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MASTER THESIS

Storability in Monopoly markets with dynamic demand

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Statement of Originality

I hereby declare that this thesis is my own work, in cooperation with my advisor prof. Fabio Antoniou, and has not previously been submitted for a degree or diploma in any university. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

Triantafyllia Moutziki

(Signed)

Dedication

To my family

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Abstract

In a dynamic storable good market, this thesis investigates how the monopolist's commitment power influences the firm's pricing policy and the welfare effects, in a framework where demand increases over time. In a classic monopoly model, where we integrate storage, it is observed that convex storage cost plays an important role in the final results. As it emerges, consumers decide to store a quantity of the good, independently of the firm's commitment power. In the case of limited commitment, prices tend to be higher than the case of full commitment and the total welfare sinks. Inventories are inserted in the framework of this thesis and it is observed that, without storage cost for the monopolist, they exist only under limited commitment.

Key words: Monopoly, Commitment, Storable Goods, Consumers Storage, Dynamic Demand

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Chapter 1

Introduction

In recent years, there is an increasing interest in the scientific community of Industrial Organization, on how the nature of goods affects consumers' and firms' behavior. There is a vast majority of the literature that investigates the behavior of consumers and firms according to the nature of goods, which are consumed or merchandised.

This dissertation investigates markets for goods which can be stored for later consumption or sale. Most economists to date tend to focus on goods durability while the research regarding goods storability has become more widespread in recent years. Accordingly, before discussing at length the storable goods literature, it will be inclusive to make a reference on the corresponding literature regarding durable goods which has a far greater scope. Hence, goods can be categorized by their durability in durable and non-durable goods.

A durable good is that kind of good that yields utility over time rather than being completely consumed in one use. Items like bricks and books may well be considered as perfectly durable goods because they literally never wear out. According to [O'Sullivan & Sheffrin \(2007\)](#) highly durable goods are those goods that usually continue to be useful for three or more years of use. Common examples are washing machines, fridges and cars. In other words, durable goods are typically characterized by long periods between successive purchases.

On the other hand, the term non-durable goods (consumables) refers to products that are consumed or are only useable for a short period of time because they wear out or become useless. Examples of non-durable goods are fast-moving consumer goods such as cleaning products, food, fuel, medication and paper products.

The research on durable goods creates the need to study storage goods. As mentioned above, this paper, following [Antoniou & Fiocco \(2019\)](#), analyses storable goods that are perishable in consumption albeit can be stored for future consumption. In an ambitious attempt to define the meaning of storable goods in the relevant literature, I cite the delineation of [Hendel et al. \(2014\)](#) "We say a good is storable if consumers can set aside units for later consumption." Storable

goods are nonperishable and do not degrade or lose their value over a time period. Typical examples of storable goods are various intermediate goods, such as oil, coffee and wheat. The goods which are used in relevant literature are those that are sold at groceries and can be purchased in advance and stored.

Literature on storage is relatively recent, so there are not many studies. Consequently this dissertation is based mainly on two papers, [Dudine et al. \(2006\)](#) and [Antoniou & Fiocco \(2019\)](#). The first one has presented a storage model, from the consumer side, where there is storage cost per unit of product and the second one was the first paper that allows simultaneous storage by the consumer and the producer.

The research gap, that this study seeks to contribute, is to examine how a change in production technology might change the results. To make it a little more specific, we change the cost of storage.

To begin with, this master thesis studies a storable good market and the role of commitment in monopoly power. In contrast with the classic monopoly problem, consumers have the capacity of stockpiling. Consumers buy and store some units of the good in the first period in order to consume them in the second period. It is anticipated that consumers decide to hold a quantity of the good because they expect that the price is going to rise.

This desire or unwillingness of consumers to store in the first period is determined by producers' ability to commit to future prices. When monopolist can commit to goods future value, is easier for the consumers to decide whether it is preferable to store or not. On the contrary, in the case of limited commitment, consumers are afraid of a possible increase in prices.

On the other hand, from the side of the monopolist, the point of view is different. The producer wants to avoid consumer storage. In most of cases that examined to date in the literature, as in [Dudine et al. \(2006\)](#), monopolist selects the price that maximizes its profits and leads consumers to store zero quantity. There are also exceptions, such as in this thesis, there is the storage of consumers which is accepted.

The herein study considers a dynamic demand curve which increases during the second period. Moreover, there is convex storage cost too. By virtue of the convex storage cost, the results differ significantly. In antithesis with [Dudine et al. \(2006\)](#), consumers' storage is not eschewed. In this study's model consumers' storage is accepted and calculated.

In addition to, the recent work [Antoniou & Fiocco \(2020\)](#), in this thesis the ability that the monopolist has to hold inventories, is taken into consideration as well. In other words, I study what happens when a monopolist and a consumer store a good's quantity at the same time. By inserting inventories into the model it is expected that the outcome is subject to change

when the monopolist cannot commit to future price.

In this dissertation the main research aim is to expand on the work of [Dudine et al. \(2006\)](#) study by incorporating a different production technology. In particular, owing to a convex storage cost, consumer's storage under full commitment is not zero.

In this attempt, to thoroughly approach the problem, I also utilize [Antoniou & Fiocco \(2019\)](#) work, who have very successfully investigated both consumers' and monopolies' storage. In an attempt of giving another dimension of the problem, only the case where there is no storage cost for the monopolist is considered here. Notwithstanding, the result modifies and inventories still exist without storage cost for the inventories. Some preliminary findings indicate that inventories may have an impact only in the case where the monopolist cannot commit on future prices because under full commitment inventories do not exist.

Chapter 2

Literature Review

The issue of storage and storable goods still attracts the economists. This thesis based on [Antoniou & Fiocco \(2019\)](#) shown that under limited commitment, a firm can use inventories to confine consumer storage incentives. In other words, they bespeak that can not commit to future prices, producer has a strategic motivation to hold inventories when they will be confronted with the imminent buyers stockpiling. They focus on storable goods, which can be stored for future consumption but are perishable in consumption. Due to inventories production cost is sunk when they are vended, inventories treated such as a strategic tactic to decrease future costs, which means lower future prices and ultimately palliates the consumer storage incentives.

An important study, for dynamic monopoly pricing of storable goods in a framework where demand changes over time, wrote by [Dudine et al. \(2006\)](#). Particularly is focused on intertemporal demand and the storage cost gets fixed, so all the agents' types have the same powerful incentive to intertemporally shift their consumption. This enables the authors to presume that both prices with and without commitment are optimal. They conclude that, if the monopolist cannot commit, then prices are increasing among all periods, and social welfare is lower than when monopolist can commit. As a result of wasteful buyer stockpiling, profits and welfare are lower than under full commitment. [Dudine et al. \(2006\)](#) results are in contrary to Coase conjecture, which is broached afterwards.

In a duopoly market, [Anton & Varma \(2005\)](#) analyze a two-period model where enables consumers to store first period purchases. They show that firms bid for consumer storage. They resorted to that when supply is oligopolistic, shoppers are patient and with relatively low storage costs then, in equilibrium, the prices are higher than when storage is unfeasible, resulting in buyer storage. This mainly happens because of firm's motivation to take a piece from rival's future market share, in contrariety with markets that don't have such dynamics, like monopoly and perfect competition.

The storage of firms and their shopper have been investigated separately in the literature so far. Thus, [Antoniou & Fiocco \(2020\)](#) which analyze a dynamic market for storable goods when production costs vary over time. Authors explain how the firm's commitment power, to future prices, influence on firm's pricing policy and its impact on total welfare under the regime of intertemporal cost variations. The expounded model, are inserted intertemporal variations in the monopolist's production costs and additionally firm have to be confronted with a continuum of consumers that are willing to store in expectation of higher future prices. Investigators ascertain that, when consumer storage costs are relatively small and production costs are anticipated to appreciate, the firm's lack of commitment commonly leads to lower prices than full commitment. As a consequence, lower prices increase consumer surplus and, under certain circumstances, entire welfare. Nevertheless, for particular costs of storage, the firm's full commitment ability has a positive impact to the consumers and in general, to the economy.

An article for durable goods is [Ortner \(2017\)](#) which investigates the occasion of a monopolist who produces a durable good and who lacks commitment power. [Ortner \(2017\)](#) based on [Coase \(1972\)](#) where production costs are constant over time and Coase culminate in that monopolist would not be able to sell at the static monopoly price. Thus Ortner studies the problem of a monopolist who cannot commit to a path of prices and whose cost of production varies stochastically over time. The author shows that stochastic costs introduce an option to the monopolist to attend on consumers with discrete valuations, at several periods and charges them different prices. The seller exercises his market power if consumer valuations are discrete and the market outcome is inefficient. When consumer types are sustained, by contrast, the monopolist cannot educe rents and the market outcome is efficient.

The trigger for the study of durable goods, and by extension of storable goods, in monopoly theory was the Coase conjecture. It is remarkable to point that Ronald Coase has received the Nobel Memorial Prize, in 1991, for his whole offer in economic science and mostly for the Coase conjecture. [Coase \(1972\)](#) describes a durable good market in which there is only one seller (monopoly), consumers have different valuations and resale is impossible. He proposes, at an unlimited time setting, if individuals' valuations are hidden than monopolist should sell its product at a low price. The reason behind is over several periods, in a monopoly situation, the seller is in price competition with itself and when consumers are relatively patient perchance wait for the lowest price. So in the first period, the monopolist will have to supply a competitive price. Finally, he suggests a commitment to a stable linear pricing strategy so that the monopolist skirt around this problem.

An early relevant contribution, on storable goods literature, is [Benabou \(1989\)](#), which

analyzes the optimal pricing policy and storage strategies of a storable good monopolist and his customers. Benabou speculates that the seller of a storable good, has a cost of changing his price but should be in coincident with inflation rate and customers guess price adjustments so that decide their willingness of store. It is remarkable that price is eroded by inflation and firms prefer to discourage consumer storage in anticipation of higher production costs. This dynamic game results in a Markov unique perfect equilibrium. It is important that [Benabou \(1989\)](#) provides a theoretical foundation for the assertion that inflation causes price uncertainty.

The role of commitment plays an exceedingly significant role in industrial organization theory. Such as [Dudine et al. \(2006\)](#) and [Coase \(1972\)](#), [Nie \(2009a\)](#) studies commitment impact on storable goods under monopoly and vertically integrated structures. He verifies the aforementioned authors that, under limited commitment prices are higher than on the occasion in which the monopolist launches commitment. Furthermore, he inspects similar results to the monopoly and under vertical integration is obtained respectively. In particular, the prices under limited commitment are also higher than that with full commitment under vertical integration. The penman extends for storable goods to duopoly structure, at [Nie \(2009b\)](#). Based on the discrete-time dynamic models, he compares full commitment versus non-commitment at the case of duopoly and he receives the corresponding results. He concludes that in the case of duopoly, the prices without commitment are higher than that under full commitment. Besides, under commitment, social welfare is improved.

In a differentiated good market where there is price competition, [Guo & Villas-Boas \(2007\)](#) investigate the impingement of consumers stockpiling and purchasing in advance for future consumption. They start with the hypothesis that consumers' stockpiling could potentially generate motivations for competing firms to offer price promotions, for the purpose of competing away potential future demand of the competitors. Firms may do it inasmuch that consumers' storage may exacerbate price competition. For this reason, researchers study price competition in a two-period model among differentiated firms facing heterogeneous shoppers. They observe that when production costs are anticipated to appreciate over time, the firms have more incentives to compete for consumer storage, which creates further squeeze to lower prices. As a conclusion the preference heterogeneity leads indirectly to exacerbating of future price competition and may displace consumer storage equilibrium, by virtue of the new differential consumer storage propensity.

Closely related to [Guo & Villas-Boas \(2007\)](#) and [Anton & Varma \(2005\)](#), is [Hong et al. \(2002\)](#). They investigate the impact of consumer heterogeneity in price searching and storage ability on price dynamics. They consider a model which divides consumers into two types, shoppers and captives, where shoppers purchase from the most cost-effective store and captives

buy from a fixed store, in both cases provided the price is relatively low. Only shoppers may store inventories of the good. This model culminates in equilibrium price dispersion. [Hong et al. \(2002\)](#) deduce that the main storage effect is to mitigate future price competition when the market is comprised only of captives consumers. In contrast with [Guo & Villas-Boas \(2007\)](#) who show that when no restriction is imposed on consumers' stockpiling capabilities, the consumers who are more likely to stockpile a product are those who have relatively higher preferences, that is, the captives in [Hong et al. \(2002\)](#).

The economists Hendel and Nevo worked together for numerous papers about Industrial Organization. One of them is [Hendel & Nevo \(2004\)](#) entertain the importance of intertemporal substitution, interpret demand elasticities and estimate the magnitude of the effect of storability. Their paper puts on the spot the predictions of a consumer inventory model and inspects the available evidence that this model is relevant. Data concerns non-durable storable products, especially on products sold in supermarkets. The dynamic effect of storability has significant consequences for both public policy and optimal firm behaviour.

In addition, at [Hendel & Nevo \(2013\)](#), study intertemporal price discrimination when consumers can anticipate demand and stockpile for future consumption. They expound a simple dynamic demand model, study empirical, by using market level data, the role of intertemporal price discrimination in storable goods markets. The intertemporal price discrimination is being studied empirically and its impact on profits and welfare. To sum up, intertemporal price discrimination can potentially move upwards the profits and likewise total welfare increases when sales are offered.

Another notable paper is [Hendel et al. \(2014\)](#) which constitute an extending of [Dudine et al. \(2006\)](#) analysis to non-linear pricing of storable goods. By developing a two-period model for consumers who can demand multiple units, is expanded the relationship of consumers' storage ability and its possible impingement on sellers ability to extract surplus via non linear prices. As it turns out, new constraints are imposed on a monopolist's ability to extract surplus and in attempt to lessening these constraints can give birth to cyclical patterns in pricing and sales, even when purchasers are homogeneous. Further to, is analyzed a monopolist who that can commit to a sequence of bundles and presented sellers' consequences from the lack of commitment. Finally, is worth mentioning that by buyer heterogeneity in storage technology into models predicted that is more likely to be on sale larger bundles.

In contrast with the former columnists, [Nava & Schiraldi \(2014\)](#) present a model for sales that does not based on consumer heterogeneity to explain periodic price fluctuation. On the contrary, it investigates the consumer storage impact on firms incentives to promote periodic price decreases in sustain collusion. Results justify why sales can encourage collusion in markets

for storable goods and render, for firms, a motivation to engage in sales.

A contribution to literature that analyzes the importance of imperfect information and search costs, do [Seiler](#) and [Pires](#). [Seiler \(2013\)](#) proposes a model under imperfect information, where consumers bear the cost of search. The search decision for storable goods is included into a dynamic demand framework and search is modeled jointly with the decision for purchase. For the estimations are used data on laundry detergent purchases. On the other hand, [Pires \(2016\)](#), in order to evaluate the importance of search cost, estimates a dynamic choice model with search frictions by using data of liquid laundry detergent. In contrast to [Pires](#) model, [Seiler](#) assumes that consumer only chooses whether to search or not. One of the main contributions of [Pires](#) relative to [Seiler](#) is to consider the across-product search process and the effects of brand choice.

A recent write-up is [Osborne \(2018\)](#) which is related to construction and estimate of a cost-of-living index. For this reason, is used a dynamic structural model for storable products, specifically, canned tuna and canned soup. For storable good markets, authors conclude that fixed-base indexes may substantially decrease cost living changes from price changes, on the contrary with an index that accurately captures movements in the cost of living.

Writers of [Berbeglia et al. \(2019\)](#) study a dynamic pricing problem which is confronted by a storable good monopolist who sells to forward-looking consumers. They analyze and compare the two major pricing mechanisms, the pre-announced pricing policy and the contingent pricing policy. It is clarified that the storable good can be purchased along a finite time horizon in indivisible atomic quantities. The results under the contingent pricing policy come to an agreement with the case of divisible storable goods ([Dudine et al., 2006](#)).

In the storable good monopoly problem [Berbeglia et al. \(2018\)](#) compare two important pricing mechanisms, by their performances, which are the pre-announced pricing mechanisms and the price contingent mechanisms. The main result is that the monopolist can obtain more profit by using a pre-announced pricing mechanism than a contingent pricing mechanism in some instances. This result suggests that, except the practical advantages of using a pre-announced pricing mechanism, there are also sometimes strong financial benefits.

The goal of [Su \(2010\)](#) is to study the optimal seller's dynamic pricing tactics and the optimum consumers' stockpiling strategies. He analyzes a dynamic pricing problem, for storable products which have inventory holding costs. He makes a dynamic game model, over an infinite time horizon, and elaborates a methodology based on rational expectations. In the opposite direction with many similar papers, which involved in sellers' inventory, [Su](#) focus on consumers' stockpiling decisions.

In [Erdem et al. \(2003\)](#) is created an empirical analysis by estimate a household demand

model for consumer goods that are branded, storable and subject to stochastic price fluctuation. Specifically, are used data for the ketchup. They analyze the effect on current period purchase decisions from inventories and expectations of future prices. The results indicate that the nature of the price process and price expectations have important ramifications on demand elasticities.

Another empirical analysis is [Perrone \(2017\)](#) about nondurable and storable goods. Her article presents a dynamic model of demand with consumer inventories and proposes a shortcut to evaluate the long-run price elasticities without the need to solve the dynamic program. Using French data on food markets, she finds elasticities in agreement with former literature's estimations.

Another paper is [Ching & Osborne \(2017\)](#), which analyzes a dynamic consumer stockpiling model so that quantifies the ascendancy of forward-looking behaviour and its consequences for pricing strategy. Authors demonstrate that, by properly modeling storage cost as inventories function, the critical variable of this model, inventory, the discount factor and natural exclusion restrictions can help to identify the model's parameters. Furthermore, they calculate a stockpiling model by using data on laundry detergents and estimate the discount factor of population distribution.

Supplemental, about storable goods, [Kano \(2018\)](#) provides new effects on consumer inventory on the purchase and stockpiling of a storable grocery product, toilet paper. He confirms the existence of correlations between the critical variables of purchase probability, purchase quantity and inventory derived. To culminate in the aforementioned correlation was studied a dynamic model of consumer inventory. Is remarkable that the evidence does not harmonise with a constant consumption rate, which, for tractability, assumed by many studies.

After an extensive study of the previous literature, there is a research gap on how the effects will change when consumers storage is not null in the full commitment case. Hence, the main research aim is to extend the work of [Dudine et al. \(2006\)](#) study by incorporating not null consumer's storage under full commitment. In this attempt, to approach the problem comprehensively, I also utilize [Antoniou & Fiocco \(2019\)](#) work, who have very successfully investigate both consumers' and monopolists' storage.

Chapter 3

Linear Example

Initially, is quoted a simple linear example to give a premonition for the basic forces at play in the following model. It is assumed that there are two periods, $\tau \in \{1, 2\}$ and the demand increases between periods one and two. Namely, in the first period demand is $D(p_1) = 80 - 10p_1$ and in the second period demand is $D(p_2) = 120 - 10p_2$

Assume that the production cost is $C = 5$. Also, presume that a consumer can store between periods 1 and 2 at a cost of $\frac{1}{2}sbDs^2$ when he stores Ds quantity. For simplicity is considered $sb = 1$.

Absent storage, the optimal monopolistic solution is $p_1 = \frac{13}{2}$ and $p_2 = \frac{17}{2}$

Full Commitment

Consider the following maximization problem for the monopolist:

$$\max_{p_1, p_2} \Pi = (p_1 - 5)[D(p_1) + Ds(p_1)] + (p_2 - 5)[D(p_2) - Ds(p_1)]$$

$$s.t. p_2 = p_1 + Ds(p_1)$$

The profits maximization show that the solution to this problem is

$$p_1^c = \frac{20}{3}$$

$$p_2^c = \frac{25}{3}$$

$$Ds^c = \frac{5}{3}$$

3.1 Limited Commitment

$$\max_{p_2} \Pi_2 = (p_2 - c)[D(p_2) - Ds(p_1)]$$

The solution is $p_2 = \frac{170 - Ds}{20}$ and given that $p_2 = p_1 + sbDs$ is calculated $Ds = \frac{170 - 20p_1}{21}$.

The aggregate profits:

$$\Pi = (p_1 - 5)[D(p_1) + Ds(p_1)] + (p_2 - 5)[D(p_2) - Ds(p_1)]$$

$$s.t. p_2 = p_1 + sbDs(p_1)$$

Become

$$\Pi = (p_1 - 5)[80 - 10p_1 + Ds(p_1)] + (p_1 + sbDs(p_1) - 5)[120 - 10(p_1 + Ds(p_1)) - Ds(p_1)]$$

By the following maximization:

$$\max_{p_1, Ds} \Pi = (p_1 - 5)[80 - 10p_1 + Ds(p_1)] + (p_1 + sbDs(p_1) - 5)[120 - 10(p_1 + Ds(p_1)) - Ds(p_1)]$$

The results are

$$p_1^{lc} = \frac{3215}{482}$$

$$p_2^{lc} = \frac{4055}{482}$$

$$Ds^{lc} = \frac{420}{241}$$

Chapter 4

The Model

4.1 Setting

Consumers

This model presents a storable good market with two-periods. In each period $\tau \in \{1, 2\}$ the monopolistic firm faces a differentiable demand. Moreover, is considered that consumers have the option of store some units of the good in the first period, with a store unit cost $sb \geq 0$, for consumption in the second period. The consumers storage demand for the first period writes as

$$D(p_1) = \alpha_1 - \beta p_1 \quad (4.1)$$

where $D(p_1)$ is the quantity that consumers buy in the first period and p_1 is the price of the good in the first period too.

Correspondingly for the second period, the consumers storage demand writes as

$$D(p_2) = \alpha_2 - \beta p_2 \quad (4.2)$$

where $D(p_2)$ is the quantity that consumers buy in the second period and p_2 is the price of the good in this period as well.

In addition to this, the other terms should be determined too. Both the fixed terms α_1 and α_2 can not be negative, namely $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$. In addition, we consider that demand increases, so that

$$\alpha_2 \geq \alpha_1 \quad (4.3)$$

From the other hand, the β presents the negative demand-price relationship. When the price increases by one unit then the quantity requested is reduced by β .

Firm

The monopolistic firm incurs a constant unit production cost. The unit cost is c in the first and in the second period.

As it is known, the monopolist's aspiration is to maximize firm's profits. The firm's aggregate profits are $\Pi \equiv \Pi_1 + \Pi_2$, where the profits in the first and second period, are respectively

$$\Pi_1 = (P_1 - c)[D(p_1) + Ds(p_1)] \quad (4.4)$$

and

$$\Pi_2 = (P_2 - c)[D(p_2) - Ds(p_1)] \quad (4.5)$$

where Ds is the quantity that consumers buy in the first period, store and consumed in the second period. Consumers store a quantity in the first period so the consumer's storage inflates the demand in the first period though it pushes down in the second period.

4.2 Timing and equilibrium concept

Each period of the game includes the following two stages.

1. The monopolist determines the price for the good.
2. Consumers buy a quantity of good and decide on the amount to be stored for next period consumption.

Chapter 5

The Static solution

Initially, is better to start this analysis when storage is not feasible and essentially we lead to the static monopoly problem. Is considered a two-period, $\tau \in \{1, 2\}$, monopoly market with

$$D(p_1) = \alpha_1 - \beta p_1$$

and

$$D(p_2) = \alpha_2 - \beta p_2$$

the first and the second period demand correspondingly.

The aggregate monopoly profits are

$$\Pi^m = (p_1 - c)D(p_1) + (p_2 - c)D(p_2) \quad (5.1)$$

After the profits maximization we can calculate the equilibrium static monopoly prices. The first period monopoly price is $p_1 = \frac{\alpha_1 + \beta c}{2\beta}$ and respectively for the second period $p_2 = \frac{\alpha_2 + \beta c}{2\beta}$

The aggregate monopoly profits are

$$\Pi^m = \frac{(\alpha_1^2 + \alpha_2^2 - 2\beta c^2 - 2\beta(\alpha_1^2 + \alpha_2^2 - 2(\alpha_1 + \alpha_2 - 2\beta c)c)}{4\beta} \quad (5.2)$$

In the static monopoly solution the consumer surplus is $CS_1 = \frac{(\alpha_1 - \beta c)^2}{8\beta}$ for the first period and and respectively for the second period $CS_2 = \frac{(\alpha_2 - \beta c)^2}{8\beta}$

In addition to, the loss of social welfare for the first period is $DWL_1 = \frac{(\alpha_1 - \beta c)^2}{8\beta}$ and $DWL_2 = \frac{(\alpha_2 - \beta c)^2}{8\beta}$ for the second time period.

Chapter 6

Full Commitment

In general, when a firm is equipped with commitment power can credibly announce both the first and the second-period prices and adhere faithfully this pricing policy. The monopolist's objective is to maximize the aggregate profits. Based on this, the firm determines a price sequence. Formally, the maximized aggregate profits is given by the sum of the first-period profits in (4.4) and the second-period profits in (4.5).

The aggregate profits under full commitment are

$$\Pi^{FC} = (P_1 - c)[D(p_1) + Ds(p_1)] + (P_2 - c)[D(p_2) - Ds(p_1)] \quad (6.1)$$

In this study is examined only the case where prices are linearly connected as follows on (6.2). Essentially, we maximize the profits subject to the following constraint of sequential optimality

$$p_2 = p_1 + sbDs(p_1) \quad (6.2)$$

Besides, it is worth mentioning that about the unit cost implies that

$$sb \geq 0 \quad (6.3)$$

Given that the firm maximizes its profits, the first order conditions of (6.1) which are scrutinized in the Appendix, is conclude to the following proposition

Proposition 1. *Under full commitment:*

(i) *The first period price is $p_1^c = \frac{\alpha_1 + \alpha_2 + 2\beta c + \alpha_1\beta sb + \beta^2 csb}{2\beta(2 + \beta sb)}$ and the second period price is*

$$p_2^c = \frac{\alpha_1 + \alpha_2 + 2\beta c + \alpha_2\beta sb + \beta^2 csb}{2\beta(2 + \beta sb)}$$

(ii) *Buyer stockpiling is $D_s^c = \frac{\alpha_2 - \alpha_1}{2(2 + \beta sb)}$*

It is remarkable that the second period price is higher than the first one which is perfectly expected. The reason is that there is storage. Storage exists because consumers were expected price rise.

Given the above prices and consumers stockpiling of Proposition 1, is easy to calculate the monopoly profits. They are formulated on the following proposition

Proposition 2. *The monopolist's aggregate profits under full commitment*

are:

$$\Pi^{FC} = \frac{(\alpha_1 + \alpha_2 - 2\beta c)^2 + \beta(\alpha_1^2 + \alpha_2^2 - 2(\alpha_1 + \alpha_2)\beta c + 2\beta^2 c^2)sb}{4\beta(2 + \beta sb)}$$

It is interesting to investigate which is the consumer surplus in every period. It is well known that consumer surplus in a monopoly market is small, is the smaller between the others and more competitive markets.

Proposition 3. *Consumers surplus under full commitment:*

$$(i) \text{ The first period consumers surplus is } CS_1^c = \frac{(\alpha_2 + \beta c(2 + \beta sb) - \alpha_1(3 + \beta sb))^2}{8\beta(2 + \beta sb)^2}$$

$$(ii) \text{ The second period consumers surplus is } CS_2^c = \frac{(\alpha_1 - \alpha_2)^2 \beta sb + (\alpha_1 + \beta c(2 + \beta sb) - \alpha_2(3 + \beta sb))^2}{8\beta(2 + \beta sb)^2}$$

Chapter 7

Limited Commitment

Under Limited Commitment, the monopolist chooses a sequence of prices in order to maximize total profits and separate the second-period profits. When the monopolistic firm does not bound for the second- period price has a motivation to raise P_2 , for the purpose of profit maximization.

The second period profits are

$$\Pi_2^{LC} = (p_2^{lc} - c)[D^{lc}(p_2^{lc}) - Ds^{lc}(p_1^{lc})] \quad (7.1)$$

Where $p_2 = p_1 + sbDs(p_1)$ just like in the Full Commitment case.

The aggregate profits are the summarize of the first an the second period profits, scilicet

$$\Pi^{LC} = (p_1^{lc} - c)[D^{lc}(p_1^{lc}) + Ds^{lc}(p_1^{lc})] + (p_2^{lc} - c)[D^{lc}(p_2^{lc}) - Ds^{lc}(p_1^{lc})] \quad (7.2)$$

Given the constraints (6.2) and (6.3) we are leaded to the following results.

Proposition 4. *Under limited commitment:*

(i) *The first period price is* $P_1^{lc} = \frac{\alpha_2 + 2\beta c + \alpha_1(1 + 2\beta sb)^2 + 4\beta sb(\alpha_2 + \beta c(2 + \beta sb))}{4\beta(1 + 2\beta sb(2 + \beta sb))}$

and the second period price is

$$P_2^{lc} = \frac{\alpha_1 + \alpha_2 + 2\beta c + 2\beta(\alpha_1 + 3\alpha_2 + 4\beta c)sb + 4\beta^2(\alpha_2 + \beta c)sb^2}{4\beta(1 + 2\beta sb(2 + \beta sb))}$$

(ii) *Buyer stockpiling is* $D_s^{lc} = \frac{(\alpha_2 - \alpha_1)(1 + 2\beta sb)}{2 + 4\beta sb(2 + \beta sb)}$

Based on the previous Proposition 4 is easy to calculate the monopolist's aggregate profits

Proposition 5. *The monopolist's aggregate profits under limited commitment*

are:

$$\Pi^{LC} = \frac{(\alpha_1 + \alpha_2 - 2\beta c)^2 + 4\beta(\alpha_1 + \alpha_2 - 2\beta c)^2 sb + 4\beta^2(\alpha_1^2 + \alpha_2^2 - 2(\alpha_1 + \alpha_2)\beta c + 2\beta^2 c^2)sb^2}{8\beta(1 + 2\beta sb(2 + \beta sb))}$$

Proposition 6. *Consumers surplus under limited commitment:*

(i) *The first period consumers surplus is* $CS_1^{lc} = \frac{(\alpha_2 + 2\beta c + 4\beta sb(\alpha_2 + \beta c(2 + \beta sb)) - \alpha_1(3 + 4\beta sb))^2}{32\beta(1 + 2\beta sb(2 + \beta sb))^2}$

(ii) *The second period consumers surplus is*

$$CS_2^{lc} = \frac{4(\alpha_1 - \alpha_2)^2 \beta sb(1 + 2\beta sb)^2 + (\alpha_1 + 2\alpha_1 \beta sb + 2\beta c(1 + 2\beta sb(1 + 2\beta sb)) - \alpha_2(3 + 2\beta sb(5 + 2\beta sb)))^2}{32\beta(1 + 2\beta sb(2 + \beta sb))^2}$$

Chapter 8

Price comparisons

Equipped with the results of the previous sections and given the former propositions, we are now in a position to contrast the equilibrium prices under the two regimes of full and limited commitment.

At first we compare the prices under full commitment case. We expect that the second period price is going to be higher than the first one, because of the storage cost.

Based on appendix results the difference between P_2 and P_1 is $\Delta P_c = \frac{(\alpha_2 - \alpha_1)sb}{4 + 2\beta sb}$ and given the fact that $\alpha_2 \geq \alpha_1$ (4.3) the former hypothesis for the price relationship is verified, namely $P_2^c \geq P_1^c$

In a similar way is easy to collate the prices under limited commitment. The difference between P_2 and P_1 now is $\Delta P_{lc} = \frac{(\alpha_2 - \alpha_1)sb(1 + 2\beta sb)}{2 + 4\beta sb(2 + \beta sb)}$

The most interesting is the comparison between Full and Limited Commitment, rather than to draw conclusions about which situation is most beneficial for the consumer and which one for the monopolist respectively.

Originally are investigated the first period prices. Is reminded that the prices are $P_1^c = \frac{\alpha_1 + \alpha_2 + 2\beta c + \alpha_1 \beta sb + \beta^2 csb}{2\beta(2 + \beta sb)}$ and $P_1^{lc} = \frac{\alpha_2 + 2\beta c + \alpha_1(1 + 2\beta sb)^2 + 4\beta sb(\alpha_2 + \beta c(2 + \beta sb))}{4\beta(1 + 2\beta sb(2 + \beta sb))}$ so the price difference, namely $P_1^{lc} - P_1^c$ is $\Delta P_{t=1} = \frac{(\alpha_2 - \alpha_1)sb}{4\beta(1 + 2\beta sb(2 + \beta sb))}$. Given that (4.3) it is positive, exactly as we expected. Under Limited Commitment, monopolist sets a higher price so as to avoid some consumers storage.

From the other hand, the second period prices are $P_2^c = \frac{\alpha_1 + \alpha_2 + 2\beta c + \alpha_2 \beta sb + \beta^2 csb}{2\beta(2 + \beta sb)}$ and $P_2^{lc} = \frac{\alpha_1 + \alpha_2 + 2\beta c + 2\beta(\alpha_1 + 3\alpha_2 + 4\beta c)sb + 4\beta^2(\alpha_2 + \beta c)sb^2}{4\beta(1 + 2\beta sb(2 + \beta sb))}$

The price difference namely $P_2^{lc} - P_2^c$ is calculated $\Delta P_{t=2} = \frac{(\alpha_2 - \alpha_1)sb(3 + 2\beta sb)}{4(2 + \beta sb)(1 + 2\beta sb(2 + \beta sb))}$. This price difference is also positive because the monopolist has an incentive to break his promise and increase the price in order to maximize the second period profits.

Chapter 9

The model with inventories

In this section, is examined the case in which both consumer and monopolist store a quantity of the good. In the way of the previous model, is speculated that consumers storage is Ds . In this model, the monopolist has the option of hold inventories. The inventories, in the model, are expressed by I .

9.1 Full Commitment

At first, is investigated a two-period model, in which both consumer and monopolist hold a quantity of good for the second period. Under full commitment, the monopolist pledges about the prices.

Subsequently, are presented the aggregate profits:

$$\Pi^{FC} = P_1(D(p_1) + Ds(p_1)) - c(D(p_1) + Ds(p_1) + I) + P_2(D(p_2) + Ds(p_1)) - c(D(p_2) - Ds(p_1) - I) \quad (9.1)$$

We assume that the monopolist store all future demand. Therefore, inventories are

$I = D(p_2) - Ds(p_1)$ Hence, the aggregate profits are:

$$\Pi^{FC} = p_1(D(p_1) + Ds(p_1)) - c(D(p_1) + Ds(p_1) + (D(p_2) - Ds(p_1))) + p_2(D(p_2) + Ds(p_1)) \quad (9.2)$$

We mark that the profits are exactly the same as the full commitment without inventories (q.v.Chapter 5). So that we expect to end up to the same results.

$$\begin{aligned} \max_{p^1, p^2} \Pi^{FC} &= p_1(D(p_1) + Ds(p_1)) - c(D(p_1) + Ds(p_1) + (D(p_2) - Ds(p_1))) + p_2(D(p_2) + Ds(p_1)) \\ & \text{s.t. } p_2 = p_1 + sbDs \end{aligned}$$

Solving the maximization problem, we conclude to the following proposition

Proposition 7. *Under full commitment:*

(i) *The first period price is $p_1^c = \frac{\alpha_1 + \alpha_2 + 2\beta c + \alpha_1 \beta s b + \beta^2 c s b}{2\beta(2 + \beta s b)}$ and the second period price is*

$$p_2^c = \frac{\alpha_1 + \alpha_2 + 2\beta c + \alpha_2 \beta s b + \beta^2 c s b}{2\beta(2 + \beta s b)}$$

(ii) *Buyer stockpiling is $D_s^c = \frac{\alpha_2 - \alpha_1}{2(2 + \beta s b)}$*

Obviously the aggregate profits under full commitment carry on to be the same.

Proposition 8. *The monopolist's aggregate profits under full commitment*

are:

$$\Pi^{FC} = \frac{(\alpha_1 + \alpha_2 - 2\beta c)^2 + \beta(\alpha_1^2 + \alpha_2^2 - 2(\alpha_1 + \alpha_2)\beta c + 2\beta^2 c^2) s b}{4\beta(2 + \beta s b)}$$

Similarly, the consumer surplus in every period is presented on the following proposition.

Proposition 9. *Consumers surplus under full commitment:*

(i) *The first period consumers surplus is $CS_1^c = \frac{(\alpha_2 + \beta c(2 + \beta s b) - \alpha_1(3 + \beta s b))^2}{8\beta(2 + \beta s b)^2}$*

(ii) *The second period consumers surplus is $CS_2^c = \frac{(\alpha_1 - \alpha_2)^2 \beta s b + (\alpha_1 + \beta c(2 + \beta s b) - \alpha_2(3 + \beta s b))^2}{8\beta(2 + \beta s b)^2}$*

Under full commitment, inventories do not play a decisive role in the monopolist decision.

We have the same results with or without monopolist storage.

9.2 Limited Commitment with inventories

In this section is investigated again a two-period model, in which both consumer and monopolist hold a quantity of good for the second period. Under limited commitment, is expected that inventories are going to modify prices.. When the monopolistic firm does not bound for the second- period price has a motivation to raise P_2 , for the purpose of profit maximization.

The second period profits are

$$\Pi_2^{LC} = p_2^{lc}(D_2^{lc} - Ds^{lc}(p_1)) - c(D_2^{lc} - Ds^{lc}(p_1) - I) \quad (9.3)$$

Where $p_2 = p_1 + sbDs(p_1)$ just like in the Full Commitment case. Moreover, we assume that the monopolist store all future demand. Therefore, inventories are $I = D(p_2) - Ds(p_1)$

The aggregate profits are the summarize of the first an the second period profits, scilicet

$$\Pi^{LC} = p_1^{lc}(D^{lc}(p_1) + Ds^{lc}(p_1)) - c(D^{lc}(p_1) + Ds^{lc}(p_1) + I) + p_2^{lc}(D^{lc}(p_2) - Ds^{lc}(p_1)) - c(D^{lc}(p_2) + Ds^{lc}(p_1) - I) \quad (9.4)$$

Given the constraints (6.2) and (6.3) we are led to the following results.

The proposition 8 provides a full characterization of consumer storage behaviour and of the equilibrium price sequence when the monopolist can not commit to future prices.

Proposition 10. *Under limited commitment:*

$$(i) \text{ The first period price is } P_1^{lc} = \frac{\alpha_2 + 4\alpha_2\beta sb + 2\beta c(1 + \beta sb)(1 + 2\beta sb) + \alpha_1(1 + 2\beta sb)^2}{4\beta(1 + 2\beta sb(2 + \beta sb))}$$

and the second period price is

$$P_2^{lc} = \frac{\alpha_1 + \alpha_2 + 2\beta c + 2\beta\alpha_1 sb + 6\alpha_2\beta sb + 2\beta^2 csb + 4\alpha_2\beta^2 sb^2}{4\beta(1 + 4\beta sb + 2\beta^2 sb^2)}$$

$$(ii) \text{ Buyer stockpiling is } D_s^{lc} = \frac{\alpha_2 - \alpha_1 - 2\beta c - 2\beta(\alpha_1 - \alpha_2 + \beta c)sb}{2 + 4\beta sb(2 + \beta sb)}$$

$$(iii) \text{ Inventories are } I = \frac{\alpha_1 + \alpha_2 + 2\beta c + 2\beta(\alpha_1 + 3\alpha_2 + \beta c)sb + 4\alpha_2\beta^2 sb^2}{4 + 8\beta sb(2 + \beta sb)}$$

Based on Proposition 10 is easy to calculate the monopolist's aggregate profits. We observed that inventories exists, under limited commitment, although the storage cost is zero. It is in contrast to the results of [Antoniou & Fiocco \(2019\)](#).

Proposition 11. *The monopolist's aggregate profits under limited commitment*

are:

$$\Pi^{lc} = \frac{(\alpha_1 + 2\alpha_1\beta sb)^2 + (\alpha_2 + 2\alpha_2\beta sb - 2\beta c(1 + \beta sb))^2 + 2\alpha_1(\alpha_2 + 4\alpha_2\beta sb - 2\beta c(1 + \beta sb(5 + 2\beta sb)))}{8\beta(1 + 2\beta sb(2 + \beta sb))}$$

Given the prices, we calculate the consumer surplus for the first and the second period respectively.

Proposition 12. *Consumers surplus under limited commitment:*

(i) *The first period consumers surplus is*

$$CS_1^{lc} = \frac{(\alpha_2 + 2\beta c - \alpha_1(3 + 2\beta sb)(1 + 4\beta sb) + 2\beta sb(\beta c + \alpha_2(3 + 2\beta sb)))^2}{32\beta(1 + 2\beta sb(2 + \beta sb))^2}$$

(ii) *The second period consumers surplus is*

$$CS_2^{lc} = \frac{8\beta sb(\alpha_1 - \alpha_2 + 2\beta c + 2\beta(\alpha_1 - \alpha_2 + \beta c)sb)^2 + (\alpha_1 + 2\alpha_1\beta sb + 2\beta c(1 + \beta sb) - \alpha_2(3 + 2\beta sb(5 + 2\beta sb)))^2}{32\beta(1 + 2\beta sb(2 + \beta sb))^2}$$

Proposition 13. *Given the prices, P_1 and P_2 is $\Delta P_c = \frac{sb(\alpha_1 - \alpha_2 + 2\beta c + 2\beta(\alpha_1 - \alpha_2 + 2\beta c)sb)}{2 + 4\beta sb(2 + \beta sb)}$.*

As it emerges P_2 is higher than P_1 given the fact that the price increases because the demand rises.

Chapter 10

Conclusion

The present study was designed to determine the effect of commitment on prices and on total welfare, in storable good markets, with a differential production technology.

This thesis studies a dynamic storable good market with only one seller. Supposed a two period model with increasing demand, we first introduce the potential of consumers' storage. The consumers can hold, at the first period, a quantity of the good for second period consumption but they have a convex storage cost. The cost is convex because for every extra quantity of the good that they save it becomes more costly, given that consumers' repository is restricted. After that the model is extended with both consumer and monopolist storage. As before, consumer has a convex storage cost but the monopolist has not storage cost.

In the above models we maximize monopoly profits in order to calculate prices and surpluses. By solving the first model, where only consumer storage is allowed, it transpires that when the demand increases, from the first to the second period, the monopolist raise the price. Because of the price increase consumer has an incentive to store. This happens in both cases of full and limited commitment.

Is observed that, under limited commitment, prices are higher than under full commitment. Monopolist sets higher prices so as to avoid some consumers storage. Another remarkable observation, at this model's result, is that under full commitment consumer store. This contradicts the results of [Dudine et al. \(2006\)](#).

In the second model, where both consumer's and monopolist's storage is allowed, it transpires that when the demand increases, from the first to the second period, the monopolist raise the price. Under full commitment, consumers have a stockpiling motivation but the monopolist has no reason to store, keeping in mind that he commits to the second period price. On the contrary, under limited commitment, both consumer and monopolist have incentives to store. It is in antithesis with the results of [Antonioni & Fiocco \(2019\)](#), under limited commitment, in this thesis the monopolist holds inventories when he has not storage cost. Both at full

and limited commitment case the prices are higher at the second period because the demand increases.

As a result, this thesis shows that in storable good markets, where demand increases over time, the monopolist's lack of commitment is unambiguously welfare detrimental. The prices are increasing during the two periods and the consumers' surplus is subsiding. By inserting inventories in the examined model, the outcome is not affected under full commitment. In contrast with, due to the absence of commitment, inventories affect the outcome.

This model's inventories could have policy implication, for example, the inventories reduce the price. For that reason they should be encouraged from the taxes.

This research has created many questions about storability, in need of further investigation. A further study could examine more closely the effects of consumer and seller storage in a monopsonistic market. To the best of my knowledge, there is not a previous study about this.

Appendix A

Appendix: Maths of Full Commitment

$$\begin{aligned} \max_{p_1, p_2} \Pi^{FC} &= (p_1 - c)[D(p_1) + Ds(p_1)] + (p_2 - c)[D(p_2) - Ds(p_1)] \\ \text{s.t. } p_2 &= p_1 + sbDs \end{aligned}$$

First Order Conditions (FOC)

$$\frac{\partial \Pi^{FC}}{\partial p_1} = 0 \quad (\text{A.1})$$

and

$$\frac{\partial \Pi^{FC}}{\partial Ds} = 0 \quad (\text{A.2})$$

From (A.1)

$$\Rightarrow \frac{\partial \Pi^{FC}}{\partial p_1} = \alpha_1 + \alpha_2 + 2\beta p_1 + 2\beta(p_1 + sbDs) = 0$$

$$\Rightarrow 4\beta p_1^c = \alpha_1 + \alpha_2 + 2\beta c - 2\beta sbDs$$

$$\Rightarrow p_1^c = \frac{\alpha_1 + \alpha_2 + 2\beta c - 2\beta sbDs}{4\beta}$$

From (A.2)

$$\Rightarrow \frac{\partial \Pi^{FC}}{\partial Ds} = -c + p_1 - c(-1 - \beta sb) + (-1 - \beta sb)(p_1 + sbDs) + sb(\alpha_2 - Ds - \beta(p_1 + sbDs)) = 0$$

$$\Rightarrow p_1 = \frac{\alpha_2 + \beta c - 2Ds - 2\beta Ds sb}{2\beta}$$

From this two results for p1

$$\frac{\alpha_1 + \alpha_2 + 2\beta c - 2\beta sbDs}{4\beta} = \frac{\alpha_2 + \beta c - 2Ds - 2\beta Ds sb}{2\beta}$$

$$\Rightarrow D_s^c = \frac{\alpha_2 - \alpha_1}{2(2 + \beta sb)}$$

$$\text{Put this on } \Rightarrow p_1^c = \frac{\alpha_1 + \alpha_2 + 2\beta c - 2\beta sb D_s}{4\beta}$$

$$\Rightarrow p_1^c = \frac{\alpha_1 + \alpha_2 + 2\beta c + \alpha_1 \beta sb + \beta^2 csb}{2\beta(2 + \beta sb)}$$

By the price relationship $p_2 = p_1 + sbD$

$$\Rightarrow p_2^c = \frac{\alpha_1 + \alpha_2 + 2\beta c + \alpha_2 \beta sb + \beta^2 csb}{2\beta(2 + \beta sb)}$$

A.1 Consumers surplus under full commitment

For the first period,

$$D(p_1) = \alpha_1 - \beta p_1$$

Find p_1^{max}

$$D(p_1) = 0$$

$$\Rightarrow \alpha_1 - \beta p_1^{max} = 0$$

$$\Rightarrow p_1^{max} = \frac{\alpha_1}{\beta}$$

$$\text{And } p_1^{mim} = p_1^c = \frac{\alpha_1 + \alpha_2 + 2\beta c + \alpha_1 \beta sb + \beta^2 csb}{2\beta(2 + \beta sb)}$$

So Consumers surplus for the first period is

$$CS_1^c = \int_{p_1^c}^{p_1^{max}} D_1, p_1$$

$$\Rightarrow CS_1^c = \int_{\frac{\alpha_1 + \alpha_2 + 2\beta c + \alpha_1 \beta sb + \beta^2 csb}{2\beta(2 + \beta sb)}}^{\frac{\alpha_1}{\beta}} \alpha_1 - \beta p_1, p_1$$

$$\Rightarrow CS_1^c = \frac{(\alpha_2 + \beta c(2 + \beta sb) - \alpha_1(3 + \beta sb))^2}{8\beta(2 + \beta sb)^2}$$

For the second period,

$$D(p_2) = \alpha_2 - \beta p_2$$

Find p_2^{max}

$$D(p_2) = 0$$

$$\Rightarrow \alpha_2 - \beta p_2^{max} = 0$$

$$\Rightarrow p_2^{max} = \frac{\alpha_2}{\beta}$$

And $p_2^{mim} = p_2^c = \frac{\alpha_1 + \alpha_2 + 2\beta c + \alpha_2\beta sb + \beta^2 csb}{2\beta(2 + \beta sb)}$

Moreover from the cost function $MC = sbDs$

Were $D_s^{min} = 0$ and $D_s^c = \frac{\alpha_2 - \alpha_1}{2(2 + \beta sb)}$

So Consumers surplus for the second period is

$$CS_2^c = \int_{p_2^c}^{p_2^{max}} D_2, p_2 + \int_0^{D_s^c} sbDs, Ds$$

$$\Rightarrow CS_2^c = \int_{\frac{\alpha_1 + \alpha_2 + 2\beta c + \alpha_2\beta sb + \beta^2 csb}{2\beta(2 + \beta sb)}}^{\frac{\alpha_2}{\beta}} \alpha_2 - \beta p_2, p_2 + \int_0^{\frac{\alpha_2 - \alpha_1}{2(2 + \beta sb)}} sbDs, Ds$$

$$\Rightarrow CS_2^c = \frac{(\alpha_1 - \alpha_2)^2 \beta sb + (\alpha_1 + \beta c(2 + \beta sb) - \alpha_2(3 + \beta sb))^2}{8\beta(2 + \beta sb)^2}$$

Appendix B

Appendix: Maths of Limited Commitment

$$\max_{p_2} \Pi_2 = (p_2 - c)[D(p_2) - Ds(p_1)]$$

First Order Conditions (FOC)

$$\frac{\partial \Pi_2}{\partial p_2} = 0 \tag{B.1}$$

From (B.1)

$$\Rightarrow \frac{\partial \Pi_2}{\partial p_2} = \alpha_2 + \beta c - Ds(p_1) - 2\beta p_2 = 0$$

$$\Rightarrow p_2 = \frac{\alpha_2 + \beta c - Ds(p_1)}{2\beta}$$

(Given that $p_2 = p_1 + sbDs(p_1)$)

$$\Rightarrow p_1 + sbDs = \frac{\alpha_2 + \beta c - Ds}{2\beta}$$

$$\Rightarrow Ds = \frac{\alpha_2 + \beta c - 2\beta p_1}{1 + 2\beta sb}$$

To the aggregate profits

$$\begin{aligned} \max_{p_1, p_2, Ds} \Pi^{lc} &= (p_1 - c)[D(p_1) + Ds(p_1)] + (p_2 - c)[D(p_2) - Ds(p_1)] \\ s.t. p_2 &= p_1 + sbDs(p_1) \end{aligned}$$

For $p_2 = p_1 + sbDs(p_1)$ the aggregate profits become:

$$\Rightarrow \Pi^{lc} = (p_1 - c)[\alpha_1 - \beta p_1 + Ds(p_1)] + (p_1 + sbDs(p_1) - c)[\alpha_2 - \beta(p_1 + sbDs(p_1)) - Ds(p_1)]$$

And by $Ds = \frac{\alpha_2 + \beta c - 2\beta p_1}{1 + 2\beta sb}$ the aggregate profits become:

$$\Rightarrow \Pi^{lc} = (p_1 - c)\left[\alpha_1 - \beta p_1 + \frac{\alpha_2 + \beta c - 2\beta p_1}{1 + 2\beta sb}\right] + (p_1 + sb\frac{\alpha_2 + \beta c - 2\beta p_1}{1 + 2\beta sb} - c)\left[\alpha_2 - \beta(p_1 + sb\frac{\alpha_2 + \beta c - 2\beta p_1}{1 + 2\beta sb}) - \frac{\alpha_2 + \beta c - 2\beta p_1}{1 + 2\beta sb}\right]$$

First Order Conditions (FOC)

$$\frac{\partial \Pi^{lc}}{\partial p_1} = 0 \quad (\text{B.2})$$

From (B.2)

$$\Rightarrow \frac{\partial \Pi^{lc}}{\partial p_1} = P_1^{lc}(4\beta(1+2\beta sb(2+\beta sb))) - (\alpha_2 + 2\beta c + \alpha_1(1+2\beta sb)^2 + 4\beta sb(\alpha_2 + \beta c(2+\beta sb))) = 0$$

$$\Rightarrow p_1^{lc} = \frac{\alpha_2 + 2\beta c + \alpha_1(1 + 2\beta sb)^2 + 4\beta sb(\alpha_2 + \beta c(2 + \beta sb))}{4\beta(1 + 2\beta sb(2 + \beta sb))}$$

Put p_1 on $Ds = \frac{\alpha_2 + \beta c - 2\beta p_1}{1 + 2\beta sb}$

$$\Rightarrow D_s^{lc} = \frac{(\alpha_2 - \alpha_1)(1 + 2\beta sb)}{2 + 4\beta sb(2 + \beta sb)}$$

By the price relationship $p_2 = p_1 + sbD$

$$\Rightarrow p_2^{lc} = \frac{\alpha_1 + \alpha_2 + 2\beta c + 2\beta(\alpha_1 + 3\alpha_2 + 4\beta c)sb + 4\beta^2(\alpha_2 + \beta c)sb^2}{4\beta(1 + 2\beta sb(2 + \beta sb))}$$

B.1 Consumers surplus under Limited Commitment

For the first period,

$$D(p_1) = \alpha_1 - \beta p_1$$

Find p_1^{max}

$$D(p_1) = 0$$

$$\Rightarrow \alpha_1 - \beta p_1^{max} = 0$$

$$\Rightarrow p_1^{max} = \frac{\alpha_1}{\beta}$$

$$\text{And } p_1^{mim} = p_1^{lc} = \frac{\alpha_2 + 2\beta c + \alpha_1(1 + 2\beta sb)^2 + 4\beta sb(\alpha_2 + \beta c(2 + \beta sb))}{4\beta(1 + 2\beta sb(2 + \beta sb))}$$

So Consumers surplus for the first period is

$$CS_1^{lc} = \int_{p_1^{lc}}^{p_1^{max}} D_1, p_1$$

$$\Rightarrow CS_1^{lc} = \int_{\frac{\alpha_1}{\beta}}^{\frac{\alpha_1 + \alpha_2 + 2\beta c + \alpha_1 \beta s b + \beta^2 c s b}{2\beta(2 + \beta s b)}} \alpha_1 - \beta p_1, p_1$$

$$\Rightarrow CS_1^c = \frac{(\alpha_2 + 2\beta c + 4\beta s b(\alpha_2 + \beta c(2 + \beta s b)) - \alpha_1(3 + 4\beta s b))^2}{32\beta(1 + 2\beta s b(2 + \beta s b))^2}$$

For the second period,

$$D(p_2) = \alpha_2 - \beta p_2$$

Find p_2^{max}

$$D(p_2) = 0$$

$$\Rightarrow \alpha_2 - \beta p_2^{max} = 0$$

$$\Rightarrow p_2^{max} = \frac{\alpha_2}{\beta}$$

$$\text{And } p_2^{mim} = p_2^{lc} = \frac{\alpha_1 + \alpha_2 + 2\beta c + 2\beta(\alpha_1 + 3\alpha_2 + 4\beta c)s b + 4\beta^2(\alpha_2 + \beta c)s b^2}{4\beta(1 + 2\beta s b(2 + \beta s b))}$$

Moreover from the cost function $MC = sbDs$

$$\text{Were } D_s^{min} = 0 \text{ and } D_s^{lc} = \frac{(\alpha_2 - \alpha_1)(1 + 2\beta s b)}{2 + 4\beta s b(2 + \beta s b)}$$

So Consumers surplus for the second period is

$$CS_2^{lc} = \int_{p_2^c}^{p_2^{max}} D_2, p_2 + \int_0^{D_s^{lc}} sbDs, Ds$$

$$\Rightarrow CS_2^{lc} = \int_{\frac{\alpha_1 + \alpha_2 + 2\beta c + 2\beta(\alpha_1 + 3\alpha_2 + 4\beta c)s b + 4\beta^2(\alpha_2 + \beta c)s b^2}{4\beta(1 + 2\beta s b(2 + \beta s b))}}^{\frac{\alpha_2}{\beta}} \alpha_2 - \beta p_2, p_2 + \int_0^{\frac{(\alpha_2 - \alpha_1)(1 + 2\beta s b)}{2 + 4\beta s b(2 + \beta s b)}} sbDs, Ds$$

$$\Rightarrow CS_2^{lc} = \frac{4(\alpha_1 - \alpha_2)^2 \beta s b(1 + 2\beta s b)^2 + (\alpha_1 + 2\alpha_1 \beta s b + 2\beta c(1 + 2\beta s b(1 + 2\beta s b)) - \alpha_2(3 + 2\beta s b(5 + 2\beta s b)))^2}{32\beta(1 + 2\beta s b(2 + \beta s b))^2}$$

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