Catalysis of proton decay by cosmic strings at finite temperature

Andrew Matheson, Leandros Perivolaropoulos, Warren Perkins [1], Anne-Christine Davis [1,2] and Robert H. Brandenberger
Physics Department, Brown University, Providence, RI 02912, USA

Received 24 April 1990

We determine the finite temperature corrections to elastic and inelastic cross sections for the scattering of fermions by cosmic strings. In particular, we obtain the proton decay catalysis cross section at finite temperature. We conclude that, as the temperature T approaches the temperature T_c at which the phase transition takes place, model dependent zero temperature enhancement factors tend to 1.

1. Introduction

Cosmic strings [1] can be formed during phase transitions in the very early Universe. They can be separated into two distinct groups: those with abelian gauge fields excited in the string core [which arise from explicit U(1) symmetry breaking], and those with non-abelian gauge fields excited in their cores.

While the two types of string are gravitationally indistinguishable, their microphysical interactions with matter allow one to discriminate between them, at least in principle. As we have recently [2] argued, in the latter case the fields excited in the core of the string can lead to baryon decay through their interaction with baryonic matter. Although it was claimed [2] that the cross section for these processes was geometrical (determined by the string width of order $M_{\rm GUT}^{-1}$), it is nevertheless possible [3] to erase a significant fraction of a primordial net baryon asymmetry in the Universe.

More recently [4,5] it has been pointed out that the analysis of ref. [2] was excessively naive. In ref. [2], we treated the fermion charges as integer multiples of the string flux, whereas in general [4] the charges will be fractional. Consequently [4–6], fermion–string scatterings of Aharonov–Bohm (A–B) type [7,8] are possible, and can occur with a strong interaction cross section.

This raises the possibility of a cosmological disaster. If the large cross section persists to GUT temperatures, then the network of cosmic strings present at early times could reduce the baryon asymmetry by many orders of magnitude below the value required by nucleosynthesis. All non-abelian string models would then be immediately ruled out (with the proviso of hot-big-bang baryogenesis).

This prospect, though appealing to some, rests on the zero temperature cross section persisting to the GUT scale. In fact, as we show here, at sufficiently high temperatures T, the relevant scale is the finite temperature mass of the fermion, $m(T) \sim T$. Instead of going as $(1\ \mathrm{GeV})^{-1}$, the cross section per unit length of string goes as T^{-1}. At GUT scale temperatures, the enhancement of the cross section reported in ref. [5] disappears. In fact, both elastic and inelastic scattering cross sections approach their geometrical values.

2. The cross section at high temperatures

We shall use the method of ref. [2], modified for finite temperature, to obtain the cross section for fermions scattered by strings. The method consists of two steps: we first use lowest order perturbation the-

[1] Permanent address: DAMTP, University of Cambridge, Cambridge CB3 9EW, UK.
[2] Also at King's College, University of Cambridge, Cambridge, UK.

ory to compute the "naive" geometrical cross section $d\sigma/dl|_{geom}$ using free spinors. Then, we correct this result by an amplification factor A obtained by solving the Dirac equation in the presence of the long range string gauge field. A is the ratio of the magnitude of the resulting spinor divided by the magnitude of the free spinor, evaluated at the core radius of the string. The final cross section per unit length of string is

$$\frac{d\sigma}{dl} = A^{2n} \frac{d\sigma}{dl}\bigg|_{geom}, \qquad (2.1)$$

where n is the number of incoming plus outgoing fermions.

In a finite temperature analysis, the following points must be taken into account [9,10]:

(a) The free spinors used in the derivation of the geometrical cross section must be replaced by free finite temperature spinors $u_\beta(p)$ [9]. These describe particles which propagate freely in the finite T heat bath and satisfy

$$S^{-1}(p,\beta)u_\beta(p)=0, \qquad (2.2)$$

where S is the renormalized finite temperature propagator evaluated at one loop order (β is the inverse temperature).

(b) In calculating the amplification factor A, the one loop renormalized T dependent effective Dirac equation [9] must be used. This takes into account that the long range string gauge fields are acting on particles in a thermal bath. In particular, the thermal bath gives the fermions a T dependent effective mass $m(T)\sim T$, for T larger than the bare fermion mass.

(c) The density of final states used in the determination of the cross section is modified by the heat bath

$$\int \frac{d^3p}{(2\pi)^3} \to \int \frac{d^3p}{(2\pi)^3}\,[1-n_{F,\beta}(E_p)], \qquad (2.3)$$

where $n_{F,\beta}(E_p)$ is the Fermi–Dirac distribution at temperature β^{-1} for energy E_p. This takes into account the states already occupied by fermions of the heat bath. Note, however, that since we are still interested in single-particle scattering, we do not take a Boltzmann average over initial states.

(d) Finally, the modifications of the string configuration at high temperature must be taken into account.

There are technical complications when working with finite temperature spinors due to the fact that the heat bath breaks the Lorentz invariance. The one loop renormalized finite temperature propagator is given by

$$S^{-1}(p,\beta)=\tilde{\not p}-\tilde m$$
$$=(1-A)E\gamma_0-(1-B)\,\boldsymbol{p}\cdot\boldsymbol{\gamma}-m(1-C), \qquad (2.4)$$

where A, B and C are p and β dependent constants whose explicit form is given in ref. [8] (in which our mC is denoted by C). Since $A\neq B$, Lorentz invariance is broken. The pole of $S(p,\beta)$ occurs when

$$\tilde p^2-\tilde m^2=0 \quad \text{or} \quad E^2-\boldsymbol{p}^2=m^2_{phys}(\boldsymbol{p}^2,\beta). \qquad (2.5)$$

Evaluating $m^2_{phys}(\boldsymbol{p}^2,\beta)$ explicitly to one loop order, we find

$$m^2_{phys}(\boldsymbol{p}^2,T)=m_0^2+\tfrac{2}{3}\alpha\pi T^2+\Delta m^2(\boldsymbol{p}^2,T), \qquad (2.6)$$

where $\Delta m^2(\boldsymbol{p}^2,T)$ contains all the $\boldsymbol{p}^2$ dependence and m_0 is the $T=0$ mass of the fermions. At high temperatures $T\gg m_0$, the dominant term on the right hand side of (2.6) is the term proportional to T^2, as can be seen by inspection of the explicit expressions for $\Delta m^2(\boldsymbol{p}^2,T)$. Hence, for an order of magnitude analysis of the cross section at high T, it will be sufficient to treat the finite T spinors as ordinary free spinors with mass

$$m_{phys}(T)\sim\alpha^{1/2}T. \qquad (2.7)$$

Now we can turn to the computation of the geometrical cross section, following the procedure of ref. [2]. For concreteness, we consider the fermions interacting with the "shifted" Higgs field ϕ which is excited in the string core and can mediate baryon number violating processes. In terms of the original Higgs field $\tilde\phi$, ϕ is given by

$$\phi=M(T)-|\tilde\phi|, \qquad (2.8)$$

where $M(T)$ is the T dependent minimum of the one loop finite temperature effective potential $V_{T,eff}(\tilde\phi)$ for $\tilde\phi$:

$$V_{T,eff}(\tilde\phi)=\tfrac{1}{4}\lambda[\tilde\phi^2-M(T)^2]^2. \qquad (2.9)$$

In terms of the scale T_c of grand unified symmetry breaking,

$$M(T)^2 = M_{T=0}^2 \left[1 - \left(\frac{T}{T_c}\right)^2\right].\qquad(2.10)$$

The cross section is evaluated to lowest order in perturbation theory, in which the scattering amplitude $\mathscr{A}$ is given by

$$\mathscr{A} = \langle \psi', s' \mid \int d^4x\, \mathscr{L}_I / \psi, s\rangle,\qquad(2.11)$$

where $|\psi\rangle$ and $|\psi'\rangle$ are the initial and final fermion states, and $|s\rangle$ and $|s'\rangle$ the corresponding string states. Neglecting back reaction, $|s'\rangle = |s\rangle$. The interaction lagrangian is

$$\mathscr{L}_I = g\bar{\psi}\phi\psi.\qquad(2.12)$$

Since the shifted Higgs field is non-vanishing only within a distance $w \sim M^{-1}$ of the center of the string, the integral over space and time reduces essentially to w^2 times the integral over the world-sheet of the string. At $T=0$, the resulting cross section obtained by integrating $|\mathscr{A}|^2$ over the available phase space for final states is

$$\left.\frac{d\sigma}{dl}\right|_{T=0} \sim g^2 \frac{m_0}{M_{T=0}^2}.\qquad(2.13)$$

This result can be understood by dimensional analysis. The m_0 is a remnant of the spinor sums which give m_0^2, and the phase space factors which give E^{-1} (or m_0^{-1} at low energies). Furthermore, $\mathscr{A}$ is proportional to $M_{T=0} w_{T=0}^2 \sim M_{T=0}^{-1}$, the first factor coming from the amplitude of ϕ in the core, the second from integrating over the string cross section. This explains the $M_{T=0}$ dependency of the result.

We now can immediately generalize the result to $T \neq 0$. At high temperature, m_0 is replaced by $m_{\rm phys}(T)$ both in the spin sums and in the phase space factors [assuming low energies $E \sim m_{\rm phys}(T)$]. From point (d) in the general discussion at the beginning of this section it follows that the finite temperature corrections to the string configuration must also be taken into account. This means replacing $M_{T=0}$ by $M(T)$ and $w_{T=0}$ by $w(T)$. Since $w(T) \simeq M(T)^{-1}$, we obtain the finite T geometrical scattering cross section per unit length

$$\left.\frac{d\sigma}{dl}\right|_{T,\rm geom} \sim g^2 \frac{m_{\rm phys}(T)}{M(T)^2} \sim g^2 \frac{T}{M(T)^2}.\qquad(2.14)$$

The T dependence of $M(T)$ is only important for $T \sim T_c$. For $T \ll T_c$,

$$\left.\frac{d\sigma}{dl}\right|_{T \ll T_c,\rm geom} \sim g^2 \frac{T}{M_{T=0}^2}.\qquad(2.15)$$

The divergence of the cross section (2.14) for $T = T_c$ is to be expected: at T_c, the symmetry is restored and hence catalysis processes occur everywhere in space.

The second step is the determination of the amplification factor A, defined by

$$A^2 = \frac{\bar{\psi}_T \psi_T(\rho = w(T))}{\bar{\psi}_{T,\rm free}\psi_{T,\rm free}(\rho = w(T))}.\qquad(2.16)$$

Here, $\psi_{T,\rm free}$ are the free finite temperature spinors discussed above and ψ_T the spinors obtained taking into account the long range gauge fields of the string, i.e. by solving the Dirac equation in the presence of the string.

However, we must first identify the correct position space Dirac equation at finite temperature. The finite temperature Dirac spinors satisfy (2.2), with $S^{-1}(p, \beta)$ given by (2.4). Assuming a z independent solution to (2.2) we find that (2.2) decomposes into separate equations for the upper and lower two-component spinors $u_\beta^{u,\ell}(p)$:

$$(\not{\tilde{p}} - \tilde{m}) u_\beta^{u}(p) = 0, \quad (\not{\tilde{p}} + \tilde{m}) u_\beta^{\ell}(p) = 0.\qquad(2.17)$$

These two two-component Dirac equations can easily be solved, and we find that both the upper and lower components $u_\beta^u(p)$ and $u_\beta^\ell(p)$ obey the finite temperature Klein–Gordon equation

$$[p^2 - m_{\rm phys}^2(\bar{p}^2, \beta)] u_\beta^{u,\ell}(p) = 0.\qquad(2.18)$$

To infer a position space Dirac equation we neglect the momentum space dependence of $m_{\rm phys}^2$, i.e. we use (2.7) instead of (2.6).

From (2.18), we infer that the position space Dirac equation, without gauging, is

$$[i\not{\partial} - m_{\rm phys}(T)]\psi_T(x) = 0.\qquad(2.19)$$

Now that we have the correct form of the Dirac equation at finite temperature, it remains to gauge it. Fortunately, $m_{\rm phys}(T)$ is the pole in the fermion propagator and hence gauge invariant. Consequently, under a gauge transformation $\psi \to U\psi U^{-1}$ we must have $\partial_\mu \to D_\mu$ in order to preserve the gauge invariance of

(2.19). The Dirac equation, modified to finite temperature, is therefore

$$[\mathrm{i}\not{D} - m_{\mathrm{phys}}(T)]\psi_T(x) = 0. \qquad (2.20)$$

The only change compared to $T=0$ is the replacement of m_0 by $m_{\mathrm{eff}}(T)$.

The amplification factor at finite T can now be determined exactly as in ref. [2] for fermions with integer charge and in ref. [10] for fermions with fractional charge (see also refs. [4,5]). For $T=0$, the amplification factor was found to be

$$A|_{T=0} = \left(\frac{M_{T=0}}{m_0}\right)^{\xi}, \qquad (2.21)$$

where ξ is a model dependent constant which ranges from 0 to $\frac{1}{2}$ depending on the fractional charge of the fermions with respect to the flux of the string [11]. The m_0 comes from the mass in the Dirac equation, and $M_{T=0}$ comes from evaluating the wave functions at the core radius. Hence, at finite T, (2.21) is replaced by

$$A = \left(\frac{M(T)}{m_{\mathrm{phys}}(T)}\right)^{\xi}. \qquad (2.22)$$

For $m_0 \ll T \ll T_c$, A is proportional to $(T_c/T)^{\xi}$.

Combining (2.14) and (2.22), we obtain, for scattering of a single particle by a cosmic string ($n=2$), the following finite temperature cross section per unit length:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}l} \sim g^2 \frac{m_{\mathrm{phys}}(T)}{M(T)^2} \left(\frac{M(T)}{m_{\mathrm{phys}}(T)}\right)^{4\xi}. \qquad (2.23)$$

In the temperature range $m_0 \ll T \ll T_c$, this scales as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}l} \sim g^2 \frac{1}{m_{\mathrm{phys}}(T)} \left(\frac{m_{\mathrm{phys}}(T)}{T_c}\right)^{2-4\xi}$$

$$\sim g^2 \frac{1}{T} \left(\frac{T}{T_c}\right)^{2-4\xi}. \qquad (2.24)$$

Here, we have used (2.10) and the fact that $M_{T=0}$ is proportional to T_c.

3. Discussion

The most obvious question is whether the conclusions of ref. [3] regarding to the destruction of baryon number by strings, are modified significantly by the enhanced string–baryon cross sections discovered in refs. [4,5]. Based on (2.24), our main result, we find that they are not changed.

In ref. [3] we concluded that, unless the coupling constants are large, catalysis by cosmic strings is too weak to wipe out the primordial baryon to entropy ratio. These calculations were performed assuming a geometrical cross section. Here, we find that at high temperatures (i.e. early times in the evolution of the Universe), the cross sections approach the geometrical ones, even if they are enhanced at zero temperature. Hence the conclusions of ref. [3] are valid even for fractional flux on the string.

The point is that the baryon decay rate is dominated by the rate at times immediately after the phase transition. From (2.24), it follows that the model dependence (ξ dependence) of the cross section disappears as T approaches T_c [but $M(T) \simeq M_{T=0}$ still remains a good approximation]. Note that by the time the approximation $M(T) \simeq M_{T=0}$ breaks down, we are so close to the critical temperature that the description of the broken phase in terms of isolated strings, and thus the entire analysis of ref. [3], will become invalid.

A completely separate question is whether in models with cosmic strings, a significant net baryon to entropy ratio can ever be generated (in ref. [3] we assumed such a ratio had been generated by some means below T_c). Here, the divergence of the cross section as T approaches T_c becomes relevant. To answer the question, it will be important to integrate the rate equations for production and destruction of a net baryon to entropy ratio together – given a particular scenario for baryogenesis.

In this paper, we have computed the catalysis cross section at finite temperatures. We have shown that the model dependent amplification factors tend to 1 as T increases towards T_c. Exactly at T_c, the cross sections diverge (as expected). Here again, model dependence appears.

Note that our analysis applies not only to catalysis cross sections. It extends to the elastic, inelastic and Aharonov–Bohm scattering processes.

Acknowledgement

Two of us (W.P. and A.-C.D.) wish to thank the Physics Department of Brown University for hospitality. This work was supported in part by the US DOE under grant DE-AC02-76ER03130 Tasks A and K.W.P. acknowledges financial assistance by the UK SERC, and R.B. by the Alfred P. Sloan Foundation.

References

[1] Fo ra review see A. Vilenkin, Phys. Rep. 121 (1985) 265.
[2] R. Brandenberger, A.-C. Davis and A. Matheson, Nucl. Phys. B 307 (1988) 909;
R. Brandenberger and P. Perivolaropoulos, Phys. Lett. B 208 (1988) 396.
[3] R. Brandenberger, A.-C. Davis and A. Matheson, Phys. Lett. B 218 (1989) 304.
[4] M. Alford and F. Wilczek, Phys. Rev. Lett. 62 (1989) 1071.
[5] M. Alford, J. March-Russell and F. Wilczek, Nucl. Phys. B 328 (1989) 140.
[6] L. Perivolaropoulos, A. Matheson, A.-C. Davis and R. Brandenberger, Brown preprint BROWN-HET-739 (1990).
[7] Y. Aharonov and D. Bohm, Phys. Rev. 119 (1959) 485.
[8] P. de Sousa Gerbert and R. Jackiw, Commun. Math. Phys. 124 (1989) 229.
[9] J. Donoghue and B. Holstein, Phys. Rev. D 28 (1983) 340.
[10] C. Bernard, Phys. Rev. D 9 (1974) 3312;
L. Dolan and R. Jackiw, Phys. Rev. D 9 (1974) 3320;
S. Weinberg, Phys. Rev. D 9 (1974) 3357.
[11] W. Perkins, L. Perivolaropoulos, A. Matheson, A.-C. Davis and R. Brandenberger, Brown preprint (1990).