# LEPTON FLAVOUR VIOLATION IN SUPERGRAVITY THEORIES 

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Received 8 December 1988


#### Abstract

The lepton flavour violating processes $\mu \rightarrow \mathrm{e} \gamma, \mu \rightarrow \mathrm{ee}^{+} \mathrm{e}^{-}$and ( $\mu, \mathrm{e}$ ) conversion in the presence of nuclei are discussed in supergravity models. The dominant contribution arises from charged s-lepton mixing, if one goes beyond the tree level and includes renormalization effects. The branching ratio for ( $\mu, \mathrm{e}$ ) conversion is obtained using reliable nuclear form factors. The resulting branching ratios are enhanced compared to those of more traditional models but they are still much smaller than the present experimental limits.


One of the inevitable consequences of all extensions of the phenomenologically successful standard model [1] is a plethora of new particles and the breakdown of various conserved quantum numbers ${ }^{\# 1}$ such as baryons, lepton flavour, leptons, etc. Among those, lepton flavour appears to be the least sacred since it is the analog of strangeness for the leptonic sector.
In the present paper we will examine in some detail lepton flavour violating processes [3-5] such as $\mu \rightarrow \mathrm{e} \gamma$, $\mu \rightarrow \mathrm{ee}^{+} \mathrm{e}^{-},(\mu, \mathrm{e})$ conversion, etc., in currently fashionable extensions of the standard model. In particular we will consider supergravity models [6] which may arise [7-9] as low energy approximations of superstring models.

Even though, as we have mentioned above, lepton flavour is not viewed as sacred, it is now understood that lepton flavour violating processes are suppressed due to the GIM mechanism. A non-vanishing contribution arises only when the intermediate particles involved are not degenerate in mass. Quite generally for light intermediate particles, e.g. neutrinos, the amplitude is proportional to
$\Delta m^{2} / M_{\mathrm{w}}^{4}$ with $\Delta m^{2}=m_{i}^{2}-m_{j}^{2}$
multiplied with suitable mixing angles. In the presence of heavy intermediate particles (e.g. neutrinos) the corresponding quantity is
$\Delta M^{2} / M^{4}, \quad \Delta M^{2}=M_{i}^{2}-M_{j}^{2}$,
where $M\left(M \gg M_{\mathrm{w}}\right)$ is the intermediate mass scale. The dependence on the mass becomes more favorable in the presence of right-handed currents if one considers the current combination $j_{\mathrm{L}}-j_{\mathrm{R}}$. In any case lepton flavour is unobservable ${ }^{\# 1}$ if $\Delta m^{2} \leqslant 10 \mathrm{eV}^{2}$ or $\Delta M^{2} \leqslant 10 \mathrm{GeV}^{2}, M \sim 10^{10} \mathrm{GeV}$. It has also been shown that the GIM mechanism is not effective if the light components of the neutrino wave function are not orthogonal [10]. It has not, however, been demonstrated that in realistic models this non-orthogonality is sizeable.
In this paper we will adopt the view that lepton flavour is more likely to be observed not via intermediate neutrinos but via charged intermediate particles with intermediate mass ( $M \sim M_{\mathrm{w}}$ ) which are non-degenerate ( $\Delta M^{2} \sim 1 \mathrm{GeV}^{2}$ ). Fashionable supergravity models naturally offer such a possibility. It has in fact been shown that flavour violation can be induced by intermediate charged s-leptons provided that one goes beyond the tree

[^0]level and includes radiative corrections to the charged s-lepton mass matrix [11,12]. At the tree level the structure of the lepton and s-lepton matrices is the same and the corresponding current is diagonal a la neutral currents [13].

In a minimal supersymmetric extension of the standard model one can write down the following Yukawa couplings [12]:
$W_{0}=\lambda_{\mathrm{u}} \mathrm{Q} \tilde{H} \mathrm{U}^{\mathrm{c}}+\lambda_{\mathrm{d}} \mathrm{QHD}^{\mathrm{c}}+\lambda_{\mathrm{e}} \mathrm{LHE}^{\mathrm{c}}$,
where

$$
Q=\left(\begin{array}{c}
u_{L} \tilde{u}  \tag{2}\\
; \\
d_{L}
\end{array}\right), \quad L=\left(\begin{array}{c}
v_{\mathrm{d} L} \\
; \\
\tilde{v}_{e} \\
e_{L} \\
\tilde{e}
\end{array}\right), \quad D^{c}=\left(d_{L}^{c} ; \tilde{d}\right), \quad U^{c}=\left(u_{L}^{c} ; \tilde{u}^{c}\right), \quad E^{c}=\left(e_{L}^{c} ; \tilde{e}^{c}\right)
$$

Then the s-lepton $6 \times 6$ matrix in the basis e , $\mathrm{e}^{*}$ takes the form

$$
\left(\begin{array}{ll}
m_{\mathrm{e}}^{+} m_{\mathrm{e}}+m_{3 / 2}^{2} 1 & A^{*} m_{\mathrm{e}}^{+} m_{3 / 2}  \tag{3}\\
A m_{\mathrm{e}} m_{3 / 2} & m_{\mathrm{e}}^{+} m_{\mathrm{e}}+m_{3 / 2}^{2} 1
\end{array}\right),
$$

where $m_{\mathrm{e}}$ is the usual lepton mass matrix, $m_{3 / 2}$ the gravitino mass and $A$ a parameter of order unity. In this approximation the lepton and s-lepton mixing mass matrices are similar which, in turn, implies that $S_{\mathrm{e}}^{+} S_{\tilde{\mathrm{e}}} \approx 1$. This means that there is no lepton flavour violation induced.

One can also show that in this model there is no contribution from neutral intermediate particles since at this level either they remain massless (neutrinos) or degenerate [13] (s-neutrinos).

The inclusion of the isosinglet right-handed neutrino $N^{c}=\left(N_{L}^{c} ; \widetilde{N}_{c}\right)$ can lead to additional terms in the superpotential of the form

$$
\begin{equation*}
W_{1}=\lambda_{\mathrm{N}} \mathrm{LHN}^{\mathrm{c}}+\frac{1}{2} M_{\mathrm{N}} \mathrm{~N}^{\mathrm{c}} \mathrm{~N}^{\mathrm{c}} . \tag{4}
\end{equation*}
$$

Now the neutrino $6 \times 6$ mass matrix can have both Dirac and isosinglet Majorana mass terms. The corresponding neutral $12 \times 12$ s-lepton mass matrix becomes analogous to that of the neutrinos. Thus even though $S_{\mathrm{e}}^{+} S_{\mathrm{v}}$ is non-diagonal, lepton flavour violating processes are suppressed due to the fact that the s-neutrinos are massive but degenerate [13].

The above results are modified if one goes beyond the tree level and includes radiative corrections arising from the first term of the superpotential (4) taking into account renormalization effects. Then the isodoublet charged s-lepton mass matrix gets corrections of the type $c m_{\mathrm{D}}^{+} m_{\mathrm{D}}$ while the isosinglet ( $\mathrm{e}^{*}$ ) gets no such corrections [12]. Thus ignoring e, $\mathrm{e}^{*}$ mixing we find that the s-lepton mixing $3 \times 3$ matrix becomes
$m^{+} m=m_{3 / 2}^{2} \mathbf{1}+m_{\mathrm{e}}^{+} m_{\mathrm{e}}+c m_{\mathrm{D}}^{+} m_{\mathrm{D}}$.
The constant $c$ is smaller than unity. Therefore the above correction is significant only if the neutrinos happen to be Majorana particles. Then the restriction which comes from the experimental bounds on the neutrino masses constrains only the quantity $m_{\mathrm{D}}^{+} m_{\mathrm{N}}^{-1} m_{\mathrm{D}}$ and not $m_{\mathrm{D}}$ itself. In fact in specific models based on grand unified theories (GUT's) $m_{\mathrm{D}}$ is quite large (of the order of the up-quark mass). We remind the reader that this, among other things, necessitated the invention of the see-saw mechanism and the introduction of the Majorana mass $m_{\mathrm{N}}$.

In our detailed study we will utilize eq. (5) in a specific GUT SO(10) model. (The analysis can be carried out in other models, e.g. the flipped [9] SU(5), in an analogous fashion but the form of the mass matrices is less understood). In this model the Dirac mass matrix takes the form
$m_{\mathrm{D}}=\left(\begin{array}{ccc}0 & P & 0 \\ P & 0 & -3 Q \\ 0 & -3 Q & V\end{array}\right)$,
with $P=0.125, Q=8.13$ and $V=43.5 \mathrm{GeV}$ resulting from a fit to the masses $m_{\mathrm{u}}=10 \mathrm{MeV}, m_{\mathrm{c}}=1.4 \mathrm{GeV}, m_{\mathrm{t}}=45$ GeV . This leads to
$m_{i}^{2}=m_{3 / 2}^{2}+c \mu_{i}^{2}, \quad i=1,2,3, \quad \mu_{1}=0, \quad \mu_{2}^{2}=120 \mathrm{GeV}^{2}, \quad \mu_{3}^{2}=3000 \mathrm{GeV}^{2}$.
Furthermore, the corresponding mixing matrix is
$S_{\tilde{\mathrm{e}}}^{(\mathrm{L})}=\left(\begin{array}{ccl}1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta\end{array}\right), \quad \frac{1}{2} \tan 2 \theta=3 Q / V=0.5606$.
This must be compared with the lepton mixing matrix [2]
$S_{\mathrm{e}}=\left(\begin{array}{cll}\cos \beta & \sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ 0 & 0 & 0\end{array}\right), \quad \frac{1}{2} \tan 2 \beta=\sqrt{m_{\mathrm{e}} / m_{\mu}}=0.070$,
Thus the relevant mixing matrix entering the leptonic left-handed current becomes
$U_{\mathrm{e}}^{(\mathrm{L})}=S_{\mathrm{e}}^{+} S_{\widetilde{\mathrm{e}}}^{(\mathrm{L})}=\left(\begin{array}{ccc}\cos \beta & -\sin \beta \cos \theta & -\sin \beta \sin \theta \\ \sin \beta & \cos \beta \cos \theta & \cos \beta \sin \theta \\ 0 & -\sin \theta & \cos \theta\end{array}\right)$.
Thus to leading order in $\delta_{j}^{2}=m_{j}^{2}-m_{3 / 2}^{2}$ we find that the amplitude for the flavour violating processes of figs. 13 is proportional to the lepton violating parameter
$\tilde{\eta}=-\sum_{j=1}^{3} U_{\mathrm{e} j} U_{\mu j}^{*} \frac{\delta_{j}^{2}}{m_{3 / 2}^{2}}=c \sin \beta \cos \beta \frac{\mu_{2}^{2}-\mu_{1}^{2}+\sin ^{2} \theta\left(\mu_{3}^{2}-\mu_{2}^{2}\right)}{m_{3 / 2}^{2}}$.
The quantity $c$ can be determined by solving the renormalization group equations in the spirit of ref. [12].
We are now going to examine in some detail the lepton violating ( $\mu, \mathrm{e}$ ) conversion and compare it with $\mu \rightarrow \mathrm{e} \gamma$ and $\mu \rightarrow \mathrm{ee}^{+} \mathrm{e}^{-}$(see figs. 1-3). In all cases the relevant intermediate fermion is the photino $\tilde{\gamma}$ which is the supersymmetric partner of the photon. In ( $\mu, \mathrm{e}$ ) conversion in addition to the charged s-lepton one needs intermediate s-quarks (see fig. 3).

Let us begin with the simplest process $\mu \rightarrow \mathrm{e} \gamma$ (see fig. 1). The amplitude for the process can be cast in the form $m=j^{\lambda} \varepsilon_{\lambda}$,
where $\varepsilon_{\lambda}$ is the photon polarization vector and $j_{\lambda}$ is given by
$j^{\lambda}=(1 / \sqrt{4 \pi \alpha}) \mathrm{u}\left(p_{1}\right)\left[\left(f_{\mathrm{MI}}+\gamma_{5} f_{\mathrm{E} 1}\right) \mathrm{i} \sigma^{\lambda \nu}\left(q_{\nu} / m_{\mu}\right)\right] \mathrm{u}\left(p_{\mu}\right)$,
where $p_{1}$ is the electron momentum and $\alpha$ the fine structure constant. We find
$f_{\mathrm{M} 1}=-f_{\mathrm{E} 1}=-\frac{1}{2} \tilde{\eta} \alpha^{2}\left(m_{\mu}^{2} / m_{3 / 2}^{2}\right) f(x)$,
where $m_{3 / 2}$ is the gravitino mass: $f(x)$ depends on the photino mass and is given by


Fig. 1. The dominant diagram leading to $\mu \rightarrow \mathrm{e} \gamma$ in the supersymmetric extension of the standard model. The effective current is left-handed (see text).


Fig. 2. The two dominant diagrams leading to $\mu \rightarrow e e^{+} e^{-}$. Antisymmetrization with respect to the outgoing identical leptons is understood.


Fig. 3. The two dominant diagrams leading to ( $\mu, \mathrm{e}$ ) conversion in the presence of nuclei.
$f(x)=\left[1 / 12(1-x)^{4}\right]\left(1+2 x^{3}+3 x^{2}-6 x-6 x^{2} \ln x\right), \quad x=m_{\tilde{\gamma}}^{2} / m_{3 / 2}^{2}$.
With the above ingredients the branching ratio takes the form
$R_{e \gamma}=(6 \pi / \alpha)\left(\left|f_{\mathrm{E} 1}\right|^{2}+\left|f_{\mathrm{M} 1}\right|^{2}\right) /\left(G_{\mathrm{F}} m_{\mu}^{2}\right)^{2}=|\tilde{\eta}|^{2} R_{0}$,
where $\tilde{\eta}$ is given by eq. (10) and $R_{0}$ is given by
$R_{0}=3 \pi \alpha^{3}|f|^{2} /\left(G_{\mathrm{F}} m_{3 / 2}^{2}\right)^{2}$.
The amplitude for ( $\mu, \mathrm{e}$ ) conversion can also be written as follows:
$m=\left[j_{(1)}^{\lambda} J_{\lambda}^{(1)} / q^{2}+\left(j_{(2)}^{\lambda} J_{\lambda}^{(2)} / m_{\mu}^{2}\right)\left(m_{3 / 2} / m_{u}\right)^{2}\right]$.
where the first term corresponds to the photonic (fig. 3a) and the second term to the non-photonic term (box diagram (3b)). One finds that
$j_{(1)}^{\lambda}=\overline{\mathrm{u}}\left(p_{1}\right)\left(f_{\mathrm{M} 1}+\gamma_{\mathrm{s}} \mathrm{f}_{\mathrm{E} 1}\right) \mathrm{i} \sigma^{\lambda \nu}\left(q_{\nu} / m_{\mu}\right)+\left(q^{2} / m_{\mu}^{2}\right)\left(F_{\mathrm{E} 0}+\gamma_{5} f_{\mathrm{M} 0}\right) \gamma^{\nu}\left(g_{\lambda \nu}-q^{\lambda} q^{\nu} / q^{2}\right) \mathrm{u}\left(p_{\mu}\right)$,
$J_{\lambda}^{(1)}=\overline{\mathrm{N}} \gamma_{\lambda} \frac{1}{2}\left(1+\tau_{3}\right) \mathrm{N} \quad \mathrm{N}=$ Nucleon,
$j_{(2)}^{\lambda}=\overline{\mathrm{u}}\left(p_{1}\right) \frac{1}{2}\left(f_{\mathrm{V}}+f_{\mathrm{A}} \gamma_{\mathrm{s}}\right) \gamma^{\lambda} \mathrm{u}\left(p_{\mu}\right)$,
$J_{\lambda}^{(2)}=\overline{\mathrm{N}}\left(f_{\mathrm{V}}+f_{\mathrm{A}} \gamma_{\mathrm{s}}\right) \frac{1}{2}\left(3 \beta_{0}+\beta_{1} \tau_{3}\right) \mathrm{N}$
with
$\beta_{0}=\frac{4}{9}+\frac{1}{9}\left(m_{\mathrm{u}} / m_{\mathfrak{d}}\right)^{2}, \quad \beta_{1}=\frac{4}{9}-\frac{1}{9}\left(m_{\tilde{\mathrm{u}}} / m_{\mathfrak{d}}\right)^{2}$.
We notice the difference in isospin dependence between $J_{\lambda}^{(1)}$ and $J_{\lambda}^{(2)}$. The form factors $f_{\mathrm{M} 1}$ and $f_{\mathrm{E} 1}$ are given by eq. (13) while
$f_{\mathrm{E} 0}=-f_{\mathrm{M} 0}=-\frac{1}{2} \tilde{\eta} \alpha^{2} g(x)\left(m_{\mu}^{2} / m_{3 / 2}^{2}\right), \quad f_{\mathrm{V}}=-f_{\mathrm{A}}=\frac{1}{2} \tilde{\eta} \alpha^{2} f_{\mathrm{b}}(x)\left(m_{\mu}^{2} / m_{3 / 2}^{2}\right)$,
with
$g(x)=\left[1 / 36(1-x)^{4}\right]\left(2-11 x^{3}+18 x^{2}-9 x+6 x^{2} \ln x\right)$,
$f_{\mathrm{b}}(x)=\left[1 / 8(1-x)^{4}\right]\left[1-5 x^{2}-4 x+2 x(x+2) \ln x\right], \quad x=\left(m_{\tilde{\gamma}}^{2} / m_{3 / 2}^{2}\right)$.
In bringing the box diagram contribution to the above form a Fierz transformation was necessary.
The dependence of ( $\mu, \mathrm{e}$ ) conversion on the gross nuclear properties $A, Z$ comes from the hadronic currents $J_{\lambda}^{(1)}$ and $J_{\lambda}^{(2)}$. We notice that since $\beta_{1} / 3 \beta_{0} \neq 1$ the box (non-photonic) and loop (photonic) terms have different $A$ and $Z$ dependence [14]. For light nuclei ( $\leqslant 60$ ) it has recently been shown [15] that most of the strength ( $\geqslant 65 \%$ ) goes to the ground state (this is not true for heavy nuclei). For ground state transitions one finds that the branching ratio is given by
$R_{\mathrm{cN}}=\left[1 /\left(G_{\mathrm{F}} m_{\mu}^{2}\right)^{2}\right]\left[\left|\left(m_{\mu}^{2} / q^{2}\right) f_{\mathrm{M} 1}+f_{\mathrm{EO}}+\frac{1}{2} \kappa f_{\mathrm{V}}\right|^{2}+\left|\left(m_{\mu}^{2} / q^{2}\right) f_{\mathrm{EI}}+f_{\mathrm{MO}}+\frac{1}{2} \kappa f_{\mathrm{A}}\right|^{2}\right] \gamma_{\mathrm{ph}}$,
$\gamma_{\mathrm{ph}}=Z\left|F_{\mathrm{Z}}\left(q^{2}\right)\right|^{2} / 6 f(A, Z)$,
$\kappa=\left\{1+\left[\left(3 \beta_{0}-\beta_{1}\right) /\left(3 \beta_{0}+\beta_{1}\right)\right](N / Z) F_{\mathrm{N}}\left(q^{2}\right) / F_{\mathrm{Z}}\left(q^{2}\right)\right\}\left(m_{3 / 2} / m_{\overline{\mathrm{u}}}\right)^{2}$.
In eq. (20) $f(A, Z)$ is the Primakoff function [16] describing the total muon capture and $F_{\mathrm{Z}}\left(q^{2}\right)$ and $F_{\mathrm{N}}\left(q^{2}\right)$ are the nuclear form factors associated with protons and neutrons, respectively. The form factors $F_{\mathrm{Z}}\left(q^{2}\right)$ can be extracted from electron scattering data or calculated in a given nuclear model [14] (e.g. shell model). The form factors $F_{\mathrm{N}}\left(q^{2}\right)$ can be calculated [14] in an analogous fashion. By noting that $q^{2}=-m_{\mu}^{2}$ and assuming $m_{\tilde{u}} \approx$ $m_{\tilde{c}} \approx m_{\tilde{d}}=m_{3 / 2}$ we get
$R_{\mathrm{eN}}=\frac{1}{2}\left[|\tilde{\eta}|^{2} \alpha^{4} /\left(G_{\mathrm{F}} m_{3 / 2}^{2}\right)^{2}\right]\left(f-g+\frac{1}{2} f_{\mathrm{b}} \kappa\right)^{2} \gamma_{\mathrm{ph}}$.
Assuming $m_{\gamma} \approx 5 \mathrm{GeV}$ and $m_{3 / 2} \approx 150 \mathrm{GeV}$, i.e. $x \ll 1$, we get
$f=1 / 12, \quad g=1 / 18, \quad f_{\mathrm{b}}=1 / 8$,
Thus the two terms of the photonic contribution tend to cancel and the non-photonic contribution becomes dominant. The quantities $F_{\mathrm{Z}}\left(q^{2}\right), F_{\mathrm{N}}\left(q^{2}\right)$ and $\kappa$ are very smooth functions of $A, Z$ and they can be computed very reliably [14]. They are presented in table 1 as functions of $A$ and $Z$. The quantity $\gamma_{\mathrm{ph}}$ shows fluctuations reflecting basically the variations of the total muon capture rate as a function of $A, Z$ but this is also under control (determined semi-empirically by Primakoff's method [16]).
The amplitude for $\mu \rightarrow \mathrm{ee}^{+} \mathrm{e}^{-}$can be cast in a form analogous to that of eq. (16) without of course the factor ( $\left.m_{3 / 2} / m_{\hat{u}}\right)^{2}$ in the second term. The (now leptonic) currents $J_{\lambda}^{(1)}$ and $J_{\lambda}^{(2)}$ are given by
$J_{\lambda}^{(1)}=\tilde{\mathbf{u}}\left(p_{2}\right) \gamma_{\lambda} \mathbf{u}\left(p_{\mathrm{e}}\right), \quad J\left({ }_{\lambda}^{2}\right)=\mathbf{u}\left(p_{2}\right) \gamma^{\lambda \frac{1}{2}}\left(1-\gamma_{\mathrm{s}}\right) \mathbf{u}\left(p_{\mathrm{e}}\right)$,

Table 1
The nuclear form factors $F_{\mathrm{Z}}$ and $F_{\mathrm{N}}$ entering in the quasielastic ( $\mu, \mathrm{e}$ ) conversion. The quantities $\gamma_{\mathrm{ph}}$ and $\kappa$ are defined in the text (see eqs. (20) and (21)).

|  | $(A, Z)$ | $F_{\mathrm{z}}$ | $F_{\mathrm{N}}$ | $\kappa$ |
| ---: | :--- | :--- | :--- | ---: |
| $(4,2)$ | 0.865 | 0.865 | 1.67 | 1.56 |
| $(12,6)$ | 0.763 | 0.763 | 1.67 | 3.64 |
| $(14,6)$ | 0.753 | 0.745 | 1.88 | 7.96 |
| $(16,8)$ | 0.736 | 0.736 | 1.67 | 4.52 |
| $(28,14)$ | 0.639 | 0.639 | 1.67 | 5.95 |
| $(32,16)$ | 0.618 | 0.618 | 1.67 | 6.37 |
| $(40,20)$ | 0.582 | 0.582 | 1.67 | 7.05 |
| $(48,20)$ | 0.563 | 0.515 | 1.85 | 16.08 |
| $(60,28)$ | 0.489 | 0.478 | 1.74 | 9.24 |
| $(72,32)$ | 0.456 | 0.435 | 1.79 | 11.54 |
| $(82,32)$ | 0.440 | 0.379 | 1.89 | 24.98 |
| $(88,38)$ | 0.412 | 0.370 | 1.79 | 12.98 |
| $(90,40)$ | 0.406 | 0.367 | 1.76 | 11.41 |
| $(114,50)$ | 0.335 | 0.306 | 1.77 | 10.35 |
| $(132,50)$ | 0.315 | 0.250 | 1.86 | 25.80 |
| $(156,64)$ | 0.263 | 0.207 | 1.76 | 11.96 |
| $(162,70)$ | 0.253 | 0.202 | 1.70 | 8.92 |
| $(168,68)$ | 0.249 | 0.191 | 1.76 | 12.47 |
| $(176,70)$ | 0.242 | 0.181 | 1.76 | 13.75 |
| $(194,82)$ | 0.198 | 0.168 | 1.77 | 7.20 |
| $(208,82)$ | 0.189 | 0.135 | 1.73 | 10.42 |

where $p_{\mathrm{e}}$ and $p_{2}$ are the momenta of the positron and electron produced in the photonic vertex. The amplitude must, of course, be antisymmetrized with respect to the two produced electrons $e_{1}$ and $e_{2}$. One can show that the branching ratio takes the form [17]
$R_{3 \mathrm{e}}=\left[|\tilde{\eta}|^{2} \alpha^{4} /\left|G_{\mathrm{F}} m_{3 / 2}^{2}\right|^{2}\right]^{\frac{1}{2}}\left\{\left[16 \ln \left(m_{\mu} / m_{\mathrm{e}}\right)-\frac{26}{3}\right] f^{2}-12 f g+3 g^{2}+2 f_{\mathrm{b}}^{2}+g f_{\mathrm{b}}-8 f f_{\mathrm{b}}\right\}$.
From eqs. (15), (22) and (25) we see that the individual branching ratios depend very sensitively on the not precisely known parameter $m_{3 / 2}$. Further uncertainties arise from the lack of information regarding the parameter $\tilde{\eta}$. The ratios of the above branching ratios, are of course, independent of these uncertain parameters $\tilde{\eta}$ and $m_{3 / 2}$. Using eqs. (23) we get
$R_{\mathrm{eN}} / R_{\mathrm{e} \gamma}=(\alpha / 6 \pi)\left(\frac{1}{3}+\frac{3}{4} \kappa\right)^{2} \gamma_{\mathrm{ph}} \approx 1.0 \times 10^{-2}$.
The numerical value was obtained using $\kappa \approx 1.7$ and $\gamma_{\mathrm{ph}}=9.3$ (see table 1). Thus $\mu \rightarrow \mathrm{e} \gamma$ is favored from the point of view of the branching ratio. The coherent effect of all nucleons is not enough to compensate for the extra power of $\alpha$ involved in ( $\mu, \mathrm{e}$ ) conversion. Similarly,
$R_{3 \mathrm{e}} / R_{\mathrm{e} \mathrm{\gamma}}=(\alpha / 24 \pi)\left[16 \ln \left(m_{\mu} / m_{\mathrm{e}}\right)-\frac{26}{3}-\frac{61}{6}\right] \approx 6.4 \times 10^{-3}$.
Again $\mu \rightarrow \mathrm{e} \gamma$ is favored. $\mu \rightarrow 3 \mathrm{e}$ and ( $\mu, \mathrm{e}$ ) conversion have, of course, certain experimental advantages in handling problems associated with the background [4,5].

As we have already mentioned, the individual branching ratios are pretty uncertain. They can only be estimated by making reasonable assumptions about $m_{3 / 2}$ and $\tilde{\eta}$. We will use the value of $c=2.4 \times 10^{-2}$ obtained from the solution of the RGE [12]. Furthermore we will employ the masses and mixing angles obtained in the SO (10) model discussed above (see eqs. (7b), (7c) and (8)). Using $m_{3 / 2}=150 \mathrm{GeV}$ we get
$\tilde{\eta}=4.4 \times 10^{-5}, \quad R_{\mathrm{cN}}=6.6 \times 10^{-18}$,
which is much below the present experimental limit [5] $R_{\mathrm{eN}}<4.6 \times 10^{-12}$.
Similarly one finds
$R_{\text {er }}=6.9 \times 10^{-16}, \quad R_{3 \mathrm{e}}=4.5 \times 10^{-18}$,
which are also much smaller than the corresponding experimental limits

$$
R_{\mathrm{ey}}(\exp .)<4.9 \times 10^{-11}, \quad R_{3 \mathrm{e}}(\exp .)<1.0 \times 10^{-12},
$$

from ref. [3] and ref. [4], respectively.
Using $m_{3 / 2}=100 \mathrm{GeV}$ we obtain

$$
R_{\mathrm{eN}}=6.1 \times 10^{-18}, \quad R_{\mathrm{ey}}=3.5 \times 10^{-15}, \quad R_{3 \mathrm{e}}=2.3 \times 10^{-17} .
$$

In conclusion, we can say that in supergravity models if one goes beyond the tree level in constructing the mass matrices one can have an enhancement of the lepton flavour violating processes $\mu \rightarrow \mathrm{e} \gamma, \mu \rightarrow \mathrm{ee}^{+} \mathrm{e}^{-}$and ( $\mu$, e) conversion as compared to that of the neutrino-mediated processes. The resulting branching ratios are, however, much smaller than the present experimental limits.

One of us (J.D.V.) would like to thank the Greek CERN committee for support and the Theory Division of CERN for their hospitality.

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