L}, \quad U_R = U^u_R U_{0R},$$
$$D_L = U^d_L D_{0L}, \quad D_R = U^d_R D_{0R},$$
$$E_L = U^e_L E_{0L}, \quad E_R = U^e_R E_{0R}. \quad (5)$$

In terms of the physical fermion fields eq. (1) can be written as

$$(4G/\sqrt{2})\epsilon_{\alpha\beta\gamma}\{(\bar{U}^c_\alpha R_1\gamma^\mu U_\beta)(\bar{E}^c R_2\gamma_\mu D_\gamma) - (\bar{U}^c_\alpha R_3\gamma^\mu D_\beta)(\bar{E}^c R_4\gamma_\mu U_\gamma) - (\bar{U}^c_\alpha R_1\gamma^\mu U_\beta)(\bar{D}^c_\gamma R_5\gamma_\mu E) + (\bar{U}^c_\alpha R_3\gamma^\mu D_\beta)(\bar{D}^c_\gamma R_6\gamma_\mu N)\}, \quad (6)$$

where

$$R_1 \equiv U^{u*}_R U^{u+}_L, \quad R_2 \equiv U^{e*}_R U^{d+}_L, \quad R_3 \equiv U^{u*}_R U^{d+}_L,$$
$$R_4 \equiv U^{e*}_R U^{u+}_L, \quad R_5 \equiv U^{d*}_R U^{e+}_L, \quad R_6 \equiv U^{d*}_R U^{\nu+}_L. \quad (7)$$

Notice that

$$R^+_1 R_3 = R^+_4 R_2 = U_{K-M}, \quad (8)$$

is the usual Kobayashi–Maskawa mixing matrix [22] appearing in the charged current in the standard model.

In order to suppress proton decay, the following conditions must be imposed [28]

$$(R_1)_{11}(R_2)_{ab} + (R_3)_{1b}(R_4)_{a1} = 0,$$
$$(R_1)_{11}(R_5)_{ab} = 0, \quad (R_3)_{1b} = 0, \quad (9)$$

where $a, b = 1, 2$.

Eqs. (9) are necessary and sufficient conditions for rotating away proton decay from the SU(5) gauge interactions. They reduce to

$$(R_1)_{11} = 0, \quad (R_3)_{11} = (R_3)_{12} = 0. \quad (10)$$

In order to illustrate our ideas we now consider an $SU(5) \times SU(3)_H$ model. The quarks and leptons belong to the $(\bar{5}, 3)$ and $(10, 3)$ representations denoted by η and θ respectively. We introduce [16] two scalar octets Φ, Φ' (1, 8) which are responsible for the horizontal symmetry breaking. These have vacuum expectation values

$$\langle\Phi\rangle \sim V(\delta_{i1} + \delta_{i4}) + V'\delta_{i6}, \quad V \neq V', \quad (11)$$

where the indices 1, 4, 6 denote directions in the $SU(3)_H$ space. The Higgs fields Φ, Φ' with the above VEVs, break completely the SU(3) horizontal symmetry. Denoting the horizontal gauge bosons by A_i, $i = 1, ..., 8$ the physical gauge bosons $\tilde{A}_i$, Z_i acquire masses

$$\mu_1^2 = \tfrac{1}{16}g_H^2 V'^2 \Rightarrow \widetilde{A}_1 \equiv 2^{-1/2}(A_4 + A_1),$$

$$\mu_2^2 = \tfrac{1}{16}g_H^2(8V^2 + V'^2) \Rightarrow \widetilde{A}_2 \equiv 2^{-1/2}(A_5 + A_2),$$

$$\mu_4^2 = \tfrac{1}{16}g_H^2(2V^2 + V'^2) \Rightarrow \widetilde{A}_4 \equiv 2^{-1/2}(A_4 - A_1),$$

$$\mu_5^2 = \mu_4^2 \Rightarrow \widetilde{A}_5 \equiv 2^{-1/2}(A_5 - A_2),$$

$$\mu_7^2 = \tfrac{1}{16}g_H^2(2V^2 + 4V'^2) \Rightarrow \widetilde{A}_7 \equiv A_7,$$

$$\mu_{Z_1}^2 = \tfrac{1}{16}g_H^2[2V^2(1 + \sin 2\phi) + 4V'^2 \sin^2\phi]$$

$$\Rightarrow Z_1 \equiv \cos\phi\, A_6 + \tfrac{1}{2}\sin\phi(\sqrt{3}\,A_8 - A_3),$$

$$\mu_{Z_2}^2 = \tfrac{1}{16}g_H^2(6V^2) \Rightarrow Z_2 \equiv \tfrac{1}{2}(\sqrt{3}\,A_3 + A_8),$$

$$\mu_{Z_3}^2 = \tfrac{1}{16}g_H^2[2V^2(1 - \sin 2\phi) + 4V'^2 \cos^2\phi]$$

$$\Rightarrow Z_3 \equiv -\sin\phi\, A_6 + \tfrac{1}{2}\cos\phi(\sqrt{3}\,A_8 - A_3), \qquad (12)$$

where $\tan(2\phi) = -V^2/V'^2$, and g_H is the coupling constant associated with the $SU(3)_H$ group.

In order to break the SU(5) gauge group down to SU(3) × SU(2) × U(1) we introduce a scalar $H_{24,1}$ (24, 1). Next we should introduce enough Higgs structure in order to break $SU(2)_L \times U(1)$ down to $U(1)_{em}$ and to produce a tree order mass matrix for all quarks and charged leptons of the form

$$(\bar{q}_0^1, \bar{q}_0^2, \bar{q}_0^3)\begin{bmatrix} 0 & 0 & m \\ m & 0 & 0 \\ 0 & m' & 0 \end{bmatrix}\begin{bmatrix} q_0^1 \\ q_0^2 \\ q_0^3 \end{bmatrix}_R , \qquad (13)$$

where q_0^i are elements of the triplet of up quarks, down quarks, and charged leptons.

We obtain the above form of mass matrix by introducing the following rich Higgs structure: Two $\Phi_{5,6}$, $\Phi'_{5,6}$ (5, 6), two $X_{5,3}$, $X'_{5,3}$ (5, 3), two $H_{45,6}$, $H'_{45,6}$ (45, 6) and two $h_{45,3}$, $h'_{45,3}$ (45, 3). The Yukawa interactions of the Higgs scalars to the fermion multiplets are given by

$$\begin{aligned}\mathcal{L}_{\text{Yuk.}} = {} & \gamma\eta_a^{\perp} C\theta^{ab}(\Phi_{5,6}^{+})_b + \delta\epsilon_{abcde}(\theta^{\perp})^{ab} C\theta^{cd}(\Phi_{5,6})^e \\ & + \lambda\eta_a^{\perp} C\theta^{bc}(H_{45,6}^{+})_{bc}^a + \zeta\epsilon_{abcef}(\theta^{\perp})^{ab} C\theta^{cd}(H_{45,6})_d^{ef} \\ & + \gamma_1\eta_a^{\perp} C\theta^{ab}(\chi_{5,3}^{+})_b + \delta_1\epsilon_{abcde}(\theta^{\perp})^{ab} C\theta^{cd}\chi_{5,3}^e \\ & + \lambda_1\eta_a^{\perp} C\theta^{bc}(h_{45,3}^{+})_{bc}^a + \zeta_1\epsilon_{abcef}(\theta^{\perp})^{ab} C\theta^{cd} h_{5,3d}^{ef} \\ & + \text{h.c.} + (\Phi, H, \chi, h \to \Phi', H', \chi', h'), \end{aligned} \qquad (14)$$

where we have suppressed the $SU(3)_H$ indices. The Higgs fields which contribute to the Yukawa terms can develop the following VEVs:

$$\langle\Phi_{5,6}\rangle = \delta_{a5}\begin{bmatrix} 0 & v_1 & v_1 \\ v_1 & & 0 \\ v_1 & & \end{bmatrix}, \quad \langle\Phi'_{5,6}\rangle = \delta_{a5}\begin{bmatrix} & & 0 \\ & & v_3 \\ 0 & v_3 & 0 \end{bmatrix}, \qquad (15)$$

$$\langle\chi_{5,3}\rangle = \delta_{a5}u_1(\delta_{1i} + \delta_{2i}), \quad \langle\chi'_{5,3}\rangle = \delta_{a5}u_3\delta_{i3}, \qquad (16)$$

$$\langle H_{45,6}\rangle = (\delta_a^b - 4\delta_4^b\delta_a^4)\delta_5^c\begin{bmatrix} 0 & v_1' & v_1' \\ v' & & 0 \\ v_1' & & \end{bmatrix},$$

$$\langle H'_{45,6}\rangle = (\delta_a^b - 4\delta_4^b\delta_a^4)\delta_5^c\begin{bmatrix} & & 0 \\ & 0 & v_3' \\ 0 & v_3' & 0 \end{bmatrix}, \qquad (17)$$

$$\langle h_{45,3}\rangle = u_1'(\delta_a^b - 4\delta_4^b\delta_a^4)\delta_5^c(\delta_{i1} + \delta_{i2}),$$

$$\langle h'_{45,3}\rangle = u_3'\delta_{i3}(\delta_a^b - 4\delta_4^b\delta_a^4)\delta_5^c. \qquad (18)$$

One can easily see that when the Higgs fields take their VEVs (15)–(18) the Yukawa terms (14), with an adjustment of the coupling constants, lead to a mass matrix for all the quarks and charged leptons of the form (13) [‡1].

[‡1] It is evident that an enlarged Higgs structure can lead to neutrino masses, however this is not relevant to our discussion of proton decay.
Our use of the 45 together with the 5 representation of Higgses is necessary in order to avoid incorrect fermion mass relation [29]. Our introduction of symmetric (antisymmetric) representations 6(3) of Higgs under $SU(3)_H$ is necessary in order to obtain the desired structure (13) of the mass matrix in the generation space.

Diagonalization of the mass matrix in this order in perturbation theory can be done by rotating the right-handed multiplets with the matrix

$$\mathbf{S}=\begin{pmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix} \tag{19}$$

and leaving unchanged the left-handed ones. Thus, the left-handed eigenstates coincide with the charged current eigenstates

$$\mathbf{J}_\mu = (\bar{u}_0, \bar{c}_0, \bar{t}_0)_L \gamma_\mu \begin{pmatrix} d_0 \\ s_0 \\ b_0 \end{pmatrix}_L \tag{20}$$

and no family mixing exists. If one takes into account one loop corrections to the fermion mass matrix due to horizontal vector boson exchange one obtains [16] the form

$$\begin{pmatrix} 0 & m_{r_3} & m \\ m & 0 & m_{r_1} \\ m_{r_3} & m' & 0 \end{pmatrix}, \tag{21}$$

where

$$m_{r_1} \simeq (g_H^2/16\pi^2) m' [\ln \mu_7^2 - \cos^2\phi \ln \mu_{Z_1}^2 - \sin^2\phi \ln \mu_{Z_3}^2],$$

$$m_{r_2} = m_{r_3} \simeq (g_H^2/32\pi^2) m [\ln(\mu_4^2/\mu_1^2) + \ln(\mu_5^2/\mu_2^2)], \tag{22}$$

where the masses of the vector bosons are given in eq. (12).

Diagonalization of the mass matrix (21) leads [16] to a weak charged current

$$\mathbf{J}_\mu = (\bar{u}, \bar{c}, \bar{t})_L \gamma_\mu \mathbf{L} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L, \tag{23}$$

with

$$\mathbf{L}=\begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix}\begin{pmatrix} c_3 & 0 & s_3 \\ 0 & 1 & 0 \\ -s_3 & 0 & c_3 \end{pmatrix},$$

where $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$. We obtain $\theta_1 = 0.2$; $\theta_2 = 10^{-4}-10^{-3}$; $\theta_3 = 10^{-3}-10^{-2}$ which should be compared with the values of the recent analysis of the experimental data [30–32]. Thus according to this philosophy [7,12,15,16] horizontal interactions are responsible for flavour mixing which disappears by switching them off.

Let us now consider proton decay in the present scheme when horizontal interactions are switched off. In this case the fermion mass matrix is of the form (13) and the matrices R_i, $i = 1,2,3$ in (7) which are of interest with respect to proton decay become:

$$R_1 = R_2 = R_3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \tag{24}$$

which satisfy the conditions (10) for no proton decay. We note that in the framework of the usual SU(5) in the limit of no flavour mixing (as we have here) proton decay appears with full strength. Of course, one cannot take too seriously the richness of the Higgs structure required in order to obtain the mass matrix (13). We believe that nature should be simple and that the mass matrix (13) emerges in a dynamical way which however can also be described by the fermion Higgs Yukawa interactions and the Higgs mechanism. On the other hand the as yet unsuccessful attempts [33–35] to explain the fermion mass matrices in a dynamical way require a richness of new structure analogous to the Higgs structure of our toy model. One could also compare our Higgs fields with the corresponding one in the SU(8) theory of ref. [26].

In conclusion, we have demonstrated in a specific example that, if flavour mixing is due to horizontal interactions, it is possible that baryon number violating interactions be rotated away before the horizontal interactions are turned on. This observation shows some intrinsic relation between flavour mixing and proton decay, at least in the present philosophy. When horizontal gauge interactions are turned on flavour mixing occurs and it is no longer possible to forbid proton decay at the usual rate with the usual dominant decay mode $e^+\pi^0$.

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