LIGHT NEUTRINOS WITH LARGE MAGNETIC MOMENTS AND THE SOLAR NEUTRINO PROBLEM

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We analyze the question of having a large enough neutrino magnetic moment in order to implement the VVO mechanism in the framework of an $SU(4) \times SU(2)_L \times SU(2)_R$ model, derivable from four-dimensional superstrings. We find that the radiative corrections to the seesaw type neutrino mass matrix can generate an appreciable magnetic moment without any seesaw type suppression.

A couple of years ago, the long-standing solar neutrino puzzle [1-3] has motivated the suggestion of Voloshin, Vysotsky and Okun [4] that the deficit of solar neutrinos is a consequence of a magnetic moment type of interaction of the neutrinos with the magnetic field of the outer layers of the sun. A v_e magnetic moment in the range

$$\mu_{\rm ve} = (10^{-11} - 10^{-10}) \mu_{\rm B}$$

 $(\mu_{\rm B} = e/2m_{\rm e}$ is the Bohr magneton) is needed in order to achieve the required conversion into inert righthanded or higher-family neutrinos which are undetectable. Presently the most stringent upper bound on $\mu_{\rm ve}$ is $0.7 \times 10^{-10} \mu_{\rm B}$ [5] and the explanation of the solar neutrino deficit via the VVO mechanism is an allowed possibility.

The standard electroweak model with an additional Higgs isodoublet that gives a Dirac mass to v_e through the coupling to a gauge singlet right-handed neutrino gives too small a value for μ_{v_e} (=10⁻¹⁸ μ_B) [6] and people have looked into extensions of the standard model that can predict a μ_{v_e} in the required value range, introducing exotic particles [7]. An important constraint that is present in all models of neutrino magnetic moment is the constraint imposed by the upper limit on the v_e mass [8]. Large neutrino magnetic moments have to coexist with small neutrino masses. A recent examination of this question in the framework of the standard seesaw model of neutrino masses has given a negative answer, i.e., too small a μ_{v_e} [9].

In the present article we reexamine the question of the neutrino magnetic moment in relation to the neutrino mass in the framework of a $SU(4) \times SU(2)_L \times$ $SU(2)_R$ model with N=1 supersymmetry which is derivable from four-dimensional fermionic superstrings. The model possesses a variety of attractive features such as naturally light doublets and fermion mass relations which have been analyzed elsewhere [10]. Our main motivation is provided by the promising fact that the neutrino mass in this model is in part provided by the coupling to gauge singlet fields that have no relation to the neutrino magnetic moment, thus allowing a degree of independence among these two parameters. In what follows, first, we briefly present the content of the model and the superpotential and, then, proceed to derive the neutrino mass

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matrix and compute the relevant diagrams that contribute to the overall neutrino magnetic moment.

The chiral superfield content of the model is the following:

$$\begin{split} F(4,2,1) &= Q(3,2,\frac{1}{6}) + \ell(1,2,\frac{1}{2}), \\ F^{c}(4,1,2) &= u^{c}(\bar{3},1,-\frac{2}{3}) + d^{c}(\bar{3},1,\frac{1}{3}) \\ &+ e^{c}(1,1,1) + N^{c}(1,1,0), \\ H(4,1,2) &= \overline{u_{H}^{c}}(3,1,\frac{2}{3}) + \overline{d_{H}^{c}}(3,1,-\frac{1}{3}) \\ &+ \overline{e_{H}^{c}}(1,1,-1) + \overline{N_{H}^{c}}(1,1,0), \\ \bar{H}(\bar{4},1,2) &= u_{H}^{c}(\bar{3},1,-\frac{2}{3}) + d_{H}^{c}(\bar{3},1,\frac{1}{3}) \\ &+ e_{H}^{c}(1,1,1) + N_{H}^{c}(1,1,0), \\ h(1,2,2) &= h(1,2,-\frac{1}{2}) + h^{c}(1,2,\frac{1}{2}), \\ D(6,1,1) &= D(3,1,-\frac{1}{3}) + D^{c}(3,1,\frac{1}{3}), \\ \Phi(1,1,0), \end{split}$$

in terms of SU(4)×SU(2)_L×SU(2)_R and SU(3)_c ×SU(2)_L×U(1)_Y representations. The matter representations F+F^c make up just the 16-representation of SO(10), while h+D is the 10-representation. In contrast, the GUT-Higgses H+H are only parts of the 16+16. The superpotential of the model will involve all existing parts of the 16 × 16 × (10)_H + 16 × (16)_H × 1 + (16)_H × (16)_H × (10)_H + (16)_H × (16)_H × (10)_H + 1 × 1 × 1 couplings.

An additional Z_2 discrete symmetry $\overline{H} \leftrightarrow -\overline{H}$ rids the model of the unwanted piece of the $16 \times \overline{16}_H \times 1$ coupling [10]. Under the given representations and the imposed discrete symmetry this is the most general cubic superpotential. In terms of the fields it reads as follows:

$$W = \lambda_i^{\prime} \mathbf{F}_i \mathbf{F}_j^{c} \mathbf{h} + \lambda_2^{\prime j} \mathbf{F}_i^{c} \mathbf{H} \Phi_j + \lambda_3 \mathbf{H} \mathbf{H} \mathbf{D} + \lambda_4 \mathbf{\tilde{H}} \mathbf{\tilde{H}} \mathbf{D} + \lambda_5^{\prime} \mathbf{h} \mathbf{h} \Phi_i + \lambda_2^{\prime \prime \prime \prime} \mathbf{\Phi}_i \Phi_j \Phi_k + \lambda_2^{\prime \prime} \mathbf{F}_i \mathbf{F}_j \mathbf{D} + \lambda_8^{\prime \prime \prime} \mathbf{F}_i^{c} \mathbf{F}_j^{c} \mathbf{D} + \lambda_9^{\prime \prime} \mathbf{D} \mathbf{D} \Phi_i .$$
(1)

The indices i, j=1, 2, 3 are generation indices. The cubic term for the three singlets ensures that they get no vacuum expectation value larger than the supersymmetry breaking. The introduction of an extra singlet Φ_0 coupled through a term $\lambda_6^{ij0}\Phi_i\Phi_j\Phi_0$ provides a mass for the Φ_i 's from the vacuum expectation value

of Φ_0 . In order to guaranty a VEV for Φ_0 the cubic Φ_0^3 term should be absent. This is only technically natural unless we complicate the imposed discrete symmetry ^{#1}.

A simpler possibility, which we adopt, is the introduction of a direct quadratic mass term for the Φ_i 's, $\mu_{ij}\Phi_i\Phi_j$. In what follows we shall assume that there is a mass μ_{ij} for the singlets Φ_i .

The symmetry breaking to $SU(3)_c \times SU(2)_L \times U(1)_Y$ occurs when the neutral scalars in H and \overline{H} acquire non-zero vacuum expectation values

$$\langle \mathbf{H} \rangle = \langle \overline{\mathbf{N}_{\mathbf{H}}^{c}} \rangle = M, \quad \langle \overline{\mathbf{H}} \rangle = \langle \mathbf{N}_{\mathbf{H}}^{c} \rangle = \overline{M}.$$
 (2)

The coloured triplets in \overline{H} and H combine with the coloured triplets in D through the couplings

λ_3 HHD+ λ_4 HHD

$$\rightarrow \lambda_3 \langle \overline{\mathrm{N}_{\mathrm{H}}^{\mathrm{c}}} \rangle \overline{\mathrm{d}_{\mathrm{H}}^{\mathrm{c}}} \mathrm{D}^{\mathrm{c}} + \lambda_4 \langle \mathrm{N}_{\mathrm{H}}^{\mathrm{c}} \rangle \mathrm{d}_{\mathrm{H}}^{\mathrm{c}} \mathrm{D} .$$
 (3)

The neutrino mass matrix turns out to be

$$\begin{pmatrix} 0 & m_{\rm u} & 0 \\ m_{\rm u} & 0 & \mathcal{M} \\ 0 & \mathcal{M} & \mu \end{pmatrix},$$
 (4)

where the entries arise from the couplings

$$\begin{aligned} \lambda_{i}^{ij} \langle \mathbf{h}_{0} \rangle \left(u_{i} u_{j}^{c} + v_{i} \mathbf{N}_{j}^{c} \right) &= m_{u}^{ij} v_{i} \mathbf{N}_{j}^{c} + \dots ,\\ \lambda_{j}^{ij} \langle \overline{\mathbf{N}_{H}^{c}} \rangle \mathbf{N}_{i}^{c} \Phi_{j} &= \lambda_{2}^{ij} M \mathbf{N}_{i}^{c} \Phi_{j} = \mathcal{M}^{ij} \mathbf{N}_{i}^{c} \Phi_{j} ,\\ \mu_{ij} \Phi_{i} \Phi_{j} . \end{aligned}$$

Being agnostic about the value of the singlet mass parameter μ , we arrive at the neutrino mass eigenvalues

$$m_{\rm u}^2\mu/M^2$$
, $\mu\pm\sqrt{\mu^2+M^2}$,

the light one corresponding dominantly to v. Even if μ were to be of order $M_{\rm Pl}$ we get tiny neutrino masses since M is forced to be superheavy from baryon decay.

In the case that μ is superheavy, the Higgs isodoublet mixing term hh Φ generates a tiny (order m_s^2/M) Higgs mixing through the diagrams of fig. 1 which is not adequate to avoid an unwanted axion state. Then, an extra source of Higgs mixing is required. This could be an extra singlet with a small, O(m_w), vacuum expectation value coupled as hh χ . This would be only

^{#1} For example, a $Z_2 \times Z_2$ symmetry with Φ_1 , $\Phi_2 \rightarrow (-1, -1)$, $\Phi_3 \rightarrow (1, -1)$ and $\Phi_0 \rightarrow (-1, 1)$ leads to $W(\Phi) = (\Phi_1 + \Phi_2)\Phi_3\Phi_0$ with supersymmetric minimum at $\Phi_1 = \Phi_2 = \Phi_3 = 0$ and $\Phi_0 \neq 0$.

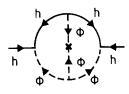


Fig. 1. Higgs mixing.

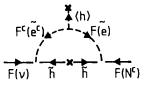


Fig. 2. νN^c contribution to the neutrino mass matrix and the magnetic moment.

technically natural. In the case that μ is small, $O(m_w)$, the radiatively generated Higgs mixing $\lambda_5^2 m_s/\pi^2$ is satisfactory for large enough values of λ_5 . This again is technically natural. In both cases we can assume that either way there exists a mixing term $m(\tilde{h})\tilde{h}\tilde{h}$ for the higgsinos with $m(\tilde{h}) \approx O(m_w)$.

The neutrino mass matrix is modified by radiative corrections. The vN^c entry is modified by contributions from the diagrams shown in fig. 2. The correction is roughly (up to logarithmic terms), suppressing indices on generation space,

$$m_{\rm R} \approx \lambda_1^2 m_{\rm e} m(\tilde{\rm h}) / 16 \pi^2 m_{\rm S}$$
,

and assuming $m(\tilde{e}) \approx m_{\rm s}$.

In this model the masses of quarks and leptons are the same at tree level but we assume that we have taken into account the radiative corrections for fermion masses.

The v Φ entries which are zero at the tree level receive radiative contributions from the diagrams of fig. 3 which are roughly of order $^{#2}$

$$m'_{\rm R} \approx (\lambda_5 \lambda_6 m'_{\rm u} m_{\rm S}/16\pi^2 \mathcal{M}) \left[\ln(\mathcal{M}^2/m_{\Phi}^2) - 1\right],$$

with m'_u denoting the value of the coupling $vN^c\lambda_1 \langle h_0 \rangle$.

The correction to the Φ mass may also be relevant in the case in which μ is not superheavy. It turns out roughly (fig. 4)

$$\mu_{\rm R} \approx \lambda_5^3 \lambda_6 \langle {\rm h} \rangle^2 m_{\rm S} / 16 \pi^2 \mu^2$$

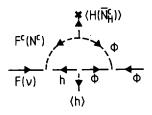


Fig. 3. $\nu\Phi$ contribution to the neutrino mass matrix and the magnetic moment.

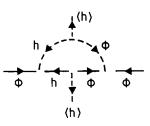


Fig. 4. $\Phi\Phi$ radiative corrections.

No radiative corrections, up to two loops at least, appear for the ΦN^c vertex.

It is also worth noticing that, in the diagrams of figs. 3 and 4 which are responsible for the m'_R and μ_R contributions, no charged particles circulate.

Thus finally the one-loop corrected neutrino mass matrix takes the form

$$\begin{pmatrix} 0 & m_{\rm u} + m_{\rm R} & m_{\rm R}' \\ m_{\rm u} + m_{\rm R} & 0 & \mathcal{M} \\ m_{\rm K}' & \mathcal{M} & \mu + \mu_{\rm R} \end{pmatrix}.$$
 (5)

Again, there are two large eigenvalues, $\mu + \mu_R \pm [(\mu + \mu_R)^2 + 4M^2]^{1/2}$, and a small neutrino mass eigenvalue,

$$(m_{\rm u} + m_{\rm R})^2 (\mu + \mu_{\rm R}) / \mathcal{M}^2 - (m_{\rm u} + m_{\rm R}) m_{\rm R}' / \mathcal{M}$$

Note that in the second term m'_R is already of order $1/\mathcal{M}$ so that the second term is

$$-\lambda_5 \lambda_6 (m_{\rm u} + m_{\rm R}) m_{\rm u} m_{\rm S} / 8\pi^2 \mathcal{M}^2$$

$$\approx -\lambda_5 \lambda_6 m_{\rm u}^2 m_{\rm S} / 8\pi^2 \mathcal{M}^2 ,$$

and its relevance depends on the relation between μ and $\lambda_5 \lambda_6 m_{\rm S} / 8\pi^2$.

A neutrino magnetic moment interaction $\bar{\nu}_R \sigma_{\mu\nu} F^{\mu\nu} \nu_L$ is also present in this model generated by the same diagrams that give rise to the neutrino mass radiative contributions with a photon insertion

^{#2} Unless $\mu \gg \mathcal{M}$ in which case we have \mathcal{M}/μ^2 instead of $1/\mathcal{M}$.

in any of the lines representing charged particles. The fact is that to this order only fig. 1 represents a diagram with charged internal lines and therefore only effective interaction terms of the type $\tilde{v}_R \sigma_{\mu\nu} F^{\mu\nu} N_L^c$ and $\bar{N}_R^c \sigma_{\mu\nu} F^{\mu\nu} v_L$ will arise. Denoting with μ the contribution to the neutrino magnetic moment matrix corresponding to the vN^c entry and with $\mu' \mu''$, the much smaller contributions to the N^c Φ and v Φ entries that might arise in higher loops ($\mu', \mu'' \ll \mu$), we have

$$L_{\text{int}} = (\bar{v}_{\text{R}} \quad \bar{N}_{\text{R}}^{\text{c}} \quad \bar{\Phi}_{\text{R}}) \sigma_{\mu\nu} F^{\mu\nu}$$

$$\times \begin{pmatrix} 0 & \mu & \mu'' \\ \mu & 0 & \mu' \\ \mu'' & \mu' & 0 \end{pmatrix} \begin{pmatrix} v_{\text{L}} \\ N_{\text{L}}^{\text{c}} \end{pmatrix}$$

$$\approx (\bar{v}_{\text{R}} \quad \bar{N}_{\text{R}}^{\text{c}} \quad \bar{\Phi}_{\text{R}}) \sigma_{\mu\nu} F^{\mu\nu}$$

$$\times \begin{pmatrix} 0 & \mu & 0 \\ \mu & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_{\text{L}} \\ N_{\text{L}}^{\text{c}} \end{pmatrix}.$$
(6)

A rough estimate gives $(\mu_{\rm B} = e/2m_{\rm e})$

$$\mu(\mathbf{v}, \mathbf{N}^{c}) \approx \mu_{\mathrm{B}}[\lambda_{1}^{2}m_{\mathrm{e}}m(\tilde{\mathbf{h}})/16\pi^{2}m_{\mathrm{S}}^{2}]$$
$$\times \{\ln[m_{\mathrm{S}}^{2}/m(\tilde{\mathbf{h}})^{2}] - 1\},\$$

where we have suppressed family indices. Taking m(h) as favourable as possible, i.e., $m(\tilde{h}) \approx m_{\rm S} \approx m_{\rm W}$ and the Yukawa couplings λ_1 to take the value 0.7×10^{-3} , we still get

 $\mu \approx \mu_{\rm B} \times 10^{-14} ,$

which is smaller by three orders of magnetic than the required value. Nevertheless, the essential point of this analysis is that the value of the neutrino magnetic moment is not directly related to the neutrino mass value.

The simplest way out of these small values is to introduce a second Higgs isodoublet $h'(1, 2, 2) = h'(1, 2, -\frac{1}{2}) + h'^{c}(1, 2, \frac{1}{2})$ with identical couplings to the standard fermions i.e. $\lambda'_1 FF^{ch'}$ which does not get a vacuum expectation value due to a direct mass m(h')h'h' that overpowers any negative mass corrections due to radiative effects. m(h') will be assumed to be near the electroweak scale. Then next to the diagram of fig. 2 there is an analogous diagram with the lower Higgs internal lines replaced by h'. The dominant contribution is then

$$\mu \approx \mu_{\rm B} [\lambda_1'^2 m_{\rm e} m(\tilde{\rm h}') / 16\pi^2 m_{\rm S}^2] \{ \ln [m^2 / m(\tilde{\rm h}')^2] - 1 \}.$$

The Yukawa couplings λ'_1 have nothing to do with the lepton mass matrix and could be assigned natural values $\lambda'_1 \approx 10^{-2}$. Then, for $m'(\tilde{\mathbf{h}}) \approx m_{\rm S} \approx m_{\rm W}$,

 $\mu \approx \mu_{\rm B} \times 10^{-11} ,$

which is the desired order of magnitude.

We can conclude that the essential points concerning neutrino masses and magnetic moments are independent of the particular $SU(4) \times SU(2)_L \times$ $SU(2)_R$ model we analyzed and hold for a general class of models in which the seesaw mechanism is realized with additional neutral singlet fields. Thus, it is possible in this class of models to have light neutrinos and sizeable magnetic moments. Supersymmetry does not play a decorative role since it guarantees the smallness of the radiative corrections and helps to obtain larger values for magnetic moments. The need for additional Higgs isodoublets that do not get vacuum expectation value is happily met by all models derived from superstrings [11].

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