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Box compactification and supersymmetry breaking

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Abstract

We discuss all possible compactifications on flat three-dimensional spaces. In particular, various fields are studied on a box with opposite sides identified, after two of them are rotated by π, and their spectra are obtained. The compactification of a general 7D supersymmetric theory in such a box is considered and the corresponding four-dimensional theory is studied, in relation to the boundary conditions chosen. The resulting spectrum, according to the allowed field boundary conditions, corresponds to partially or completely broken supersymmetry. We briefly discuss also the breaking of gauge symmetries under the proposed box compactification.

In almost all extensions beyond the Standard Model, supersymmetry plays a central role. In particular, Superstring Theory [1], as well as related theories of extended objects [2], provide a framework for a quantum theory of gravity. Nevertheless, since supersymmetry is not a low-energy symmetry of Nature, and has to be broken, supersymmetry breaking should be a key ingredient of the final theory. This important issue is still open. The tree-level *Scherk–Schwarz Supersymmetry Breaking* (SSSB) mechanism [3–8] is one of the proposals put forward, linking supersymmetry breaking to compactification. The smallness of supersymmetry breaking scale in comparison to the other scales, like the traditional unification or Planck scales, if it is to be associated with compactification, requires the presence of large extra dimensions [9,10]. Many models of this type have been proposed in the last few years [8] and, although, none is phenomenologically waterproof, it is generally admitted that the possibility of extra dimensions at the TeV scale is open. In SSSB one takes advantage of the R-symmetry of the supersymmetric theory to shift appropriately the masses of bosons and fermions lifting in this way the degeneracy and, thus, breaking supersymmetry. Alternative ways of breaking supersymmetry include gaugino condensation in the hidden sector [11] or, in brane scenarios [12], bulk to brane and brane to brane super-

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symmetry breaking [13]. Supersymmetry may also be broken by background fluxes [14,15]. In the case of background magnetic fields, the occurring tadpoles of which, will presumably be removed in the full quantum theory [14].

In the present Letter we elaborate on the possibility of breaking supersymmetry at the compactification process employing a novel compactification scheme. Gauge symmetry breaking as a result of compactification is also studied. Thus, as far as supersymmetry breaking is concerned, although we work along the lines of SSSB, it should be stressed that there is a fundamental difference with it, since in SSSB the boundary conditions for R-symmetry singlets, like vectors, are always periodic, in contrast to our *box compactification*, where they can be non-trivial even for R-singlets. In addition to that, the profile of our supersymmetry breaking is always that of a vanishing supertrace, resembling spontaneous breaking, in contrast to the SSSB patterns. We shall discuss our main differences with SSSB later on. At the moment, let us recall that according to a theoretical proposal, we are living in a $(4+n)$-dimensional space–time, n dimensions of which have been compactified to form a orientable compact space X^n. By turning off all fields except gravity, Einstein equations require the vacuum to be Ricci-flat and, thus, it is of the form $M^4 \times X^n$, where M^4 is the four-dimensional Minkowski space–time. The internal manifold X^n is assumed to be a complete, connected and compact Ricci-flat manifold like a Calabi–Yau space (in the case of String Theory). Nevertheless, one may assume that X^n *is flat and not just Ricci-flat*. In that case, the possible vacua are orientable compact euclidean space-forms. The most well studied case is that of an n-dimensional torus T^n. Other cases involve orbifolds of T^n by some discrete group, which although are singular spaces, strings can consistently propagate on them. These kind of orbifolds can also be obtained as limiting cases of smooth Calabi–Yau space. In this case, all curvature of the Calabi–Yau space is concentrated at the orbifold points. However, here we shall be interested in smooth, compact and flat n-dimensional spaces.

Unfortunately, existing classifications [16] of orientable compact euclidean space-forms do not go beyond 3D. In particular, in two dimensions, the only orientable compact euclidean space-form is the torus T^2. In three dimensions we have the following possibilities by making identifications on possible fundamental polyhedra in $\mathbb{R}^3$:

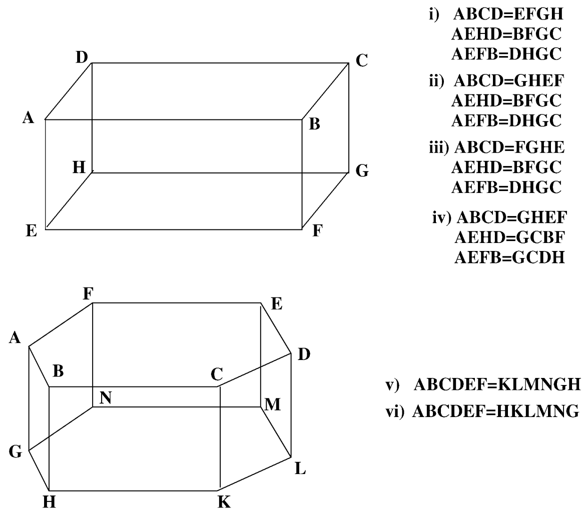

Fig. 1. Possible identification on $\mathbb{R}^3$ which produce compact orientable three-spaces.

(i) On a parallelepiped by identifying opposite sides;
(ii) On a parallelepiped by identifying opposite sides, one pair rotated by π;
(iii) On a parallelepiped by identifying opposite sides, one pair rotated by $\pi/2$;
(iv) On a parallelepiped by identifying opposite sides, all pairs rotated by π;
(v) On a hexagonal prism by identifying opposite sides, the top rotated by $2\pi/3$ with respect to the bottom;
(vi) On a hexagonal prism by identifying opposite sides, the top rotated by $\pi/3$ with respect to the bottom.

In addition to the above, there exist four non-compact orientable Euclidean space-forms, four non-compact and non-orientable and four compact and non-orientable Euclidean space-forms. This makes a total of 18 distinct types of locally euclidean spaces. Of them, only $\mathbb{R}^3$ is simply connected while the rest of the spaces are connected to the 17 crystallographic groups. It should be noted that the non-orientable cases are obtained by including "glide reflections", i.e., a reflection in a plane through the origin followed by a translation parallel to the plane.

In what follows we will assume a 7D theory which is spontaneously compactified to 4D on a compact and smooth internal space. According to the above discussion then, any flat 7D vacuum will be of the form $M^4 \times X^3$, where X^3 is any of the spaces (i)–(vi). One may easily recognize that (i) is just T^3 while the rest of the cases are orbifolds of T^3 by a freely acting isometry.

To make the discussion concrete let us assume that the internal space is the 3D box which is obtained after having identified its opposite sides with one pair rotated by π, i.e., the case (ii) on $\mathbb{R}^3$ with coordinates (x, y, z) subject to the identifications

$$\begin{aligned}
&(x, y, z) \approx (x + R_1, y, z), \\
&(x, y, z) \approx (x, y + R_2, z), \\
&(x, y, z) \approx (-x, -y, z + R_3). \qquad (1)
\end{aligned}$$

So, we have the normal identifications under translations in the x, y directions, while points in the z directions are identified after a π-rotation in the perpendicular x, y plane. We will call this space B^3. Corresponding efforts for compactifications on squares [17] produce orbifold singularities.

There is a $\mathbb{Z}_2$ symmetry, which acts as on the coordinates as[1]

$$g\colon \left(x^1, x^2, x^3\right) \approx \left(-x^1, -x^2, x^3 + R_3\right). \qquad (2)$$

We observe that $g^2 = 1$ since

$$g^2\colon \left(x^1, x^2, x^3\right) \approx \left(x^1, x^2, x^3 + 2R_3\right), \qquad (3)$$

and (x, y, z), $(x, y, z + 2R_3)$ are identified. Thus, B^3 is a double cover of T^3.

After having defined the geometry, we are now ready to study the behaviour of fields in the box of Eq. (1). It should be noted that we are mainly interested in the k_3-periodicity as the periodicity in k_1, k_2 are determined as usual by the identification $x \approx x + R_1$, $y \approx y + R_2$.

1. Scalar

A scalar field Φ is periodic on T^3 and on B^3. It should, therefore, satisfy

$$\begin{aligned}
\Phi\left(x^1, x^2, x^3\right) &= \alpha\Phi\left(-x^1, -x^2, x^3 + R_3\right) \\
&= \alpha^2\Phi\left(x^1, x^2, x^3 + 2R_3\right) \qquad (4)
\end{aligned}$$

so that $\alpha^2 = 1$. Thus, on B^3, a scalar field may have periodic or antiperiodic boundary conditions, i.e.,

$$\Phi\left(x^1, x^2, x^3\right) = \pm\Phi\left(-x^1, -x^2, x^3 + R_3\right). \qquad (5)$$

The eigenvalues of the scalar Laplace operator $\nabla^2 = -\partial_i\partial^i$ on B^3 are as usual $k^2 = k_1^2 + k_2^2 + k_3^2$ and the corresponding eigenstates $\cos(k_1x^1)\cos(k_2x^2)e^{ik_3x^3}$. As x^1, x^2 are periodic with periods R_1, R_2, respectively, we will always have (for the first eigenstates)

$$\begin{aligned}
&k_1 = \frac{2\pi n_1}{R_1}, \\
&k_2 = \frac{2\pi n_2}{R_2}, \quad n_1, n_2 = 0, 1, \ldots. \qquad (6)
\end{aligned}$$

On the other hand, the value of k_3 depends on the boundary conditions (5). In particular, we get

$$\begin{aligned}
&k_3^{(+)} = \frac{2\pi n_3}{R_3}, \\
&k_3^{(-)} = \frac{(2n_3 + 1)\pi}{R_3}, \quad n_3 = 0, 1, \ldots, \qquad (7)
\end{aligned}$$

for the periodic $(+)$ and antiperiodic $(-)$ choice, respectively.

2. Fermion

Similarly, for a fermion Ψ we should have

$$\begin{aligned}
&\Psi\left(x^1, x^2, x^3\right) \\
&\quad = \beta e^{i\phi\sigma_3}\Psi\left(-x^1, -x^2, x^3 + R_3\right) \\
&\quad = \beta^2 e^{2i\phi\sigma_3}\Psi\left(x^1, x^2, x^3, x^3 + 2R_3\right), \qquad (8)
\end{aligned}$$

where σ_3 is a Pauli matrix. For periodic Ψ on T^3 we get that $\beta^2 e^{2i\phi\sigma_3} = 1$ so that $\beta = \pm 1$, $\phi = \pi$. Therefore, the boundary conditions for fermion fields on B^3 are

$$\Psi\left(x^1, x^2, x^3\right) = \pm e^{i\pi\sigma_3}\Psi\left(-x^1, -x^2, x^3 + R_3\right) \qquad (9)$$

and we get

$$\begin{aligned}
&k_3^{(+)} = \frac{2\pi n_3}{R_3} + \frac{\pi}{R_3}\sigma_3, \\
&k_3^{(-)} = \frac{2\pi n_3}{R_3} + \frac{\pi}{R_3}(1 + \sigma_3). \qquad (10)
\end{aligned}$$

Clearly, the "periodic" $(+)$ condition makes the fermion massive with mass $m^2 = \pi^2/R_3^2$. In contrast, the

[1] The space B^3 may be viewed as $T^3/\mathbb{Z}_2$. It is not an orbifold as $\mathbb{Z}_2$ acts freely on T^3 (there are no fixed points under the action of $\mathbb{Z}_2$).

second, “antiperiodic” $(-)$, boundary condition, due to the projection operator $(1+\sigma_3)$, makes the upper component of Ψ massive, while its lower component has a zero mode.

3. Vector

For a vector A_i we will have

$$\begin{aligned} A_i\left(x^1,x^2,x^3\right) &= \gamma\left(e^{i\theta J_3}\right)_i^j A_j\left(x^1,x^2,x^3+R_3\right) \\ &= \gamma^2\left(e^{i\theta J_3}\right)_i^j\left(e^{i\theta J_3}\right)_j^k A_k\left(x^1,x^2,x^3+2R_3\right), \end{aligned} \tag{11}$$

where $J_3=\mathrm{diag}(\sigma_2,0)$ is the generator of rotations in the x^1,x^2 plane and so

$$\gamma^2\left(e^{i\theta J_3}\right)_i^j\left(e^{i\theta J_3}\right)_j^k=\delta_i^k. \tag{12}$$

It is not difficult then to verify that $\theta=\pi$ and

$$A_i\left(x^1,x^2,x^3\right)=\pm R_i^j A_j\left(-x^1,-x^2,x^3+R_3\right), \tag{13}$$

where $R=\mathrm{diag}(-1,-\sigma_3)$. Then, the eigenvalues for the components of A_i should be

$$A_1, A_2\colon\quad k_3^{(+)}=\frac{(2n_3+1)\pi}{R_3},\qquad k_3^{(-)}=\frac{2\pi n_3}{R_3}, \tag{14}$$

$$A_3\colon\quad k_3^{(+)}=\frac{2\pi n_3}{R_3},\qquad k_3^{(-)}=\frac{(2n_3+1)\pi}{R_3}, \tag{15}$$

for the periodic $(+)$ and antiperiodic $(-)$ boundary conditions, respectively.

4. Symmetric two-tensor

For a symmetric two-tensor h_{ij} we will have

$$\begin{aligned} h_{ij}\left(x^1,x^2,x^3\right) &= \pm R_i^\ell R_j^k h_{ij}\left(-x^1,-x^2,x^3+R_3\right). \end{aligned} \tag{16}$$

As a result, its k_3 eigenvalues will be

h_{ij} $(i,j\neq 3)$, h_{33}:

$$k_3^{(+)}=\frac{(2n_3+1)\pi}{R_3},\qquad k_3^{(-)}=\frac{2\pi n_3}{R_3}, \tag{17}$$

h_{i3} $(i\neq 3)$:

$$k_3^{(+)}=\frac{2\pi n_3}{R_3},\qquad k_3^{(-)}=\frac{(2n_3+1)\pi}{R_3}, \tag{18}$$

for the periodic $(+)$ and antiperiodic $(-)$ boundary conditions of Eq. (16), respectively.

It is clear that the components A_1, A_2 and A_3 of a vector A_M, as well as the components of a tensor, have different k_3. This is due to the fact that the box we are employing here is a non-homogeneous space.

Let us now see how we can use the above to break supersymmetry. We will consider a 7D supersymmetric $\mathcal{N}=1$ theory [18,19] with a vector supermultiplet which contains a vector A_M, 3 scalars ϕ^i, $i=1,2,3$, and one symplectic-Majorana spinor λ^a, $a=1,2$. We would like to see the theory when we dimensionally reduce on the space B^3. The effective 4D theory then contains the following fields $(A_\mu, A_i, \phi^i, \lambda_1^a, \lambda_2^a)$, i.e., a vector A_μ, 6 scalars $\Phi^I=(A_i,\phi^i)$, $I=1,\dots,6$, and 4 spinors $\Psi^A=(\lambda_1^a,\lambda_2^a)$, $A=1,\dots,4$. This is simply a vector multiplet of a 4D $\mathcal{N}=4$ theory. All these fields depend on the internal x^1,x^2,x^3 coordinates so we need to expand in terms of harmonics on B^3. The harmonics for the latter are

$$Y_{\{n_1n_2n_3\}}=\frac{1}{\sqrt{V}}\cos\left(k_1x^1\right)\cos\left(k_2x^2\right)e^{ik_ix^i}, \tag{19}$$

where $k_i=2\pi n_i/R_i$, $n_i=0,1,\dots$, and V the volume of B^3. Then, the expansion of the 4D fields is

$$\begin{aligned} A_\mu &= A_\mu(x)Y_{\{n\}}, \qquad & A_i &= A_i(x)Y_{\{n\}}, \\ \phi &= \phi(x)Y_{\{n\}}, \qquad & \lambda^a &= \lambda^a(x)Y_{\{n\}}. \end{aligned} \tag{20}$$

We have, thus, a tower of massive states with the masses of the vectors, scalars and fermions given by

$$\begin{aligned} M_V^2 &= k_1^2+k_2^2+k_3^2 \\ &= \left(\frac{2\pi n_1}{R_1}\right)^2+\left(\frac{2\pi n_2}{R_2}\right)^2+\left(\frac{2\pi n_3}{R_3}\right)^2, \end{aligned} \tag{21}$$

$$M_S^2=M_F^2=M_V^2. \tag{22}$$

It can easily be checked that $\mathrm{Str}\,M^2=0$.

For the box (ii) we are considering, depending on the boundary conditions, we have a basis

$$Y_{\{n\}}^{(\pm)}\Rightarrow k_3^{(\pm)}$$

as in (19), but with $k_3=k_3^{(\pm)}$, respectively. For instance, we may take for the bosons

$$\begin{aligned} A_\mu &= A_\mu(x)Y_{\{n\}}^{(+)}, \qquad & A_{1,2} &= A_{1,2}(x)Y_{\{n\}}^{(-)}, \\ A_3 &= A_3(x)Y_{\{n\}}^{(-)}, \qquad & \phi^i &= \phi^i(x)Y_{\{n\}}^{(-)}. \end{aligned} \tag{23}$$

The corresponding mass spectrum is then presented in Table 1.

For the 7D spinors we recall that in $SO(7)\supset SU_L(2)\times SU_R(2)\times SU(2)$, we have $\mathbf{8}=(\mathbf{2},\mathbf{1};\mathbf{2})+$

Table 1

A_μ	$M_V^2 = k_1^2 + k_2^2 + k_3^{(+)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{2\pi n_3}{R_3})^2$
$A_{1,2}$	$M_S^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{2\pi n_3}{R_3})^2$
A_3	$M_S^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{(2n_3+1)\pi}{R_3})^2$
ϕ^i	$M_S^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{(2n_3+1)\pi}{R_3})^2$

Table 2

χ_L^1	$M_F^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{n_3\pi}{R_3})^2$
χ_R^1	$M_F^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{2\pi n_3}{R_3})^2$
χ_L^2	$M_F^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{(2n_3+1)\pi}{R_3})^2$
χ_R^2	$M_F^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{(2n_3+1)\pi}{R_3})^2$

(**1**, **2**; **2**). As a result, a 7D spinor λ is decomposed into two left- and two right-handed 4D spinors. We may take

$$\lambda = \chi_L^\alpha(x) \otimes \epsilon^\alpha Y_{\{n\}}^{(-)} + \chi_R^\alpha(x) \otimes \theta^\alpha Y_{\{n\}}^{(-)}, \quad \alpha = 1, 2, \tag{24}$$

where ϵ^a, θ^a are two-component spinors and $\chi_{1,2}^a$ are 4D spinors. The mass spectrum of the 4D spinor is presented in Table 2.

Thus, from Tables 1, 2 we see that we get *one massless vector, two massless scalars and two massless fermions of opposite chirality*, all corresponding to $n_i = 0$. On the other hand, *four scalars and two spinors of opposite chirality do not have zero modes*. The massless spectrum in 4D is then a vector of a $\mathcal{N} = 2$ theory. As a result, compactification on this particular box with the above boundary conditions leads to the supersymmetry breaking

$$\mathcal{N} = 4 \Rightarrow \mathcal{N} = 2.$$

Note that the profile of the breaking is that of spontaneous supersymmetry breaking, since the supertrace still vanishes.

A complete supersymmetry breaking can be also achieved by assuming the following expansion of the 7D spinor

$$\lambda = \chi_L^\alpha(x) \otimes \epsilon^\alpha Y_{\{n\}}^{(-)} + \chi_R^\alpha(x) \otimes \theta^\alpha Y_{\{n\}}^{(+)}, \quad \alpha = 1, 2. \tag{25}$$

In this case the spectrum of the 4D spinors is presented in Table 3.

Table 3

χ_L^1	$M_F^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{n_3\pi}{R_3})^2$
χ_R^1	$M_F^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{(2n_3+1)\pi}{R_3})^2$
χ_L^2	$M_F^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{(2n_3+1)\pi}{R_3})^2$
χ_R^2	$M_F^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{(2n_3+1)\pi}{R_3})^2$

Table 4

A_μ	$M_V^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{(2n_3+1)\pi}{R_3})^2$
$A_{1,2}$	$M_S^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{2\pi n_3}{R_3})^2$
A_3	$M_S^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{(2n_3+1)\pi}{R_3})^2$
ϕ^i	$M_S^2 = k_1^2 + k_2^2 + k_3^{(-)2}$	$(\frac{2\pi n_1}{R_1})^2 + (\frac{2\pi n_2}{R_2})^2 + (\frac{(2n_3+1)\pi}{R_3})^2$

We see that from Tables 1, 3 that the massless spectrum is a vector A_μ, two scalars $A_{1,2}$ and a left-handed 4D spinor, which is not-supersymmetric. Thus, adopting the expansion in Eq. (25), we have completely break supersymmetry

$$\mathcal{N} = 4 \Rightarrow \mathcal{N} = 0.$$

We can also break $\mathcal{N} = 4$ to $\mathcal{N} = 1$ by considering different boundary conditions for the bosons of the 7D multiplet as well. For example, let us take

$$A_\mu = A_\mu(x) Y_{\{n\}}^{(+)}, \qquad A_{1,2} = A_{1,2}(x) Y_{\{n\}}^{(-)},$$
$$A_3 = A_3(x) Y_{\{n\}}^{(-)}, \qquad \phi^i = \phi^i(x) Y_{\{n\}}^{(-)}. \tag{26}$$

Then, the mass spectrum for the $4D$ fields is presented in Table 4.

The massless sector then for the $4D$ fields expanded as in Eqs. (25), (26) is given in Tables 3, 4 and consists of two scalars $A_{1,2}$ and one left-handed spinor. This is the massless representation of a chiral $\mathcal{N} = 1$ supersymmetry.

We may also study the effective 4D theory after the DR over $B_2 = T^3/\mathbb{Z}_2$. Consider a 7D supersymmetric theory which contains a vector A_M, 3 scalars ϕ^i, $i = 1, 2, 3$, and one symplectic-Majorana spinor λ^a, $a = 1, 2$, all in the adjoint representation of a semisimple group G. After DR on T^3 with normal boundary conditions to 4D, the effective action turns out to be

$$S_{\rm eff} = \int d^4x\, {\rm Tr}\biggl(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\bar\lambda^i\gamma^\mu D_\mu\lambda^i + \frac{1}{2}\partial_\mu\varphi_\alpha\partial^\mu\varphi^\alpha + i\lambda^i\bigl[\lambda^j,(\sigma_\alpha)^{ij}\varphi^a\bigr] + i\bar\lambda^i\bigl[\bar\lambda^j,\bigl(\sigma_\alpha^*\bigr)^{ij}\varphi^a\bigr] + \frac{1}{4}[\varphi_\alpha,\varphi_\beta]\bigl[\varphi^\alpha,\varphi^\beta\bigr] + \sum_{n_i=1}\mathcal{L}^{KK}_{n_1\ldots n_4}\biggr), \qquad (27)$$

where by $\mathcal{L}^{KK}_{n_1\ldots n_4}$ we collectively denote all massive Kaluza–Klein contributions. In addition, we have combined the 3 original scalars ϕ^i and the 3 scalars (A_4, A_5, A_6) originating from the DR of A_M in $\varphi_\alpha = (\phi_i, A_{3+i})$.

Now let us consider the $B_2 = T^3/\mathbb{Z}_2$ compactification. This amounts in shifting certain modes from the massless to the massive sector of the 4D theory. With an expansion of the form (23), (24), the 4D theory turns out to be as above but with an additional mass term

$$S^{(1)}_{\rm eff} = S_{\rm eff} + \int d^4x\,\frac{1}{2}\,{\rm Tr}\,M_{\alpha\beta}\varphi^a\varphi^\beta. \qquad (28)$$

The existence of the mass term clearly breaks susy. Indeed, there are interactions missing from the 4D effective theory (28) on $B_2 = T^3/\mathbb{Z}_2$. Written in $\mathcal{N}=1$ language, the superpotential is

$$W = \frac{1}{3}\epsilon^{ijk}\Phi_i\Phi_j\Phi_k + M_{ij}\Phi^i\Phi^j, \quad i,j,k = 1,2,3, \qquad (29)$$

where we have define $\Phi_i = A_{3+i} + i\phi_i$. Then clearly, the interactions from $\lambda\partial^2W/\partial\Phi^2\lambda$

$$\lambda\lambda M\Phi \qquad (30)$$

are missing from the effective action (28). Depending on the form of the mass term in (28), the $\mathcal{N}=4$ supersymmetry can either break to $\mathcal{N}=1,0$. Thus, the $B_2 = T^3/\mathbb{Z}_2$ compactification of the 7D $\mathcal{N}=2$ theory is described by an effective 4D theory with non-supersymmetric interactions among the fields.

At this point let us compare supersymmetry breaking described above to the one obtained through the Scherk–Schwarz mechanism. According to the latter, employing the R-symmetry of the theory, one may give masses to certain fields such that supersymmetry may be broken. In a S^1 compactification, one may impose the condition

$$\Phi(x^\mu, y+2\pi L) = e^{2\pi i Q_\Phi}\Phi\bigl(x^\mu\bigr), \qquad (31)$$

where Q_Φ is the R-charge of the field Φ. This leads to splitting of the $4D$ masses of the various fields according to their R-charge. Fermions and bosons, having different Q_Φ, obtain different contributions to their masses and supersymmetry is broken. This looks much like our boundary conditions (5) or (8). However, as gauge fields A_M are always R-singlets, (vectors never carry R-charge, except when the R-symmetry is gauged), it is not possible to acquire modified boundary conditions. Vector fields, as well as higher-rank tensors, have $Q_\Phi = 0$ and obey periodic boundary conditions under translations in the extra dimension. This should be contrasted to our case, where, due to the rotation in the x, y plane involved, vectors, as well as higher-rank tensors, do not necessarily obey periodic boundary conditions, as we have already seen. As a result, in spite of their similarities, box compactification and SSSB are different. It should also be noted that the profile of our *box compactification* is that of spontaneous breaking with a vanishing supertrace, a feature not shared by SSSB as the latter breaks global supersymmetry explicitly where the mass-square supertrace is not necessarily zero. We have also to stress that there is no way to make all components of a vector periodic due to non-homogeneity of the box, which is manifest exactly in the different k_3-periodicity of the A_M components.

Although in this Letter the emphasis has been given to the breaking of supersymmetry, box compactification can equally well lead to gauge symmetry breaking. This may be discussed independently from supersymmetry and, thus, we will consider, for example, an $SU(5)$ gauge theory in 7D. After compactifying on B^3, we may expand the 7D gauge fields A^I_M, $I = 1,\ldots,24$, in terms of the B^3 harmonics as we did above. We can exploit our freedom to choose the boundary conditions and take

$$A^I_\mu = A^I_\mu(x)Y^{(+)}_{\{n\}} \quad \text{for } I \text{ in } SU(3)\times SU(2)\times U(1),$$
$$A^I_\mu = A^I_\mu(x)Y^{(-)}_{\{n\}} \quad \text{otherwise.} \qquad (32)$$

Then, clearly, the fields $A^i_\mu(x)$ have a massless mode, identified with the usual 4D gauge bosons, while all the rest X, Y bosons are massive. However, we also get the scalars A_m^I which we should make massive by choosing $A_m^I = A_m^I(x)Y_{\{n\}}^{(-)}$.

Similarly for a Higgs in the fundamental H^A, $A = 1, \ldots, 5$, we may take

$$\begin{aligned} H^A &= H^A(x)Y_{\{n\}}^{(+)} \quad \text{for } A \text{ in } SU(2), \\ H^A &= H^A(x)Y_{\{n\}}^{(-)} \quad \text{otherwise.} \end{aligned} \tag{33}$$

The above expansions at this stage look rather ad hoc. The following can serve as a hint of how they could arise. Assume that the $\mathbb{Z}_2$ symmetry acts also in the gauge sector as

$$\mathbb{Z}_2 \subset U(1) \subset SU(5): \quad g\mathbf{5} = -\mathbf{5}, \quad g\mathbf{24} = +\mathbf{24}, \tag{34}$$

for the fundamental (**5**) and adjoint (**24**) of $SU(5)$. In other words, we embed $\mathbb{Z}_2$ in the $U(1)$ subgroup of $SU(5) \supset SU(3) \times SU(2) \times U(1)$ and we assign periodic and antiperiodic $\mathbb{Z}_2$-"parity" to the adjoint and fundamental reps, respectively. Then, in the branching

$$\begin{aligned} \mathbf{5} &= (\mathbf{2}, \mathbf{3})_3 + (\mathbf{1}, \mathbf{3})_{-2}, \\ \mathbf{24} &= (\mathbf{1}, \mathbf{1})_0 + (\mathbf{3}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{8})_0 + (\mathbf{2}, \mathbf{3})_{-5} + (\mathbf{2}, \bar{\mathbf{3}})_5, \end{aligned} \tag{35}$$

we have to choose periodic $(+)$, or antiperiodic $(-)$ boundary conditions according to their $(U(1) \bmod 2)$-charge. Thus, for a Higgs in the fundamental, the triplet will have antiperiodic boundary conditions and, thus, it will have no massless mode, while the doublet will be periodic and will have a massless mode. In contrast, for the adjoint, the $(\mathbf{2}, \mathbf{3})_{-5}$ and $(\mathbf{2}, \bar{\mathbf{3}})_5$ will have no massless mode, as they have odd $(U(1) \bmod 2)$-charge and the $\mathbb{Z}_2$-"parity" of the adjoint is $+1$.

The recent activity on theories and models characterized with large extra dimensions provides a framework that can accommodate a connection between the phenomenologically required small supersymmetry breaking and compactification. In the present Letter we analyzed the basic features of a novel compactification scheme on a flat three-dimensional torus, where opposite sides are identified after two of them have undergone a rotation by π. Although the scheme superficially resembles orbifold compactification it is not an orbifold compactification, since it does not involve any fixed points. Starting with a supersymmetric theory, the chosen boundary conditions for component fields can be such that lead to a compactified theory with reduced or completely broken supersymmetry. Examples of boundary conditions that, for a 7D theory, lead to $N = 4 \to N = 2$, $N = 1$, $N = 0$ breakings were worked out. It remains to be seen in future work whether this framework can be used for the construction of realistic models. The spectrum profile of the supersymmetry breaking scheme discussed is analogous to the one associated with spontaneous supersymmetry breaking, characterized by a vanishing supertrace. We should also stress once more the difference of the present scheme to the Scherk–Schwarz supersymmetry breaking scheme in which component fields acquire non-trivial boundary conditions through their different R-symmetry charges. In this scheme vector fields cannot be affected. In contrast, here the compactification scheme allows for non-trivial gauge field boundary conditions. Although, we did not elaborate much on gauge symmetry breaking, it is clear that box compactification can naturally serve as a way to break gauge symmetries as well in ways analogous to the ones employed in orbifold theories [20]. An intriguing question not touched by the present first short presentation of box compactification is that of the arbitrariness of the chosen boundary conditions. The answer is linked to the quantum dynamics that will ultimately discriminate between the various available compactification solutions.

References

[1] M.B. Green, J.H. Schwarz, E. Witten, Superstring Theory, Cambridge Univ. Press, Cambridge, 1987.

[2] J. Polchinski, hep-th/9611050.

[3] J. Scherk, J.H. Schwarz, Phys. Lett. B 82 (1979) 60;
J. Scherk, J.H. Schwarz, Nucl. Phys. B 153 (1979) 61;
E. Cremmer, J. Scherk, J.H. Schwarz, Phys. Lett. B 84 (1979) 83.

[4] P. Fayet, Nucl. Phys. B 246 (1984) 89.

[5] P. Fayet, Nucl. Phys. B 263 (1986) 649.

[6] R. Rohm, Nucl. Phys. B 237 (1984) 553;
H. Itoyama, T.R. Taylor, Phys. Lett. B 186 (1987) 129.

[7] S. Ferrara, C. Kounnas, M. Porrati, Phys. Lett. B 206 (1988) 25;
C. Kounnas, M. Porrati, Nucl. Phys. B 310 (1988) 355;
C. Kounnas, B. Rostand, Nucl. Phys. B 341 (1990) 641.

[8] A. Pomarol, M. Quiros, Phys. Lett. B 438 (1998) 255, hep-ph/9806263;
I. Antoniadis, S. Dimopoulos, A. Pomarol, M. Quiros, Nucl. Phys. B 544 (1999) 503, hep-ph/9810410;

A. Delgado, A. Pomarol, M. Quiros, Phys. Rev. D 60 (1999) 095008, hep-ph/9812489.
[9] I. Antoniadis, Phys. Lett. B 246 (1990) 377;
J.D. Lykken, Phys. Rev. D 54 (1996) 3693, hep-th/9603133.
[10] I. Antoniadis, S. Dimopoulos, G.R. Dvali, Nucl. Phys. B 516 (1998) 70, hep-ph/9710204;
N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B 429 (1998) 263, hep-ph/9803315.
[11] D. Amati, K. Konishi, Y. Meurice, G.C. Rossi, G. Veneziano, Phys. Rep. 162 (1988) 169;
H.P. Nilles, Int. J. Mod. Phys. A 5 (1990) 4199.
[12] I. Antoniadis, E. Dudas, A. Sagnotti, Phys. Lett. B 464 (1999) 38, hep-th/9908023;
C. Angelantonj, I. Antoniadis, G. D'Appollonio, E. Dudas, A. Sagnotti, Nucl. Phys. B 572 (2000) 36, hep-th/9911081;
I. Antoniadis, K. Benakli, M. Quiros, Nucl. Phys. B 583 (2000) 35, hep-ph/0004091.
[13] A. Anisimov, M. Dine, M. Graesser, S. Thomas, Phys. Rev. D 65 (2002) 105011, hep-th/0111235;
P. Mayr, Nucl. Phys. B 593 (2001) 99, hep-th/0003198;
I. Antoniadis, K. Benakli, A. Laugier, T. Maillard, Nucl. Phys. B 662 (2003) 40, hep-ph/0211409;
R. Rattazzi, C.A. Scrucca, A. Strumia, Nucl. Phys. B 674 (2003) 171, hep-th/0305184.
[14] C. Bachas, hep-th/9503030.
[15] T.R. Taylor, C. Vafa, Phys. Lett. B 474 (2000) 130, hep-th/9912152;
K. Becker, M. Becker, M. Haack, J. Louis, JHEP 0206 (2002) 060, hep-th/0204254;
P.G. Camara, L.E. Ibanez, A.M. Uranga, hep-th/0311241.
[16] J. Wolf, Spaces of Constant Curvature, McGraw–Hill, New York, 1967;
G.F.R. Ellis, Gen. Relativ. Gravit. 2 (1971) 7;
M. Lachieze-Rey, J.P. Luminet, Phys. Rep. 254 (1995) 135, gr-qc/9605010.
[17] B.A. Dobrescu, E. Ponton, hep-th/0401032.
[18] E. Bergshoeff, I.G. Koh, E. Sezgin, Phys. Rev. D 32 (1985) 1353.
[19] T. Gherghetta, A. Kehagias, Phys. Rev. Lett. 90 (2003) 101601, hep-th/0211019;
T. Gherghetta, A. Kehagias, Phys. Rev. D 68 (2003) 065019, hep-th/0212060.
[20] L.J. Dixon, J.A. Harvey, C. Vafa, E. Witten, Nucl. Phys. B 261 (1985) 678;
L.J. Dixon, J.A. Harvey, C. Vafa, E. Witten, Nucl. Phys. B 274 (1986) 285.