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Hybrid MSW + VO solution of the solar neutrino problem in string-motivated unified theories

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Abstract

It is shown that the hybrid MSW + VO solution of the solar neutrino problem, according to which the solar ν_e undergo matter-enhanced transitions into $\nu_{\mu,\tau}$ in the Sun followed by long wave length ($\sim 1.5 \times 10^8$ km) $\nu_e \leftrightarrow \nu_{\mu,\tau}$ oscillations in vacuum between the Sun and the Earth, can occur naturally in string-motivated grand unified theories. We consider the supersymmetric version of a string-type $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ theory with $U(1)_X$ family symmetry, which was shown to successfully describe the charged fermion masses and the quark mixing, and extend the earlier fermion mass analysis to the neutrino sector. We show that the four oscillation parameters Δm_{31}^2 , Δm_{21}^2 and $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{13}$, characterising the combined matter-enhanced transitions and vacuum oscillations of the solar ν_e , naturally get values in the ranges of the hybrid MSW + VO solutions found recently. © 1998 Elsevier Science B.V. All rights reserved.

1. Introduction

In the present article we show that the hybrid matter-enhanced transitions + vacuum oscillations solutions of the solar neutrino problem, possible in the case of three flavour neutrino mixing and found in Ref. [1], can appear naturally in a class of string-type unified theories. As is well-known, the solar neutrino observations provide strong evidences for the existence of non-zero neutrino masses and lepton mixing. The signals measured in all six experiments which have obtained data on the solar neutrino flux so far, Homestake, Kamiokande II and III, SAGE, GALLEX and Super-Kamiokande, are considerably smaller than the signals predicted by the standard solar models. The case for a significant solar neutrino deficit was reinforced recently [2] with the publication of new more precise helioseismological data (see [2] for the relevant references). These data made possible to perform a number of critical tests of the solar models, including the models which have been constructed with the purpose of solving the solar neutrino problem without invoking “unconventional” neutrino behaviour (matter-enhanced transitions, vacuum oscillations, etc.). The analyses completed so far showed [2] that only the standard solar models which include the diffusion of heavy elements in the solar interior are

compatible with the most recent helioseismological observations, and that none of these models can describe the current solar neutrino data. The indicated results make the possibility of an “unconventional” behaviour of the solar ν_e on their way from the central part of the Sun to the Earth surface look at present more plausible than ever in the past. However, the crucial unambiguous experimental proof for such a behaviour is still lacking and it is hoped that the presently operating and the future solar neutrino experiments will be able to determine the true cause of the solar neutrino problem.

The hybrid resonance transitions + vacuum oscillations (MSW + VO) solution is based on the assumption that the solar ν_e undergo matter-enhanced transitions into ν_μ and/or ν_τ when they propagate from the central part to the surface of the Sun and that these transitions are followed by long wave length ($\sim 1.5 \times 10^8$ km) vacuum oscillations of the ν_e and/or $\nu_{\mu(\tau)}$ when the flavour neutrinos travel from the surface of the Sun to the Earth. The solution of interest is a genuine three-flavour-neutrino mixing solution since it is possible only if the three flavour neutrinos mix in vacuum:

$$|\nu_l\rangle = \sum_{k=1}^3 U_{lk}^* |\nu_k\rangle, \quad l = e, \mu, \tau, \quad (1)$$

where $|\nu_l\rangle$ is the state vector of the (left-handed) flavour neutrino ν_l having a definite momentum (\mathbf{p}), $|\nu_k\rangle$ is the state vector of a neutrino ν_k possessing a definite mass m_k and a definite momentum (\mathbf{p}), $m_k \neq m_j$, $k \neq j = 1, 2, 3$, $m_1 < m_2 < m_3$ ¹, and U is a 3×3 unitary matrix – the lepton mixing matrix. The (average) solar ν_e survival probability, $P(\nu_e \rightarrow \nu_e)$, which is a basic quantity in the analysis of the solar neutrino data in the case of interest and for which a simple analytic expression was derived in [1], depends on four neutrino mass and mixing parameters: two basically responsible for the matter-enhanced transitions and another two responsible for the long wave length vacuum oscillations. In view of our further discussion we will assume these parameters to be respectively Δm_{31}^2 , $\sin^2 2\theta_{13}$ and Δm_{21}^2 , $\sin^2 2\theta_{12}$, where as usual $\Delta m_{ij}^2 = m_i^2 - m_j^2$, and

$$\cos 2\theta_{13} = 1 - 2|U_{e3}|^2 > 0, \quad \sin 2\theta_{13} = 2|U_{e3}|\sqrt{1 - |U_{e3}|^2} > 0, \quad (2)$$

$$\sin^2 2\theta_{12} = 4 \frac{|U_{e1}|^2 |U_{e2}|^2}{(|U_{e1}|^2 + |U_{e2}|^2)^2}. \quad (3)$$

As was shown in Ref. [1] on the basis of an analysis of the recent solar neutrino data, the MSW + VO solution is possible for

$$4.0 \times 10^{-12} \text{ eV}^2 \lesssim \Delta m_{21}^2 \lesssim 5.0 \times 10^{-10} \text{ eV}^2, \quad (4)$$

$$0.15 \lesssim \sin^2 2\theta_{12} \lesssim 1.0, \quad (5)$$

and

$$5.0 \times 10^{-6} \text{ eV}^2 \lesssim \Delta m_{31}^2 \lesssim 1.5 \times 10^{-4} \text{ eV}^2, \quad (6)$$

$$3.1 \times 10^{-4} \lesssim \sin^2 2\theta_{13} \lesssim 0.5. \quad (7)$$

More specifically, in [1] five different sub-regions in the region of the space of parameters, defined by Eqs. (2)–(7), corresponding to five different types of hybrid MSW + VO solutions, A, B, C, D, and E, have been identified (see Figs. 1a–1c). Let us add that, what actually specifies the solutions A, B, C, D and E and makes them physically very different is the distinct way the various solar neutrino flux components (pp , ${}^7\text{Be}$, ${}^8\text{B}$, etc.) are affected by the matter-enhanced transitions and the vacuum oscillations in the case of each particular solution (see [1] for further details).

¹ It is always possible to choose $m_1 < m_2 < m_3$ without loss of generality and we will work with this convention in what follows.

As it follows from Eqs. (2)–(7), the combined MSW + VO solution requires rather small neutrino mass squared differences and very specific mixing in the lepton sector and ratio of Δm_{21}^2 and Δm_{31}^2 . In the context of GUT's, the light neutrino masses can be generated, as is well-known, by the see-saw mechanism: $m_\nu \sim m_Q^2/M_{\text{GUT}}$, where m_Q is the up quark mass and M_{GUT} is the unification scale. Moreover, making use of simple discrete or continuous $U(1)$ symmetries at the unification scale, specific textures for the fermion mass matrices can be constructed with a minimum number of parameters which leads, in particular, to rather specific predictions for the mixing in the lepton sector. Effective unified or partially unified GUTs which incorporate naturally the indicated properties can be built within the string theories. In the free fermionic formulation for example, a number of such models has been constructed [3–5]. A typical feature of the latter is that in addition to the gauge group containing the $SU(3) \otimes SU(2)_L \otimes U(1)_Y$ as a subgroup, a number of surplus $U(1)$ symmetry factors which distinguish between the three fermion families appear. However, models based on these constructions do not contain Higgs fields in the adjoint or higher dimensional representations. As a consequence, the unification symmetry of the traditional GUT's like $SU(5)$ and $SO(10)$, cannot be broken down to the symmetry of the Standard Theory.

Attempts to overcome this difficulty led to constructions of partially unified GUT's in which only small-dimensional Higgs representations are needed to break the unification symmetry [3,4]. A partially unified theory which fulfils all basic requirements for a string-type GUT [4], is based on the Pati–Salam [6] gauge symmetry group $SU(4) \otimes O(4)$. The $SU(4) \otimes O(4)$ unification symmetry can be broken down [4] to the standard model gauge symmetry without using adjoint or any higher dimensional Higgs representations. The model possesses a number of attractive features. Colour triplets and Higgs doublets arise in different representations. Thus, it is free from doublet–triplet splitting complications as the triplets become massive from simple trilinear couplings. There are no dangerous proton decay mediating gauge bosons. As a consequence, the $SU(4) \otimes O(4)$ breaking scale M_{GUT} can in principle be lower than the symmetry breaking scale of other unified theories, (e.g., the $SO(10)$ theory). In fact, the basic constraints on M_{GUT} are the low energy measurements of the strong coupling constant α_s and of $\sin^2\theta_w$. Thus, in the case when only the minimal supersymmetric standard model (MSSM) spectrum appears below M_{GUT} these parameters are correctly predicted if $M_{\text{GUT}} \sim 10^{16}$ GeV. Furthermore, a recent non-renormalisable operator analysis [7] of the model's supersymmetric version has revealed quite remarkable features of the charged fermion mass matrices.

In this work we demonstrate that the hybrid MSW + VO scenario, shown recently to provide an alternative solution of the solar neutrino problem [1] (see also, e.g., [8]), can be naturally implemented in the string-type partially unified $SU(4) \otimes O(4)$ theory. Our analysis will be performed in the context of the supersymmetric version of the theory with the $SU(4) \otimes O(4)$ gauge symmetry augmented by a $U(1)$ family symmetry.

2. The $SU(4) \otimes O(4)$ model

Here we briefly summarise the basic features of the model which are relevant for our analysis [7]. The gauge group is $SU(4) \otimes O(4) \otimes U(1)_X$, or equivalently its isomorphic (enhanced) Pati-Salam symmetry

$$SU(4) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_X, \quad (8)$$

where $U(1)_X$ is a gauged family symmetry of the same type as discussed in Ref. [9]. The left- and right-handed fermion fields are accommodated in the following representations:

$$F^{i\alpha a} = (4, 2, 1) = \begin{pmatrix} u^R & u^B & u^G & \nu \\ d^R & d^B & d^G & e^- \end{pmatrix}^i, \quad (9)$$

$$\bar{F}_{x\alpha}^i = (\bar{4}, 1, \bar{2}) = \begin{pmatrix} \bar{d}^R & \bar{d}^B & \bar{d}^G & e^+ \\ \bar{u}^R & \bar{u}^B & \bar{u}^G & \bar{\nu} \end{pmatrix}^i, \quad (10)$$

where $\alpha = 1, \dots, 4$ is an $SU(4)$ index, $a, x = 1, 2$ are $SU(2)_{L,R}$ indices, and $i = 1, 2, 3$ is a family index. The $U(1)_X$ family symmetry is broken spontaneously below the string scale M_S by the superfields $\theta, \bar{\theta}$, both singlets under the $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ symmetry, but with $U(1)_X$ charges $+1, -1$ respectively. This breaking cancels anomalous D-terms provided the vacuum expectation values of θ and $\bar{\theta}$ satisfy ² [7]

$$\langle \theta \rangle \sim \langle \bar{\theta} \rangle \sim (0.1 - 0.3) M_S. \tag{11}$$

The Higgs fields are contained in the following representations:

$$h_a^x = (1, \bar{2}, 2) = \begin{pmatrix} h_2^+ & h_1^0 \\ h_2^0 & h_1^- \end{pmatrix} \tag{12}$$

(where h_1 and h_2 are the low energy Higgs superfields associated with the MSSM.) The two heavy Higgs representations are

$$H^{\alpha b} = (4, 1, 2) = \begin{pmatrix} u_H^R & u_H^B & u_H^G & \nu_H \\ d_H^R & d_H^B & d_H^G & e_H^- \end{pmatrix}, \tag{13}$$

and

$$\bar{H}_{\alpha x} = (\bar{4}, 1, \bar{2}) = \begin{pmatrix} \bar{d}_H^R & \bar{d}_H^B & \bar{d}_H^G & e_H^+ \\ \bar{u}_H^R & \bar{u}_H^B & \bar{u}_H^G & \bar{\nu}_H \end{pmatrix}. \tag{14}$$

At a scale $M_{\text{GUT}} \sim 10^{16}$ GeV, these Higgs fields are assumed to develop VEVs, $\langle H \rangle = \langle \tilde{\nu}_H \rangle \sim M_{\text{GUT}}$, $\langle \bar{H} \rangle = \langle \tilde{\bar{\nu}}_H \rangle \sim M_{\text{GUT}}$, leading to the symmetry breaking at M_{GUT}

$$SU(4) \otimes SU(2)_L \otimes SU(2)_R \rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_Y, \tag{15}$$

in the usual notation.

Under the symmetry breaking Eq. (15), the bidoublet Higgs field h in Eq. (12) splits into two Higgs doublets h_1, h_2 whose neutral components subsequently develop weak scale VEVs,

$$\langle h_1^0 \rangle = v_1, \langle h_2^0 \rangle = v_2, \tag{16}$$

with $\tan \beta \equiv v_2/v_1$.

In addition to the Higgs fields in Eqs. (13) and (14) one introduces also an $SU(4)$ sextet field $D = (6, 1, 1)$. Assuming now a Z_2 invariance with respect to $H(\bar{H}) \rightarrow (-1) \times H(\bar{H})$, the tree level mass terms of the superpotential of the model read

$$W = \lambda_1^i \bar{F}_i F_j h + \lambda_2 H H D + \lambda_3 \bar{H} \bar{H} D + \mu h h, \tag{17}$$

Note that we have banned terms which might lead to unacceptably large neutrino-higgsino mixing [10]. Additional terms not included in Eq. (17) may be forbidden by imposing suitable discrete or continuous symmetries, the details of which need not concern us here. The D field carries colour and therefore does not develop a VEV, but the terms in Eq. (17) $H H D$ and $\bar{H} \bar{H} D$ combine the colour triplet parts of H, \bar{H} and D into acceptable GUT-scale mass terms. When the H fields attain their VEVs at $M_{\text{GUT}} \sim 10^{16}$ GeV, the superpoten-

² Note that in order to avoid anomalies of the fermionic components and preserve flatness conditions of the scalar VEVs, we introduce always $R + \bar{R}$ representations.

tial of Eq. (17) reduces to that of the MSSM augmented by neutrino masses. Below M_{GUT} the part of the superpotential involving matter superfields is just

$$W = \lambda_U^{ij} \bar{U}_i Q_j h_2 + \lambda_D^{ij} \bar{D}_i Q_j h_1 + \lambda_E^{ij} \bar{E}_i L_j h_1 + \lambda_N^{ij} \bar{\nu}_i L_j h_2 + \dots, \quad (18)$$

where Q_j and L_j are the quark and lepton $SU(2)_L$ doublet superfields and \bar{U}_i , \bar{D}_i , \bar{E}_i and $\bar{\nu}_i$ are the charge $2/3$ and $(-1/3)$ quark, the charged lepton and the neutrino $SU(2)_L$ singlet superfields. The Yukawa couplings in Eq. (18) satisfy the boundary conditions

$$\lambda_U^{ij}(M_{\text{GUT}}) \equiv \lambda_D^{ij}(M_{\text{GUT}}) = \lambda_E^{ij}(M_{\text{GUT}}) = \lambda_N^{ij}(M_{\text{GUT}}). \quad (19)$$

Thus, Eq. (19) retains the successful relation $m_\tau = m_b$ at M_{GUT} .

The boundary conditions listed in Eq. (19) lead to unacceptable mass relations for the light two families. In addition, the large family hierarchy in the Yukawa couplings appears to be unnatural since one would naively expect the dimensionless couplings all to be of the same order. This leads us to the conclusion that the λ_U^{ij} in Eq. (17) may not originate from the usual renormalisable tree level dimensionless coupling. The $U(1)_X$ symmetry will allow a renormalisable Yukawa coupling in the 33-term only and generate the rest of the effective Yukawa couplings by non-renormalisable operators that are suppressed by some higher mass scale. This suppression provides an explanation for the observed fermion mass hierarchy and occurs as powers of two expansion parameters ϵ and δ defined as follows:

$$\epsilon \equiv \langle \theta \rangle / M_S \sim \langle \bar{\theta} \rangle / M_S \sim M_X / M_S, \quad \delta \equiv \langle H \rangle \langle \bar{H} \rangle / M_Y^2 = M_{\text{GUT}}^2 / M_Y^2, \quad (20)$$

where $M_X \sim \langle \theta \rangle \sim \langle \bar{\theta} \rangle$ is the $U(1)_X$ symmetry breaking scale and M_Y is a heavy mass which should be of order M_X if the operators have a field theoretic origin, or M_S if the operator's origin is from the string.

The $U(1)_X$ symmetry used is anomalous³. However, the $U(1)$ charges Q_X are such that all of the mixed anomalies are equal, so that we can appeal to Green-Schwarz string anomaly cancellation. Moreover, the spontaneous breaking of the $U(1)_X$ via the $\theta, \bar{\theta}$ -VEVs at a high scale should proceed along a flat direction to avoid large vacuum energy contributions from D-terms.

Using the $\theta, \bar{\theta}$ fields, we generate all effective Yukawa terms (except the (33) elements) by operators of the form

$$O_{ij} \sim (\bar{F}_i F_j) h \left(\frac{H\bar{H}}{M_Y^2} \right)^r \left(\frac{\theta^r \bar{\theta}^m}{M_S^{n+m}} \right) \sim F_i F_j h \delta^r \epsilon^{n+m}, \quad (21)$$

where $r, n, m \geq 0$ are set so that the operator is invariant under the $U(1)_X$ family symmetry. The (33) operator is kept to be of the renormalisable (trilinear) form. When H and \bar{H} develop their VEVs, the operators in Eq. (14) will become effective Yukawa couplings of the form $\bar{F}Fh$ with a small coefficient of order $(M_{\text{GUT}}^2/M_Y^2)^r$. The see-saw mechanism for suppressing the mass of the three lighter neutrinos is realized in a natural way, provided that non-renormalisable terms of the form

$$\bar{F}_i \bar{F}_j \left(\frac{H\bar{H}}{M_X} \right) \left(\frac{\theta^o \bar{\theta}^p}{M_S^{o+p}} \right) \sim \bar{F}_i \bar{F}_j M_{\text{GUT}} \sqrt{\delta} \epsilon^{o+p} \quad (22)$$

are included in the superpotential. It should be noted that when the exponents in Eq. (22) are not integers, the corresponding element is zero due to a residual discrete symmetry. Gauge unification occurs at M_{GUT} , and so the gauge couplings must run together between M_{GUT} and M_S . This is possible, for example, when some extra states are present in the above range [12].

³ In fact, in the context of string derived models, one linear combination among the various $U(1)$ symmetries accompanying the non-abelian gauge group factors, is anomalous [3–5].

3. Textures

In Ref. [7], we showed that textures exist which fit the data *and* explain the presence of the zeroes through highly suppressed operators. The family mass hierarchy itself is explained through suppression via $U(1)_X$ family symmetry: all of the fundamental Yukawa couplings are of order 1 in the model. Rather than perform a complicated systematic analysis of all possible models, here we simply examine the predictions for the neutrino masses and the lepton mixing of one successful model. We choose an example of a texture (model 1 of Ref. [7]), that provides a very good quality of the fit of the data on charged fermion masses ($\chi^2/\text{d.o.f.} = 0.34$ for 3 d.f.). The texture is

$$\lambda = \begin{pmatrix} 0 & O^M + \text{s.d.} & 0 \\ O^M + O^A & O^W + \text{s.d.} & 0 \\ 0 & O^C & O_{33} \end{pmatrix}, \quad (23)$$

where O_{33} is the renormalisable operator and *s.d.* stands for small sub-dominant operators which are negligible in the down quark and charged lepton sectors, but which are responsible for the charm and up quark masses. The C-G structure of the operators $O^{A,C,M,W}$ is listed in Table 1. They are arrived at by taking different contractions of the $SU(4) \otimes SU(2)_R \otimes SU(2)_L$ group indices [7]. Operators listed in the table are the ones which were found to be useful phenomenologically, and which are utilised in this paper. The $U(1)_X$ symmetry is responsible for high powers of ϵ , reproducing the approximate zeroes assumed in Eq. (23). We pick an assignment of $U(1)_X$ charges corresponding to family dependent charges of case 2 of Ref. [7]. The charge assignment for this case appears in Table 2, where we have added family independent components to the $U(1)_X$ charges in order to make the mixed anomalies of $SU(4)^2 U(1)_X$, $SU(2)_R^2 U(1)_X$ and $SU(2)_L^2 U(1)_X$ equal. This allows anomaly cancellation under the GSW mechanism. The operators resulting from the charge assignments in Table 2 reproduce the mass/mixing hierarchies with order 1 Yukawa couplings in the charged fermion sector if $\epsilon = 0.14$, $\delta = 0.21$. We shall take these values of expansion parameters for the analysis of the neutrino masses below. If we take only the field content of Table 2, the D-flatness constraint sets the natural scale of $|\langle \theta \rangle|^2$ and $|\langle \bar{\theta} \rangle|^2$ to be of order $g_u^2 M_{Pl}^2 / (4\pi^2)$, g_u being the unified gauge coupling constant at M_S .

At M_{GUT} , the Yukawa matrices are of the form

$$\lambda^I = \begin{bmatrix} 0 & H_{12} e^{i\phi_{12}} x_{12}^I & 0 \\ H_{21} x_{21}^I e^{i\phi_{21}} + \tilde{H}_{21} \tilde{x}_{21}^I e^{i\tilde{\phi}_{21}} & H_{22} x_{22}^I e^{i\phi_{22}} & 0 \\ 0 & H_{32} x_{32}^I e^{i\phi_{32}} & H_{33} e^{i\phi_{33}} \end{bmatrix}, \quad (24)$$

where only the dominant operators are listed. The *I* superscript labels the charge sector, x_{ij}^I refers to the Clebsch-Gordan coefficient relevant to the charge sector *I* in the ij^{th} position, ϕ_{ij} are unknown phases and H_{ij} is the magnitude of the effective dimensionless Yukawa coupling in the ij^{th} position. In our case H_{ij} represent

Table 1

When the Higgs fields develop their VEVs at M_{GUT} , the $r=1$ operators utilised lead to the effective Yukawa couplings with Clebsch-Gordan coefficients as shown. We have included the relative normalisation for each of the operators

	$Q\bar{U}h_2$	$Q\bar{D}h_1$	$L\bar{E}h_1$	$L\bar{N}h_2$
O^A	1	1	1	1
O^C	$\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$
O^M	0	$\sqrt{2}$	$\sqrt{2}$	0
O^W	0	$\sqrt{\frac{2}{5}}$	$-3\sqrt{\frac{2}{5}}$	0

Table 2
 $U(1)_X$ charges of fields in the model

F_1	F_2	F_3	\bar{F}_1	\bar{F}_2	\bar{F}_3	h	H	\bar{H}
4	0	-1	3	-1	1	0	1	-1

the effective small Yukawa couplings generated by operators from Eq. (21). Any sub-dominant operators that we introduce will be denoted below by a prime and it should be borne in mind that these will only affect the up-quark and Dirac neutrino matrices. Once unphysical phases have been rotated away by re-phasing the fermion fields, only one physical phase remains. The charged fermion analysis did not significantly constrain the remaining physical phase ϕ and for our purposes it shall be set to zero.

Table 1 displays the Clebsch-Gordan coefficients derived from the operators utilised for this model, a subset of operators used for this and other models in Ref. [7]. Putting in the Clebsch-Gordan coefficients from Table 1 we arrive at the component Yukawa matrices, at the GUT scale of

$$\lambda^U = \begin{pmatrix} 0 & \eta_{12} & 0 \\ H'_{21} & H'_{22} & 0 \\ 0 & \sqrt{2} H_{32}/\sqrt{5} & H_{33} \end{pmatrix}, \quad (25)$$

$$\lambda^D = \begin{pmatrix} 0 & \sqrt{2} H_{12} & 0 \\ \sqrt{2} H_{21} & H_{22}/\sqrt{5} & 0 \\ 0 & -\sqrt{2} H_{32}/\sqrt{5} & H_{33} \end{pmatrix}, \quad (26)$$

$$\lambda^E = \begin{pmatrix} 0 & \sqrt{2} H_{12} & 0 \\ \sqrt{2} H_{21} & 3H_{22}/\sqrt{5} & 0 \\ 0 & -3\sqrt{2} H_{32}/\sqrt{5} & H_{33} \end{pmatrix}, \quad (27)$$

$$\lambda^{\nu}_{\text{Dirac}} = \begin{pmatrix} 0 & \eta_{12} & 0 \\ H'_{21} & H'_{22} & 0 \\ 0 & -3\sqrt{2} H_{32}/\sqrt{5} & H_{33} \end{pmatrix}, \quad (28)$$

where η_{12} is expected to be generated from a higher order non-renormalisable term.

The following GUT scale input parameters minimised the fermion mass/mixing global χ^2 for the case considered here [7]:

$$\begin{aligned} H_{22} &= 2.88 \times 10^{-2}, & H_{12} &= 2.81 \times 10^{-3}, & H_{21} &= 1.30 \times 10^{-3}, & H_{33} &= 1.18, \\ H'_{22} &= 1.91 \times 10^{-3}, & H'_{21} &= 1.94 \times 10^{-3}, & H_{32} &= 7.28 \times 10^{-2}, & \eta_{12} &\cong 10^{-3}. \end{aligned} \quad (29)$$

These values of the parameters lead to the predictions

$$\begin{aligned} \alpha_s(M_Z) &= 0.119, & \tan\beta &= 59.5, & m_t &= 175, & m_d &= 6.25, & m_s &= 158, & m_b &= 4.24, & m_c &= 1.30, \\ |V_{us}| &= 0.2211, & |V_{ub}| &= 3.71 \times 10^{-3}, & m_u &= 5, & |V_{cb}| &= 0.038, \end{aligned} \quad (30)$$

where all masses are quoted in the $\overline{\text{MS}}$ renormalisation scheme, $m_{c,b,t}$ are running masses quoted in GeV and $m_{u,d,s}$ are evaluated at 1 GeV and measured in units of MeV. The predictions for $|V_{us}|$, $|V_{ub}|$ and $|V_{cb}|$ are at the scale M_Z .

4. Neutrino masses

Up to now, we have shown that our ansatz generates successfully the quark and charged lepton mass hierarchy. In addition, the entries of the Dirac neutrino mass matrix are completely determined at the unification scale since they are related to those of the up-quark mass matrix:

$$m_D(M_{\text{GUT}}) = \lambda_{\text{Dirac}}^\nu \nu_2. \quad (31)$$

Thus, the overall scale of the matrix is determined by the Higgs field VEV which gives mass to the up quark, $\nu_2 = \langle h_2 \rangle$, while the Dirac Yukawa couplings can be encoded in terms of the parameters δ, ϵ as can be seen from (28).

We turn now to the discussion of the heavy right-handed (RH) neutrino Majorana mass matrix M_R . Since the RH neutrinos are members of the multiplets $\bar{F}_{x\alpha}^i$ (Eq. (10)) accommodating right-handed quarks and leptons, their charges are already fixed. Thus, within a given charge assignment, we have no freedom of choosing the relative orders of magnitude for the elements of M_R . The $U(1)_X$ charge of H (opposite to the charge of \bar{H}) was not fixed by the charged fermion analysis. Once the $U(1)_X$ charges are chosen, the structure of M_R is directly determined by the non-renormalisable terms of Eq. (22). Therefore, the M_R entries are also encoded in terms of the same parameters δ and ϵ up to Yukawa coefficients of order 1. On the other hand, the overall scale is related to the unification mass M_{GUT} . When the charges from Table 2 are substituted into Eq. (22), it is found that the effective RH neutrino Majorana mass matrix is

$$M_R = M_{\text{GUT}} \sqrt{\delta} \begin{pmatrix} a \cdot \epsilon^8 & \epsilon^4 & d \cdot \epsilon^6 \\ \epsilon^4 & b & e \cdot \epsilon^2 \\ d \cdot \epsilon^6 & e \cdot \epsilon^2 & c \cdot \epsilon^4 \end{pmatrix}. \quad (32)$$

In Eq. (32), a, b, c, d, e are the dimensionless Yukawa couplings, mentioned above. In the case of the string derived version of the model, they are calculable and determined in terms of the gauge coupling at the unification scale. In the field theory version, however, they cannot be determined, so they will be treated as free real parameters. The suppression factors appearing in the heavy RH neutrino Majorana mass matrix will finally give $M_R < M_{\text{GUT}}$. This implies that the light neutrino masses which will be obtained through the see-saw mechanism, will turn out to be larger than might be naively expected for $M_R \sim M_{\text{GUT}}$.

In the following, we will show that four of the five hybrid solutions found in Ref. [1] can be obtained for natural values of a, b, c, d, e . We will describe first the procedure of our analysis. As has already been explained, the neutrino Dirac mass matrix is completely set by the charged fermion data. A decisive role is played by the involved Yukawa couplings at the GUT scale. Since these are related to the up-quark couplings, they are obtained by using the 3-loop QCD \otimes 1-loop QED- β functions below m_t , and the 1-loop MSSM β -functions between m_t and M_{GUT} , as in Ref. [7]. The renormalisation effect of the Dirac Yukawa couplings of the neutrinos is negligible because they are integrated out at the heavy Majorana mass scale. Only the τ -neutrino Dirac coupling is large enough to affect the results, and this is integrated out within a couple of orders of magnitude below M_{GUT} , i.e., at the scale where the see-saw mechanism is expected to operate. The effective light left-handed (LH) neutrino Majorana mass matrix at M_{GUT} is then

$$m_{\nu_L}^{\text{eff}} \equiv -m_D^T M^{-1} m_D.$$

The light neutrino masses are given by

$$\mathcal{O} m_{\nu_L}^{\text{eff}} \mathcal{O}^T = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad (33)$$

where \mathcal{O} is an orthogonal matrix that diagonalises $m_{\nu_L}^{\text{eff}}$ and m_i are the light neutrino masses ($m_1 \ll m_2 \ll m_3$). The lepton mixing matrix is calculated by $U = \mathcal{O} V_{E_L}$, where the matrix V_{E_L} appears as a result of the diagonalisation of the charged lepton mass matrix. The mixing elements and $m_{1,2,3}$ are then renormalised from M_{GUT} down to m_i , again using the 1-loop MSSM RGEs [11]. The renormalisation of mixing angles and masses between m_i and M_Z is small and neglected.

Our string motivated criteria for the naturalness of the Yukawa couplings a, b, c, d, e , is that they lie in the range 0.1–2.0. By varying a, b, c, d, e , four predictions corresponding to hybrid oscillation solutions A, B, C, D were found that were consistent with this constraint. These are summarized in Table 3 and should be compared with the regions shown in Fig. 1a–c.

The solutions in Table 3 correspond to the following M_{GUT} , masses of the heavy (approximately RH) neutrinos $M_{R_j}^D$, $j = 1, 2, 3$, and lepton mixing matrices (defined in the basis (ν_1, ν_2, ν_3) , $m_1 < m_2 < m_3$):

$$\begin{aligned} \text{Solution A: } M_{\text{GUT}} &= 0.9 \times 10^{16}, M_R^D = (6.8 \times 10^{14}, 4.1 \times 10^7, 3.0 \times 10^{13}), \\ U^A &\cong \begin{pmatrix} 0.69 & -0.72 & -0.025 \\ -0.72 & -0.69 & 0.038 \\ -0.045 & -0.0080 & -1.00 \end{pmatrix}, \end{aligned} \quad (34)$$

$$\begin{aligned} \text{Solution B: } M_{\text{GUT}} &= 0.8 \times 10^{16}, M_R^D = (2.9 \times 10^{15}, 7.7 \times 10^7, 1.5 \times 10^{13}), \\ U^B &\cong \begin{pmatrix} -0.26 & 0.96 & 0.031 \\ 0.96 & 0.26 & 0.057 \\ 0.047 & 0.044 & -1.00 \end{pmatrix}, \end{aligned} \quad (35)$$

$$\begin{aligned} \text{Solution C: } M_{\text{GUT}} &= 2.5 \times 10^{16}, M_R^D = (1.4 \times 10^{15}, 3.0 \times 10^7, 7.2 \times 10^{12}), \\ U^C &\cong \begin{pmatrix} -0.52 & 0.85 & 0.020 \\ 0.85 & 0.52 & 0.066 \\ 0.046 & 0.052 & -1.00 \end{pmatrix}, \end{aligned} \quad (36)$$

$$\begin{aligned} \text{Solution D: } M_{\text{GUT}} &= 1.9 \times 10^{16}, M_R^D = (6.8 \times 10^{14}, 1.4 \times 10^7, 1.8 \times 10^{13}), \\ U^D &\cong \begin{pmatrix} -0.69 & 0.72 & -0.0086 \\ 0.72 & 0.69 & 0.062 \\ 0.051 & 0.037 & -1.00 \end{pmatrix}, \end{aligned} \quad (37)$$

where the $M_{\text{GUT},R}$ masses are in GeV.

The MSW + VO solutions A, B, C, and D of the solar neutrino problem of interest will be tested in the currently operating Super-Kamiokande experiment as well as in the future solar neutrino experiments SNO, BOREXINO, HELLAZ, etc. As discussed in [1], one of the distinctive predictions of the indicated hybrid MSW + VO solutions is the existence of strong and very characteristic distortions of the spectrum of ^8B neutrinos, which should be observable in the Super-Kamiokande and/or SNO experiments. The seasonal (time) variation

Table 3

Examples of values of Δm_{31}^2 , Δm_{21}^2 , $\sin^2 2\theta_{13}$ and $\sin^2 2\theta_{12}$ obtained in the model discussed by us and corresponding to the hybrid MSW + VO solutions A, B, C and D found in Ref. [1] (Δm_{ij}^2 are in units of eV^2). The dimensionless real parameters a, b, c, d and e from the heavy RH neutrino Majorana mass matrix are supposed to take values in the interval (0.1 - 2.0)

	a	b	c	d	e	Δm_{31}^2	Δm_{21}^2	$\sin^2 2\theta_{13}$	$\sin^2 2\theta_{12}$
A	1.0	0.25	0.11	0.125	0.25	$1.2 \cdot 10^{-4}$	$5.5 \cdot 10^{-12}$	$2.5 \cdot 10^{-3}$	1.00
B	0.75	0.625	0.407	1.2	0.457	$9.6 \cdot 10^{-6}$	$8.7 \cdot 10^{-11}$	$3.8 \cdot 10^{-3}$	0.25
C	1.0	0.345	0.7	2.0	0.3	$1.4 \cdot 10^{-5}$	$1.2 \cdot 10^{-10}$	$1.6 \cdot 10^{-3}$	0.80
D	1.7	0.25	0.24	1.91	0.4	$1 \cdot 10^{-4}$	$3.9 \cdot 10^{-11}$	$3.0 \cdot 10^{-4}$	1.00

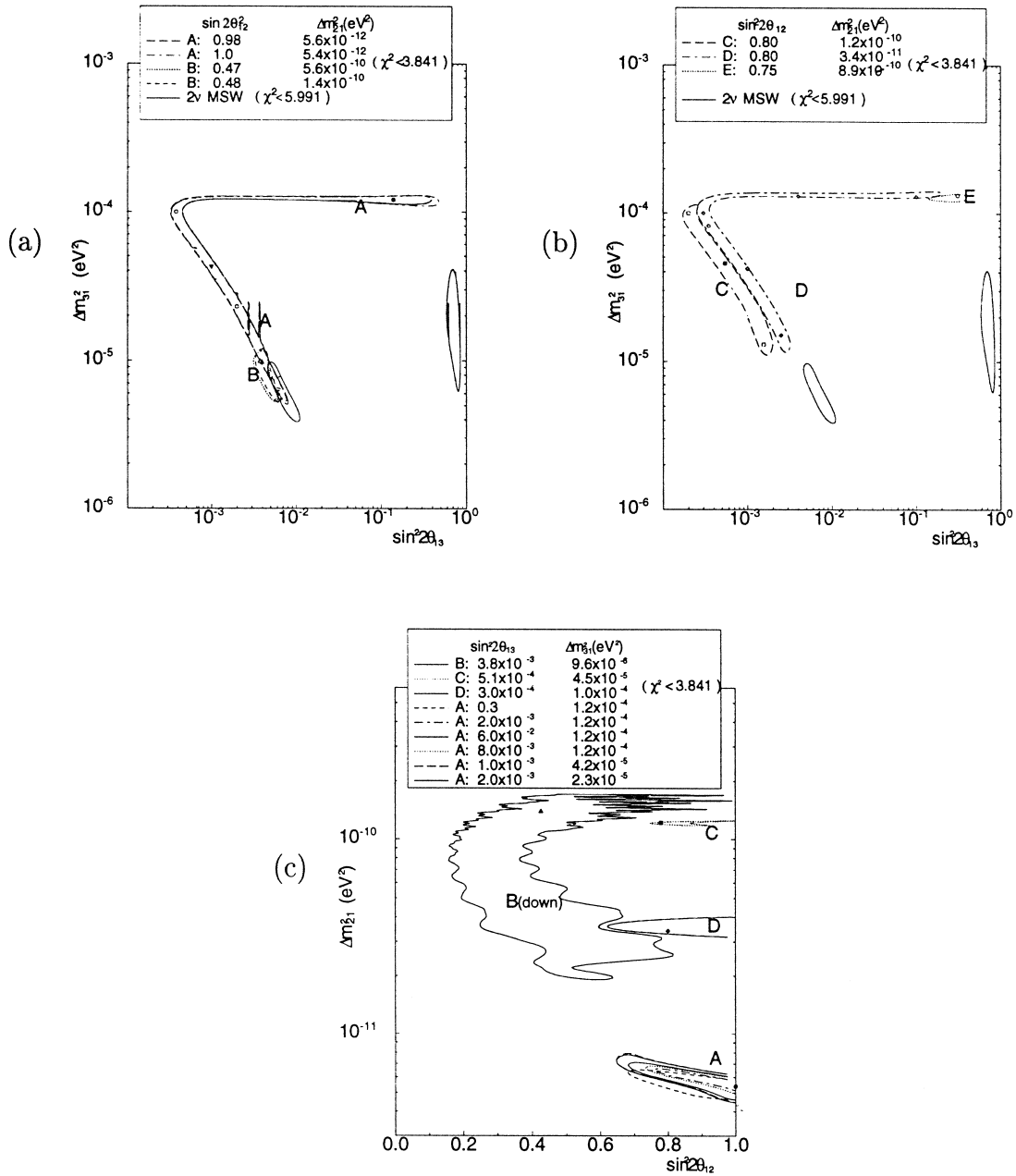


Fig. 1. MSW + VO solutions of the solar neutrino problem [1] - regions of values of the MSW transition and vacuum oscillation parameters. (a) and (b) show Δm_{31}^2 and $\sin^2 2\theta_{13}$, corresponding to solutions A,B,C,D and E and obtained for fixed values of the vacuum oscillation parameters Δm_{21}^2 and $\sin^2 2\theta_{12}$ indicated in the figures. Shown are also the 95% C.L. 2ν MSW non-adiabatic and adiabatic solution regions. (c) shows Δm_{21}^2 and $\sin^2 2\theta_{12}$ for solutions A, B, C, and D. The regions correspond to fixed values of the MSW parameters Δm_{31}^2 and $\sin^2 2\theta_{13}$ indicated in the figure.

of the ^8B , ^7Be , pp , etc. solar neutrino fluxes caused by ⁴ the vacuum oscillations (see, e.g., [12] and the references quoted therein) in the case of the MSW + VO solutions, cannot exceed for given values of Δm_{21}^2 and $\sin^2 2\theta_{12}$, the seasonal variation caused by purely two-neutrino vacuum oscillations with the same Δm_{21}^2 and $\sin^2 2\theta_{12}$.⁵ Nevertheless, the vacuum oscillation induced seasonal variations of the ^7Be and ^8B neutrino fluxes predicted for the MSW + VO solutions B and C, can be observable in the Super-Kamiokande, SNO and BOREXINO experiments for certain regions of the solution values of the parameters [1]. For solutions A and D this effect will be observable for the ^7Be and pp -neutrinos [13], if the pp -neutrino flux is measured with detectors like HELLAZ or HERON. Finally, for the MSW + VO solution parameters we have obtained in the model studied here, the day-night asymmetry caused by ν_e and $\nu_{\mu(\tau)}$ MSW transitions in the Earth is estimated to be rather small in the signals of the detectors sensitive only to ^8B or ^7Be neutrinos [1,14].

5. Conclusions

In the present paper we have investigated the possibility to accommodate the hybrid MSW + VO solutions of the solar neutrino problem, found recently in [1], in a string motivated partially unified model. The model is based on the $SU(4) \otimes O(4)$ gauge group and was shown to provide successful predictions in the charged fermion mass sector and fit the data well. In addition, a $U(1)_X$ family symmetry allows the generation of effective Yukawa couplings which are naturally small, without having to put very small fundamental Yukawa couplings in the model by hand. The charged fermion data fit constrains the Dirac neutrino mass matrix and the magnitudes of the elements of the RH-neutrino Majorana mass matrix are then set by the $U(1)_X$ family symmetry.

We have shown in the present work that the new hybrid MSW + VO solutions to the solar neutrino problem, found in [1], appear naturally in this rather compelling model. The horizontal $U(1)_X$ symmetry plays a vital role in suppressing certain elements of the RH-neutrino Majorana mass matrix, allowing some of the light neutrinos to be heavier than one would be naively expecting and therefore to lie in the mass ranges required. The vertical symmetry relates the Dirac neutrino masses to the up quark masses, allowing more predictivity. This is evident in the fact that the whole 3×3 lepton mixing matrix is unambiguously determined. The predictions of the model for the neutrino masses and the lepton mixing will be tested in the currently operating and the future solar neutrino experiments (Super-Kamiokande, SNO, BOREXINO, HELLAZ, etc.).

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⁴ The solar neutrino flux at the Earth surface changes with the time of the year due to the ellipticity of the Earth orbit around the Sun. The change of the flux from December to June due to the standard geometrical effect is 6.68%.

⁵ This result follows directly from Eq. (9) or Eq. (17) in Ref. [1].

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