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Bulk Higgs with 4D gauge interactions

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Abstract

We consider a model with an extra compact dimension in which the Higgs is a bulk field while all other Standard Model fields are confined on a brane. We find that four-dimensional gauge invariance can still be achieved by appropriate modification of the brane action. This changes accordingly the Higgs propagator so that, the Higgs, in all its interactions with Standard Model fields, behaves as an ordinary 4D field, although it has a bulk kinetic term and bulk self-interactions. In addition, it cannot propagate from the brane to the bulk and, thus, no charge can escape into the bulk but it remains confined on the brane. Moreover, the photon remains massless, while the dependence of the Higgs vacuum on the extra dimension induces a mixing between the graviphoton and the $\mathcal{Z}$-boson. This results in a modification of the sensitive ρ-parameter.

1. Introduction

The quest for a unification of gravity with the rest of the fundamental interactions has led to the idea of extra spatial dimensions, first introduced in the Kaluza–Klein (KK) theory. Presently, the only consistent theoretical framework for the development of such a unified theory is String Theory or some form of it [1], which also requires extra spacetime dimensions. In this higher-dimensional context, gravity describes the geometry of a $D = 4 + d$ spacetime possessing d extra spatial dimensions. Many aspects of particle physics have been considered in this general framework, giving the opportunity of a new and fresh look in old and challenging problems. In particular, the introduction of D-branes has led in a reformulation and reevaluation of the original hierarchy problem, by considering compact internal spaces of *large* radius R, possibly corresponding to a fundamental higher-dimensional Planck mass of $O(\text{TeV})$ [2], or even of infinite radius [3–5]. Many other issues have also been reexamined in this general setup, in which, as a general rule, all degrees of freedom of the Standard Model (SM) are assumed to

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be confined on a 4-dimensional subspace (*brane*), while only gravity propagates in the full space (*bulk*). The construction of models in which additional fields besides the graviton field can propagate in the bulk has been proven to be quite challenging. It is common wisdom that, in a world with gauge field on a brane, charged fields may exist only on the brane as a result of gauge invariance. Exception to this is neutral particles, as for example in brane models with SM on the brane and a bulk right-handed neutrino [6–9]. The rule is that only fields which are neutral under the Standard Model gauge interactions can be introduced as bulk fields with brane-interactions.

In the present Letter we reexamine the question of fields in the bulk. Considering the Standard Model Higgs, we will show that, Higgs fields propagating in the bulk but with *localized* gauge interactions may exist as well. We restrict ourselves in the case of one extra compact dimension. The Higgs is introduced as a bulk field while the gauge bosons and the rest of the Standard Model are strictly confined on the brane. We find that the constraint of four-dimensional gauge invariance modifies the Higgs propagator so that, the Higgs, in all its interactions with Standard Model behaves as an ordinary four-dimensional field and although it propagates within the bulk, it cannot propagate from the brane to the bulk. Despite the fact that no charge can escape into the bulk but it remains confined on the brane, the Higgs has a bulk kinetic term and bulk self-interaction. Moreover, a bulk Higgs field can have a vacuum expectation value dependent on the extra dimension. Such a VEV induces an interaction between the *graviphoton* and the $\mathcal{Z}$-boson. This interaction amounts to a mixing of $\mathcal{Z}$ with the graviphoton, which has already acquired a mass due to the presence of the brane [10].

In Section 2, we introduce a model with a bulk 5D scalar field with 4D $U(1)$ gauge interactions on a brane and we present a gauge invariant action. In Section 3, we discuss the graviphoton-gauge field mixing by a vacuum 5D scalar field configuration. In Section 4, we introduce the SM Higgs in the 5D bulk interacting with the rest of the SM fields, which are localized on the 4D brane, and we find that, although the photon remains massless, there exist a graviphoton-$\mathcal{Z}$ mixing and an associated modification of the ρ-parameter. In Section 5 we discuss quantum effects due to the 5D Higgs, and finally, in Section 6, we summarize our findings.

2. Bulk Higgs and 4D gauge invariance

Let us consider a 5D space $M^4 \times S^1$ with the extra compact dimension $x^5 = y$ that takes values in the circle $0 \leqslant y \leqslant R$. A brane is present at the location $y = 0$ of this space. The five-dimensional metric is taken to the flat metric

$$G_{MN} = \begin{pmatrix} \eta_{\mu\nu} & 0 \\ 0 & 1 \end{pmatrix}.$$

Consider now a complex scalar field $\Phi(x^\mu, y)$ with the 5D canonical dimension $3/2$. In addition to that, there is a four-dimensional $U(1)$ gauge field $B_\mu(x)$ propagating on the brane. The action for the model is

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_{\rm Br} + \mathcal{S}_{\rm int}, \tag{1}$$

where

$$\mathcal{S}_0 = -\int d^4x \int dy \, |\partial_M \Phi|^2$$

is the free-action for the bulk scalar field

$$\mathcal{S}_{\rm Br} = -\frac{1}{4}\int d^4x \, B_{\mu\nu}(x) B^{\mu\nu}(x),$$

with $B_{\mu\nu}(x) = \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x)$ is the action for the $U(1)$ gauge field and

$$\mathcal{S}_{\rm int} = -iag \int d^4x \, B^\mu(x) \big(\Phi^*(x,0)\partial_\mu \Phi(x,0) - \partial_\mu \Phi^*(x,0)\Phi(x,0) - igB_\mu(x)\big|\Phi(x,0)\big|^2\big),$$

is their interaction. In the interaction term g is the dimensionless four-dimensional gauge coupling while the parameter a has canonical dimension -1 and corresponds to a length. The action (1) describes a 5D scalar interacting with a 4D gauge field. The scalar field propagates in the bulk of the spacetime, while its interactions are localized on the brane at $y=0$. Clearly, $\mathcal{S}_{\rm Br}$ is invariant under the standard $U(1)$ transformation $\delta B_\mu(x)=\partial_\mu\omega(x)$. However, the bulk and the interaction Lagrangian are not gauge invariant as they stand.

At this point let us define the transformation properties of the scalar field. An obvious guess is that the scalar field transforms only at the position of the brane, i.e., where it experiences the gauge interactions. Let us consider then the gauge transformations

$$\delta\Phi(x,y)=iag\omega(x)\delta(y)\Phi(x,y),\qquad \delta B_\mu(x)=\partial_\mu\omega(x). \tag{2}$$

Note that these transformations are singular on the brane, i.e.,

$$\delta\Phi(x,0)=iag\omega(x)\delta(0)\Phi(x,0).$$

However, they can be regularized by taking $a^{-1}=\delta(0)$. Then, we can write[1]

$$\delta\Phi(x,y)=ig\omega(x)\hat{\delta}(y)\Phi(x,y), \tag{3}$$

so that the gauge transformations on the brane are just

$$\delta\Phi(x,0)=ig\omega(x)\Phi(x,0),\qquad \delta B_\mu(x)=\partial_\mu\omega(x).$$

Note that on the brane we can define a four-dimensional field with the correct canonical dimension

$$\phi(x)=a^{1/2}\Phi(x,0)=\delta(0)^{-1/2}\Phi(x,0)$$

in terms of which $\mathcal{S}_{\rm int}$ is just

$$\mathcal{S}_{\rm int}=-ig\int d^4x\,B^\mu(x)\big(\phi^*(x)\partial_\mu\phi(x)-\partial_\mu\phi^*(x)\phi(x)-igB_\mu(x)\big|\phi(x)\big|^2\big).$$

Under the gauge transformation (3), the action (1) transforms as

$$\delta\mathcal{S}=iag\int d^4x\,\omega(x)\big(\Phi^{\dagger\prime\prime}(x,0)\Phi(x,0)-\Phi''(x,0)\Phi^\dagger(x,0)\big),$$

and thus, it is not gauge invariant. However, gauge invariance can be maintained by adding appropriate terms. Indeed, let us consider the term

$$\begin{aligned}\mathcal{S}_1&=-a\int d^4x\,\big(\Phi^{\dagger\prime\prime}(x,0)\Phi(x,0)+\Phi''(x,0)\Phi^\dagger(x,0)\big)\\&=-a\int d^4x\int dy\,\delta(y)\big(\Phi^{\dagger\prime\prime}(x,y)\Phi(x,y)+\Phi''(x,y)\Phi^\dagger(x,y)\big).\end{aligned}$$

It is easy to verify then that

$$\delta\mathcal{S}_1=-iag\int d^4x\,\omega(x)\big(\Phi^{\dagger\prime\prime}(x,0)\Phi(x,0)-\Phi''(x,0)\Phi^\dagger(x,0)\big)=-\delta\mathcal{S},$$

and thus, the total action

$$\mathcal{S}_{\rm tot}=\mathcal{S}+\mathcal{S}_1$$

is gauge invariant. The above discussion may easily be generalized to the non-Abelian case.

[1]
$$\hat{\delta}(y)=\lim_{\epsilon\to0}\left\{\frac{\delta(y-\epsilon)}{\delta(\epsilon)}\right\},\qquad \hat{\delta}(0)=\lim_{\epsilon\to0}\left\{\frac{\delta(0-\epsilon)}{\delta(\epsilon)}\right\}=1.$$

3. Bulk Higgs with brane gauge interactions

Let us consider now a Higgs field H living in a 5D bulk. All its gauge interactions, as well as some of its self-interactions, are localized at $y = 0$. Taking into account gravity with 5D Planck mass M_5, we have the action[2]

$$\mathcal{S} = M_5^3 \int d^5x \sqrt{-G}\mathcal{R} - \int d^5x \sqrt{-G}\big(G^{MN} D_M H^\dagger D_N H + V^{(5)}(H)\big) - \int d^4x \sqrt{-G^{(i)}}\left(T + V^{(4)}(H) + \frac{1}{4} B_{\mu\nu} B^{\mu\nu}\right), \tag{4}$$

where the gauge field strength is $B_{\mu\nu}(x) = \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x)$ and the covariant derivative

$$D_M \equiv \begin{cases} D_\mu = \partial_\mu - iga\delta(y) B_\mu(x), \\ D_5 = \partial_5. \end{cases} \tag{5}$$

Note the presence of both a bulk self-interaction potential $V^{(5)}(H)$ as well as a potential on the brane $V^{(4)}(H)$. The canonical dimensions of these terms are different, being $[V^{(5)}] = 5$ and $[V^{(4)}] = 4$.

If we add to $\mathcal{S}$ the extra local term

$$\mathcal{S}_1 = -a \int d^4x \left(H^{\dagger\prime\prime}(x,0) H(x,0) + H''(x,0) H^\dagger(x,0)\right), \tag{6}$$

the total action $\mathcal{S} + \mathcal{S}_1$ becomes invariant under the set of gauge transformations

$$\delta H(x,y) = iga\omega(x)\delta(y) H(x,y), \qquad \delta B_\mu(x) = \partial_\mu \omega(x). \tag{7}$$

Let us now consider the general KK-form of the 5D metric

$$G_{MN} = \begin{pmatrix} g_{\mu\nu} + R^2 A_\mu A_\nu & R^2 A_\mu \\ R^2 A_\mu & R^2 \end{pmatrix}, \qquad G^{MN} = \begin{pmatrix} g^{\mu\nu} & -A_\mu \\ -A_\mu & R^{-2} + A^\mu A_\mu \end{pmatrix}, \tag{8}$$

where the radion field G_{55} has been friezed to its VEV R, the radius of the extra S^1 dimension. With this form of the 5D metric, $g_{\mu\nu}$ describes the 4D graviton and A_μ the 4D *graviphoton*. In the absence of the brane, the graviphoton A_μ is a massless boson corresponding to the translational invariance along S^1. However, in the presence of the brane, translational invariance is broken and the graviphoton becomes massive. This has been shown explicitly in [10] for a fat brane formed by a kink soliton. This is also the case for a brane with delta-function profile. To see this, let us turn off all fields except gravity. In this case, the 5D action in the presence of the brane is

$$\mathcal{S} = M_5^3 \int d^5x \sqrt{-G}\mathcal{R} - T \int d^4x \sqrt{-G^{(i)}}, \tag{9}$$

where $G^{(i)}_{\mu\nu}$ is the induced metric at the position of the brane

$$G^{(i)}_{\mu\nu} = g_{\mu\nu} + R^2 A_\mu A_\nu. \tag{10}$$

Then, we have for the determinant of the induced metric

$$\det\big(-G^{(i)}\big) = \det(-g_{\mu\nu}) \det\big(\delta_\mu^\nu + R^2 A_\mu A^\nu\big) = \det(-g) e^{\mathrm{Tr}\ln(1+R^2 A\otimes A)} = \det(-g) e^{\sum_{n=1}^\infty \frac{R^{2n} A^{2n}}{n}} = \det(-g) e^{\ln(1+R^2A^2)} = \det(-g)\big(1 + R^2 A_\mu A^\mu\big). \tag{11}$$

[2] $G^{(i)}$ is the determinant of the induced metric on the brane. The parameter T is the *brane tension* and has canonical dimensions $[T] = 4$.

Moreover, with the KK form (8) of the metric, where $g_{\mu\nu} = g_{\mu\nu}(x)$ and $A_\mu = A_\mu(x)$, the 5D Ricci scalar $\mathcal{R}$ turns out to be

$$\mathcal{R} = \bar{\mathcal{R}} - \frac{R^2}{4} F_{\mu\nu} F^{\mu\nu}. \tag{12}$$

$\bar{\mathcal{R}}$ is the 4D Ricci scalar of the 4D metric $g_{\mu\nu}$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the graviphoton field. Substituting, (11), (12) in (9), we get

$$\mathcal{S} = 2\pi R M_5^3 \int d^4x \sqrt{-g}\mathcal{R} + 2\pi R^3 M_5^3 \int d^4x \sqrt{-g}\left\{-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right\} - T' \int d^4x \sqrt{-g}\big(1 + R^2 A_\mu A^\mu\big)^{1/2}, \tag{13}$$

where $T' \equiv T + \langle V^{(4)} + R V^{(5)} + R|\partial_5 H|^2\rangle$ includes the Higgs contribution to the vacuum energy. Linearizing the above expression, we arrive at the effective 4D action ($2\pi R M_5^3 \equiv M_P^2$)

$$\mathcal{S} = M_P^2 \int d^4x \sqrt{-g}\mathcal{R} - T' \int d^4x \sqrt{-g} + (R M_P)^2 \int d^4x \sqrt{-g}\left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2}\frac{T'}{M_P^2} A_\mu A^\mu\right) + \cdots \tag{14}$$

from where we read off the graviphoton mass

$$M_A^2 = \frac{T'}{M_P^2} = \frac{1}{2\pi}\left(\frac{T'}{R M_5^3}\right).$$

Thus, we see that the brane breaks the $U(1)$ symmetry of the background and gives a non-zero mass to the graviphoton. This mass depends on the brane tension as in the case where the brane is a kink of finite width, formed by a scalar field [10].

As we are only interested in the Higgs-gauge sector, we may ignore gravity and the brane tension. In this case, we may disregard the first two terms in (14). The complete action turns out to be

$$\begin{aligned}
&-(R M_P)^2 \int d^4x \sqrt{-g}\left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} M_A^2 A_\mu A^\mu\right) - \int d^5x \sqrt{-g}\big(D_\mu H^\dagger D^\mu H + V^{(5)}\big) \\
&\quad + R \int d^5x \sqrt{-g} A^\mu \big(D_\mu H^\dagger \partial_5 H + \partial_5 H^\dagger D_\mu H\big) - \int d^5x \sqrt{-g}\big(1 + R^2 A_\mu A^\mu\big)\partial_5 H^\dagger \partial_5 H \\
&\quad - \int d^4x \sqrt{-g} V^{(4)} - \frac{1}{4} \int d^4x \sqrt{-g}\big(1 + R^2 A_\kappa A^\kappa\big) B_{\mu\nu} B^{\mu\nu}.
\end{aligned} \tag{15}$$

It is obvious that due to the Higgs field, the graviphoton and the brane gauge field are mixed. We will discuss this mixing and its consequences below. For this we define the 4D Higgs field on the brane with the correct canonical dimensions

$$\phi(x) \equiv H(x,0) a^{1/2} = H(x,0)\delta(0)^{-1/2}$$

and the canonical graviphoton field

$$A_\mu(x) \to (R M_P)^{-1} A_\mu(x).$$

Now, let us note that there are terms quadratic to the gauge and graviphoton fields coming from the bulk scalar covariant derivatives, as well as a coupling $A_\mu(D^\mu H)\partial_5 H^\dagger$. We shall assume a y-dependent vacuum $H(x,y)$. By setting

$$\mathrm{Im}\left\{\frac{H'(0,0)}{H(0,0)}\right\} \equiv M$$

the vector mass-terms in (15) turns out to be

$$-\int d^4x\,\sqrt{-g}\left(g^2 B_\mu B^\mu|\phi|^2 + \frac{1}{2}M_A^2 A_\mu A^\mu + 2\frac{gM}{M_P}A_\mu B^\mu|\phi|^2\right). \tag{16}$$

The vacuum is determined from the Higgs field equation

$$\left[\partial_5^2 + \mu_0^2 - a\left\{\partial_5^2, \delta(y)\right\} + \delta(y)\left(\mu_4^2 - \lambda_4|H|^2\right)\right]H(0, y) = 0,$$

where we have taken

$$V^{(4)} = -\mu_4^2|H|^2 + \frac{\lambda_4}{2}|H|^4, \qquad V^{(5)} = -\mu_5^2|H|^2 + \frac{\lambda_5}{2}|H|^4 \approx \mu_0^2|H|^2 + \cdots.$$

The above equation possesses solutions of the form[3]

$$H(0, y) = C_1\delta(y) + C_2\sinh(\mu_0 y). \tag{17}$$

By substituting, we obtain

$$\nu_0^2 + a\delta''(0) - \delta(0)\mu_4^2 + \lambda_4\delta^3(0)|C_1|^2 = 0.$$

Since $[\mu_4^2] = 1$ and $[\lambda_4] = -2$, we may introduce the canonical 4D parameters

$$\bar{\mu}_4^2 = a\mu_4^2, \qquad \bar{\lambda}_4 = a^2\lambda_4.$$

Introducing the renormalized mass $\mu^2 \equiv \mu_0^2 + a\delta''(0)$, we get

$$|C_1|^2 = \frac{a}{\bar{\lambda}_4}\left(\bar{\mu}_4^2 - \mu^2\right).$$

The condition

$$\left\langle H(0,0)\right\rangle = a^{-1/2}\frac{v}{\sqrt{2}}$$

gives

$$C_1 = a^{-1/2}\frac{v}{\sqrt{2}}, \qquad \frac{v^2}{2} = \frac{\bar{\mu}_4^2 - \mu^2}{\bar{\lambda}_4}. \tag{18}$$

The graviphoton mixing parameter M introduced above is given by

$$M = \sqrt{2a}\,\frac{\mu_0}{v}\,\mathrm{Im}\{C_2\}.$$

Taking for simplicity C_2 to be purely imaginary, we may rewrite the vacuum solution as

$$H(0, y) = a^{-1/2}\frac{v}{\sqrt{2}}\left(a\delta(y) + i\frac{M}{\mu_0}\sinh(\mu_0 y)\right). \tag{19}$$

The solution can be periodic in y by choosing a purely imaginary μ_0. This is not a problem as the physical Higgs mass, as we will see, is shifted by $a\delta''(0)$.

[3] An alternative solution, with $\mu_0^2 < 0$, is

$$H(0, y) = C_1\delta(y) + C_2'\sin(|\mu_0|y).$$

Denoting by $M_B^2 = g^2 v^2$ the gauge boson mass in the absence of the graviphoton, we obtain for the mixed mass-matrix of vector bosons

$$\mathcal{M}_V^{(0)^2} = \begin{pmatrix} M_B^2 & M_B^2 \tan\zeta \\ M_B^2 \tan\zeta & M_A^2 \end{pmatrix}, \tag{20}$$

where we have defined

$$\tan\zeta = \frac{1}{g}\left(\frac{M}{M_P}\right). \tag{21}$$

The mass eigenstates are the gauge fields

$$A_\mu^{(1)} = \cos\xi\, B_\mu - \sin\xi\, A_\mu, \qquad A_\mu^{(2)} = \sin\xi\, B_\mu + \cos\xi\, A_\mu, \tag{22}$$

where

$$\tan 2\xi = \frac{2\tan\zeta}{\left[\left(\frac{M_A}{M_B}\right)^2 - 1\right]} \tag{23}$$

and the corresponding masses of $A_\mu^{(1,2)}$ are

$$M_{(1)}^2 = \frac{1}{2}\left(M_A^2 + M_B^2 + \sqrt{\left(M_B^2 - M_A^2\right)^2 + 4M_B^4 \tan^2\zeta}\right), \tag{24}$$

$$M_{(2)}^2 = \frac{1}{2}\left(M_A^2 + M_B^2 - \sqrt{\left(M_B^2 - M_A^2\right)^2 + 4M_B^4 \tan^2\zeta}\right). \tag{25}$$

4. The Standard Model with a bulk Higgs

Let us consider now the same 5D spacetime $M^4 \times S^1$ with a 4D brane embedded in it. All Standard Model fields, except the Higgs $SU(2)_L$ doublet, are localized on the brane. The Higgs field experiences the full 5D bulk. Nevertheless, gauge interactions are strictly four-dimensional. The same is true for Yukawa interactions as well. The model is actually a minimal embedding of the Standard Model in extra dimensions with the Higgs doublet as the only field that lives in the 5D bulk.

Ignoring, for the moment, gravity and the graviphoton, the action for this model may be written as

$$\mathcal{S}_{\rm SM} = \int d^5x\, \sqrt{-g}\left(-|D_M H|^2 - V^{(5)}(H) - \mathcal{L}_{\rm SM}\delta(y)\right), \tag{26}$$

where $H(x, y)$ is an $SU(2)_L$ isodoublet with hypercharge $1/2$

$$H = \begin{pmatrix} H^{(+)} \\ H^{(0)} \end{pmatrix}, \tag{27}$$

and the covariant derivatives are given by

$$D_M = \begin{cases} D_\mu \equiv \partial_\mu - \frac{i}{2} g' \hat{\delta}(y) B_\mu - \frac{i}{2} g \hat{\delta}(y) \vec{W}_\mu \cdot \vec{\tau}, & \mu = 0, \ldots, 3. \\ D_5 \equiv \partial_5. \end{cases} \tag{28}$$

The Higgs potential can be taken to be

$$V^{(5)} = -\mu_5^2 \left|H(x, y)\right|^2 + \lambda_5 \left|H(x, y)\right|^4 \approx \mu_0^2 \left|H(x, y)\right|^2 + \cdots. \tag{29}$$

Localized Higgs self-interactions $V^{(4)}$ of analogous form are present in $\mathcal{L}_{\rm SM}$ as well, namely

$$V^{(4)} = -\mu_4^2 \left|H(x, 0)\right|^2 + \frac{\lambda_4}{2} \left|H(x, 0)\right|^4. \tag{30}$$

The theory is invariant under the set of 4D-*gauge transformations* of the $SU(2)_L \times U(1)_Y$ gauge group

$$\delta H(x,y) = \frac{i}{2}\hat{\delta}(y)\big(g\vec{\omega}(x)\cdot\vec{\tau} + g'\omega(x)\big)H(x,y),$$
$$\delta\vec{W}_\mu(x) = \partial_\mu\vec{\omega}(x) + g\vec{\omega}(x)\times\vec{W}_\mu(x), \qquad \delta B_\mu = \partial_\mu\omega(x), \tag{31}$$

provided we add to the action the local term

$$\mathcal{S}_1 = -a\int d^4x\big(H^{\dagger\prime\prime}(x,0)H(x,0) + H''(x,0)H^\dagger(x,0)\big). \tag{32}$$

The variation of this term cancels the variation

$$\delta\mathcal{S}_{\rm SM} = -\frac{i}{2}a\int d^4x\big\{H^{\dagger\prime\prime}(x,0)(g\vec{\omega}\cdot\vec{\tau} + g'\omega)H(x,0) - H^\dagger(x,0)(g\vec{\omega}\cdot\vec{\tau} + g'\omega)H''(x,0)\big\}$$

so that the total action $\mathcal{S} = \mathcal{S}_{\rm SM} + \mathcal{S}_1$ is $SU(2)_L \times U(1)_Y$-invariant.

The graviphoton mixing to the neutral gauge fields proceeds as before. The relevant mixing terms are

$$\begin{aligned}\mathcal{S}_m = {}& -(RM_P)^2\int d^4x\sqrt{-g}\left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M_A^2A_\mu A^\mu\right) - \int d^5x\sqrt{-g}\big(D_\mu H^\dagger D^\mu H + V\big)\\ & - \int d^5x\sqrt{-g}\big(1 + R^2A_\mu A^\mu\big)\partial_5H^\dagger\partial_5H + R\int d^5x\sqrt{-g}A^\mu\big(D_\mu H^\dagger\partial_5H + \partial_5H^\dagger D_\mu H\big)\\ & - \frac{1}{4}\int d^4x\sqrt{-g}\big(1 + R^2A_\kappa A^\kappa\big)B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}\int d^4x\sqrt{-g}\big(1 + R^2A_\kappa A^\kappa\big)\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu},\end{aligned} \tag{33}$$

where $B_{\mu\nu}$, $\vec{W}_{\mu\nu}$ are the $U(1)$ and $SU(2)$ field strengths, respectively and V stands for $V^{(5)} + \delta(y)V^{(4)}$.

Let us now consider the Higgs vacuum solution (19)

$$H(0,y) = a^{-1/2}\left(a\delta(y) + i\frac{M}{\mu_0}\sinh(\mu_0 y)\right)\langle\phi\rangle \tag{34}$$

and introduce a four-dimensional Higgs field isodoublet $\phi(x)$ as

$$\phi(x) \equiv H(x,0)a^{1/2} = H(x,0)\delta(0)^{-1/2}.$$

Then, the emerging mass terms for the graviphoton and the gauge vectors turns out to be

$$\begin{aligned}\mathcal{S}_{\rm mass} = {}& -\int d^4x\sqrt{-g}\left(\frac{1}{2}\frac{T'}{M_P^2}A_\mu A^\mu + \frac{g^2}{4}\vec{W}_\mu\cdot\vec{W}^\mu|\phi|^2 + \frac{g'^2}{4}B_\mu B^\mu|\phi|^2 + \frac{gg'}{2}B^\mu\vec{W}_\mu\cdot(\phi^\dagger\vec{\tau}\phi)\right.\\ & \left. + g'\frac{M}{M_P}|\phi|^2A_\mu B^\mu + g\frac{M}{M_P}A^\mu\vec{W}_\mu\cdot\big(\phi^\dagger\vec{\tau}\phi\big) + R^2M^2|\phi|^2A_\mu A^\mu\right).\end{aligned} \tag{35}$$

We have introduced a canonically normalized graviphoton field through the rescaling $A_\mu \to A_\mu/(RM_P)$. Substituting the VEV

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$$

we obtain the mass terms

$$\mathcal{S}_{\rm mass} = -\int d^4x\sqrt{-g}\left(\frac{1}{2}M_A^2A_\mu A^\mu + M_W^2W_\mu^+W^{-\mu} + \frac{1}{2}M_Z^2Z_\mu Z^\mu - \frac{vM_W}{\cos\theta_W}\left(\frac{M}{M_P}\right)A_\mu Z^\mu\right),$$

where

$$M_A^2 = \frac{T'}{M_P^2} + R^2M^2v^2, \qquad M_W^2 = \frac{g^2v^2}{4}, \qquad M_Z^2 = \frac{M_W^2}{\cos^2\theta_W}. \tag{36}$$

As usual, we have introduced $\tan\theta_W = \frac{g'}{g}$ and

$$W^\mu_\pm = \frac{1}{\sqrt{2}}\big(W^\mu_1 \pm i W^\mu_2\big), \qquad Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu. \tag{37}$$

The photon $\sin\theta_W W^3_\mu + \cos\theta_W B_\mu$ stays massless while the graviphoton is mixed with the neutral Z-boson. The neutral vector mass matrix is

$$\mathcal{M}^2 = \begin{pmatrix} M_Z^2 & -RMvM_Z \\ -RMvM_Z & M_A^2 \end{pmatrix} = \begin{pmatrix} M_Z^2 & -\tan\zeta M_Z^2 \\ -\tan\zeta M_Z^2 & M_A^2 \end{pmatrix} \tag{38}$$

and we have introduced the mixing angle

$$\tan\zeta \equiv \frac{v}{M_Z}\left(\frac{M}{M_P}\right). \tag{39}$$

The mass eigenstates are

$$Z_{1\mu} = \cos\xi Z_\mu - \sin\xi A_\mu, \qquad Z_{2\mu} = \sin\xi Z_\mu + \cos\xi A_\mu, \tag{40}$$

where

$$\tan 2\xi = \frac{2\tan\zeta}{\frac{M_A^2}{M_Z^2} - 1}. \tag{41}$$

The corresponding masses of Z_1, Z_2 are

$$\begin{aligned} M^2_{Z_1} &= \frac{1}{2}\Big(M_A^2 + M_Z^2 - \sqrt{\big(M_A^2 - M_Z^2\big)^2 + 4\tan^2\zeta M_Z^4}\Big), \\ M^2_{Z_2} &= \frac{1}{2}\Big(M_A^2 + M_Z^2 + \sqrt{\big(M_A^2 - M_Z^2\big)^2 + 4\tan^2\zeta M_Z^4}\Big). \end{aligned} \tag{42}$$

For $M_A \gg M_Z$, we get

$$M^2_{Z_1,Z_2} \approx \frac{1}{2}\left\{M_A^2 + M_Z^2 \mp M_A^2\left(1 - \left(\frac{M_Z}{M_A}\right)^2 + 2\tan^2\zeta\left(\frac{M_Z}{M_A}\right)^4\right)\right\} \approx \begin{cases} M_Z^2\big(1 - \tan^2\zeta \frac{M_Z^2}{M_A^2}\big) + \cdots, \\ M_A^2 + \cdots. \end{cases}$$

Identifying $Z_{1\mu}$ with the neutral gauge boson produced at LEP, we may write

$$\frac{M_W^2}{M_{Z_1}^2} \approx \cos^2\theta_W\left(1 + \frac{\tan^2\zeta}{\cos^2\theta_W}\left(\frac{M_W^2}{M_A^2}\right)\right).$$

As a result, the graviphoton–Z boson mixing, gives a departure for the Standard Model value ($\rho = 1$) of the parameter $\rho \equiv M_W^2/\cos^2\theta_W M_{\mathcal{Z}}^2$

$$\delta\rho = \frac{\tan^2\zeta}{\cos^2\theta_W}\left(\frac{M_W^2}{M_A^2}\right) = \frac{4}{g^2}\left(\frac{M}{M_P}\right)^2\left(\frac{M_W}{M_A}\right)^2 \tag{43}$$

which is positive.

Moreover, we may consider the neutral current Lagrangian of Z_μ

$$\mathcal{L}_{\mathrm{NC}} = \frac{g}{\cos\theta_W}\frac{1}{2}\sum_i \bar\psi_i\gamma_\mu\big(T_{3i}(1-\gamma_5) - 2Q_i\sin^2\theta_W\big)\psi_i Z^\mu. \tag{44}$$

Changing to mass eigenstates, we get

$$\mathcal{L}_{\rm NC} = \frac{g}{\cos\theta_W}\frac{1}{2}\sum_i \bar{\psi}_i\gamma_\mu\big(T_{3i}(1-\gamma_5) - 2Q_i\sin^2\theta_W\big)\psi_i\big(\cos\xi Z_1^\mu + \sin\xi Z_2^\mu\big) \tag{45}$$

which includes a coupling of the physical graviphoton Z_2^μ to matter proportional to $\sin\xi$. For $M_A \gg M_Z$, this coupling is of order

$$\xi \approx (\delta\rho)^{1/2}\left(\frac{M_Z}{M_A}\right).$$

Then, constraints on the ratio M/M_A may be obtained by considering the Z_1-partial width to fermions $Z_1 \to f\bar{f}$ [12–14]. Indeed, with a shift of the ρ parameter, after taking account higher Higgs representations and m_t effects, of order $\lesssim 10^{-3}$ [15], we get

$$\frac{M}{M_A} \lesssim 10^{15} \implies M_A \gtrsim 10^{-15}(g\tan\zeta)M_P. \tag{46}$$

Thus, for $g\tan\zeta \sim O(1)$, we may have

$$M_A > 10\ \text{TeV}.$$

5. Quantum effects on the brane

Radiative processes of Standard Model particles will also, in general, involve virtual bulk fields that interact with them. In the present model the Higgs field has been introduced as a bulk field and we would expect that its KK excitations will contribute to loop processes on the brane. For example, the gauge couplings are expected to receive contributions from the Higgs that involve the bulk [11]. In the simplified $U(1)$ model of Section 1, the lowest order Higgs contributions to the vacuum polarization are[4]

$$g^2\frac{a}{R^2}\int\frac{d^4q}{(2\pi)^2}\tilde{A}_\mu(q)\tilde{A}^\mu(-q)\left\{\int\frac{d^4p}{(2\pi)^2}\sum_n\sum_{n'}\mathcal{D}(p;\omega_n,\omega_{n'})\right\}$$
$$+\,g^2\frac{a^2}{R^4}\int\frac{d^4q}{(2\pi)^2}\tilde{A}^\mu(q)\tilde{A}^\nu(-q)\int\frac{d^4p}{(2\pi)^2}\sum_{n,n',n'',n'''}(2q_\mu+p_\mu)(2q_\nu+p_\nu)\mathcal{D}(p;\omega_n,\omega_{n'})$$
$$\times\,\mathcal{D}(q+p;\omega_{n''},\omega_{n'''}),$$

where $\mathcal{D}(p;\omega_n,\omega_{n'})$ is the Fourier transform of the Higgs propagator

$$\mathcal{G}(x-x';y,y') = \frac{1}{(2\pi)^2}\int d^4p\,e^{ip\cdot(x-x')}\frac{1}{R}\sum_n e^{i\omega_n y}\frac{1}{R}\sum_{n'}e^{i\omega_{n'}y'}\mathcal{D}(p;\omega_n,\omega_{n'}). \tag{47}$$

Notice that only the propagator $G(x-x';0,0)$ with end points on the brane appears in these graphs. The above sum can be written as

$$g^2\int\frac{d^4q}{(2\pi)^2}\tilde{A}_\mu(q)\tilde{A}^\mu(-q)\int\frac{d^4p}{(2\pi)^2}\bar{\mathcal{D}}(p)$$
$$+\,g^2\int\frac{d^4q}{(2\pi)^2}\tilde{A}^\mu(q)\tilde{A}^\nu(-q)\int\frac{d^4p}{(2\pi)^2}(2q_\mu+p_\mu)(2q_\nu+p_\nu)\bar{\mathcal{D}}(p)\bar{\mathcal{D}}(q+p) \tag{48}$$

[4] The frequencies ω_n are $2\pi n/R$.

in terms of

$$\bar{\mathcal{D}}(p) = \frac{a}{R^2} \int \sum_{n,n'} \mathcal{D}(p; \omega_n, \omega_{n'}) = a \int \frac{d^4x}{(2\pi)^2} e^{-ip\cdot(x-x')} G(x - x'; 0, 0). \tag{49}$$

The expression (48) is a standard four-dimensional expression for the vacuum polarization. The existence of the fifth dimension is encoded in the Higgs propagator $G(x - x'; y, y')$. Note however that this is not the free 5D propagator, satisfying[5]

$$\left(-\mu_0^2 + \partial_\mu \partial^\mu + \partial_5^2\right) \mathcal{G}_0(x - x'; y, y') = \delta^{(4)}(x - x')\delta(y - y').$$

In order to maintain four-dimensional gauge invariance, we have introduced the extra local piece $\mathcal{S}_1$ in the brane action. This has the effect to modify the Higgs propagator. The modified propagator satisfies the equation[6]

$$\left[-\mu_0^2 + \partial_\mu \partial^\mu + \partial_5^2 - a\left(\partial_5^2 \delta(y) + \delta(y)\partial_5^2\right)\right] \mathcal{G}(x - x'; y, y') = \delta^{(4)}(x - x')\delta(y - y'). \tag{50}$$

The solution of this equation, although lengthy,[7] proceeds in a straightforward fashion and leads to

$$\bar{D}(p) = -\frac{1}{(2\pi)^2}\left(\frac{1}{p^2 + \mu^2}\right) \tag{51}$$

with $\mu^2 = \mu_0^2 + a\delta''(0)$. This is just a free four-dimensional propagator. It is in sharp contrast to the free propagator which should go as

$$-\frac{a}{(2\pi)^3} \frac{1}{R} \sum_n \frac{1}{p^2 + \mu_0^2 + \omega_n^2}$$

and contains the contributions of the infinite tower of KK states. The constraint of 4D-gauge invariance has removed all these contributions from processes on the brane.

We should also note that for $y, y' \neq 0$, we have

$$\tilde{\mathcal{G}}_1(p; y, y') = -\frac{1}{(2\pi)^2}\left\{K(y - y') - \frac{K(y)K(y')}{K(0)}\right\}$$

[5] The parameter μ_0^2 plays the role of an effective mass.

[6] Note that $a\delta(0) = 1$.

[7] As a shortcut, we may consider the Fourier transform

$$\tilde{\mathcal{G}}_1(p; y, y') = \frac{1}{(2\pi)^2} \int d^4x\, e^{-ip\cdot(x-x')} \mathcal{G}(x - x'; y, y')$$

which satisfies the equation

$$\left\{-\mu_0^2 - p^2 + \frac{\partial^2}{\partial y^2} - a\left(\delta(y)\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial y^2}\delta(y)\right)\right\} \tilde{\mathcal{G}}_1(p; y, y') = \frac{\delta(y - y')}{(2\pi)^2}$$

and verify by substitution that its solution is

$$\tilde{\mathcal{G}}_1(p; y, y') = -\frac{1}{(2\pi)^2}\left\{\frac{a}{p^2 + \mu^2}\delta(y)\delta(y') + K(y - y') - \frac{K(y)K(y')}{K(0)}\right\},$$

where $(-R/2 \leqslant z \leqslant R/2)$

$$K(z) \equiv \frac{1}{R} \sum_{n=-\infty}^{+\infty} \frac{e^{i\omega_n z}}{\bar{p}^2 + \omega_n^2} = \frac{1}{2\bar{p}} \frac{\cosh(\bar{p}(R/2 - |z|))}{\sinh(\bar{p}R/2)},$$

$\bar{p}^2 \equiv p^2 + \mu_0^2$ and $\mu^2 = \mu_0^2 + a\delta''(0)$. The Green's function $\tilde{\mathcal{G}}_1$ is related to $\bar{D}(p)$ as $\bar{D}(p) = a\tilde{\mathcal{G}}_1(p, 0, 0)$.

and the Higgs can propagate within the bulk. However, in addition to processes on the brane, a Higgs field cannot propagate from the brane to the bulk. This is clear from the propagator $\tilde{\mathcal{G}}_1$ for which

$$\tilde{\mathcal{G}}_1(p;0,y') = -\frac{1}{(2\pi)^2}\frac{\delta(y')}{p^2+\mu^2}.$$

This is consistent with charge conservation, as charge cannot escape to the bulk through process like, i.e., $Z_1 W^+ \to H^+ H^0$.

6. Brief summary

The present Letter was motivated by the (im)possibility of propagation of fields (scalars or fermions) in the bulk if gauge fields are localized on a brane. Indeed, it seems that gauge invariance requires the presence of gauge fields (photons) in the bulk as well. Then, from the brane-point of view, gauge invariance would be violated and charge would escape to the bulk. However, one may try to define gauge invariance strictly as a local symmetry of four dimensions and keep gauge fields absolutely trapped on the brane. In such a formulation, gauge invariance turns out to modify the action of the charged bulk fields so that, although they can propagate freely within the bulk, they cannot propagate from the brane towards the bulk. Apart from their bulk self-interactions, in all other interactions occurring on the brane they behave as ordinary four-dimensional fields. From the point of view of the brane, such a theory would be indistinguishable from a theory in which all charged fields are confined on the brane. Nevertheless, in the particular example of the Standard Model with a bulk Higgs isodoublet field that we have considered, a bulk Higgs field opens the possibility of a vacuum expectation value that depends on the fifth dimension. Such a VEV induces new interaction terms of gravitational origin that can lead to observable results. The five dimensional metric of the compactified $M^4 \times S^1$ space of the model includes the *graviphoton* $A_\mu(x)$, a massive vector field that owes its mass to the breaking of translational invariance of the fifth dimension by the brane. The solution (19) for the Higgs vacuum allows for the coupling $A_\mu(D^\mu H)\partial_5 H^\dagger$. This coupling induces a mixing between the graviphoton and the massive neutral vector boson of the Standard Model. This coupling modifies slightly the mass eigenvalues and the sensitive mass ratio $M_W/M_{\mathcal{Z}}$.

An important issue of the model presented is the issue of singularities. It should be noted that the singular object $\delta(0) = a^{-1}$ does not appear in observable quantities. Similarly, $a\delta''(0)$ gets absorbed in a mass redefinition. Although, we have not introduced a regularization scheme, we do not expect to encounter any further problem associated with singularities arising due to the zero thickness of the brane. In the framework of a regularization scheme the brane would effectively acquire a non-zero thickness. Nevertheless, we feel confident that the main results obtained within our treatment would not change. It should be noted that our treatment is analogous to the treatment of singularities in the Horava–Witten theory [1,16] where a number of cancellations among singular terms result in unambiguous physical expressions. Another issue is of course renormalizability of the model. In the presence of bulk Higgs interaction given by a potential $V^{(5)}$, the model is clearly non-renormalizable. However, we may consider just a bulk mass term for the Higgs with no bulk interactions at all, while all interactions, including the self-interactions, to be strictly localized on the brane. This model should be renormalizable as in 4D is renormalizable in the usual sense and in the 5D bulk, it is a free field.

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