22 September 1994

PHYSICS LETTERS B

ELSEVIER

Physics Letters B 336 (1994) 194–199

Large top and bottom Yukawa couplings in minimal supersymmetry

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Received 3 July 1994
Editor R Gatto

Abstract

We present analytic expressions for the top and bottom Yukawa couplings in the context of the minimal supersymmetric standard model when both couplings $h_{t,0}$, $h_{b,0}$ are large at the unification scale For sufficiently large $h_{t,0}$, $h_{b,0}$, using as input the central value of the bottom mass $m_b(m_b) = 4\,25$ GeV, we find that the top mass lies in the range $m_t \approx (174–178)$ GeV, while $\tan\beta \approx (55–58)$ Implications on the evolution of the scalar masses and the radiative symmetry breaking scenario are discussed

It is widely believed that the minimal supersymmetric standard model (MSSM) is the most natural extention of the standard theory of strong and electromagnetic interactions. Furthermore, the MSSM can be naturally embedded in all unified supergravity and superstring constructions. Detailed calculations [1] taking into account the most recent data on the low energy values of gauge couplings and other measurable parameters, have shown that the above theoretical expectations are correct when supersymmetry breaks at the order of 1 TeV and the unification of gauge couplings takes place at the scale of 10^{16} GeV provided that the MSSM fermion and higgs content is used It was therefore recently realized [2] that it is time to take a step further and – in addition to the gauge coupling unification – explore the nature of Yukawa couplings which, in the MSSM, are treated as free parameters Grand unified theories [3] predict various relations among them, depending on the precise group chosen by the particular unification scenario Furthermore, string theories relate the fate of Yukawa couplings with that of the gauge constants It is expected that some of them might be of the order of the common gauge coupling constant at the unification point. In fact, in all string scenarios there appears a hierarchical form of the quark and lepton Yukawa couplings, due to additional $U(1)$ symmetries which usually allow only one generation of fermions to receive masses from the trilinear superpotential terms. Concentrating on the quark sector, this would mean that the only tree level Yukawa couplings which may be large and comparable to the unified gauge coupling are h_t and h_b Knowing their initial values at the GUT scale, we may determine their low energy values by evolving them down using the renormalization group equations This has been done in many recent works [4,5,2] using numerical methods. In the present letter, we wish to present analytic forms for the above couplings, when both are large at the unification scale Particular solutions as in the interesting case of $h_{t,0} \approx h_{b,0}$ have already appeared in the

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SSDI 0370-2693(94)00929-5

literature [5]. Such forms might prove extremely useful, for example, in the calculation of the scalar masses and the Higgs mass parameters. In particular, the latter play a very important role in the radiative symmetry breaking scenario [6]. In particular in the large tan β scenario where both couplings are large, the higgs mass-squared parameters may both be driven negative and the stability of the neutral higgs potential will be questionable Therefore, analytic forms for the mass parameters can simplify the analysis of the minimization conditions and provide a better insight of the role of the Yukawa coupling contributions to the scalar masses.

In the following, we will assume that only h_t and h_b Yukawa couplings are large. We will assume that all other Yukawa couplings, including h_τ, are small and will ignore them. Thus, we do not implement the minimal GUT relation $h_{\tau,0}\equiv h_{b,0}$ at the GUT scale. We should point out that this happens very often in the case of string GUTs (as in the case of the flipped $SU(5)$ [8]), and therefore it is of particular interest. Nevertheless, our solutions are still a good approximation even in the case of appreciable initial $h_{\tau,0}$ values.

Ignoring all other Yukawa couplings, the coupled differential system of h_t-h_b couplings is written as follows

$$\frac{d}{dt}h_t^2=\frac{1}{8\pi^2}\{6h_t^2+h_b^2-G_Q\}h_t^2\,, \tag{1}$$

$$\frac{d}{dt}h_b^2=\frac{1}{8\pi^2}\{h_t^2+6h_b^2-G_B\}h_b^2\,, \tag{2}$$

with

$$G_Q=\sum_{i=1}^{3}c_Q^i g_i^2\,,\quad G_B=\sum_{i=1}^{3}c_B^i g_i^2\,, \tag{3}$$

where $c_Q^i=\{\frac{16}{3},3,\frac{13}{15}\}$ $c_B^i=\{\frac{16}{3},3,\frac{7}{15}\}$ for $SU(3)$, $SU(2)$ and $U(1)$, respectively. In order to solve (1), (2), we choose first to absorbe the gauge coefficients G_Q, G_B by making the transformations·

$$h_t^2=\gamma_Q^2 x,\quad h_b^2=\gamma_B^2 y\,, \tag{4}$$

where

$$\gamma_I^2=\exp\left\{\frac{-1}{8\pi^2}\int_{t_0}^{t}G_I(t')\,dt'\right\},\quad I=Q,B \tag{5}$$

with $t_0=\ln M_{GUT}$ Then, by observing that $\gamma_Q\approx\gamma_B$, since their only difference is only a small coefficient in the $U(1)$ factor, it can be seen easily that the D.E.'s (1), (2), in the approximation $\gamma_Q=\gamma_B$ transform as follows

$$\frac{d}{dt}x=\frac{1}{8\pi^2}\gamma_Q^2\{6x+y\}x\,, \tag{6}$$

$$\frac{d}{dt}y=\frac{1}{8\pi^2}\gamma_Q^2\{x+6y\}y \tag{7}$$

This last coupled system can give a solution for x in terms of y and vice versa, leading to the algebraic equation

$$\left(\frac{x-y}{x_0-y_0}\right)^7=\left(\frac{xy}{x_0y_0}\right)^6. \tag{8}$$

Furthermore subtracting (6) from (7), we may obtain the following D.E.:

$$\frac{d}{dt}(x-y)=\frac{6}{8\pi^2}\gamma_Q^2(x+y)(x-y)\,, \tag{9}$$

while $(x+y)$ can be substituted from (7) to give a differential equation for the difference $\omega=x-y$ of the form

$$\frac{d}{dt}(x-y)=\frac{6}{8\pi^2}\gamma_Q^2(x-y)\sqrt{k_0(x-y)^{7/6}+(x-y)^2}\,, \tag{10}$$

where the parameter $k_0=4x_0y_0/(x_0-y_0)^{7/6}$ depends on the initial conditions $x_0\equiv h_{t,0}^2$ and $y_0\equiv h_{b,0}^2$.

In order to solve Eq. (9) we make the following transformation

$$u=\frac{k_0}{(x-y)^{5/6}}\equiv\frac{k_0}{\omega^{5/6}}\quad dI=\frac{6}{8\pi^2}\gamma_Q^2\,dt\,. \tag{11}$$

Eq. (9) can be put in the form

$$\frac{u^{1/5}du}{\sqrt{1+u}}=-\tfrac{5}{6}k_0^{6/5}\,dI\,, \tag{12}$$

which can be integrated to give the solution in terms of hypergeometric functions ${}_2F_1(a,b,c;z)$, i e

$$u^{7/10}\,{}_2F_1\left(\tfrac{1}{2},-\tfrac{7}{10},\tfrac{3}{10},\frac{1}{-u},\right)-u_0^{7/10}\times{}_2F_1\left(\tfrac{1}{2},-\tfrac{7}{10},\tfrac{3}{10},\frac{1}{-u_0},\right)=\tfrac{7}{12}\,k_0^{12/10}\tilde{I}(t)\,. \tag{13}$$

As $a+b-c=-\frac{1}{2}<0$ and $z=-1/u$, the above solution is valid in the entire circle $|z|=1$, i.e as long as $|u|\geq 1$ Once the initial values, x_0, y_0 are chosen, the last equation can determine the value of the difference $\omega=x-y$ ($I(t)$ can be calculated for any given scale μ) Then, x and y (and therefore h_t and h_b) can be found with the use of Eq (8) One finds

$$x=\tfrac{1}{2}\omega(1+\sqrt{1+k_0\omega^{-5/6}}) , \tag{14}$$

$$y=\tfrac{1}{2}\omega(-1+\sqrt{1+k_0\omega^{-5/6}}) \tag{15}$$

In the following, we choose to distinguish two separate cases, depending on the value of k_0 where the results exhibit a rather interesting simplicity

(i) $k_0 \ll 1$ In this case (9) is easily integrated to give

$$x=y+\frac{x_0-y_0}{1-\frac{6}{8\pi^2}(x_0-y_0)I(t)}\equiv y+Q(t) , \tag{16}$$

with $I(t)\equiv\int_{t_0}^{t}\gamma_Q^2(t')\,dt'$ Substitution of (16) into (3) and integration leads to the result

$$h_t^2=\gamma_Q^2h_{t,0}^2\frac{q(t)}{1-7h_{t,0}^2J(t)} , \tag{17}$$

with

$$q(t)=\exp\left[-\frac{1}{8\pi^2}\int_{t_0}^{t}\gamma_Q^2(t')Q(t')\,dt'\right] \tag{18}$$

and

$$J(t)=\frac{1}{8\pi^2}\int_{t_0}^{t}\gamma_Q^2(t')q(t')\,dt' \tag{19}$$

Eq. (7), can also be integrated in the same way resulting to a similar formula for the bottom quark mass

(ii) $k_0 \gg 1$ In this case we can ignore the second term in Eq (9) A straightforward integration then gives

$$x=y+\frac{x_0-y_0}{(1-\frac{7}{8\pi^2}\sqrt{x_0y_0}I(t))^{12/7}}\equiv y+\Omega(t) \tag{20}$$

Returning to the initial differential system, we may obtain the formulae for the top and bottom Yukawa couplings For the top-quark we obtain

$$h_t^2=\gamma_Q^2h_{t,0}^2\frac{p(t)}{1-7h_{t,0}^2L(t)} , \tag{21}$$

while for the bottom quark

$$h_b^2=\gamma_B^2h_{b,0}^2\frac{p(t)^{-1}}{1-7h_{b,0}^2N(t)} , \tag{22}$$

where

$$p(t)=\exp\left[-\frac{1}{8\pi^2}\int_{t_0}^{t}\gamma_Q^2(t')\Omega(t')\,dt'\right], \tag{23}$$

$$L(t)=\frac{1}{8\pi^2}\int_{t_0}^{t}\gamma_Q^2(t')p(t')\,dt' , \tag{24}$$

$$N(t)=\frac{1}{8\pi^2}\int_{t_0}^{t}\gamma_Q^2(t')p(t')^{-1}\,dt' \tag{25}$$

In order to compare our analytic results with numerical methods, we have also solved the RGEs (1), (2) numerically for some particular cases As an example we present here the results obtained for initial value of the top Yukawa coupling close to its fixed point, i e $h_{t,0}=3$ 5 and $h_{b,0}=1$ 5. The numerical solution gives $h_t=1$ 02 and $h_b=0.955$ at the scale $m_t\approx 170$ GeV, while the analytic expressions obtained above give $h_t=1.025$ and $h_b=0$ 957 We have checked that this accuracy holds for all the regions of validity of the above analytic expressions

In Table 1, we present values of the top mass, when both $h_{t,0}$ and $h_{b,0}$ couplings are large. In particular, we choose two characteristic values of the top-Yukawa coupling, (one of them very close to its fixed point) and calculate m_t (running mass) and tan β, assuming a central value for the bottom quark, i e $m_b(m_b)=n_b(v/\sqrt{2})h_b$ cos $\beta=4$ 25 GeV Here n_b includes the running from the scale $\sim m_t$ down to the scale m_b and is taken to be $n_b\approx 1.4$.

A very interesting infer from this table is that the top quark has a mass around 175 GeV which is close to the central value predicted by CDF [13], while tan $\beta\approx(55-58)$. We note however, that sparticle exchange corrections on m_b [9,5] or thresholds and other uncertainties [10] may result to small m_t corrections for specific $h_{t,0}$, $h_{b,0}$ regions

Table 1
m_t and $\tan\beta$ predictions for two initial values of $h_{t,0}$ and various h_{b0} Yukawa couplings, using expressions (21) and (22), while fixing $m_b = n_b(v/\sqrt{2})h_b \cos\beta = 4.25$ GeV, with $n_b \approx 1.4$ being the renormalization group factor from $\mu \sim m_t$ down to $\mu = m_b$

h_{t0}	h_{b0}	m_t	$\tan\beta$
2.5	1.75	175.6	56.25
	2.00	175.0	56.73
	2.25	174.5	57.07
	2.50	174.0	57.32
3.5	1.75	177.3	54.85
	2.00	176.6	56.26
	2.25	176.1	56.65
	2.50	175.7	56.93
	3.00	175.0	57.31

Analytic solutions for the large $h_{b,t}$ Yukawa couplings are of particular interest in supersymmetric theories. The masses of the higgses responsible for the electroweak breaking as well as the scalar masses of the third generation receive large negative contributions when h_b, h_t are also large. In the radiative electroweak symmetry breaking scenario [6], one of the higgs mass-squared parameters should become negative This is in fact possible because of the large negative Yukawa corrections. However, when both Yukawa couplings are large, both higgs mass parameters receive large contributions Thus, when dealing with the RGE of the scalar masses it is very useful to have analytic expressions for the Yukawa functions which appear in the role of scale-dependent coefficients in the differential equations of the former.

In the following, we discuss briefly the effects of h_t, h_b couplings to the Higgs and scalar quark masses of the third generation, disentangling the coupled differential system of them and reducing it down to a simple differential equation of second order which may be solved either numerically or by standard analytic mathematical methods

We start first (as a reminder for reminding the reader) with the exact solution of the scalar masses $m_{\tilde{t}_L} \equiv \tilde{m}_{U_1}$, $m_{\tilde{t}_R} = \tilde{m}_{U_2}$, and $m_{\tilde{H}_2} \equiv \tilde{m}_{U_3}$ in the case where $h_t \gg h_b$, and $\tan\beta \simeq \mathcal{O}(1)$. It has been found in this case that the above scalar masses are given by a simple formula [11]

$$\tilde{m}^2_{U_n} = m_0^2 + C_{U_n}(t) m^2_{1/2} - n\delta^2_{\tilde{m}}(t) - n\delta^2_A(t) , \qquad (26)$$

where $\delta^2_A(t)$ has been estimated to be much smaller than $\delta^2_{\tilde{m}}$ and can be ignored for the present purposes, while

$$\delta^2_m(t) = \left(\frac{m_t(t)}{2\pi v \gamma_Q(t) \sin\beta}\right)^2 (3m_0^2 I(t) + m^2_{1/2} J_0(t)) , \qquad (27)$$

where $I(t)$ has already been defined in solving Eq (16), while

$$J_0(t) = \sum_{n=1}^{3} \int_{t_0}^{t} C_{U_n}(t') \gamma_Q^2(t') \mathrm{d}t' . \qquad (28)$$

The coefficients C_{U_n} can be found in the literature [6,4]. Notice that in the derivation of the above, it was assumed that $h_{t,0} \gg h_{b,0}$ while $h_{b,0}$ was omitted from the equations. However, in the case where $h_{b,0} \sim h_{t,0}$ this is no longer valid Since both Yukawa couplings are $h_{t/b,0} \sim \mathcal{O}(1)$, both higgs mass-squared parameters $m^2_{H_1}$, $m^2_{H_2}$ receive large negative contributions and play a very important role in the stability of the neutral higgs potential This can be easily seen from the minimization conditions $\partial \nu_H / \partial v_i = 0$, where $v_i \equiv \langle H_i \rangle$, which result in well known equations

$$\tfrac{1}{2} M_Z^2 = \frac{\mu_1^2 - \mu_2^2 \tan^2\beta}{\tan^2\beta - 1} , \qquad (29)$$

$$\tfrac{1}{2} \sin 2\beta = -\frac{m_3^2}{\mu_1^2 + \mu_2^2} , \qquad (30)$$

where $\mu_i^2 = m^2_{H_i} + \mu^2 + \sigma_i^2$, with σ_i^2 being the corrections [12] to the Higgs potential from the one-loop contributions and μ the higgs mixing term. Therefore, these contributions should be calculated with great care. Analytic expressions, if possible, may be extremely useful in the minimization conditions of the potential

In the following, we use the same techniques as in Ref. [11] to calculate the Higgs and scalar masses in the case $\tan\beta \gg 1$, or, equivalently, when $h_{b,0}$, $h_{t,0}$ are large The relevant RGEs can be found in the literature [4,6]. We define

$$U = m^2_{U_1} + m^2_{U_2} + m^2_{U_3} , \qquad (31)$$

$$D = m^2_{D_1} + m^2_{D_2} + m^2_{D_3} , \qquad (32)$$

where the m_{U_i} have been defined previously, while $m_{\tilde{b}_L} \equiv \tilde{m}_{D_1}$, $m_{\tilde{b}_R} = \tilde{m}_{D_2}$, and $m_{\tilde{H}_1} \equiv \tilde{m}_{D_3}$. By recalling the same arguments used in the solution of scalar masses for the case $h_{t,0} \gg h_{b,0}$ we can conclude that the contributions of the trilinear parameters A_U, A_D do not play an important role in the final solutions for the scalar masses in the present case too. Therefore, to simplify the subsequent analysis, we drop A_U, A_D terms (the extension of the solution to the most general case is straightforward). Then it is easily observed that one can write the equations for the sums of scalar masses in the following form

$$\frac{dU}{dt} = \frac{1}{8\pi^2}\{6Uh_t^2 + Dh_b^2 - G_U m_{1/2}^2\}, \qquad (33)$$

$$\frac{dD}{dt} = \frac{1}{8\pi^2}\{Uh_t^2 + 6Dh_b^2 - G_D m_{1/2}^2\}, \qquad (34)$$

where $G_U = G_Q + G_{H_2} + G_{U^c}$ and $G_D = G_Q + G_{H_1} + G_{B^c}$. To simplify the above coupled equations we make the following transformations

$$U = \tau x, \quad \tau = \exp\left\{-\frac{3}{4\pi^2}\int_{t_0}^{t} h_t^2\, dt'\right\},$$

$$D = \sigma y, \quad \sigma = \exp\left\{-\frac{3}{4\pi^2}\int_{t_0}^{t} h_b^2\, dt'\right\} \qquad (35)$$

Then Eqs. (33), (34) can be written in the form

$$\tau\frac{dx}{dt} = \frac{1}{6}\frac{d\sigma}{dt}y - \frac{G_U}{8\pi^2}m_{1/2}^2, \qquad (36)$$

$$\sigma\frac{dy}{dt} = \frac{1}{6}\frac{d\tau}{dt}x - \frac{G_D}{8\pi^2}m_{1/2}^2 \qquad (37)$$

It is trivial to check that D E (36) in the case $h_t \gg h_b$ can be solved independently giving the solution (25) for the up-squarks and the higgs H_2 Moreover, using the fact that $G_U \approx G_D \approx 2G_Q$, one can easily obtain an exact solution of Eqs. (36), (37). In the general case, (36), (37) can combine to two simple, second order differential equations of the form

$$\frac{d^2}{dQ^2}y - \frac{\alpha}{\beta}y = -\frac{d}{dQ}\left(\frac{g}{\beta}\right) - \frac{f}{\beta}, \qquad (38)$$

$$\frac{d^2}{dP^2}x - \frac{\beta}{\alpha}x = -\frac{d}{dP}\left(\frac{f}{\alpha}\right) - \frac{g}{\alpha}, \qquad (39)$$

with

$$\alpha = \frac{\sigma}{\tau}\frac{h_b^2}{8\pi^2} \equiv \frac{1}{\tau}\frac{d\sigma}{dt}, \quad dP = \alpha\, dt, \qquad (40)$$

$$\beta = \frac{\tau}{\sigma}\frac{h_t^2}{8\pi^2} = \frac{1}{\sigma}\frac{d\tau}{dt}, \quad dQ = \beta\, dt \qquad (41)$$

and

$$f(t) = \frac{G_U}{8\pi^2\tau}m_{1/2}^2, \quad g(t) = \frac{G_D}{8\pi^2\sigma}m_{1/2}^2 \qquad (42)$$

These are both decoupled and have a standard second order form with a nonhomogeneous part on the RHS Since the coefficients are known functions of the analytic solutions $h_b(t)$, $h_t(t)$ they can be solved either numerically or in particular cases by standard methods yielding expressions for the sums of m_{U_i}, m_{D_i} Then the expressions for the individual masses may be obtained in the way described in Ref [11] Detailed results of the above procedure will be presented elsewhere

We have been informed that C. Kounnas and F Zwirner have obtained similar expressions for $h_{t/b}$ couplings We would like to thank them for discussions and communicating to us their results and M Carena and C. Wagner for discussions on their work. G K.L would like to acknowledge a useful discussion with I. Antoniadis

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