THE NEUTRINO MASSES IN SO(10) GRAND UNIFIED THEORY

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The neutrino masses and mixing are investigated in an SO(10) model in which the ten-dimensional and 126-dimensional representations are allowed to obtain vacuum expectation values. The parameters specifying the heavy Majorana neutrino mass matrix are constrained from the cosmological bound of light neutrino masses and the limits from $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. The implications of our model on 0ν-$\beta\beta$ decay and muon-number violating processes are explored.

In the standard model [1–3], the neutrinos are massless and all lepton flavours are conserved. There is no satisfactory explanation, however, for such a conservation law since it is associated with global, (non-local) gauge symmetries. It may thus be simply a low-energy phenomenon. One can obtain massive neutrinos if one suitably extends the standard model. The most general neutrino mass matrix is of the form [4]

$$\mathcal{M} = \begin{array}{c|cc} & \nu_R^{0c} & N_R^0 \\ \hline \bar{\nu}_L^0 & \mathcal{M}_\nu & \mathcal{M}_D \\ \bar{N}_L^{0c} & \mathcal{M}_D^T & \mathcal{M}_N \end{array}, \tag{1}$$

where

$$\nu_L = (\nu_e^0, \nu_\mu^0, ..., \nu_f^0)_L, \quad N_R = (N_1^0, N_2^0, ..., N_F^0)_R .$$

The submatrix $\mathcal{M}_\nu$ is called Majorana mass matrix and occurs in the presence of $|\Delta L| = 2$ interactions. It can occur if one expands only the Higgs sector of the standard model: (i) at the tree level if one introduces an isotriplet Higgs scalar whose neutral member acquires a vacuum expectation value $\Delta_L^0 = 0$; (ii) at the one-loop level if $\Delta_L^0 = 0$ via the collaborative effort of exotic Higgs scalars [5] (isotriplet, singly and doubly charged isosinglets, additional isodoublets). These scenarios can be implemented in suitable extensions of the minimal GUT SO(5) model [6].

The submatrices $\mathcal{M}_D$ and $\mathcal{M}_N$ necessarily require the extension of the fermionic sector to include the isosinglet right-handed neutrino. Then the submatrix $\mathcal{M}_D$ arises naturally à la up-quark, but it leads to unacceptably high neutrino masses. It is therefore necessary to generate the Majorana submatrix $\mathcal{M}_N$ with very large matrix elements. This can occur at tree level if one introduces an isosinglet Higgs scalar which can acquire a vacuum expectation value. This scheme can be implemented in the GUT SO(10) models [7]. In fact Witten has shown that in such models the inclusion of the singlet, which belongs to the 126-dimensional representation of SO(10), is not necessary for generating the mass $\mathcal{M}_N$ at the two-loop level [8]. In any case, if the matrix $\mathcal{M}_N$ is not singular with large eigenvalues one can get an effective light Majorana neutrino mass matrix of the form

$$\mathcal{M}_\nu^{\text{eff}} \approx \mathcal{M}_\nu - \mathcal{M}_D \mathcal{M}_N^{-1} \mathcal{M}_D^T . \tag{2}$$

Thus even if $\mathcal{M}_\nu = 0$ one can generate light Majorana neutrinos with masses of the order m_D^2/M_N which can be in agreement with observation. This is known as the "see-saw" mechanism.

The above features persist in supersymmetric extensions of the above models [9]. They are, however, modified in superstring theories [10] since the number of GUT groups and their allowed representations are severely restricted. Even though the "see-saw" mechanism is known not to be operative one can make the neutrinos sufficiently light but they tend to be Dirac particles.

In the present paper we will investigate the question of neutrino masses and mixing in the context of the GUT SO(10) model along the lines of the work of Harvey, Ramond and Reiss (HRR) [11]. More specifically, we will test the stability of the neutrino spectrum to reasonable variations of the phenomenologically otherwise unconstrained matrix $\mathcal{M}_N$. Even though the ingredients of this model are well known, for the reader's convenience we will summarize its main features here. For the generation of the fermion mass matrix, the ten-dimensional and 126-dimensional representations of SO(10) are introduced. The first one contains the **5** and **5*** representations of SU(5) and acquires vacuum expectation values in the **5** and **5*** directions. The 126-dimensional one decomposes as follows [4]:

$$126 \to 1 + 5^* + 10 + 15^* + 45 + 40, \tag{3}$$

and is allowed to acquire VEV in the 1, 5* and 45 directions. Notice that now $\mathcal{M}_\nu \simeq 0$. The first of these VEVs gives rise to the Dirac-like mass matrices [4,12] which can be chosen to be real:

$$\mathcal{M}_{(\kappa)} = \begin{pmatrix} 0 & R & 0 \\ R & S & 0 \\ 0 & 0 & T \end{pmatrix}, \quad \mathcal{M}_{(\alpha)} = \begin{pmatrix} 0 & P & 0 \\ P & 0 & Q \\ 0 & Q & V \end{pmatrix}, \tag{4a}$$

$$\mathcal{M}_{(e)} = \begin{pmatrix} 0 & R & 0 \\ R & -3S & 0 \\ 0 & 0 & T \end{pmatrix}, \quad \begin{pmatrix} 0 & P & 0 \\ P & 0 & -3Q \\ 0 & -3Q & V \end{pmatrix}, \tag{4b}$$

where κ = down quarks, α = up quarks, e = charged leptons. There are three parameters in each of the matrices so they can be fixed by the corresponding mass spectra. Thus for our purposes the matrix $\mathcal{M}_D$ can be considered completely determined as a function of the top-quark mass m_t. No information exists for the matrix $\mathcal{M}_N$. HRR, for simplicity, take it to be of the form

$$\mathcal{M}_N = \begin{pmatrix} 0 & A & 0 \\ A & 0 & 0 \\ 0 & 0 & B \end{pmatrix}, \tag{5}$$

where A and B can be chosen to be real. Then they proceed to diagonalize (2) (with $\mathcal{M}_\nu = 0$) for various values of the parameters A, B and m_t. The essential features of their calculation are

(i) the electronic neutrino is very light ($\sim 10^{-5}$ eV) and uncoupled to the other two;

(ii) the other two neutrinos admix. The lighter has a stable mass ($\sim$0.25 eV) while the heavier one has a mass which depends on the parameters of the model and varies between 2 and 180 eV;

(iii) only two of the heavy neutrinos get admixed and they become degenerate.

As a result, $\nu_e \leftrightarrow \nu_\mu$ oscillations, muon-number violating processes and $\beta\beta$ decay are suppressed. Furthermore decay experiments which attempt to measure neutrino masses become very hard.

We would like therefore to test the stability of the HRR results. One could expand their model by introducing the 120-dimensional representation [13] of SO(10) which has the novel feature that couples antisymmetrically in flavour space and contains the isosinglet S [5]. This, however, will make the whole thing intractable. In any case, introducing new Higgs scalars is of no help since there is no symmetry to guide us in choosing the Yukawa couplings. We notice that the matrices $\mathcal{M}_\alpha$ and $\mathcal{M}_D$ are of Fritzsch type. We also recall that in Witten's mechanism the matrix $\mathcal{M}_N$ is of Fritzsch type (proportional to $\mathcal{M}_\alpha$). We therefore feel that it is reasonable to assume a form which is a compromise between HRR and Witten's form and take the matrix $\mathcal{M}_N$ to be of the form

$$\mathcal{M}_N = \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix}. \tag{6}$$

The parameters A, B and C are related to the right-hand neutrino eigenmasses M_1, M_2 and M_3 as follows:

$$A = \left(\frac{M_1 M_2 M_3}{M_3 - M_2 + M_1}\right)^{1/2}, \quad C = \left(\frac{(M_3 - M_2)(M_2 - M_1)(M_3 - M_1)}{M_3 - M_2 + M_1}\right)^{1/2}, \quad B = M_3 - M_2 + M_1 \tag{7}$$

(we have assumed that $M_3 > M_2 > M_1$).We thus get

$$A \simeq (M_1 M_2)^{1/2} < C \simeq (M_1 M_3)^{1/2} < B \simeq M_3. \tag{8}$$

Thus B cannot exceed the value of 10^{14} GeV which is the GUT scale.

The effective light neutrino mass matrix now becomes

$$-\mathcal{M}_\nu^{\mathrm{eff}} \approx \mathcal{M}_D \mathcal{M}_N^{-1} \mathcal{M}_D^{\mathrm{T}} = \begin{pmatrix} 0 & \alpha & 0 \\ \alpha & \delta & \beta \\ 0 & \beta & \gamma \end{pmatrix}, \tag{9a}$$

where

$$\alpha = P^2/A, \quad \beta = -3Q(P/A + V/A) - VPC/BA, \quad \gamma = V^2/B, \quad \delta = 3(Q/B)(PC/A + 3Q). \tag{9b}$$

We will vary the parameters A, B and C subject to the conditions

(i) the cosmological bound on neutrino masses [14]

$$m_{\nu_1} + m_{\nu_2} + m_{\nu_3} < 40 \text{ eV}, \tag{10}$$

(ii) the neutrino oscillation limits [15]

$$P(\nu_e \leftrightarrow \nu_\mu) \approx 0. \tag{11}$$

The $\nu_\mu \leftrightarrow \nu_\tau$ oscillation experiments can be interpreted as arising from fairly large ν_μ, ν_τ mixing. One thus finds

$$m_{\nu_3}^2 - m_{\nu_2}^2 < 7.9\ (\text{eV})^2 = 7.9 \times 10^{-18}\ (\text{GeV})^2. \tag{12}$$

The eigenvalues of the matrix $(m_\nu)_{\mathrm{eff}}$ given by eq. (9) are the roots of the equation

$$\lambda^3 - (\gamma + \delta)\lambda^2 + (\delta\gamma - \beta^2 - \alpha^2)\lambda + \alpha^2\gamma = 0.$$

These roots satisfy the conditions

$$\lambda_1 + \lambda_2 + \lambda_3 = \gamma + \delta, \quad \lambda_1\lambda_2\lambda_3 = -\alpha^2\gamma, \quad \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = \gamma\delta - \alpha^2 - \beta^2.$$

Assuming that $|\lambda_1| \ll |\lambda_{2,3}|$ we find

$$\lambda_1 = -\alpha^2\gamma/(\gamma\delta - \beta^2 - \alpha^2) \simeq \alpha^2\gamma/(\beta^2 - \gamma\delta), \quad \lambda_{2,3} \approx \{\gamma + \delta \pm [(\gamma - \delta)^2 + 4\beta^2]^{1/2}\}/2. \tag{12a,b}$$

Noting that one of the $\lambda_{2,3}$ is negative and the masses are chosen positive we rewrite the eqs. (10) and (11)

$$[(\gamma - \delta)^2 + 4\beta^2]^{1/2} < 40 \text{ eV}, \quad (\gamma + \delta)[(\gamma - \delta)^2 + 4\beta^2]^{1/2} < 7.9\ (\text{eV})^2. \tag{13,14}$$

Using relation (13) we can cast (14) in the form

$$\gamma + \delta < 0.2\ \text{eV}. \tag{15}$$

Eq. (15) can be cast in the form

$$V^2 + 9Q^2 + 3QPC/A < 2 \times 10^{-10} B. \tag{16}$$

The quantities P, Q and V can be determined from the observed quark spectrum. Using $m_u = 0.1$, $m_c = 1.4$ and $m_t = 45$ GeV, we obtain $P = 0.125$, $Q = 8.19$ and $V = 43.51$ GeV. Thus eq. (14) becomes

$$C/A < 6.5 \times 10^{-11} B - 800. \tag{17}$$

For C and A to have the same sign we get $B \gtrsim 1.2 \times 10^{13}$ GeV. We will adopt the value $B \approx 3 \times 10^{13}$ GeV which is close to the GUT scale. For this choice we get

$$C/A < 10^3, \quad C < B. \tag{18}$$

Noticing now that $|\beta| \gg |\gamma - \delta|$ and inserting the limit (18) in eq. (13) we obtain

$$3QBP/A \lesssim 2 \times 10^{-8} B + V^3/3Q, \tag{19}$$

from which we obtain the limit

$$A \gtrsim 1.5 \times 10^8\ \text{GeV}. \tag{20}$$

We thus construct the neutrino mass matrix varying the parameters A, B and C subject to the conditions (18) and (20). After diagonalizing this matrix we obtain the mass eigenstates ν and N and the relationship [4]

$$\begin{pmatrix} \nu^0 \\ \mathrm{N}^{0c} \end{pmatrix}_{\mathrm{L}} = \begin{pmatrix} S^{(11)} & S^{(12)} \\ S^{(21)} & S^{(22)} \end{pmatrix} \begin{pmatrix} \nu \\ \mathrm{N} \end{pmatrix}_{\mathrm{L}}, \quad \begin{pmatrix} \nu^{0c} \\ \mathrm{N}^0 \end{pmatrix}_{\mathrm{R}} = \begin{pmatrix} S^{*(11)} & S^{*(12)} \\ S^{*(21)} & S^{*(22)} \end{pmatrix} \begin{pmatrix} \exp(-\mathrm{i}\alpha)\,\nu \\ \exp(-\mathrm{i}\phi)\,\mathrm{N} \end{pmatrix}_{\mathrm{R}}, \tag{21}$$

where $\exp(-\mathrm{i}\alpha_j)$ and $\exp(-\mathrm{i}\phi_j)$ are the *CP* eigenvalues of the neutrino eigenstates ν_j and N_j, respectively, which in our model coincide with the sign of the eigenmasses. In fact for all values of the parameters the submatrices $S^{(11)}$ and $S^{(22)}$ are almost unitary and take the form

$$S^{(11)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_\nu & \sin\theta_\nu \\ 0 & -\sin\theta_\nu & \cos\theta_\nu \end{pmatrix}, \quad S^{(22)} = \begin{pmatrix} \cos\theta_{\mathrm{N}} & \sin\theta_{\mathrm{N}} & 0 \\ -\sin\theta_{\mathrm{N}} & \cos\theta_{\mathrm{N}} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{22}$$

where $\theta_\nu \approx \pi/4$, while θ_{N} takes values around $\pi/4$ when B lies between 10^9 and 10^{10} GeV. The light neutrino spectrum scales with $\sqrt{m_t}$ in agreement with the HRR model. For a given fixed B it is pretty much independent of C and scales with A as shown in fig. 1. m_{ν_1} is very small in the range of 10^{-4}–10^{-5} eV which is in agreement with the HRR model. We find, however, that the other two eigenmasses are almost degenerate and in the range of 1–20 eV. This must be contrasted with the HRR model prediction mentioned above. We thus see that the introduction of the parameter C has a strong effect in the light neutrino sector. Its effects on the heavy neutrino sector are less dramatic. Its essential effect is to remove the degeneracy of M_{N_1} and M_{N_2}.

Before examining some further phenomenological implications of our model we remind the reader that the charged currents take the form

$$J^{\mathrm{L}}_\mu = 2(\bar{\mathrm{e}}_{\mathrm{L}}\gamma^\mu U^{(11)}\nu_{\mathrm{L}} + \bar{\mathrm{e}}_{\mathrm{L}}\gamma^\mu U^{(12)}\mathrm{N}_{\mathrm{L}}) + \text{h.c.}, \quad J^{\mathrm{R}}_\mu = 2(\bar{\mathrm{e}}_{\mathrm{R}}\gamma^\mu U^{(21)}\nu_{\mathrm{R}} + \bar{\mathrm{e}}_{\mathrm{R}}\gamma^\mu U^{(22)}\mathrm{N}_{\mathrm{R}}) + \text{h.c.},$$

where, in our model, $U^{(kl)} = S^{(e)+}S^{(kl)}$, $k, l = (11, 12, 21, 22)$ and $S^{(e)}$ is the charged lepton mixing mass matrix which diagonalizes the matrix $\mathcal{M}_{(e)}$ of eq. (4b) which takes the form

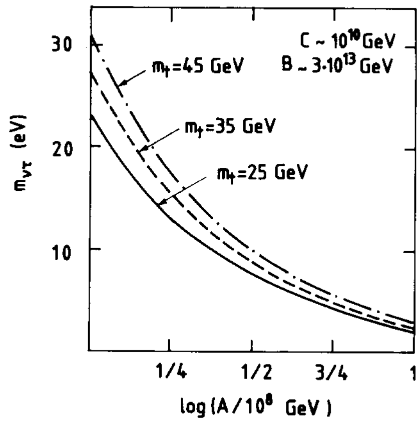

Fig. 1.

$$S^{(e)} = \begin{pmatrix} \cos\theta_e & \sin\theta_e & 0 \\ -\sin\theta_e & \cos\theta_e & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \tan\theta_e \approx \sin\theta_e = \sqrt{m_e/m_\mu} \approx 0.07.$$

We thus find that

$$U^{(11)} = \begin{pmatrix} \cos\theta_e & \sin\theta_e\cos\theta_\nu & -\sin\theta_e\sin\theta_\nu \\ -\sin\theta_e & \cos\theta_e\cos\theta_\nu & -\cos\theta_e\sin\theta_\nu \\ 0 & \sin\theta_\nu & \cos\theta_\nu \end{pmatrix},$$

$$U^{(22)} = \begin{pmatrix} \cos(\theta_e+\theta_N) & \sin(\theta_e+\theta_N) & 0 \\ -\sin(\theta_e+\theta_N) & \cos(\theta_e+\theta_N) & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The expressions for $U^{(12)}$ and $U^{(21)}$ can easily be obtained but they are not going to be given here (see ref. [4]). For our best choice of parameters $A \sim 2 \times 10^8, B \sim 3 \times 10^{13}$ and $C \sim 10^{11}$ we can get

$$\theta_\nu \approx \pi/4, \quad m_{\nu_1} \approx 1.5 \times 10^{-6}\ \text{eV}, \quad m_{\nu_2} \approx 17.27\ \text{eV}, \quad m_{\nu_3} \approx 17.40\ \text{eV},$$

$$\theta_N \approx \pi/7.5, \quad M_1 \simeq 0.798 \times 10^8\ \text{GeV}, \quad M_2 \simeq 3.96 \times 10^8\ \text{GeV}, \quad M_3 \simeq 3 \times 10^{13}\ \text{GeV}.$$

Thus we can make the following phenomenological predictions.

(i) Neutrino oscillations. Following the formalism described elsewhere [4] we find

$$P(\nu_e \leftrightarrow \nu_\mu) \lesssim 2 \times 10^{-2}, \quad P(\nu_e \leftrightarrow \nu_\tau) \lesssim 8 \times 10^{-3}, \quad P(\nu_\tau \leftrightarrow \nu_\mu) \simeq \sin^2(5\,L\,(\text{km})/E_\nu\,(\text{GeV})).$$

For the value $L/E_\nu \approx 0.03$ km/GeV of ref. [15] we get

$$P(\nu_\tau \leftrightarrow \nu_\mu) \simeq 10^{-2}.$$

(ii) Neutrinoless ββ decay. We find the following eigenmasses:

$$\langle m_\nu \rangle_L \approx \sin^2\theta_e \cos^2\theta_\nu\,(m_2 - m_3) \simeq 3.2 \times 10^{-4}, \quad \langle M_N \rangle_R \approx 3 \times 10^{10}\ \text{GeV}, \quad \langle M_N \rangle_L \approx 10^{27}\ \text{GeV},$$

Table 1

	Our model	HRR model	Experimental upper limits
η_N^{LL}	3.3×10^{-21}	2.5×10^{-38}	2.0×10^{-10}
η_N^{RR}	3.7×10^{-14}	0	2.0×10^{-10}
η_N^{RL}	3.8×10^{-18}	2.0×10^{-24}	1.0×10^{-12}
η_ν^{LR}	5.0×10^{-16}	2.0×10^{-17}	2.0×10^{-3}

which must be compared with values [16,17]

$$\langle m_\nu \rangle < 2 \text{ eV}, \quad \langle M_N \rangle_L = \langle M_N \rangle_R > 10^7 \text{ GeV}$$

extracted from experiment in conjunction with nuclear calculations. We thus see that the 0ν-$\beta\beta$ decay continues to be suppressed in our model. The smallness of $\langle m_\nu \rangle$ was also small due to the smallness of both m_{ν_1} and m_{ν_2}. A similar cancellation occurs in the average mass which appears in the (μ^-, e^+) reaction, i.e.,

$$\langle m'_\nu \rangle = \tfrac{1}{2} \sin 2\theta_e (m_1 - m_2 \cos^2\theta_\nu + m_3 \sin^2\theta_\nu) \approx 5.6 \times 10^{-3} \text{ eV}.$$

(iii) Muon-number non-conserving processes. ($\mu \to e\gamma$, $\mu \to 3e$, $\mu^- - e^-$, conversion, $K \to \pi\mu e$, muonium–antimuonium oscillations, etc.) The relevant parameters are those associated with heavy neutrinos η_N^{LL}, η_N^{LR}, η_N^{RR} defined in ref. [4] (the light neutrino contribution is unobservable). They are presented in table 1. We notice that these coefficients are much larger than those calculated from the HRR model but still very far from the experimental limits.

We have made a study of the neutrino mass spectrum based on the standard SO(10) model. We find that the lightest neutrino is unobservably small and coupled very weakly to the other two. The other two neutrinos are degenerate with masses in the 10 eV region completely admixed. All physical processes, however, with the possible exception of neutrino oscillations, appear to be strongly suppressed.

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