



Top quark mass in exophobic Pati–Salam heterotic string model

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ABSTRACT

We analyse the phenomenology of an exemplary exophobic Pati–Salam heterotic string vacuum, in which no exotic fractionally charged states exist in the massless string spectrum. Our model also contains the Higgs representations that are needed to break the gauge symmetry to that of the Standard Model and to generate fermion masses at the electroweak scale. We show that the requirement of a leading mass term for the heavy generation, which is not degenerate with the mass terms of the lighter generations, places an additional strong constraint on the viability of the models. In many models a top quark Yukawa may not exist at all, whereas in others two or more generations may obtain a mass term at leading order. In our exemplary model a mass term at leading order exist only for one family. Additionally, we demonstrate the existence of supersymmetric F - and D -flat directions that give heavy mass to all the colour triplets beyond those of the Standard Model and leave one pair of electroweak Higgs doublets light. Hence, below the Pati–Salam breaking scale, the matter states in our model that are charged under the observable gauge symmetries, consist solely of those of the Minimal Supersymmetric Standard Model.

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1. Introduction

The Standard Model of particle physics remains unscathed by contemporary experiments. Its augmentation with the right-handed neutrinos, as envisioned by Pati and Salam nearly four decades ago [1], is mandated by solar and terrestrial neutrino observations. The Pati–Salam model naturally leads to the embedding of the Standard Model in $SO(10)$ representations. Most strikingly the matter embedding in three 16 spinorial representations correlates the 54 gauge charges of the Standard Model states into the single number of spinorial multiplets. The reduction in the number of experimental parameters from fifty four to one provides the most important clue for the fundamental origins of the Standard Model. The remaining parameters, and in particular the flavour parameters, must find their origin in a theory that unifies gauge theories with gravity. It is then of further appeal that heterotic-string theory accommodates the $SO(10)$ embedding of the Standard Model matter spectrum. Three generation heterotic-string models that preserve the $SO(10)$ embedding of the Standard Model states were constructed since the late eighties [2–7].

Absence of higher order Higgs representations in heterotic-string models that are based on level one Kac–Moody current algebras necessitates that the $SO(10)$ symmetry is broken directly at the string level by discrete Wilson lines. A well-known theorem due to Schellekens [8] states that any such string model that preserves the canonical $SO(10)$ -GUT embedding of the weak hypercharge, and in which the non-Abelian GUT symmetries are broken by discrete Wilson lines, necessarily contain states that carry charges that do not obey the original GUT quantisation rule [8].¹ In terms of the Standard Model charges these exotic states carry fractional electric charge. Electric charge conservation implies that the lightest of these states is stable, and their existence in nature is severely constrained by experiments [10].

While the existence of fractionally charged states in string models that preserve the canonical $SO(10)$ embedding of the Standard Model states, and in which the $SO(10)$ symmetry is broken by Wilson lines, is mandated by Schellekens theorem, they may appear only in vector-like representations, rather than in chiral representations. Superpotential terms for the vector-like states can then generate an intermediate or string scale mass to the exotic states, through the VEVs of Standard Model singlet fields [11,6]. However, as the generation of the VEVs is obtained in an effective field theory analysis a more appealing solution is to find string models in which the exotic fractionally charged

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¹ A similar observation was made in the context of Calabi–Yau compactification models with E_6 gauge group broken by Wilson lines [9].

states are confined to the massive spectrum. Recently, we demonstrated the existence of Pati–Salam vacua in which exotic fractionally charged states do not exist in the massless spectrum [12]. We dubbed such models as exophobic string vacua. We further showed that there exist such exophobic Pati–Salam string models that contain three generations and the required Higgs states to produce realistic mass spectrum. We demonstrated the existence of exophobic string vacua by utilising the free fermionic classification techniques. These methods were developed in Ref. [13] for type II string $N = 2$ supersymmetric vacua. They were extended in Refs. [14,15] for the classification of heterotic $Z_2 \times Z_2$ free fermionic orbifolds, with unbroken $SO(10)$ and E_6 GUT symmetries, and in Ref. [12] heterotic-string vacua in which the $SO(10)$ symmetry is broken to the Pati–Salam subgroup.

The classification method used in Refs. [13–15,12] utilises symmetric boundary conditions for the set of internal world-sheet fermions that correspond to the six-dimensional compactified lattice. The symmetric boundary conditions correspond to Z_2 shifts in the compactified six-dimensional torus and enable the scan of large sets of vacua. Such symmetric assignments in Pati–Salam heterotic string models lead to the projection of the untwisted Higgs bi-doublets and preservation of the corresponding colour triplets [16]. In quasi-realistic free fermionic models untwisted Higgs doublets couple to twisted matter states. The leading coupling is identified with the top quark mass term in the superpotential [17]. Hence, this coupling is not present in the exophobic Pati–Salam models of Ref. [12]. The question arises whether a top quark mass term exists in these string vacua. A viable top quark Yukawa term is one of the first criteria that a realistic string vacuum should admit.

An alternative to the twisted–twisted–untwisted coupling that is used in the quasi-realistic free fermionic models is a twisted–twisted–twisted coupling. The existence of a viable coupling is model dependent. The three states appearing in the trilevel term must arise from the three distinct twisted sectors. Hence, for example, if all the vectorial and spinorial twisted states would arise from a single sector, the vacuum would not be viable. In this Letter we examine this question in the exophobic string vacuum of Ref. [12]. We show in one concrete model that the required coupling does exist. Additionally, we calculate the entire cubic level superpotential and show the existence of flat directions that leave a light pair of electroweak Higgs doublets and give heavy mass to all vector-like colour triplets. Hence, below the Pati–Salam breaking scale the spectrum of our model coincides with that of the Minimal Supersymmetric Standard Model (MSSM).

2. Exophobic Pati–Salam heterotic-string model

Our exophobic Pati–Salam heterotic-string model is constructed in the free fermionic formulation [18]. In this formulation a string model is specified in terms of a set of boundary condition basis vectors v_i , $i = 1, \dots, N$

$$v_i = \{\alpha_i(f_1), \alpha_i(f_2), \alpha_i(f_3), \dots\},$$

for the 64 world-sheet real fermions [18], and the one-loop generalised GGSO projection coefficients, $c \left[\begin{smallmatrix} v_i \\ v_j \end{smallmatrix} \right]$. The basis vectors span a space \mathcal{E} which consists of 2^N sectors that give rise to the string spectrum. Each sector, $\eta \in \mathcal{E}$, is given by

$$\eta = \sum N_i v_i, \quad N_i = 0, 1 \quad (2.1)$$

The spectrum is truncated by a generalised GSO projection whose action on a string state $|S\rangle$ is

$$e^{i\pi v_i \cdot F_S} |S\rangle = \delta_S c \left[\begin{smallmatrix} S \\ v_i \end{smallmatrix} \right] |S\rangle, \quad (2.2)$$

where F_S is the fermion number operator and $\delta_S = \pm 1$ is the space–time spin statistics index. The world-sheet free fermions in the light-cone gauge in the usual notation are: ψ^μ , χ^i , y^i , ω^i , $i = 1, \dots, 6$ (left-movers) and \bar{y}^i , $\bar{\omega}^i$, $i = 1, \dots, 6$, ψ^A , $A = 1, \dots, 5$, $\bar{\eta}^B$, $B = 1, 2, 3$, $\bar{\phi}^\alpha$, $\alpha = 1, \dots, 8$ (right-movers). The exophobic Pati–Salam model is generated by a set of thirteen basis vectors $B = \{v_1, v_2, \dots, v_{13}\}$, where

$$\begin{aligned} v_1 = 1 &= \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}, \\ v_2 = S &= \{\psi^\mu, \chi^{1,\dots,6}\}, \\ v_{2+i} = e_i &= \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\}, \quad i = 1, \dots, 6, \\ v_9 = b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \\ v_{10} = b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \\ v_{11} = z_1 &= \{\bar{\phi}^{1,\dots,4}\}, \\ v_{12} = z_2 &= \{\bar{\phi}^{5,\dots,8}\}, \\ v_{13} = \alpha &= \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}. \end{aligned} \quad (2.3)$$

The first two basis vectors generate a model with $N = 4$ space–time supersymmetry and $SO(44)$ gauge group in four dimensions. The next six basis vectors correspond to freely acting shifts on the internal six-dimensional compactified torus and reduce the gauge symmetry to $SO(32)$. The basis vectors z_1 and z_2 are freely acting as well, and reduce the gauge symmetry arising from the Neveu–Schwarz (NS) sector to $SO(16) \times SO(8) \times SO(8)$. Additional space–times vector bosons may arise from the sectors [14,15,12]

$$\mathbf{G} = \left\{ \begin{array}{cccccc} z_1, & z_2, & \alpha, & \alpha + z_1, & & \\ x, & z_1 + z_2, & \alpha + z_2, & \alpha + z_1 + z_2, & \alpha + x, & \alpha + x + z_1 \end{array} \right\} \quad (2.4)$$

and enhance the four-dimensional gauge group. In (2.4) we defined the vector combination

Table 1
Untwisted matter spectrum and $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^3$ quantum numbers.

Sector	Field	$SU(4) \times SU(2)_L \times SU(2)_R$	$U(1)_1$	$U(1)_2$	$U(1)_3$
S	D_1	(6, 1, 1)	+1	0	0
	D_2	(6, 1, 1)	0	+1	0
	D_3	(6, 1, 1)	0	0	+1
	\bar{D}_1	(6, 1, 1)	-1	0	0
	\bar{D}_2	(6, 1, 1)	0	-1	0
	\bar{D}_3	(6, 1, 1)	0	0	-1
	Φ_{12}	(1, 1, 1)	+1	+1	0
	$\bar{\Phi}_{12}$	(1, 1, 1)	+1	-1	0
	Φ_{12}	(1, 1, 1)	-1	-1	0
	$\bar{\Phi}_{12}$	(1, 1, 1)	-1	+1	0
	Φ_{13}	(1, 1, 1)	+1	0	+1
	$\bar{\Phi}_{13}$	(1, 1, 1)	+1	0	-1
	Φ_{13}	(1, 1, 1)	-1	0	-1
	$\bar{\Phi}_{13}$	(1, 1, 1)	-1	0	+1
	$\Phi_i, i = 1, \dots, 6$	(1, 1, 1)	0	0	0
	Φ_{23}	(1, 1, 1)	0	+1	+1
	$\bar{\Phi}_{23}$	(1, 1, 1)	0	+1	-1
	Φ_{23}	(1, 1, 1)	0	-1	-1
	$\bar{\Phi}_{23}$	(1, 1, 1)	0	-1	+1

$$x = 1 + S + \sum_{i=1}^6 e_i + z_1 + z_2,$$

which may enhance the observable $SO(16)$ gauge symmetry to E_8 . For suitable choices of the GGSO projection coefficients all the space-time vector bosons arising from the sectors in Eq. (2.4) are projected out. The basis vectors b_1 and b_2 correspond to the $Z_2 \times Z_2$ twists of a $Z_2 \times Z_2$ orbifold. Each Z_2 twist reduces the number of supersymmetry generators from $N = 4$ to $N = 2$. In combination b_1 and b_2 break $N = 4$ to $N = 1$ space-time supersymmetry, and reduce the NS gauge symmetry to $SO(10) \times U(1)^3 \times SO(8) \times SO(8)$.

In the quasi-realistic heterotic string models the gauge symmetries are realised as level one Kac–Moody algebras. The massless spectrum of such models does not contain scalar Higgs multiplets in the adjoint representation that can be used to break the non-Abelian $SO(10)$ GUT symmetry. Consequently, the GUT gauge group must be broken at the string level, by a boundary condition basis vector in the free fermionic formalism, or a discrete Wilson line in the orbifold formalism. The basis vector α reduces the $SO(10)$ symmetry to the Pati–Salam subgroup. The gauge group in our model is therefore:

$$\text{observable: } SO(6) \times SO(4) \times U(1)^3,$$

$$\text{hidden: } SO(4)^2 \times SO(8).$$

The matter states in our model are embedded in $SU(4) \times SU(2)_L \times SU(2)_R$ representations as follows:

$$F_L(\mathbf{4}, \mathbf{2}, \mathbf{1}) \rightarrow q\left(\mathbf{3}, \mathbf{2}, -\frac{1}{6}\right) + \ell\left(\mathbf{1}, \mathbf{2}, \frac{1}{2}\right),$$

$$\bar{F}_R(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \rightarrow u^c\left(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3}\right) + d^c\left(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3}\right) + e^c(\mathbf{1}, \mathbf{1}, -1) + \nu^c(\mathbf{1}, \mathbf{1}, 0),$$

$$h(\mathbf{1}, \mathbf{2}, \mathbf{2}) \rightarrow h^d\left(\mathbf{1}, \mathbf{2}, \frac{1}{2}\right) + h^u\left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right),$$

$$D(\mathbf{6}, \mathbf{1}, \mathbf{1}) \rightarrow d_3\left(\mathbf{3}, \mathbf{1}, \frac{1}{3}\right) + \bar{d}_3\left(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3}\right),$$

where F_L and \bar{F}_R contain a single Standard Model generation; h^d and h^u are electroweak Higgs doublets; and D contains vector-like colour triplets. The decomposition of the Pati–Salam breaking Higgs fields in terms of the Standard Model group factors is:

$$\bar{H}(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \rightarrow u_H^c\left(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3}\right) + d_H^c\left(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3}\right) + \nu_H^c(\mathbf{1}, \mathbf{1}, 0) + e_H^c(\mathbf{1}, \mathbf{1}, -1),$$

$$H(\mathbf{4}, \mathbf{1}, \mathbf{2}) \rightarrow u_H\left(\mathbf{3}, \mathbf{1}, -\frac{2}{3}\right) + d_H\left(\mathbf{3}, \mathbf{1}, \frac{1}{3}\right) + \nu_H(\mathbf{1}, \mathbf{1}, 0) + e_H(\mathbf{1}, \mathbf{1}, 1).$$

The electric charge in the Pati–Salam models is given by:

$$Q_{em} = \frac{1}{\sqrt{6}}T_{15} + \frac{1}{2}I_{3_L} + \frac{1}{2}I_{3_R} \tag{2.5}$$

where T_{15} is the diagonal generator of $SU(4)$ and I_{3_L}, I_{3_R} are the diagonal generators of $SU(2)_L, SU(2)_R$, respectively.

The second ingredient that is needed to define the string vacuum are the GGSO projection coefficients that appear in the one-loop partition function, $c\left[\begin{smallmatrix} V_i \\ V_j \end{smallmatrix}\right]$, spanning a 13×13 matrix. Only the elements with $i > j$ are independent, and the others are fixed by modular

Table 2
Twisted matter spectrum (observable sector) and $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^3$ quantum numbers.

Sector	Field	$SU(4) \times SU(2)_L \times SU(2)_R$	$U(1)_1$	$U(1)_2$	$U(1)_3$
$S + b_2 + e_1 + e_6$	F_{1L}	(4, 2, 1)	0	-1/2	0
$S + b_2 + e_6$	\bar{F}_{1R}	($\bar{4}$, 1, 2)	0	-1/2	0
$S + b_3 + e_1 + e_2 + e_3$	F_{2L}	(4, 2, 1)	0	0	-1/2
$S + b_1 + e_4 + e_5$	\bar{F}_{2R}	($\bar{4}$, 1, 2)	1/2	0	0
$S + b_1 + e_3 + e_4 + e_5 + e_6$	F_{1R}	(4, 1, 2)	-1/2	0	0
$S + b_3 + e_1 + e_2 + e_4$	\bar{F}_{3R}	($\bar{4}$, 1, 2)	0	0	1/2
$S + b_3 + e_2 + e_4$	F_{3L}	(4, 2, 1)	0	0	1/2
$S + b_3 + e_2 + e_3$	\bar{F}_{4R}	($\bar{4}$, 1, 2)	0	0	-1/2
$S + b_2 + x + e_2 + e_5$	h_1	(1, 2, 2)	-1/2	0	-1/2
$S + b_1 + x + e_3 + e_5$	h_2	(1, 2, 2)	1/2	0	1/2
$S + b_1 + x + e_3 + e_5 + e_6$	h_3	(1, 2, 2)	0	1/2	1/2
$S + b_3 + x + e_2$	ζ_1	(1, 1, 1)	1/2	-1/2	0
	$\bar{\zeta}_1$	(1, 1, 1)	-1/2	1/2	0
$S + b_3 + x + e_1 + e_2 + e_3 + e_4$	ζ_2	(1, 1, 1)	1/2	1/2	0
	$\bar{\zeta}_2$	(1, 1, 1)	-1/2	-1/2	0
$S + b_2 + x + e_1 + e_2 + e_5$	D_4	(6, 1, 1)	-1/2	0	-1/2
	$\zeta_a, a = 3, 4$	(1, 1, 1)	1/2	0	-1/2
	$\bar{\zeta}_a, a = 3, 4$	(1, 1, 1)	-1/2	0	1/2
	χ_+	(1, 1, 1)	1/2	1/2	1
	χ_-	(1, 1, 1)	1/2	1/2	-1
$S + b_1 + x + e_3 + e_4 + e_5$	ζ_5	(1, 1, 1)	0	1/2	1/2
	$\bar{\zeta}_5$	(1, 1, 1)	0	-1/2	-1/2
$S + b_1 + x + e_4 + e_5 + e_6$	ζ_6	(1, 1, 1)	0	1/2	1/2
	$\bar{\zeta}_6$	(1, 1, 1)	0	-1/2	-1/2
$S + b_2 + x$	ζ_7	(1, 1, 1)	1/2	0	-1/2
	$\bar{\zeta}_7$	(1, 1, 1)	-1/2	0	1/2

invariance. A priori there are therefore 78 independent coefficients corresponding to 2^{78} distinct string vacua. Eleven coefficients are fixed by requiring that the models possess $N = 1$ supersymmetry. Additionally, imposing the condition that the only space-time vector bosons that remain in the spectrum are those that arise from the untwisted sector restricts the number of phases to a total of 51 independent GGSO phases. Each distinct configuration of these phases corresponds to a distinct vacuum. Some degeneracy in this space of models may still exist due to additional symmetries over the entire space. This is not relevant for our purposes here as our aim in this work is to extract from the total space an exemplary model with the required phenomenological properties. Statistical analysis over the entire space was presented in Ref. [12].

The breaking of the $SO(10)$ GUT symmetry by the α boundary condition basis vector results in combinations of the basis vectors that can produce a priori massless states with fractional electric charge. All these sectors, and the type of states that they a priori can give rise to, are enumerated in Ref. [12].

By employing an algorithm to generate random selection of the GGSO projection coefficient the Pati–Salam free fermionic heterotic-string vacua were classified in Ref. [12]. For suitable choices of the GGSO projection coefficients all the massless fractionally charged states are projected out. Fractionally charged states in this case only exist in the massive string spectrum, which is compatible with experimental constraints. An explicit choice of GGSO projection coefficients that produces a model with this property is given by:

$$(v_i | v_j) = \begin{pmatrix} 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & b_1 & b_2 & z_1 & z_2 & \alpha \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ S & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ e_1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ e_2 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ e_3 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ e_4 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ e_5 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ e_6 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ b_1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ b_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ z_1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ z_2 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ \alpha & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \quad (2.6)$$

where we introduced the notation $c \left[\begin{smallmatrix} v_i \\ v_j \end{smallmatrix} \right] = e^{i\pi(v_i | v_j)}$.

The twisted massless states generated in the string vacuum of Eq. (2.6) produce the needed spectrum for viable phenomenology. It contains three chiral generations; one pair of heavy Higgs states to break the Pati–Salam gauge symmetry along a flat direction; light

Table 3
Twisted matter spectrum (hidden sector) and $SU(2)^4 \times SO(8) \times U(1)^3$ quantum numbers.

Sector	Field	$SU(2)^4 \times SO(8)$	$U(1)_1$	$U(1)_2$	$U(1)_3$
$S + b_3 + x + e_1 + e_4$	H_{12}^1	(2, 2, 1, 1, 1)	-1/2	-1/2	0
$S + b_3 + x + e_1 + e_2 + e_3$	H_{12}^2	(2, 2, 1, 1, 1)	1/2	-1/2	0
$S + b_2 + x + e_2 + e_5 + e_6$	H_{12}^3	(2, 1, 2, 1, 1)	1/2	0	-1/2
$S + b_3 + x + e_2 + e_3$	H_{34}^1	(1, 1, 2, 2, 1)	1/2	-1/2	0
$S + b_3 + x + e_1 + e_2 + e_4$	H_{34}^2	(1, 1, 2, 2, 1)	-1/2	-1/2	0
$S + b_2 + x + e_1 + e_2 + e_5 + e_6$	H_{34}^3	(1, 1, 2, 2, 1)	1/2	0	-1/2
$S + b_1 + x + e_3 + e_4 + e_5 + e_6$	H_{34}^4	(1, 1, 2, 2, 1)	0	1/2	1/2
$S + b_1 + x + e_4 + e_5$	H_{34}^5	(1, 1, 2, 2, 1)	0	-1/2	-1/2
$S + b_3 + x + z_1$	H_{13}^1	(2, 1, 2, 1, 1)	-1/2	-1/2	0
$S + b_3 + x + z_1 + e_1 + e_3 + e_4$	H_{13}^2	(2, 1, 2, 1, 1)	-1/2	1/2	0
$S + b_2 + x + z_1 + e_2$	H_{13}^3	(2, 1, 2, 1, 1)	1/2	0	1/2
$S + b_2 + x + z_1 + e_2 + e_6$	H_{14}^1	(2, 1, 1, 2, 1)	1/2	0	1/2
$S + b_1 + x + z_1 + e_3$	H_{14}^2	(2, 1, 1, 2, 1)	0	1/2	1/2
$S + b_1 + x + z_1 + e_6$	H_{14}^3	(2, 1, 1, 2, 1)	0	-1/2	-1/2
$S + b_3 + x + z_1 + e_3 + e_4$	H_{24}^1	(1, 2, 1, 2, 1)	-1/2	1/2	0
$S + b_3 + x + z_1 + e_1$	H_{24}^2	(1, 2, 1, 2, 1)	-1/2	-1/2	0
$S + b_2 + x + z_1 + e_1 + e_2$	H_{24}^3	(1, 2, 1, 2, 1)	-1/2	0	-1/2
$S + b_1 + x + z_1 + e_3 + e_4$	H_{24}^4	(1, 2, 1, 2, 1)	0	-1/2	1/2
$S + b_1 + x + z_1 + e_4 + e_6$	H_{24}^5	(1, 2, 1, 2, 1)	0	1/2	-1/2
$S + b_2 + x + z_1 + e_1 + e_2 + e_6$	H_{23}^1	(1, 2, 2, 1, 1)	1/2	0	1/2
$S + b_2 + x + z_2 + e_2 + e_5 + e_6$	Z_1	(1, 1, 1, 1, 8 _c)	-1/2	0	1/2
$S + b_1 + x + z_2 + e_3 + e_4$	Z_2	(1, 1, 1, 1, 8 _s)	0	-1/2	-1/2
$S + b_1 + x + z_2 + e_3 + e_5$	Z_3	(1, 1, 1, 1, 8 _c)	0	-1/2	1/2
$S + b_1 + x + z_2 + e_4 + e_6$	Z_4	(1, 1, 1, 1, 8 _s)	0	-1/2	-1/2
$S + b_1 + x + e_5 + e_6$	Z_5	(1, 1, 1, 1, 8 _c)	0	1/2	-1/2

Higgs bi-doublets needed to break the electroweak symmetry and generate fermion masses; one vector sextet of $SO(6)$ needed for the missing partner mechanism; it is completely free of massless exotic fractionally charged states. States in vectorial representation are obtained in the free fermionic models from the untwisted Neveu–Schwarz sector and from twisted sectors that contain four periodic world-sheet right-moving complex fermions. Massless states are obtained in such sectors by acting on the vacuum with a Neveu–Schwarz right-moving fermionic oscillator. The model of Eq. (2.6) contains three pairs of untwisted $SO(6)$ sextets, and an additional sextet from a twisted sector. These can obtain string scale mass along flat directions. Additionally, it contains a number of $SO(10)$ singlet states, some of which transform in non-trivial representations of the hidden sector gauge group. The full massless spectrum of the model is shown in Tables 1, 2 and 3, where we define the vector combination $b_3 \equiv b_1 + b_2 + x$.

3. The superpotential and the top quark Yukawa

Using the methodology of Ref. [19] for the calculation of renormalisable and nonrenormalisable terms, we calculate the cubic level superpotential of our exophobic Pati–Salam string model. In particular, we seek to extract models that produce a cubic level mass term for the heavy generation, but not for the lighter generations, which should arise from higher order nonrenormalisable terms. These requirements impose additional non-trivial constraints on the viable string vacua. Many models do not produce any coupling of the form $\bar{F}_R F_L h$. Such models do not admit viable phenomenology as the models should produce at least a top quark mass term at leading order. Similarly, models that produce leading mass terms for two or more families are not viable. The model presented in Ref. [12] produces the cubic level terms $(\bar{F}_{1R} F_{3L} + \bar{F}_{4R} F_{2L}) h_3$. In this model therefore two heavy families may be degenerate in mass. More appealing are therefore models that produce only a single mass term at leading order. The model produced by Eq. (2.6) is an example of such a model. The trilevel superpotential is given by

$$\begin{aligned}
\frac{W_{SM}}{g\sqrt{2}} = & \bar{F}_{2R} F_{3L} h_1 + \{h_1 h_1 \Phi_{13} + h_2 h_2 \Phi_{23} + h_3 h_3 \bar{\Phi}_{23} + h_1 h_3 \zeta_1\} + \{D_1 D_2 \bar{\Phi}_{12} + \bar{D}_1 D_2 \Phi_{12}^- + D_1 \bar{D}_2 \bar{\Phi}_{12}^- + \bar{D}_1 \bar{D}_2 \Phi_{12} + D_1 D_3 \bar{\Phi}_{13} \\
& + \bar{D}_1 D_3 \Phi_{13}^- + D_1 \bar{D}_3 \bar{\Phi}_{13}^- + \bar{D}_1 \bar{D}_3 \Phi_{13} + D_2 D_3 \bar{\Phi}_{23} + \bar{D}_2 D_3 \Phi_{23}^- + D_2 \bar{D}_3 \bar{\Phi}_{23}^- + \bar{D}_2 \bar{D}_3 \Phi_{23}\} + \{D_1 F_{1R} F_{1R} + \bar{D}_1 \bar{F}_{2R} \bar{F}_{2R} \\
& + D_2 (\bar{F}_{1R} \bar{F}_{1R} + F_{1L} F_{1L}) + D_3 (\bar{F}_{4R} \bar{F}_{4R} + F_{2L} F_{2L}) + \bar{D}_3 (\bar{F}_{3R} \bar{F}_{3R} + F_{3L} F_{3L}) + D_4 (\bar{F}_{2R} \bar{F}_{3R} + D_2 \chi_- + \bar{D}_2 \chi_+ + D_4 \Phi_{13})\} \\
& + \bar{\Phi}_{13} \chi_- \chi_+ + \Phi_{23} \bar{\Phi}_{12} \Phi_{13}^- + \Phi_{13} \bar{\Phi}_{12} \Phi_{23}^- + \Phi_{23} \bar{\Phi}_{13} \Phi_{12}^- + \Phi_{12}^- \Phi_{23}^- \bar{\Phi}_{13}^- + \Phi_{13} \bar{\Phi}_{23} \bar{\Phi}_{12}^- + \Phi_{12} \bar{\Phi}_{23} \bar{\Phi}_{13}^- + \Phi_{13}^- \bar{\Phi}_{12}^- \bar{\Phi}_{23}^- \\
& + \Phi_{12} \bar{\Phi}_{13} \bar{\Phi}_{23}^- + \zeta_1^2 \bar{\Phi}_{12}^- + \bar{\zeta}_1^2 \Phi_{12}^- + (\zeta_3^2 + \zeta_4^2 + \zeta_7^2) \bar{\Phi}_{13}^- + (\bar{\zeta}_3^2 + \bar{\zeta}_4^2 + \bar{\zeta}_7^2) \Phi_{13}^- + \frac{1}{2} \bar{\zeta}_2 \zeta_5 \chi_+ + \zeta_2^2 \bar{\Phi}_{12} \\
& + (\zeta_5^2 + \zeta_6^2) \bar{\Phi}_{23} + \Phi_{12} \bar{\zeta}_2^2 + \Phi_5 (\zeta_1 \bar{\zeta}_1 + \zeta_2 \bar{\zeta}_2) + \Phi_2 (\zeta_5 \bar{\zeta}_5 + \zeta_6 \bar{\zeta}_6) + \Phi_{23} (\bar{\zeta}_5^2 + \bar{\zeta}_6^2) + \Phi_4 \zeta_7 \bar{\zeta}_7 + \frac{\zeta_4 \zeta_5 \bar{\zeta}_2}{\sqrt{2}} + \frac{\zeta_2 \bar{\zeta}_3 \bar{\zeta}_5}{\sqrt{2}}.
\end{aligned} \tag{3.1}$$

The string vacuum contains three anomalous $U(1)$ s

$$\text{Tr} U(1)_1 = -12; \quad \text{Tr} U(1)_2 = -24; \quad \text{Tr} U(1)_3 = -12 \tag{3.2}$$

redefining we obtain two anomaly-free

$$U(1)'_1 = U(1)_1 - U(1)_3, \quad (3.3)$$

$$U(1)'_2 = U(1)_1 - U(1)_2 + U(1)_3 \quad (3.4)$$

and one anomalous combination

$$U(1)'_A = U(1)_1 + 2U(1)_2 + U(1)_3, \quad \text{Tr } U(1)_A = -72. \quad (3.5)$$

The electroweak Higgs doublets come in pairs and are accommodated in the Pati–Salam bi-doublets h_1, h_2, h_3 . Their mass matrix is

$$M_h \sim \begin{matrix} & h_1 & h_2 & h_3 \\ \begin{matrix} h_1 \\ h_2 \\ h_3 \end{matrix} & \begin{pmatrix} \Phi_{13} & \frac{\zeta_1}{\sqrt{2}} & 0 \\ \frac{\zeta_1}{\sqrt{2}} & \bar{\Phi}_{23} & 0 \\ 0 & 0 & \Phi_{23} \end{pmatrix} \end{matrix}. \quad (3.6)$$

In order to keep h_1 massless we need to impose the condition

$$\Phi_{13}\bar{\Phi}_{23} - \frac{\zeta_1^2}{2} = 0. \quad (3.7)$$

Next, we discuss the colour-triplet mass matrix in our string derived Pati–Salam model. Three pairs of colour-triplets arise in the model from the untwisted Neveu–Schwarz sector, and are accommodated in the sextet of the Pati–Salam $SU(4)$. We denote these by $D_i = d_i(3, 1, 1/3) + d_i^c(\bar{3}, 1, -1/3)$, $\bar{D}_i = \bar{d}_i(\bar{3}, 1, -1/3) + \bar{d}_i^c(3, 1, 1/3)$. An additional sextet arises in the model from a twisted sector. A further pair of colour triplets is obtained from the heavy Higgs states, \bar{F}_{1R} and F_{1R} that are used to break the Pati–Salam symmetry, and must get a VEV of the order of the GUT scale. We denote the colour triplets in these fields by $F_{\alpha R} = d_{\alpha H} + \dots$. At the cubic level the colour triplet mass matrix then takes the form,

$$M_D = \begin{matrix} & d_1 & d_2 & d_3 & \bar{d}_1 & \bar{d}_2 & \bar{d}_3 & d_4 & d_{1H} \\ \begin{matrix} d_1^c \\ d_2^c \\ d_3^c \\ \bar{d}_1^c \\ \bar{d}_2^c \\ \bar{d}_3^c \\ d_4^c \\ \bar{d}_{1H}^c \end{matrix} & \begin{pmatrix} 0 & \bar{\Phi}_{12} & \bar{\Phi}_{13} & 0 & \bar{\Phi}_{12}^- & \bar{\Phi}_{13}^- & 0 & F_{1R} \\ \bar{\Phi}_{12} & 0 & \bar{\Phi}_{23} & \Phi_{12}^- & 0 & \bar{\Phi}_{23}^- & \chi_- & 0 \\ \bar{\Phi}_{13} & \bar{\Phi}_{23} & 0 & \Phi_{13} & \Phi_{23}^- & 0 & 0 & 0 \\ 0 & \Phi_{12}^- & \Phi_{13} & 0 & \Phi_{12} & \Phi_{13} & 0 & 0 \\ \Phi_{12}^- & 0 & \Phi_{23}^- & \Phi_{12} & 0 & \Phi_{23} & \chi_+ & 0 \\ \bar{\Phi}_{13}^- & \bar{\Phi}_{23}^- & 0 & \Phi_{13} & \Phi_{23} & 0 & 0 & 0 \\ 0 & \chi_- & 0 & 0 & \chi_+ & 0 & \Phi_{13} & 0 \\ 0 & \bar{F}_{1R} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}. \quad (3.8)$$

We have $\det(M_D) \sim \phi_{13}^2$ so in order to keep triplets heavy and h_1 light we need $\{\Phi_{13}, \zeta_1, \bar{\Phi}_{23}\} \neq 0$.

Next, we examine the pattern of symmetry breaking. The anomalous $U(1)_A$ is broken by the Green–Schwarz–Dine–Seiberg–Witten mechanism [20] in which a potentially large Fayet–Iliopoulos D -term ξ is generated by the VEV of the dilaton field. Such a D -term would, in general, break supersymmetry, unless there is a direction $\hat{\phi} = \sum \alpha_i \phi_i$ in the scalar potential for which $\sum Q_A^i |\alpha_i|^2 < 0$ and that is D -flat with respect to all the non-anomalous gauge symmetries along with F -flat. If such a direction exists, it will acquire a VEV, cancelling the Fayet–Iliopoulos ξ -term, restoring supersymmetry and stabilising the vacuum. Assuming VEVs for the non-Abelian gauge singlets and a pair of PS breaking Higgs, $F_{1R} = \bar{F}_{1R} = M_G$, the D -flatness constraints in our model are given by:

$$U(1)'_1: (|\Phi_{12}|^2 - |\bar{\Phi}_{12}|^2) + (|\Phi_{12}^-|^2 - |\bar{\Phi}_{12}^-|^2) + 2(|\Phi_{13}^-|^2 - |\bar{\Phi}_{13}^-|^2) - (|\Phi_{23}|^2 - |\bar{\Phi}_{23}|^2) + (|\Phi_{23}^-|^2 - |\bar{\Phi}_{23}^-|^2) \\ + \frac{1}{2} \sum_{i=1,2} (|\zeta_i|^2 - |\bar{\zeta}_i|^2) - \frac{1}{2} \sum_{i=5,6} (|\zeta_i|^2 - |\bar{\zeta}_i|^2) + \sum_{i=3,4,7} (|\zeta_i|^2 - |\bar{\zeta}_i|^2) - \frac{1}{2} |F_{1R}|^2 = 0, \quad (3.9)$$

$$U(1)'_2: 2(|\Phi_{12}^-|^2 - |\bar{\Phi}_{12}^-|^2) + 2(|\Phi_{13}|^2 - |\bar{\Phi}_{13}|^2) - 2(|\Phi_{23}^-|^2 - |\bar{\Phi}_{23}^-|^2) + (|\zeta_1|^2 - |\bar{\zeta}_1|^2) + 2|\chi_-|^2 \\ + \frac{1}{2} (|\bar{F}_{1R}|^2 - |F_{1R}|^2) = 0, \quad (3.10)$$

$$U(1)'_A: 3(|\Phi_{12}|^2 - |\bar{\Phi}_{12}|^2) - (|\Phi_{12}^-|^2 - |\bar{\Phi}_{12}^-|^2) + 2(|\Phi_{13}|^2 - |\bar{\Phi}_{13}|^2) + 3(|\Phi_{23}|^2 - |\bar{\Phi}_{23}|^2) + (|\Phi_{23}^-|^2 - |\bar{\Phi}_{23}^-|^2) \\ - \frac{1}{2} (|\zeta_1|^2 - |\bar{\zeta}_1|^2) + \frac{3}{2} \sum_{i=2,5,6} (|\zeta_i|^2 - |\bar{\zeta}_i|^2) + 3|\chi_+|^2 - |\chi_-|^2 - \frac{1}{2} |F_{1R}|^2 - |\bar{F}_{1R}|^2 = + \frac{3g^2}{16\pi^2} M^2 \equiv \xi. \quad (3.11)$$

In Eq. (3.11) g is the gauge coupling in the effective field theory, and M is the so-called reduced Planck mass $M \equiv M_{\text{Planck}}/\sqrt{8\pi}$. In setting ξ we followed the conventions of [21]. The set of F -flatness constraints are obtained by requiring

$$\left\langle F_i \equiv \frac{\partial W}{\partial \eta_i} \right\rangle = 0 \quad (3.12)$$

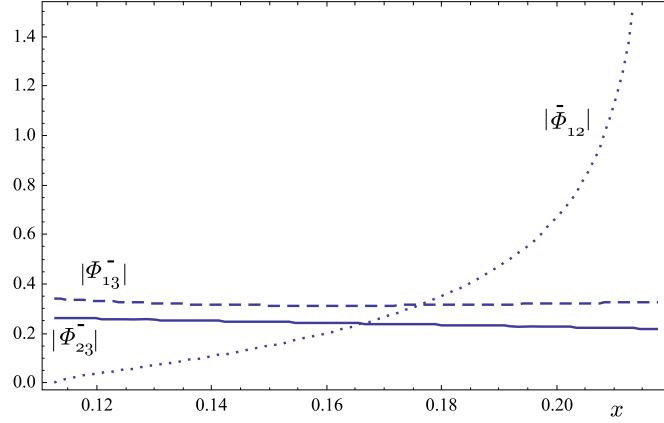


Fig. 1. Solution of the D -flatness equations for $|\bar{\Phi}_{13}^-|$, $|\bar{\Phi}_{23}^-|$ and $|\bar{\Phi}_{12}^-|$ as a function of $\chi = |\bar{\Phi}_{23}^-| = |\bar{\Phi}_{13}^-| = \frac{1}{2}|\bar{\Phi}_{23}^-|$ (all VEVs are in units of $\sqrt{\xi}$).

where η_i are all the fields that appear in the model. The solution (i.e. the choice of fields with non-vanishing VEVs) to the set of Eqs. .9.9.9.10.11.12(3.9)–(3.12), though non-trivial, is not unique. Therefore in a typical model there exist a moduli space of solutions to the F and D flatness constraints, which are supersymmetric and degenerate in energy [22]. Assuming VEVs for the non-Abelian gauge singlets and a pair of PS breaking Higgs, $F_{1R} = \bar{F}_{1R} = M_G$, the following 9 parameter exact solution

$$\{\Phi_3, \Phi_4, \Phi_6, \bar{\Phi}_{23}, \Phi_{23}^-, \bar{\Phi}_{23}^-, \Phi_{13}^-, \bar{\Phi}_{13}^-, \bar{\Phi}_{12}^-\} \quad (3.13)$$

satisfies all F -flatness equations while keeping one linear combination of the bi-doublets (h_1, h_2) massless:

$$0 = \Phi_1 = \Phi_2 = \chi_+ = \chi_- = \zeta_i = \bar{\zeta}_i, \quad i = 3, \dots, 7, \quad (3.14)$$

$$\Phi_5 = -\frac{2i}{\sqrt{3}} \frac{\bar{\Phi}_{12}^-}{\bar{\Phi}_{23}^-} \sqrt{\frac{\Phi_{13}^- \Phi_{23}^- \bar{\Phi}_{23}^-}{\bar{\Phi}_{13}^-}}, \quad (3.15)$$

$$\Phi_{23} = \frac{\Phi_{23}^- \bar{\Phi}_{23}^-}{\bar{\Phi}_{23}^-}, \quad \Phi_{13} = -\frac{\Phi_{13}^- \bar{\Phi}_{23}^-}{3\bar{\Phi}_{23}^-} \quad (3.16)$$

$$\bar{\Phi}_{13} = -\frac{3\bar{\Phi}_{23}^- \bar{\Phi}_{13}^-}{\bar{\Phi}_{23}^-}, \quad \Phi_{12} = -\frac{\bar{\Phi}_{12}^- \Phi_{13}^- \Phi_{23}^- \bar{\Phi}_{23}^-}{3\bar{\Phi}_{23}^{-2} \bar{\Phi}_{13}^-}, \quad (3.17)$$

$$\Phi_{12}^- = \frac{\bar{\Phi}_{12}^- \Phi_{13}^- \bar{\Phi}_{23}^-}{3\bar{\Phi}_{23}^- \bar{\Phi}_{13}^-}, \quad \bar{\Phi}_{12}^- = -\frac{\bar{\Phi}_{12}^- \bar{\Phi}_{23}^-}{\bar{\Phi}_{23}^-}, \quad (3.18)$$

$$\zeta_1 = i\sqrt{\frac{2\Phi_{13}^- \bar{\Phi}_{23}^-}{3}}, \quad \bar{\zeta}_1 = -\sqrt{2\Phi_{23}^- \bar{\Phi}_{13}^-}, \quad (3.19)$$

$$\zeta_2 = i\sqrt{\frac{2\Phi_{23}^- \Phi_{13}^- \bar{\Phi}_{23}^-}{3\bar{\Phi}_{23}^-}}, \quad \bar{\zeta}_2 = \sqrt{2\bar{\Phi}_{23}^- \bar{\Phi}_{13}^-}. \quad (3.20)$$

The triplet mass matrix (3.8) determinant is

$$\det M_D = -\frac{64}{27} \frac{F_{1R} \bar{F}_{1R} \bar{\Phi}_{12}^- \Phi_{13}^{-3} \Phi_{23}^{-2} \bar{\Phi}_{23}^{-3}}{\bar{\Phi}_{23}^{-3}} \quad (3.21)$$

and thus all triplets are massive.

For this F -flatness solution, the three D -flatness Eqs. .9.9.9.10.11(3.9)–(3.11) depend on seven parameters, $|\bar{\Phi}_{23}^-|$, $|\Phi_{23}^-|$, $|\bar{\Phi}_{13}^-|$, $|\Phi_{13}^-|$, $|\bar{\Phi}_{12}^-|$, and $|F_{1R}| = |\bar{F}_{1R}|$. Setting $|F_{1R}| = |\bar{F}_{1R}| = M_G = 0.02\sqrt{\xi}$ the D -flatness equations can be solved numerically in terms of three parameters. Choosing, for example, $|\bar{\Phi}_{23}^-| = |\bar{\Phi}_{13}^-| = \frac{1}{2}|\bar{\Phi}_{23}^-| = \chi$ we can solve numerically for $|\Phi_{13}^-|$, $|\Phi_{23}^-|$ and $|\bar{\Phi}_{12}^-|$. The results are shown in Fig. 1. In Fig. 2 we plot the mass of the two lightest colour triplets for the one parameter solution displayed in Fig. 1. From the figure we note that for singlet VEVs of the order of $0.1\sqrt{\xi}$ the lightest triplet mass is of the order of $0.4M_{\text{GUT}}$. Thus the additional colour triplets are heavy enough to protect proton from decaying through dangerous triplet mediated dim-5 operators [23]. Additionally, we note that the three $U(1)$ symmetries in Eqs. 3.3.3.4.5(3.3)–(3.5) are broken in the F - and D -flat vacuum. We further remark that the superpotential vanishes in the vacuum as well.

4. Conclusions

In this Letter we analysed the phenomenology of an exemplary exophobic Pati–Salam heterotic string vacuum, in which exotic fractionally charged states exist in the massive spectrum, but not among the massless states. In that respect the exophobic models are distinguished from other models in which exotic states gain heavy mass by vacuum expectation values of Standard Model singlet fields.

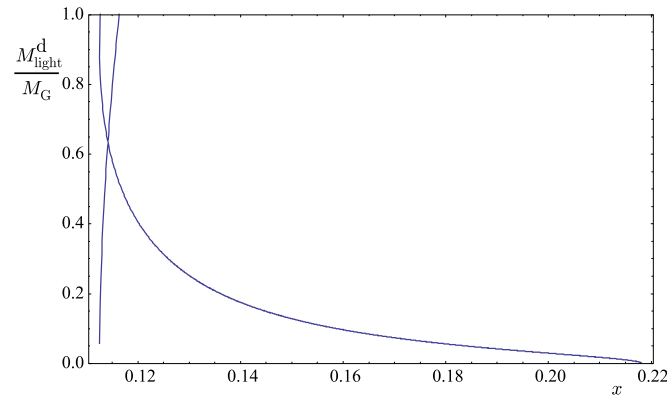


Fig. 2. The ratio of the two lightest colour triplet mass over M_{GUT} as a function of $\chi = |\bar{\phi}_{23}| = |\bar{\phi}_{13}| = \frac{1}{2}|\bar{\phi}_{23}^-|$ (in units of $\sqrt{\xi}$).

Our exophobic model also contains the Higgs representations that are needed to break the gauge symmetry to that of the Standard Model and to generate fermion masses at the electroweak scale. One can then start to probe the phenomenology of such models in more detail. We showed in particular that the presence of a top quark Yukawa coupling at leading order places an additional strong constraint on the viability of the models. In many models a top quark Yukawa may not exist at all, whereas in others two or more generations may obtain a mass term at leading order. In our exemplary model a mass term at leading order exists only for one family. Additionally, we demonstrated the existence of supersymmetric F - and D -flat directions that give heavy mass to all the colour triplets beyond those of the Standard Model and leave one pair of electroweak Higgs doublets light. Hence, below the Pati–Salam breaking scale the spectrum of our model consists solely of that of the Minimal Supersymmetric Standard Model. We remark that while there exist other models in which the exotic states are decoupled along flat directions, in many of these models the mass scale of the exotic states is ambiguous as the relevant mass terms arise from higher order superpotential terms that are expected to be suppressed compared to the leading string scale mass terms [24]. The novelty in our model is that the exotic states are absent from the massless spectrum to begin with and hence necessarily have string scale masses. In this respect the model is superior to earlier constructions. Further analysis of higher order terms in the superpotential can now be pursued to confront the model with the detailed Standard Model mass and mixing data. We note that the interplay between statistical searches and detailed analysis of specific models takes us a step further toward the construction of string models that reproduce the phenomenological Standard Model data. We will return to these issues in future publications.

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References

- [1] J.C. Pati, A. Salam, Phys. Rev. D 10 (1974) 275;
I. Antoniadis, G.K. Leontaris, Phys. Lett. B 216 (1989) 333.
- [2] I. Antoniadis, J. Ellis, J. Hagelin, D.V. Nanopoulos, Phys. Lett. B 231 (1989) 65.
- [3] A.E. Faraggi, D.V. Nanopoulos, K. Yuan, Nucl. Phys. B 335 (1990) 347.
- [4] I. Antoniadis, G.K. Leontaris, J. Rizos, Phys. Lett. B 245 (1990) 161;
G.K. Leontaris, J. Rizos, Nucl. Phys. B 554 (1999) 3.
- [5] A.E. Faraggi, Phys. Lett. B 278 (1992) 131;
A.E. Faraggi, Nucl. Phys. B 387 (1992) 239.
- [6] G.B. Cleaver, A.E. Faraggi, D.V. Nanopoulos, Phys. Lett. B 455 (1999) 135.
- [7] G.B. Cleaver, A.E. Faraggi, C. Savage, Phys. Rev. D 63 (2001) 066001.
- [8] A.N. Schellekens, Phys. Lett. B 237 (1990) 363.
- [9] X.G. Wen, E. Witten, Nucl. Phys. B 261 (1985) 651;
G. Athanasiou, J. Atick, M. Dine, W. Fischler, Phys. Lett. B 214 (1988) 55.
- [10] See e.g. V. Halyo, et al., Phys. Rev. Lett. 84 (2000) 2576.
- [11] A.E. Faraggi, Phys. Rev. D 46 (1992) 3204;
S. Chang, C. Coriano, A.E. Faraggi, Nucl. Phys. B 477 (1996) 65;
C. Coriano, A.E. Faraggi, M. Plümacher, Nucl. Phys. B 614 (2001) 233.
- [12] B. Assel, C. Christodoulides, A.E. Faraggi, C. Kounnas, J. Rizos, Phys. Lett. B 683 (2010) 306;
B. Assel, C. Christodoulides, A.E. Faraggi, C. Kounnas, J. Rizos, Nucl. Phys. B 844 (2011) 365.
- [13] A. Gregori, C. Kounnas, J. Rizos, Nucl. Phys. B 549 (1999) 16.
- [14] A.E. Faraggi, C. Kounnas, S.E.M. Nooij, J. Rizos, Nucl. Phys. B 695 (2004) 41, hep-th/0311058.
- [15] A.E. Faraggi, C. Kounnas, J. Rizos, Phys. Lett. B 648 (2007) 84;
A.E. Faraggi, C. Kounnas, J. Rizos, Nucl. Phys. B 774 (2007) 208;
T. Catelin-Julian, A.E. Faraggi, C. Kounnas, J. Rizos, Nucl. Phys. B 812 (2009) 103.
- [16] A.E. Faraggi, Nucl. Phys. B 428 (1994) 111;
A.E. Faraggi, Phys. Lett. B 520 (2001) 337.
- [17] A.E. Faraggi, Phys. Lett. B 274 (1992) 47;
A.E. Faraggi, Phys. Rev. D 47 (1993) 5021.
- [18] I. Antoniadis, C. Bachas, C. Kounnas, Nucl. Phys. B 289 (1987) 87;
H. Kawai, D.C. Lewellen, S.H.-H. Tye, Nucl. Phys. B 288 (1987) 1.

- [19] S. Kalara, J.L. Lopez, D.V. Nanopoulos, Nucl. Phys. B 353 (1991) 650;
J. Rizos, K. Tamvakis, Phys. Lett. B 262 (1991) 227;
A.E. Faraggi, Nucl. Phys. B 487 (1997) 55.
- [20] M.B. Green, J.H. Schwarz, Phys. Lett. B 149 (1984) 117;
M. Dine, N. Seiberg, E. Witten, Nucl. Phys. B 289 (1987) 589;
J. Atick, L. Dixon, A. Sen, Nucl. Phys. B 292 (1987) 109.
- [21] G. Cleaver, et al., Phys. Rev. D 59 (1999) 115003.
- [22] F. Buccella, et al., Phys. Lett. B 115 (1982) 375;
A. Font, L.E. Ibanez, H.P. Nilles, F. Quevedo, Nucl. Phys. B 307 (1988) 109;
J.A. Casas, E.K. Katehou, C. Munoz, Nucl. Phys. B 317 (1989) 171;
T. Kobayashi, H. Nakano, Nucl. Phys. B 496 (1997) 103.
- [23] G.K. Leontaris, Z. Phys. C 53 (1992) 287.
- [24] T. Kobayashi, S. Raby, R.J. Zhang, Nucl. Phys. B 704 (2005) 3;
O. Lebedev, et al., Phys. Rev. D 77 (2008) 046013.