# Structure-dependent electronic properties of nanocrystalline cerium oxide films

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We investigate the electronic properties of nanocrystalline cerium oxide  $(\text{CeO}_x)$  films, grown by various techniques, and we establish universal relations between them and the film structure, composition, and morphology. The nanocrystalline  $\text{CeO}_x$  films mainly consist of  $\text{CeO}_2$  grains, while a considerable concentration of trivalent  $\text{Ce}^{3+}$  is distributed at the  $\text{CeO}_2$  grain boundaries forming amorphous  $\text{Ce}_2\text{O}_3$ . A small portion of  $\text{Ce}^{3+}$  is also located around O-vacancy sites. The optical properties of the  $\text{CeO}_x$  films are considered, taking into account the reported band-structure calculations. The fundamental gap  $E_g$  of  $\text{CeO}_x$  is due to the indirect  $\text{O2}p \rightarrow \text{Ce4}f$  electronic transition along the *L* high-symmetry lines of the Brillouin zone and it is correlated with the  $[\text{Ce}^{3+}]$  content, explaining the redshift of  $E_g$  in nanostructured  $\text{CeO}_x$ , which is due to the  $\text{CeO}_2^{3+}$  at the grain boundaries and not due to the quantum-size effect itself. We also correlate the energy position of the  $\text{O2}p \rightarrow \text{Ce4}f$  electronic transition, which varies up to 160-meV wide, with the lattice constant of the  $\text{CeO}_2$  grains. We also show that the higher-order transitions are more sensitive to film composition. The refractive index, far below  $E_g$ , is explicitly correlated with the film density, independently of the  $\text{Ce}^{3+}/\text{Ce}^{4+}$  and O concentrations, grain size, and lattice parameter. The density is also found to be the major factor affecting the absolute value of the  $\varepsilon_2$  peak, which corresponds to the  $\text{O2}p \rightarrow \text{Ce4}f$  electronic transition.

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### I. INTRODUCTION

Cerium oxides have recently attracted much interest due to their unique properties, which make them suitable for various applications. The valence of Ce is very important for the structure of cerium oxides; tetravalent Ce forms cerium dioxide (CeO<sub>2</sub>),<sup>1,2</sup> commonly called ceria, which has a cubic fluorite lattice (*Fm*3*m* space group), while trivalent Ce forms the sesquioxide Ce<sub>2</sub>O<sub>3</sub>, which has a hexagonal lattice (*P*3*m*1 space group).<sup>1,3</sup> Both cerium oxides are refractory compounds, which are optically transparent in the visible spectral region and highly absorbing in the ultraviolet region.<sup>1,4</sup> However, in bulk cerium oxides the existence of trivalent Ce<sup>3+</sup> has been considered to reduce the band gap.<sup>5</sup>

The most common of cerium oxides is  $CeO_2$ , which is stable even in substoichiometric form (CeO<sub>2-x</sub>,  $0 \le x$  $\leq 0.4$ ) (Ref. 6) and thus has been easily produced by several growth techniques. Among them are electron-beam evaporation (EBE),<sup>7-9</sup> pulsed laser deposition,<sup>10-15</sup> metalorganic chemical vapor deposition,<sup>16</sup> ion-beam-assisted deposition (IBAD),<sup>6,17</sup> and reactive magnetron sputtering (RMS).<sup>18-20</sup> Thin films of CeO<sub>2</sub> exhibit unique physical properties, such as a lattice constant ( $\alpha = 0.541$  nm) similar to that of Si (making it suitable for epitaxial insulating layers for Si-device technology), a high refractive index, and a high dc dielectric constant. Therefore, CeO<sub>2</sub> films are appropriate for applications in optical,<sup>4,7,21</sup> electro-optical,<sup>8,9</sup> microelectronic,<sup>7,12,16,18,22,23</sup> and optoelectronic<sup>19</sup> devices. Their exceptional optical performance has led to the extensive study of the electronic properties of thin-film and nanostructured CeO<sub>2</sub>.<sup>4,10,20,24-28</sup> However, the relative results in the literature are still controversial. For example, nanostructured CeO<sub>2</sub> is reported to exhibit a lower fundamental gap than crystalline CeO<sub>2</sub>, although the quantum-size effects are expected to induce a blueshift of the band gap<sup>29,30</sup> due to the more localized bands. In addition, most of the reports correlate the values of optical constants, such as the refractive index (*n*) (reported to vary between 1.6 and 2.5 at 633 nm), fundamental gap (2.5 to 3.3 eV), and energy position of the  $O2p \rightarrow Ce4f$  electronic transition, only with the growth technique and conditions. Therefore, a detailed understanding of the effect of physical properties and microstructure on the optical performance and electronic properties of nanostructured cerium oxides has not yet been established.

In this work we present an effort to investigate the microstructural features, which affect the electronic properties of nanostructured cerium oxide  $(CeO_x)$  films. For this purpose a very detailed complementary study of the crystal structure (CeO<sub>x</sub> phases and cell sizes), morphology (density, grain size, and defect distribution such as oxygen vacancies, grain boundaries, and voids), and composition (oxygen, trivalent Ce3+, and tetravalent Ce4+ contents) has been carried out using well-established characterization techniques such as x-ray diffraction (XRD), x-ray photoelectron spectroscopy (XPS), and x-ray reflectivity (XRR), the latter being a modern technique for the study of thin films in terms of density, surface, and interface roughness.<sup>10,31-33</sup> For our study we have grown CeO<sub>x</sub> films using two different growth techniques, EBE and IBAD, and we compare our results with the optical data of epitaxial CeO<sub>2</sub> films produced by RMS and reported by Guo et al.<sup>20</sup> and with the reported band-structure calculations for CeO<sub>2</sub> and Ce<sub>2</sub>O<sub>3</sub>,<sup>1</sup> in order to establish universal relations between the optical properties and the microstructure of the  $CeO_r$  films, which are general and valid for any growth process.

The electronic properties of the various  $CeO_r$  films are studied by spectroscopic ellipsometry (SE) data analysis. Three methods of analysis are used. The first is the use of the three-phase model (air/film/substrate) (Ref. 34 and 35) in combination with the Tauc-Lorentz model<sup>36</sup> in order to describe and quantify the contributions of the major O2p $\rightarrow$  Ce4f and O2p $\rightarrow$ Ce5d electronic transitions of CeO<sub>2</sub> and the Ce4 $f^0 \rightarrow$  Ce4 $f^1$  transition of Ce<sub>2</sub>O<sub>3</sub> (Ref. 5) and to determine the values of refractive index n of  $CeO_x$  for energies below the fundamental gap  $E_g$ .<sup>10</sup> The second method is based on Bruggeman's effective medium theory,<sup>34,37</sup> which is used to investigate the effect of the film's density and voids content to *n* (for  $E < E_g$ ) and to the absolute values of the dielectric function's peak, which corresponds to the O2p $\rightarrow$  Ce4 f electronic transition. The third method of analysis of the optical data is through the formulation developed by Tauc, Grigrovici, and Vancu<sup>38</sup> for the description of the dielectric function at energies close to the band gap, using the so-called Tauc plots, in order to determine accurately the fundamental gap of various CeO<sub>x</sub> films. Additional ultraviolet photoelectron spectroscopy (UPS) experiments were carried out to study the density of states (DOS) of the valence band of representative CeO<sub>x</sub> films.

We found that the nanostructured CeO<sub>x</sub> films mainly consist of CeO<sub>2</sub> grains, while a considerable concentration of trivalent  $Ce^{3+}$  has been also observed. The trivalent  $Ce^{3+}$  is distributed at the CeO<sub>2</sub> grain boundaries forming amorphous  $Ce_2O_3$ . A small portion (<5%) of  $Ce^{3+}$  is also located around O-vacancy sites. We explicitly correlate the CeO<sub>2</sub> (and  $Ce_2O_3$ ) content with the value of the fundamental gap  $E_g$  of CeO<sub>x</sub>. Since cerium oxides exhibit both a direct and an indirect gap, we identify as  $E_g$  the indirect gap of the  $O2p \rightarrow Ce4f$  transition, which is located  $\sim 0.5$ -eV below the direct gap. The correlation of  $E_g$  with the film composition explains the redshift of  $E_g$  in nanostructured  $\text{CeO}_x$ ,<sup>4</sup> which is due to the  $Ce^{3+}$  at the grain boundaries and not to the grain-size (quantum-size) effect itself. We also correlate the energy position of the  $O2p \rightarrow Ce4f$  interband transition, which varies up to 160-meV wide, with the lattice constant of the  $CeO_2$  grains. Finally, the refractive index, far below  $E_g$ , is explicitly correlated with the film density according to classical dispersion theory,<sup>39</sup> independently of the  $Ce^{3+}/Ce^{4+}$  and O concentrations, grain size, and lattice parameter, revealing that density is the key factor affecting n. In addition, we show that density is also the major factor affecting the absolute value of the  $\varepsilon_2$  peak, which corresponds to the  $O2p \rightarrow Ce4f$  electronic transition of  $CeO_2$ .

# **II. EXPERIMENT**

Three sets of CeO<sub>x</sub> films were grown on *c*-Si(001) substrates in an ultrahigh vacuum (UHV) chamber (base pressure  $<1\times10^{-9}$  Torr). The first set consisted of CeO<sub>x</sub> samples, 110–170-nm thick, grown by electron-beam evaporation at different substrate temperatures  $T_s$  (RT–950 °C). A double electromagnet assembly was used for the surface scan of CeO<sub>2</sub> chunks by a 40–50-mA/7-kV electron beam. The vapor pressure of CeO<sub>2</sub> under these conditions was 1–3  $\times10^{-5}$  Torr and was monitored and controlled by a quadrapole mass spectrometer. The second set of samples was grown by EBE with similar growth conditions as the samples of the first set, but with higher thickness (~0.5  $\mu$ m), in order to better study their bulk properties and compare them directly with the IBAD-produced samples. The third set consisted of CeO<sub>x</sub> films, about 0.5- $\mu$ m thick, grown by IBAD at RT. IBAD growth was realized by evaporation of CeO<sub>2</sub>, using a 150–200-mA/7-kV electron beam (providing a vapor pressure of 1–4×10<sup>-4</sup> Torr), and subsequent bombardment with a focused Ar<sup>+</sup>-ion beam generated at a Kauffman broad-beam ion source. The varying parameter at this set of experiments was the Ar<sup>+</sup>-ion energy, which was either 0.75 or 1.25 keV. The evaporation rate and the Ar<sup>+</sup> flow rate were varying in order to retain a relative flux of ions arriving (ion/neutral ratio) at  $\Phi_{Ar^+}/\Phi_{CeO_r} \approx 0.5$ .

The optical properties of the deposited films were measured *ex situ* at RT using a JOBIN-YVON phase-modulated spectroscopic ellipsometer, in the energy range 1.5-5.5 eVwith a step of 10 meV at a 70° angle of incidence. UPS experiments were carried out in a UHV chamber equipped with a hemispherical electron energy analyzer (SPECS LH-10) and a discharge lamp (He II radiation with photon energy 20.66 eV). The spectrometer was calibrated using the Fermi edge position of a sputter-cleaned Au foil, which was defined as zero binding energy of the spectra.

XRR/XRD experiments were conducted in a Siemens D-5000 diffractometer equipped with a Goebel mirror,<sup>40</sup> which transforms the divergent x-ray beam parallel to 200- $\mu$ m width. The XRD scans were performed in Bragg-Brentano geometry between  $5^{\circ}$  and  $60^{\circ}$ . The source was a conventional Cu  $K_{\alpha}$  2.2-kW x-ray tube. The generator current and voltage were 40 mA and 40 kV, respectively, the step was  $0.02^{\circ}$ , and the scan speed was  $0.1^{\circ}$ / min. By taking rocking curves of the reflected x-ray beam from the specimen (at angles lower than the critical angle for total external reflection) we calibrated the goniometer with an accuracy of 0.001°. The XRR measurements were performed in the form of  $\theta$ -2 $\theta$  locked couple scans between 0 and 3° using the same x-ray apparatus and a special reflectivity sample stage. XRR scans with a 0.1° offset of the detector were conducted to subtract the contribution of the diffuse scattering from the specular XRR scan.

The XPS experiments were performed in a stainlesssteel turbopumped UHV chamber (base pressure ~5  $\times 10^{-10}$  Torr). The chamber was equipped with a Leybold LHS-12 hemispherical electron energy analyzer and a nonmonochromatized dual anode x-ray source for XPS. The data were obtained by using the Al  $K_{\alpha}$  radiation at 1486.6 eV and detection normal to the surface with a constant analyzer pass energy of 100 eV, which resulted in an analyzed area of 2  $\times 3$  mm<sup>2</sup>. The binding-energy scale of the spectrometer was referenced to the Au4 $f_{7/2}$  value at 84.05±0.05 eV. In all speciments the wide scan spectrum shows that the main peaks are O, Ce, and carbon contamination due to air exposure before introduction to the UHV chamber.

### **III. RESULTS AND DISCUSSION**

#### A. Film microstructure

XRD and transmission-electron microscopy (TEM) (TEM results are not shown here) were used to study the micro-



FIG. 1. Representative XRD patterns from representative  $\text{CeO}_x$  films (EBE and IBAD produced), showing the nanocrystalline character of the films and the cubic fluorite structure of  $\text{CeO}_2$ . The Miller indices, which correspond to the XRD peaks of  $\text{CeO}_2$ , are also shown.

structure of the various  $\text{CeO}_x$  films. Both techniques revealed the polycrystalline character of the  $\text{CeO}_x$  films, showing broad diffraction peaks (XRD) and diffraction rings (TEM), which are the characteristic patterns of nanocrystalline materials. The films exhibit columnar structure. The columns are polycrystalline and consist of nanoscale grains (9–28 nm). For the quantitative analysis of the film microstructure we used the XRD diffractograms in order to ensure the nondestructive character of the study, because the use of energetic electron beams (such as in TEM) may cause desorption of oxygen from CeO<sub>2</sub> and formation of Ce<sub>2</sub>O<sub>3</sub>.<sup>2</sup>

The XRD diffractograms revealed the cubic fluorite structure of CeO<sub>2</sub> (Ref. 41) in all films (deposited either by EBE or IBAD). For example, Fig. 1 shows XRD patterns from representative CeO<sub>x</sub> films with the assignment of the corresponding Miller indices of the CeO<sub>2</sub> XRD reflections. The XRD peaks have been fitted by pseudo-Voigt curves in order to determine their angular position, broadening, and intensity. The different growth techniques and conditions affect the broadenings and the relative intensities of the various XRD peaks due the variations of grain size and orientation, respectively. No sign of crystalline Ce<sub>2</sub>O<sub>3</sub> was detected by XRD. However, XPS had detected a considerable portion of Ce<sup>3+</sup>, as we show below, suggesting that the Ce<sub>2</sub>O<sub>3</sub> phase is amorphous.

The mean grain size *G* of CeO<sub>2</sub> can be determined from the broadening full width at half maximum<sub>*hkl*</sub> (FWHM)<sub>*hkl*</sub> of the XRD peaks, which correspond to the lattice planes with Miller indices (*hkl*), through Scherrer's formula:<sup>42</sup>

$$G = \frac{0.9 \cdot \lambda}{\text{FWHM}_{hkl} \cdot \cos \theta_{hkl}},\tag{1}$$



FIG. 2. The lattice constant of the  $CeO_2$  grains, calculated using Eq. (2), vs the major growth conditions of EBE and IBAD (the straight lines are guides to the eye).

where  $\theta_{hkl}$  is the Bragg angle that corresponds to the (*hkl*) lattice planes and  $\lambda$  is the x-ray wavelength. For the determination of *G* we used the broadening of the (111) XRD reflection, which is manifested at lower angular values and thus is less affected by the  $k_{\alpha 1}$  and  $k_{\alpha 2}$  splitting of the incident beam [except for the film deposited by EBE at 950 °C where we used the (002) reflection because this film was grown along this direction and exhibited a weak (111) peak]. However, we note here that Scherrer's formula is valid for unstrained grains because the strain effects may broaden the XRD peaks, as well. Therefore, before applying Eq. (1) to determine the mean grain size of the CeO<sub>2</sub> nanostructures, we should calculate the lattice constant  $\alpha$ .

The lattice constant  $\alpha$  of the cubic fluorite-type CeO<sub>2</sub> grains can be determined through the spacing of the (*hkl*) lattice planes ( $d_{hkl}$ ), and taking into account Bragg's law:

$$d_{hkl} = \frac{\alpha}{\sqrt{h^2 + k^2 + l^2}} \Rightarrow \alpha = \frac{\lambda \cdot \sqrt{h^2 + k^2 + l^2}}{2 \cdot \sin \theta_{hkl}}.$$
 (2)

Figure 2 shows the calculated lattice constant values vs  $T_s$  and ion energy for the EBE and IBAD films, respectively. In Fig. 2(b) the point at E=0 keV corresponds to a ~500-nm-thick film produced by EBE at RT (i.e., with conditions and thicknesses similar to IBAD for E=0 keV). The lattice constant of all the films has been found to be slightly larger than the corresponding value of bulk ceria,<sup>41</sup> showing that all the films are in-plane compressed. The relative strain *s* values can be estimated from the calculated lattice constants: *s* 

 $=(\alpha - \alpha_{\text{bulk}})/\alpha_{\text{bulk}}$ , where  $\alpha_{\text{bulk}}$  is the lattice constant of the bulk, unstrained ceria.<sup>41</sup> The relative strain for all EBE and IBAD films has been determined to be lower than 0.7% and 1.6%, respectively. The observed strain in CeO<sub>2</sub> grains is due to growth-related processes such as the development of thermal stresses (for films grown by EBE at  $T_s > \text{RT}$ ) and the compressive stresses developed by the intense ion bombardment during ion-assisted growth of refractory compounds (for IBAD).<sup>43</sup> Nevertheless, the *s* values are very low and thus their effect on FWHM<sub>*hkl*</sub> is minor and Scherrer's formula can be used to estimate the grain size of CeO<sub>2</sub>.

The grain size of EBE-grown films increases with  $T_s$  (from 9 to 24 nm) due to the higher mobility of the adatoms during growth.<sup>43</sup> The IBAD-grown films exhibit relatively bigger grains (23–28 nm) either due to their thickness (~500 nm, instead of the 110–170-nm thickness of the EBE-grown films) or the energy transport from the inert gas ions to the Ce and O adatoms at the film surface during growth, which results in similar effects as those of increased  $T_s$ .<sup>43</sup> Nevertheless, the origin of the grain-size values does not affect the following study, because we correlate directly the electronic properties with the structural features (composition, grain size itself, etc.) of CeO<sub>x</sub> films, independently of the growth technique and film thickness.

Another microstructural feature, which considerably affects the optical properties of  $\text{CeO}_x$ , is the film density.<sup>10</sup> The film density can be determined by XRR through the critical angle  $\theta_c$  for total external reflection of x rays, which can be expressed in terms of the mass density  $\rho$ :<sup>31</sup>

$$\theta_c^2 = \left(\frac{e^2 \lambda^2}{\pi m c^2}\right) \cdot \left(N_o \frac{Z_t}{A_t}\right) \cdot \rho, \qquad (3)$$

where *m*, *e* are the electron mass and charge, respectively,  $N_o$  is Avogadro's number,  $A_t$  is the mean atomic mass,  $\lambda$  the x-ray wavelength, and  $Z_t$  the mean number of electrons (sum of core, valence, and conduction electrons) per atom.

The XRR measurements were analyzed using Siemens REFSIM software and a Monte Carlo (MC) algorithm.<sup>31</sup> Figure 3(a) shows the XRR curve and the results of the MC fitting for a CeO<sub>2</sub> film, 112-nm thick, deposited at 950 °C. The calculated density  $(6.7 \text{ g/cm}^3)$  is smaller than the density of the bulk CeO<sub>2</sub> (7.21 g/cm<sup>3</sup>),<sup>44</sup> due to the polycrystalline character of the films and the existence of voids. The voids content in the film may be estimated from the density, taking into account, its value for bulk  $CeO_2$ .<sup>41,43</sup> The XRR curves of two CeO<sub>x</sub> films deposited by EBE at RT and 950 °C are compared in Fig. 3(b). The film deposited at RT is much less dense (4.5 g/cm<sup>3</sup>). Figure 4 illustrates the densification and elimination of voids with  $T_s$ , due to the higher mobility of the adatoms during growth, which results in more compact stacking and close-packed structures.<sup>31,43</sup> The films grown by IBAD are much denser ( $\rho$  up to 6.1 g/cm<sup>3</sup>) than the films grown by EBE at RT. This densification is a typical effect of ion bombardment during growth of refractory compounds.<sup>31,43</sup> XRR has also determined the surface roughness of the films, which varies in the range 3-4 nm. These values are much lower than those reported in the literature.<sup>20</sup> The surface-roughness values of our films are not correlated



FIG. 3. (a) XRR data from a  $\text{CeO}_x$  film grown by EBE at 950 °C and the results of the MC fitting and (b) comparison of XRR data of two  $\text{CeO}_x$  films deposited by EBE at RT and 950 °C.

with any other film characteristic (thickness, density, grain size), and they may possibly be attributed to a top nucleation layer as has been observed in Si growth.<sup>44</sup>

The film composition has been studied by XPS. XPS observed the characteristic O1*s* and Ce3*d* peaks, from which the film stoichiometry and bonding type can be determined. Figure 5 displays the core levels of Ce3*d* of two representative films grown by EBE and IBAD; the characteristic peaks in which the Ce3*d* states are analyzed are indicated by the vertical dotted lines. The Ce3*d* spectrum of CeO<sub>x</sub> can be fully described by six peaks of three spin-orbit doublets.<sup>45–48</sup> The assignments of these peaks and the reference values for their binding energies have been a subject of intense research and much data are available in the literature.<sup>45–49</sup>

Figure 5 also shows the fitted deconvolution of the Ce3*d* levels to the various states, in addition to the experimental XPS data. The observed XPS peaks are attributed to trivalent Ce<sup>3+</sup> and tetravalent Ce<sup>4+</sup>. The peaks, which correspond to



FIG. 4. The variation of density and voids content with  $T_s$  for the EBE-grown films.



FIG. 5. The Ce3*d* core levels for representative CeO<sub>x</sub> films grown by IBAD and EBE. The fractions of Ce<sup>3+</sup> and Ce<sup>4+</sup> can be determined from the Ce3*d* XPS peaks, showing that the IBAD-produced films exhibit a higher Ce<sup>4+</sup> content.

Ce<sup>4+</sup> are the U''' (916.7 eV) and V''' (898.4 eV), which result from the Ce3 $d^9$  O2 $p^6$  Ce4 $f^0$  final states. The additional states of Ce<sup>4+</sup> [U (901.0 eV), V (882.5 eV), U'' (907.3 eV), V'' (888.8 eV] result from a mixture of Ce3 $d^9$  O2 $p^5$  Ce4 $f^1$ and Ce3 $d^9$  O2 $p^4$  Ce4 $f^2$  final states. The transfer of electrons from the O2p to the Ce4f orbital and the decrease in the Ce3d binding energy are due to the interaction of the Ce4flevel with the Ce3d core hole, which pulls the Ce4f level to lower energy. Finally, the contribution of Ce<sup>3+</sup> to the Ce3dspectrum consists of two doublet pairs: (i) U' (903.5 eV)/V'(884.9 eV) and (ii)  $U_o$  (898.8 eV)/ $V_o$  (880.3 eV). These doublets correspond to a mixture of the Ce3 $d^9$  O2 $p^5$  Ce4 $f^2$ and Ce3 $d^9$  O2 $p^6$  Ce4 $f^1$  final states.

For the quantitative determinations of the  $[Ce^{3+}]/[Ce^{4+}]$  ratio we used two methods: (i) the method that proposed by Shyu *et al.*,<sup>50</sup> in which the area of the whole Ce3*d* multiplet

is linearly correlated with the ratio  $Ce^{3+}/(Ce^{3+}+Ce^{4+})$ ; in this case the decrease of U''' indicates the increase in  $Ce^{3+}$ species due to the  $Ce^{+4}$  reduction. (ii) The second method uses directly the ratio of the intensities of  $Ce^{4+}$  and  $Ce^{3+}$ peaks. The results of the two methods are in very good agreement and reveal the existence of a considerable concentration of  $Ce^{3+}$ , although XRD identified only the tetravalent  $CeO_2$ . The results of the XPS analysis are summarized in Table I. The trivalent  $Ce^{3+}$  can be distributed either in regions of sesquioxide  $Ce_2O_3$  (Ref. 1) or around O vacancies in  $CeO_2$ .<sup>5</sup>

In order to investigate if the trivalent  $Ce^{3+}$  has the form of  $Ce_2O_3$  or is due to O vacancies, we calculated the O content in the film assuming that the total O content is the sum of the required O to fully oxidize  $Ce^{4+}$  and  $Ce^{3+}$  and to form  $CeO_2$  and  $Ce_2O_3$ , respectively. Then, taking into account that the stoichiometry x=[O]/[Ce] is equal to 2 for  $CeO_2$  and 1.5 for  $Ce_2O_3$ , the ratio of the required oxygen to fully oxidize  $Ce^{4+}$  and  $Ce^{3+}$  to the sum  $[Ce^{4+}]+[Ce^{3+}]$  is determined from the concentrations  $[Ce^{4+}]$  and  $[Ce^{3+}]$  according to the following equation:

$$x = \frac{[O]}{[Ce]} = \frac{3}{2} \times [Ce^{3+}] + 2 \times [Ce^{4+}].$$
(4)

The calculated stoichiometry from Eq. (4) has been compared with the stoichiometry determined directly from the intensities of the O1s (with binding energy 529.5 eV) and Ce3d XPS peaks. Figure 6 shows the stoichiometry variations with the concentration of  $[Ce^{3+}]$  determined by both methods (requiring O to fully oxidize  $Ce^{3+}$  and  $Ce^{4+}$  and direct comparison of the O1s and Ce3d XPS peak intensities). The two methods are in excellent agreement for films exhibiting low  $[Ce^{3+}]$  content, meaning that in these films the entire  $[Ce^{3+}]$  content is consumed in forming  $Ce_2O_3$ . On the other hand, we observe a slight O deficiency (with respect to the required O needed to fully oxidize Ce) in the films with high  $[Ce^{3+}]$  content, suggesting that  $Ce_2O_3$  and O vacancies coexist in these films. However, the maximum O deficiency is less than 5% and thus the majority (>95%) of  $[Ce^{3+}]$  is consumed in Ce<sub>2</sub>O<sub>3</sub> in any case.

XPS provided evidence that  $Ce_2O_3$  exists in the nanostructured  $CeO_x$  films, while XRD identified only  $CeO_2$ .

TABLE I. The composition, density, and energy positions of  $E_g$  and the O2 $p \rightarrow$ Ce4f electronic transition ( $E_{01}$ ) for the various CeO<sub>x</sub> films with respect to the growth technique and conditions.

$T_d$ (°C)	$E_i$ (keV)	$[Ce^{4+}]/[Ce^{3+}]$	$[O]_{O1s}/[Ce]_{Ce3d}^{a}$	$[O]/[Ce]_{Ce3d}^{b}$	$E_g (eV)^c$	$E_{01}$ (eV)	$ ho~({ m gcm^{-3}})$
950		4.9	1.91	1.92	$3.05 \pm 0.18$	$3.91 \pm 0.04$	6.7
600					$2.99 \pm 0.18$	$3.97 \pm 0.04$	5.5
300		3.8	1.85	1.90	$2.94 \pm 0.15$	$3.94 \pm 0.05$	4.9
RT		3.0	1.80	1.88	$2.92 \pm 0.04$	$4.02 \pm 0.04$	4.5
RT	1.25	4.3	1.89	1.91	$2.97 \pm 0.06$	$3.85 \pm 0.02$	5.9
RT	0.75	6.4	1.96	1.94	$3.12 \pm 0.06$	$3.94 \pm 0.02$	6.1
	<i>T<sub>d</sub></i> (°C) 950 600 300 RT RT RT RT	$T_d$ (°C) $E_i$ (keV)         950	$T_d$ (°C) $E_i$ (keV) $[Ce^{4+}]/[Ce^{3+}]$ 9504.96003003.8RT3.0RT1.25A.3RTRT0.756.4	$T_d$ (°C) $E_i$ (keV) $[Ce^{4+}]/[Ce^{3+}]$ $[O]_{O1s}/[Ce]_{Ce3d}^a$ 9504.91.916003003.81.85RT3.01.80RT1.254.31.89RT0.756.41.96	$T_d$ (°C) $E_i$ (keV) $[Ce^{4+}]/[Ce^{3+}]$ $[O]_{O1s}/[Ce]_{Ce3d}^{a}$ $[O]/[Ce]_{Ce3d}^{b}$ 9504.91.911.926003003.81.851.90RT3.01.801.88RT1.254.31.891.91RT0.756.41.961.94	$T_d$ (°C) $E_i$ (keV) $[Ce^{4+}]/[Ce^{3+}]$ $[O]_{O1s}/[Ce]_{Ce3d}^a$ $[O]/[Ce]_{Ce3d}^b$ $E_g$ (eV) <sup>c</sup> 9504.91.911.92 $3.05 \pm 0.18$ 6002.99 \pm 0.183003.81.851.903003.81.852.94 \pm 0.15RT3.01.801.882.92 \pm 0.04RT1.254.31.891.912.97 \pm 0.06RT0.756.41.961.943.12 \pm 0.06	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

<sup>a</sup>Using directly the intensities of the O1s and Ce3d XPS peaks.

<sup>b</sup>From Eq. (4), using the deconvolution of the Ce3*d* XPS peak.

<sup>c</sup>Using Eq. (10).



FIG. 6. The CeO<sub>x</sub> stoichiometry (for both EBE- and IBADgrown films) calculated (a) directly from the intensities of the O1s and Ce3d XPS peaks (open circles) and (b) from the concentrations of Ce<sup>3+</sup> and Ce<sup>4+</sup> (solid squares), assuming that Ce<sup>3+</sup> and Ce<sup>4+</sup> exclusively form Ce<sub>2</sub>O<sub>3</sub> and CeO<sub>2</sub>, respectively.

Therefore, the Ce<sub>2</sub>O<sub>3</sub> phase is amorphous. The amorphous character of Ce<sub>2</sub>O<sub>3</sub> is an indication that this phase is located at the grain surface and at the grain boundaries. Following simple dimensional analysis we determine that if Ce<sub>2</sub>O<sub>3</sub> and CeO<sub>2</sub> are located at the grain surface and volume, respectively, then the square of  $[Ce^{3+}]$  (surface distribution) will be linearly correlated with the third power of  $[Ce^{4+}]$  (grain volume distribution). Indeed, Fig. 7 shows the correlation of the square of  $[Ce^{3+}]$  ( $\cong$ [Ce<sub>2</sub>O<sub>3</sub>]) with the third power of  $[Ce^{4+}]$  (=[CeO<sub>2</sub>]) and the third power of *G* (which is proportional to grain volume *V<sub>g</sub>*). The observed linear relation



FIG. 7. The correlation of the square of the  $Ce^{3+}$  concentration with the third power of the  $Ce^{4+}$  concentration and the grain volume  $(V_g \propto G^3)$ , showing that  $Ce^{3+}$  and  $Ce^{4+}$  are located at the grains' surface and volume, respectively.

between  $[Ce^{3+}]^2$  and  $[Ce^{4+}]^3$  confirms the dimensional analysis and the distribution of CeO<sub>2</sub> and Ce<sub>2</sub>O<sub>3</sub> at the grain volume and surface, respectively. The experimental points of  $G^3$  vs  $[Ce^{3+}]^2$  are more scattered around the straight line of the dimensional analysis. This is attributed to the strain of the grains that affects the broadening of the XRD peaks and the determined grain-size values.

#### **B.** Electronic properties

After studying in detail the microstructure and morphology of  $CeO_r$  films, we proceed to the study of their optical properties. The optical properties of the  $CeO_r$  films have been investigated by SE in the visible UV spectral region (1.5-5.5 eV). The main benefit of the ellipsometric measurements (compared to other optical measurements) is the direct evaluation of both the real and imaginary parts of the complex dielectric function  $\varepsilon(\omega) = [\varepsilon_1(\omega) + i\varepsilon_2(\omega)]$ , through the ellipsometric angles  $\Psi$  and  $\Delta$ , with no need for Kramers-Kronig integration.<sup>35</sup> The determination of the complex dielectric function enables the investigation of the material's electronic structure  $[\varepsilon_2(\omega)]$  is directly related to the joined density of states (JDOS) for interband absorption. The material's electronic structure and consequently the determined  $\varepsilon(\omega)$  is very sensitive to the materials' microstructure, since small grains may induce quantum-size effects,<sup>29,30</sup> the point defects may create localized states within the band gap,<sup>5</sup> and in mixed materials (such as  $CeO_2/Ce_2O_3$ ) the various phases may have different electronic structures band gaps,<sup>1</sup> etc.

In the case of transparent films the measured quantity is the pseudodielectric function  $\langle \varepsilon(\omega) \rangle$ , which takes into account contributions from the substrate and the film's thickness due to the multiple reflections originating from the filmsubstrate interface.<sup>35</sup> Such intense multiple reflections are manifested in the  $\langle \varepsilon(\omega) \rangle$  spectra (Fig. 8) of CeO<sub>x</sub> films for photon energy  $E = \hbar \omega < E_g$ , meaning that all the CeO<sub>x</sub> films are highly transparent for E < 3 eV, regardless of their thickness values (in Fig. 8, the dashed and dotted lines correspond to EBE-grown thin films of the first set, while the solid line corresponds to a thick ~500 nm IBAD-grown film). For photon energy  $E > E_g$  ( $\cong$ 3 eV) optical absorption begins and the Si substrate contribution to  $\langle \varepsilon(\omega) \rangle$  is reduced, even for the thinner film (EBE grown, 950 °C, dotted line).

The second part (E>3 eV) of the SE spectra is dominated by an absorption peak at ~4 eV, which corresponds to the  $O2p \rightarrow Ce4f$  transition<sup>1,5,10,20,51</sup> of  $CeO_2$ , according to the calculated band structure and DOS (Ref. 1) shown in Fig. 9. In particular, the fundamental gap of the  $CeO_x$  films is due to indirect  $O2p \rightarrow Ce4f$  transitions of  $CeO_2$  along the *L* highsymmetry lines of the Brillouin zone denoted by arrows in Fig. 9. This point is discussed in more detail in the following paragraphs. The  $O2p \rightarrow Ce4f$  transition is very sharp due to the low energy broadening of the Ce4f band.<sup>1,5</sup> Many authors refer to the Ce4f state as an atomiclike localized band into the band gap between the O2p valence band and the extended Ce5d conduction band,<sup>1,2,19,49</sup> defining as the CeO<sub>2</sub> band gap the energy separation between the O2p and Ce5d states, which is about 6 eV.<sup>2,5,19,49</sup> Nevertheless, the O2p



FIG. 8. SE measurements of  $\langle \varepsilon(\omega) \rangle$  from representative CeO<sub>x</sub> films grown by IBAD and EBE (to different thicknesses). All films are highly transparent below 3 eV, where intense multiple reflections are manifested. An absorption peak appears at ~4 eV, which is due to a O2*p*→Ce4*f* electronic transition of CeO<sub>2</sub>.

in Ce<sub>2</sub>O<sub>3</sub> the Ce4 $f^0$  band is partially filled with valence electrons<sup>1</sup> and thus only higher-order transitions occur in Ce<sub>2</sub>O<sub>3</sub>. The maximum value of the  $\varepsilon_2$ , which corresponds to the O2 $p \rightarrow$ Ce4f interband transition, has been a subject of intense debate<sup>1,5,20,51</sup> and varies considerably in our films (see Fig. 8). This point is also discussed thoroughly in the following text. Finally, for E > 4 eV the dielectric function is also affected by the contribution of the low-energy tails of the O2 $p \rightarrow$ Ce4 $f^1$  transition of Ce<sub>2</sub>O<sub>3</sub>.<sup>2,49</sup>

In order to evaluate the spectral dependence of the dielectric function of  $\text{CeO}_x$  films, we fitted the experimental  $\langle \varepsilon(\omega) \rangle$  using the modified Tauc-Lorentz (TL) model,<sup>36</sup> in combination with the three-phase model (air/CeO<sub>x</sub> film/*c*-Si substrate). In the TL model the imaginary part [ $\varepsilon_2(\omega)$ ] of the dielectric function is determined by multiplying the Tauc JDOS (Ref. 38) by the  $\varepsilon_2$  obtained from the Lorentz oscillator model.<sup>39</sup> Thus, the TL model provides the ability to de-



FIG. 9. The band structure and the DOS for pure CeO<sub>2</sub> (after Ref. 1). The arrows indicate the indirect transitions, which correspond to the fundamental gap of CeO<sub>2</sub> and the nanostructured CeO<sub>x</sub> films.

termine the fundamental optical gap  $E_g$  of the electronic transitions, in addition to the energy  $E_0$ , broadening *C*, and strength *A* of each Lorentz oscillator.<sup>39</sup> The energy  $E_0$  of this model corresponds to the Penn gap, where the strong absorption of the material takes place. The imaginary part  $\varepsilon_2$  of a TL oscillator is described by the following expressions:<sup>36</sup>

$$\varepsilon_2(E) = \frac{AE_0C(E-E_g)^2}{(E^2-E_0^2)^2+C^2E^2} \cdot \frac{1}{E}, \quad E > E_g.$$
(5a)

$$\varepsilon_2(E) = 0, \quad E \leq E_g,$$
 (5b)

and the real part  $\epsilon_1$  is obtained by Kramers-Kronig integration.  $^{36,39}$ 

$$\varepsilon_1(E) = \varepsilon_\infty + \frac{2\hbar^2}{\pi} P \int_{\omega_g}^{\infty} \frac{x\varepsilon_2(x)}{x^2 - E^2} dx.$$
 (6)

The parameter  $\varepsilon_{\infty}$  is larger than unity and accounts for the contributions from electronic transitions occurring at energies higher than 8 eV (i.e., Ce5 $p \rightarrow$ Ce5d and Ce5 $p \rightarrow$ Ce6s transitions<sup>1</sup>) and they have not been taken into account in our analysis with two TL oscillators. The quality of the relative model fit can be judged by the comparison of the experimental and calculated  $\Psi$  and  $\Delta$  and it is based on the minimization of the mean square error  $\chi^2$ :

$$\chi^{2} = \frac{1}{(2N - P - 1)} \sum_{i=1}^{N} \{ [\Psi_{\text{calc}}(E_{i}) - \Psi_{\text{exper}}(E_{i})]^{2} + [\Delta_{\text{calc}}(E_{i}) - \Delta_{\text{exper}}(E_{i})]^{2} \},$$
(7)

where 2*N* is the number of independent measurements ( $\Psi$  and  $\Delta$  are at *N* energy positions  $E_i$ ), *P* is the number of the free, independent TL-model parameters, and  $\Delta_{calc}$  ( $\Psi_{calc}$ ) and  $\Delta_{exper}$  ( $\Psi_{exper}$ ) are the calculated and experimental  $\Delta$  ( $\Psi$ ) angles at each energy  $E_i$ . For the TL fit a Lavender-Marquant minimization algorithm was used to minimize  $\chi^2$  with confidence limits 95%. The typical values of  $\chi^2$  for the TL fit on the studied CeO<sub>x</sub> vary,  $0.1 < \chi^2 < 0.4$ .

For the TL fits we used two TL oscillators. The first one describes the  $O2p \rightarrow Ce4f$  transition (at ~4 eV), which is characteristic of  $CeO_2$  (see Fig. 9). We note here that the so-called  $O2p \rightarrow Ce4f$  transition should be forbidden because it violates the angular momentum selection rule ( $\Delta l$ =0, $\pm$ 1, where *l* is the angular momentum quantum number). However, this transition is manifested in all  $\varepsilon(\omega)$  spectra of CeO<sub>2</sub> in the literature,<sup>5,10,20,24</sup> because the highest occupied valence band (denoted as the O2p band in Fig. 9) is not exclusively due to O2p electrons because it also contains a contribution from Ce4d electrons, which provide partial d character.<sup>1,5</sup> Therefore, the electronic transition at  $\sim 4$  eV is a nominal  $O2p \rightarrow Ce4f$  transition (and we follow this terminology to be consistent with previous literature) whereas it has actually strong  $d \rightarrow f$  (Ce4 $d \rightarrow$ Ce4f) character, which fulfills the selection rules. We also used a second TL oscillator (at 6.5-8 eV) to describe the low-energy tales of higher-order transitions such as the  $O2p \rightarrow Ce5d$  interband transition of CeO<sub>2</sub> and the Ce4 $f^0 \rightarrow$  Ce4 $f^1$  transition of  $Ce_2O_3$ .<sup>2,49</sup> The second TL oscillator averages the contribu-



FIG. 10. The experimental ( $\langle \varepsilon \rangle$  exper, solid triangles) and fitted curves ( $\langle \varepsilon \rangle$  two TL, dotted lines), within the three-phase model, for a representative CeO<sub>x</sub> film (EBE at RT), as well as the corresponding  $\varepsilon_1$ ,  $\varepsilon_2$  ( $\varepsilon$ , two TL, solid lines) curves for the bulk material, which were calculated using the best-TL-fit parameters.

tions of the above-mentioned transitions and their fine structure (which was observed for the  $O2p \rightarrow Ce5d$  transition of  $CeO_2$ , due to the crystal-field splitting of the Ce5d conduction band to the  $Ce5de_g$  and  $Ce5dt_{2g}$  subbands<sup>1,5,49,52</sup>). Figure 10 shows representative experimental (exp) and fitted (2 TL), within the three-phase model,  $\langle \varepsilon(\omega) \rangle$  curves for a representative EBE-grown  $CeO_x$  film, as well as the  $\varepsilon_1$ ,  $\varepsilon_2$ curves for the corresponding bulk material, which were calculated based on the best-TL-fit parameters.

The results of the two-TL fits were used to predict and extend the  $\varepsilon(\omega)$  curves to lower (0.16 eV) and higher (up to 12 eV) energies than those within the experimental spectral region. The low limit of E = 0.16 eV is much lower than  $E_o$ but higher than the optical absorption at 275  $\text{cm}^{-1}$ , which is due to the triply degenerate polar lattice vibrations.<sup>5</sup> Thus, in this analysis we account exclusively for the effect of the electronic transitions and we can evaluate some properties such as the value of the refractive index,  $\varepsilon_1$  for  $E \ll E_g$  (e.g., the solid line in Fig. 9), and the energy position of the higher-order transitions. However, the predicted  $\varepsilon(\omega)$  curves, outside the experimental spectral region, should be considered with particular care, especially in the case of the higherorder transitions, since only the low-energy tails of these transitions contribute to the experimental spectra. Table I summarizes the main results of the two-TL fit analysis,



FIG. 11. The calculated  $\varepsilon(\omega)$  spectra, based on the two-TL bestfit parameters, for representative CeO<sub>x</sub> films. The optical data reported by Marabelli and Wachter for bulk CeO<sub>2</sub> (Ref. 5) and by Guo *et al.* for epitaxial CeO<sub>2</sub> (Ref. 20) are also shown for comparison.

namely, the fundamental gap  $E_g$  and the energy position  $E_{01}$  of the O2 $p \rightarrow$ Ce4f transition, which are included in the experimental spectral region.

Figure 11 shows the calculated  $\varepsilon(\omega)$  spectra based on the two-TL best-fit parameters of representative EBE- and IBAD-grown  $CeO_x$  films, in comparison with the optical data reported previously for the bulk CeO<sub>2</sub> (Ref. 5) and for epitaxial CeO<sub>2</sub> films.<sup>20</sup> In all  $\varepsilon(\omega)$  spectra a strong absorption peak with maximum at  $E_{01} \sim 4 \text{ eV}$ , due to an electronic transition, is prominent. According to TL calculations, this transition has been located up to 160 meV (from 3.85 to 4.01 eV). In addition, we did not observe any correlation between the energy positions of this transition with the film composition (CeO<sub>2</sub> and Ce<sub>2</sub>O<sub>3</sub> content), proving that this transition originates exclusively from CeO2, and thus, the Ce2O3 regions do not contribute at all, in agreement with the bandstructure calculations (Ref. 1 and Fig. 9). On the other hand, we observed a correlation between the energy position  $E_{01}$  of the maximum optical absorption of the first TL oscillator with the lattice constant of the CeO<sub>2</sub> grains, which is shown in Fig. 12. This correlation is well understood, since the Brillouin zone (and consequently the energy positions of electronic transitions) reflects the symmetry of the crystal cell, which is its reciprocal. The scattering of the experimental points is due to the very small variations of  $E_{01}$  and  $\alpha$  as well as to other factors that may affect the energy positions of the interband transitions, such as film thickness, surface roughness, and CeO<sub>2</sub> grain size.



FIG. 12. The energy position of the  $O2p \rightarrow Ce4f$  electronic transition of CeO<sub>2</sub> vs the lattice constant of the CeO<sub>2</sub> grains.

The energy position  $E_{02}$  of the second TL oscillator is very sensitive to the film composition and varies within the range 6.5–8 eV. This is clearly illustrated comparing the  $\varepsilon_2$ curves of the EBE- and IBAD-produced films in Fig. 11. The second TL oscillator is located at  $\sim 6.5$  eV in the case of the Ce<sub>2</sub>O<sub>3</sub>-rich EBE film and at  $\sim$ 8 eV in the case of the more stoichiometric CeO<sub>2</sub> film grown by IBAD. Comparing these results with the energy position of the  $O2p \rightarrow Ce5d$  interband transition, which occurs at  $\sim 9 \text{ eV}$  for the bulk CeO<sub>2</sub> (Fig. 11, solid line, and Ref. 5), we identify a trend of a strong redshift of the second TL oscillator with increasing  $Ce_2O_3$  content. The observed redshift with increasing  $Ce_2O_3$ content can be very well understood, if we take into account that the second TL oscillator averages the contributions of interband transitions of CeO2 and Ce2O3, and it is in very good agreement with the band-structure calculations, which show that  $Ce_2O_3$  exhibits a lower band gap than  $CeO_2$ .<sup>1,52</sup> However, we should mention that the variations (6.5-8 eV)of the second TL oscillator with Ce2O3 content are comparable with the error of the  $E_{02}$  values calculated by the TL fit (~1.5 eV). The error of the  $E_{02}$  values is due to the large broadening of this oscillator in addition to the fact that only the low-energy tail of this oscillator contributes to the experimental spectrum. Therefore, the observed trend cannot be considered as clear evidence but only as a serious indication for the redshift of the second oscillator; further experimental work at higher photon energies is required to confirm this observation.

According to previous studies<sup>2,49</sup> the O2 $p \rightarrow$ Ce5d interband transition of CeO<sub>2</sub> is expected to exhibit a Penn gap >8 eV, while the corresponding value of the Ce4 $f^0 \rightarrow$ Ce4 $f^1$  transition of Ce<sub>2</sub>O<sub>3</sub> is around 6 eV. Therefore, the observed redshift of the second TL oscillator should be correlated with the occupancy of the Ce4f states, which are characteristic of Ce<sub>2</sub>O<sub>3</sub>.

In order to investigate the validity of this argument, we carried out UPS measurements to study the valence band of representative  $\text{CeO}_x$  films. Figure 13 shows UPS measurements illustrating the valence-band spectra of representative  $\text{CeO}_x$  films grown by EBE (solid line,  $[\text{Ce}^{3+}] \cong 0.23$ ) and IBAD (dashed line,  $[\text{Ce}^{3+}] \cong 0.13$ ). The characteristic O2*p* 



FIG. 13. Valence-band spectra of representative  $\text{CeO}_x$  films grown by EBE (solid line,  $[\text{Ce}^{3+}] \cong 0.23$ ) and IBAD (dashed line,  $[\text{Ce}^{3+}] \cong 0.13$ ). The characteristic O2*p* band is manifested in both spectra and a shoulder attributed to  $\text{Ce}^{3+}4f^0$  states is more pronounced in the spectrum of the EBE-grown film. The upper edge of the valence band is quite affected by the  $\text{Ce}^{3+}4f^0$  states.

band is manifested in both spectra [this band is reported to be located around 6-eV below the Fermi level  $(E_F)$  (Refs. 2 and 49)] and a shoulder attributed to  $Ce^{3+}4f^0$  states [located at about 3-eV below  $E_F$  (Refs. 2 and 49)] is more pronounced in the spectrum of the EBE-grown film. The upper edge of the valence band is quite affected by the  $Ce^{3+}4f^0$  states.

Indeed, the energy separation between the upper edge of the valence band and  $E_F$  is about 1 and 2.6 eV for the EBEand IBAD-grown films, respectively, due to the more pronounced  $Ce^{3+}4f^0$  states in the first film. The valence-band spectra are associated with the DOS and the interband transitions, which, determined by SE data analysis, are associated with JDOS; therefore, no quantitative correlation between the UPS and SE results can be established. However, the UPS results support the SE data analysis, considering that the redshift of the second TL oscillator is attributed to the Ce4 $f^0 \rightarrow$  Ce4 $f^1$  transition of Ce<sub>2</sub>O<sub>3</sub> and it is more pronounced when the valence band exhibits more enhanced Ce<sup>3+</sup>4 $f^0$  features.

In addition to the energy positions of the electronic transitions, another issue that has produced major disagreement in the literature is the maximum value of  $\varepsilon_2$  that corresponds to the O2 $p \rightarrow$ Ce4f transition at ~4 eV.<sup>1,5,20,51</sup> It has been proposed that this value is majorly affected by the broadening of this transition,<sup>1</sup> however, we have previously shown that the absolute value of the extinction coefficient k (and consequently  $\varepsilon_2 = 2nk$ ) at ~4 eV for CeO<sub>x</sub> films is considerably affected by the CeO<sub>x</sub> film density.<sup>10</sup> Therefore, the voids content (which degrades the film density) may be the major factor that affects the absolute  $\varepsilon_2$  value of the O2p  $\rightarrow$  Ce4f transition. In order to investigate the role of voids, we used Bruggeman's effective medium theory (BEMT). For the BEMT analysis we assumed that the dielectric function of CeO<sub>x</sub> films consist of contributions of the dielectric functions of pure  $CeO_2$  and voids. This assumption is justified in the spectral region 1.5-4.5 eV, where the dominant O2p



FIG. 14. The optical data reported in Ref. 20 for the epitaxial CeO<sub>2</sub> films grown by RMS (solid line) The results of BEMT simulations for CeO<sub>x</sub> films consisted of a material similar to that RMS grown and voids, in comparison with the optical data of the marginal cases of EBE-grown CeO<sub>x</sub> (950 °C and RT) determined through the two-TL fittings.

 $\rightarrow$  Ce4*f* transition comes exclusively from CeO<sub>2</sub> and, thus, the contribution of Ce<sub>2</sub>O<sub>3</sub> is minor.

The higher  $\varepsilon_2$  values in the litterature have been reported by Guo et al.<sup>20</sup> and they correspond to an epitaxial CeO<sub>2</sub> film; they are presented in Fig. 11 in comparison with the  $\varepsilon_2$ values of bulk CeO<sub>2</sub> (Ref. 5) and representative EBE- and IBAD-produced CeO<sub>x</sub> films of our study. For the BEMT analysis we used their data as the reference dielectric function of pure, single-crystal  $CeO_2$ . We determined the dielectric functions of hypothetical CeO<sub>x</sub> materials, which consist of single-crystal regions and voids (with various volume fractions of voids), using BEMT. Then, we compared the simulated BEMT results with the dielectric functions of the EBE- and IBAD-grown films, determined through the two-TL fittings. Such dielectric functions simulated by BEMT are presented in Fig. 14 in the spectral region 1.5-4.5eV. The solid line (refered as RMS) corresponds to the pure RMS-grown CeO<sub>2</sub> of Guo *et al.*,<sup>20</sup> whereas the dotted and dashed lines correspond to hypothetical composite materials  $(CeO_2 + voids)$  with different voids volume fractions. In addition, we show the optical data of the marginal cases of EBE-grown CeO<sub>x</sub> films (950 °C and RT, presented with open triangles and circles exhibiting the highest, 6.7 g/cm<sup>3</sup>, and lowest, 4.5 g/cm<sup>3</sup>, densities, respectively) determined through the two-TL fittings.

We notice that there is very good agreement between the simulated dielectric functions of the hypothetical composite  $\text{CeO}_x$  and the dielectric functions of the EBE-grown films for  $E < E_g$ , suggesting that  $\varepsilon_1$  for  $E < E_g$  is mainly affected by the film density and less so by other factors such as composition and grain size. The maximum absolute values of the  $\varepsilon_1$  and  $\varepsilon_2$  for the  $O2p \rightarrow Ce4f$  transition at  $\sim 4$  eV, of the two hypothetical, composite  $CeO_x$  materials (with 10% and 40% voids volume fractions), are similar to the corresponding values of the two EBE-grown films. The slight differences in the



FIG. 15. The square of the refractive index of  $\text{CeO}_x$  films follows a linear correlation with the film density, which is valid for all  $\text{CeO}_x$  films grown by EBE, IBAD, and RMS.

line shapes of the dielectric functions of the hypothetical, composite ( $CeO_2 + voids$ ), and the EBE-grown  $CeO_x$  are due to broadening variations, which are attributed to other factors such as film stress.

The voids volume fractions of the two hypothetical composite materials are consistent with the voids calculated by XRR and presented in Fig. 4. The agreement and consistency between BEMT simulations and XRR results suggest that the epitaxial CeO<sub>2</sub> film of Guo *et al.*<sup>20</sup> exhibits density equal to the bulk CeO<sub>2</sub> single crystal.<sup>41</sup> It is very interesting that the values of  $\varepsilon_1$  (for  $E < E_g$ ) and the maximum values of  $\varepsilon_1$ ,  $\varepsilon_2$ of the various nanocrystalline CeO<sub>x</sub> films may be described so well, based exclusively on the dielectric functions of the single-crystal CeO<sub>2</sub>. This confirms that the O2*p*→Ce4*f* transition comes exclusively from CeO<sub>2</sub>, is not affected by the Ce<sub>2</sub>O<sub>3</sub> content in the films at all, and that the voids content (and consequently the film density) is the major factor that determines the maximum value of  $\varepsilon_2$  in the O2*p* →Ce4*f* transition.

Based on the previous BEMT analysis we identified a correlation of  $\varepsilon_1$  (for  $E < E_g$ ) with the voids volume fraction (equivalent to film density). This is very important for the determination of the refractive index n, which is correlated with  $\varepsilon_1$  through the relation  $\varepsilon_1 = n^2 - k^2$ . In order to establish an explicit relation between n (for  $E < E_g$ ) and the film density we correlated the n values at 0.16 eV determined by the TL-fit results with the density measured by XRR. The value of  $n^2$  at low energy is equivalent to  $\varepsilon_1$ , taking into account that k=0 for  $E < E_{g}$ . The correlation between  $\varepsilon_1(E \sim 0) = n^2$  and film density  $\rho$  is clearly illustrated in Fig. 15. The square of *n* follows a universal linear relation with  $\rho$ for films deposited by EBE, IBAD, and RMS and confirms whether the films are nanocrystalline or single crystal. The RMS point has been determined by fitting the optical data of epitaxial, crystalline  $CeO_2$  reported by Guo *et al.*<sup>20</sup> with the two-TL model, assuming that the density of the single crystal is equal to that of bulk  $CeO_2$  (7.21 g/cm<sup>3</sup>), as we have determined using the combined BEMT/XRR analysis.

The linear correlation between  $n^2$  and  $\rho$  can be well ex-

plained according to the classical theory of light dispersion.<sup>39</sup> The real part of the dielectric function of a material, which exhibits q electronic transitions, is given by the expression

$$\varepsilon_1(\omega) = 1 + \frac{4\pi N_e e^2}{m} \sum_{j=1}^q \frac{f_j \cdot (\omega_j^2 - \omega^2)}{(\omega_j^2 - \omega^2)^2 + \gamma_j^2 \omega^2}, \qquad (8)$$

where  $N_e$  is the total density of electrons and  $f_j$ ,  $\omega_j$ , and  $\gamma_j$ are the strength, frequency ( $\omega_j = E_{0j}/\hbar$ ), and broadening of the *j*th transition, respectively. The  $f_j$  follows the sum rule  $\sum_{j=1}^{q} f_j = 1$ . For  $E \ll \hbar \omega_k$ ,  $\gamma_k$ , and taking into account that the electron density is associated with the mass density  $\rho$ through the relation  $N_e = N_o (Z_t \rho / A_t)$ ,<sup>53</sup> Eq. (8) becomes

$$n^2 = \varepsilon_1(E \ll E_g) = 1 + \left(\frac{4\pi N_o Z e^2}{mA} \sum_{j=1}^q \frac{f_j}{\omega_j^2}\right) \cdot \rho.$$
(9)

 $\Sigma_{j=1}^{q}(f_j/\omega_j^2)$  reflects the band structure of the material, through the energy values of the electronic transitions. It is very interesting that the linear relation between  $n^2$  and  $\rho$ suggests, according to Eq. (9), that  $\Sigma_{j=1}^{q}(f_j/\omega_j^2)$  remains almost the same for all CeO<sub>x</sub> films whether they are nanocrystalline and substoichiometric (such as the EBE- and IBAD-grown films) or single crystal and stoichiometric (such as the RMS-grown film).

The most interesting and important electronic property of the materials, in order to be used in optics and optoelectronics, is the value of the fundamental gap  $E_g$  (opticalabsorption edge). Therefore, the  $E_g$  values of  $\operatorname{CeO}_x$  have attracted considerable interest.<sup>1,10,11,20,24</sup> The fundamental gap of  $\text{CeO}_x$  is the indirect gap of the  $\text{O2}p \rightarrow \text{Ce4}f$  transition along the L high-symmetry lines of the Brillouin zone of  $CeO_2$  (shown in Fig. 9), which is narrower than the direct gap of the same transition<sup>20,24</sup> and the gaps of the O2p $\rightarrow$  Ce5*d* and Ce4*f*<sup>0</sup> $\rightarrow$  Ce4*f*<sup>1</sup> transitions of CeO<sub>2</sub> and Ce<sub>2</sub>O<sub>3</sub>, respectively.<sup>1,2,49</sup> Using Tauc's formulation<sup>20,38</sup> we determined that the indirect gap of the  $O2p \rightarrow Ce4f$  transition in our samples is located  $\sim 0.5$ -eV below the direct gap of the same transition. The fundamental gap of  $CeO_x$  films can be determined through the two-TL fittings according to Eq. (7).  $E_g$  was found to vary between 2.3 and 2.9 eV, according to the two-TL fittings. These values are much smaller than those reported in the literature.<sup>20,24</sup> This is a typical underestimation of  $E_{o}$  when the TL model is used, due to the Urbach tails in optical absorption, which are caused by defect states in the gap<sup>36</sup> and they are falsely taken into account by the TL fitting. However, a consistent trend of decreasing  $E_g$  with increasing Ce<sup>3+</sup> content has been observed.

Another way to determine  $E_g$  is directly from the experimental  $\varepsilon_2$  spectra following the formalism of Tauc, Grigrovici, and Vancu.<sup>38</sup> According to this formalism, for energies *E* close to  $E_g$  and for indirect electronic transitions,  $\varepsilon_2$  follows the formula

$$E \cdot \sqrt{\varepsilon_2} \propto (E - E_g). \tag{10}$$

This formalism is more straightforward and provides the value of  $E_g$  more accurately, without taking into account the Urbach tails. However, for thin films (such as those EBE



FIG. 16. Tauc plots of representative IBAD- and EBE-grown  $\text{CeO}_x$  films.

grown at 950 °C) the multiple reflections introduce higher error in the determination of  $E_g$ . Figure 16 presents the Tauc plots of representative IBAD- and EBE-grown CeO<sub>x</sub> films. The straight lines are linear fits with Eq. (10), through which  $E_g$  is determined. The more stoichiometric IBAD film exhibits higher  $E_g$  than the EBE film, confirming the trend of decreasing  $E_g$  with increasing Ce<sup>3+</sup>, which was observed in the two-TL-fit results. The  $E_g$  values determined by the Tauc plots are higher (they vary between 2.92 and 3.12 eV) than those determined by the two-TL fits and they are in better agreement with the literature, which reports that the  $E_g$  of single crystal CeO<sub>2</sub> is 3.31 eV.<sup>20</sup>

Figure 17 summarizes the variations of  $E_g$  with the [Ce<sup>3+</sup>] concentration, determined by both methods [two-TL fittings based on Eqs. (6) and (7) and Tauc plots based on Eq. (10)]. The RMS point is the value reported by Guo *et al.*<sup>20</sup> using Tauc plots. Both methods of  $E_g$  calculation show the same overall trend of decreasing  $E_g$  with increasing [Ce<sup>3+</sup>].



FIG. 17. The variation of the fundamental gap with the  $[Ce^{3+}]$  concentration determined from SE data analysis by the two-TL fittings (TL, open symbols) and by Tauc plots (TP, solid symbols), for various growth techniques (the solid and dotted lines are guides to the eyes).

This trend is a universal behavior, since it is valid for all the used growth techniques. This is in agreement with previous works reporting that the O deficiency in CeO<sub>2</sub> induces an optical-absorption tail below 3 eV.<sup>5,19</sup> The two methods of  $E_g$  determination diverge with increasing [Ce<sup>3+</sup>] due to the corresponding increase of vacancy content (see Fig. 6), which enhances the gap states (due to defects) and the Urbach tails. These gap states have been proposed to be associated with O vacancies forming F centers.<sup>5,19</sup> In addition, in the CeO<sub>x</sub> films of this study there exists a considerable amount of defect states due to interfaces, determined by capacitance-voltage (C-V) electrical measurements (not shown here), which may also contribute to the degradation of  $E_g$ . The strong correlation of  $E_g$  with  $[\text{Ce}^{3+}]$  explains the observed redshift of  $E_g$  in nanocrystalline  $\text{CeO}_x$ .<sup>4</sup> The quantum-size effect is expected to enhance the band gap of the materials<sup>29,30</sup> due to the higher localization of energy bands with decreasing particle size (ultimately the bands are reduced to atomic states). Therefore, the redshift of  $E_{g}$  in nanocrystalline  $\text{CeO}_x$  films is not due to the quantum-size effect itself, but to the existence of  $\text{Ce}^{3+}$  at the grain boundaries, as we have shown in this study. Thus, the  $Ce^{3+}$  content increases, with decreasing grain size, eliminating the results of the quantum-size effect and causing the redshift of  $E_g$ .

# **IV. CONCLUSIONS**

We have studied a variety of nanocrystalline CeO<sub>x</sub> films grown by EBE and IBAD. Their structure, morphology, and composition have been studied in detail using XRD, XRR, and XPS. The nanocrystalline CeO<sub>x</sub> films consist mainly of CeO<sub>2</sub> grains, while a considerable concentration (23%-12%)at) of trivalent Ce<sup>3+</sup> is distributed at the CeO<sub>2</sub> grain boundaries, forming amorphous Ce<sub>2</sub>O<sub>3</sub>. A small portion of Ce<sup>3+</sup> (<5%) is also located around O-vacancy sites. The bulk (surface) distribution of CeO<sub>2</sub> (Ce<sub>2</sub>O<sub>3</sub>) is a common feature of the EBE- and IBAD-grown films. The Ce<sub>2</sub>O<sub>3</sub> content is explicitly correlated with the grain size due to the corresponding reduction of the surface/bulk ratio. The film density varies between 4.5 and 6.7  $g/cm^3$  due to the existence of voids.

We investigated the electronic properties of the various nanocrystalline  $CeO_x$  films, compared them with the data reported for epitaxial CeO2, and established universal relations (valid for both nanocrystalline CeO<sub>x</sub> and epitaxial  $CeO_2$ ) between them and the film structure, composition, and morphology. The evaluation of the films' optical properties has been also based on previous band-structure calculations for CeO<sub>2</sub> and Ce<sub>2</sub>O<sub>3</sub>. The fundamental gap  $E_g$  of CeO<sub>x</sub> is due to the indirect  $O2p \rightarrow Ce4f$  transition along the L high-symmetry lines of the Brillouin zone and varied between 2.9 and 3.3 eV. It was correlated with the  $[Ce^{3+}]$ concentration, explaining the redshift of  $E_g$  in nanostructured  $CeO_x$ , which is due to the  $Ce^{3+}$  at the grain boundaries and not the quantum-size effect itself. In addition, we observed the enhancement of the Urbach tails in optical absorption (which are associated with the vacancy content in  $CeO_r$  and therefore they are due to defect states in the gap) with increasing  $[Ce^{3+}]$  and vacancy content. We also correlated the energy position of the maximum optical absorption  $E_{01}$ , due to the  $O2p \rightarrow Ce4f$  electronic transition, which varies up to 160-meV wide, with the lattice constant of the  $CeO_2$  grains. The refractive index, far below  $E_{o}$ , has been explicitly correlated (through the classical theory of light dispersion) with the film density, independently of the  $[Ce^{3+}]$ ,  $[Ce^{4+}]$ , and [O] concentrations, grain size, and the lattice parameter, showing that the density of the films is the major factor affecting *n*. The density was also found to be the major factor affecting the absolute, maximum value of the  $\varepsilon_2$  peak, which corresponds to the  $O2p \rightarrow Ce4f$  electronic transition. Finally, we investigated the energy location of the second Tauc-Lorentz oscillator, which averages the higher-order transitions of  $CeO_2$  and  $Ce_2O_3$ , through fitting of its low-energy tail, and we found that its energy position is very sensitive to the film's composition, as opposed to the energy position of the  $O2p \rightarrow Ce4f$  transition, which originates exclusively from CeO<sub>2</sub> and it is not correlated with the film composition at all.

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