ON THE FERMION MASS SPECTRUM IN THE SUPERSYMMETRIC SU(4)×O(4) MODEL

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Received 23 November 1989

In a class of superstring derived models, only the fermions of the third generation receive masses at the tree level. We describe realistic mechanisms to provide masses to the lighter generations, and apply them to the SU(4)×O(4) model.

Recently supersymmetric GUTs [1] have regained considerable attention. The reason is that a wide class of them [2–6] has been derived as an effective $N=1$ supersymmetric theory from superstring theories living in higher dimensions [7].

SUSY GUTs from superstrings are quite appealing and challenging. String consistencies in higher dimensions appear as certain constraints on the supersymmetric GUTs. It is hoped, however, that these constraints finally will pick out only a few models which are consistent with the low energy phenomena.

Indeed, in the string derived models [2–6] one does not have the absolute freedom to choose the gauge group and the Higgs and matter field representations. Furthermore, in supersymmetric GUTs with string origin, the spectrum of the theory, the couplings and the global symmetries are also determined by the string properties.

In a wide class of SUSY GUT models, there remains a discrete symmetry in the low energy superpotential which allows only the Higgs which couples to the heaviest fermion generation to develop a VEV. Therefore only the top, the bottom quark and the tau lepton will receive masses at the tree level.

In practice the following situation appears: when the original symmetry at the string level breaks to the low energy gauge group, due to some additional discrete symmetries remnant from the higher one at the string level, each fermion family couples to a different Higgs field. On the other hand, due to phenomenological reasons, only those Higgses that couple to the heaviest generation are allowed to develop a VEV.

We know, however, that in the standard model, fermion generations not having Yukawa couplings to the Higgs which develops a VEV never get a mass. On the other hand we could consider contributions of non-renormalizable terms [3] and one loop contributions [8] from graphs involving their Yukawa couplings. These contributions, however, are at most of the order O(100 MeV). Therefore they could be sufficient to provide masses to the leptons and the light quarks [8], but not to the heavy members of the quark sector. Nevertheless, in superstring model building we end up with an $N=1$ supersymmetric model which, unlike the standard nonsupersymmetric one, can give radiatively masses to all generations of quarks. Indeed, the soft supersymmetry breaking can provide us with flavour mixing trilinear couplings which combine quark and s-quarks with gauginos. The presence of these terms can lead to flavour changing interactions on one hand [9] and to radiative contributions in the quark mass matrix on the other [10].

It is interesting to apply the above ideas to a specific model and see how they can work. In the present work, we will take the SU(4)×O(4) $N=1$ supersymmetric model and explore the possibility of obtaining the correct quark spectrum, assuming that only the third fermion generation gets a mass at the tree level. In order to be more specific in our subsequent discussion, let us give at this point the basic ingredients of the model. (For a detailed description see refs. [5,11]). The 16 fermion fields belong to the $F=(4,2,1)$ and $\bar{F}=(\bar{4},1,2)$ representations of the

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$SU(4)\times SU(2)_L\times SU(2)_R$ symmetry, i.e.

$$F=(u, d, \nu, e), \quad \bar{F}=(u^c, d^c, \nu^c, e^c) . \tag{1}$$

The Higgs fields are those needed to break the SU(4) and $SU(2)_R$ symmetry

$$H=(4, 1, 2), \quad \bar{H}=(\bar{4}, 1, 2) , \tag{2}$$

where $\langle H\rangle=\langle\bar{H}\rangle=O(M_{GUT})$, and the one needed to break $SU(2)_L$ symmetry

$$h=(1, 2, 2)=\begin{pmatrix} h^0 & \bar{h}^+ \\ h^- & \bar{h}^0 \end{pmatrix} \tag{3a}$$

with VEV

$$\langle h\rangle=\begin{pmatrix} v & 0 \\ 0 & \bar{v} \end{pmatrix} . \tag{3b}$$

The model also employs the sextet fields $D=(6, 1, 1)$, $\bar{D}=(\bar{6}, 1, 1)$ as well as the singlet fields ϕ_i, $i=0$, 1, 2, 3, where $\langle\phi_i\rangle=0$ while $\langle\phi_0\rangle=\mu$, where μ is of the order of the electroweak scale. The terms of the superpotential relevant to our discussion are [5, 11]

$$w=\lambda_1 F\bar{F}h+\lambda_2 F\bar{H}\phi_0+\lambda_3 HDD+\lambda_4\bar{H}\bar{H}\bar{D} +\lambda_5 hh\phi_i+\lambda_6\phi_i^3+\dots . \tag{4}$$

We note here that the first term according to our assumptions gives masses to the fermions of the third generation while the second term provides a superheavy mass to the right handed neutrino. The rest of the terms give superheavy masses to the colour triplets which arise from the decomposition of the sextets $[D\to(3, 1, 1)+(\bar{3}, 1, 1)]$.

The model predicts $m_t=\lambda_1\bar{v}$, $m_b=\lambda_1 v$ at the GUT scale, therefore the Yukawa couplings of the top and bottom quarks are equal at M_{GUT} ($\lambda_1=\lambda_t=\lambda_b$). Using renormalization group arguments it can be shown that large Yukawa couplings (and therefore the top quark coupling) tend to approach an infrared fixed point [12]. Therefore starting with $\lambda_t=\lambda_b$ at M_{GUT} we should not expect large differences between them at the weak scale. This means that the splitting of the top and bottom quark mass should mainly rely on the large ratio of the VEVs $\rho=\bar{v}/v$. Thus if we want to have a top quark mass of the order of 90 GeV, this ratio must be at least greater than 10. In the minimal supersymmetric standard model this value can hardly be acceptable since such a choice of the VEVs cannot also minimize the tree level potential. In more complicated models, however, the situation is somewhat different. In particular for string derived models the appearance of extra U(1) factors allows the possibility of choosing the value ρ sufficiently larger [10] than the one allowed in the minimal supersymmetric standard model [13].

We note in passing that there is another possibility [14] to obtain a large mass for the top quark in our model, which we would like to mention. The adjoint of SU(4) under SU(3) decomposes into $\mathbf{15}\to\mathbf{8}+\bar{\mathbf{3}}+\mathbf{3}+\mathbf{1}$, i.e. the usual gluon-octet, a coloured triplet T and antitriplet T^c, and a neutral singlet. The fermionic parts of the triplet and antitriplet superfields have the same charges with the up quark–antiquark fields, the only difference being that the former (T, T^c), do not have $SU(2)_L$ quantum numbers. The Yukawa lagrangian then can mix the above triplets with the ordinary quarks through the terms $g_4(T^cQL+TQ^cL^c)$, where g_4 is the SU(4) gauge coupling constant. Assuming now VEVs for the scalar partners of the neutrinos in the third direction (i.e. $\langle\tilde{\nu}_3\rangle$, $\langle\tilde{\nu}_3^c\rangle\neq 0$), one can form two massive "quarks" of the up type. One eigenstate is heavy enough and plays the role of gaugino whilst the other one is interpreted as the top quark which is estimated [14] to approach its upper experimental bound (180 GeV). we note, however, that this case introduces new phenomenological consequences which should be explored in detail. In the following we will assume that the ratio ρ can be sufficiently large in superstring models; therefore we will not need to complicate the analysis by the introduction of gaugino triplets.

As we mentioned already, the two lighter fermion generations of fermions will receive masses from one loop gaugino exchange graphs. Thus, in the quark sector, one can have large radiative contributions from gluino exchange diagrams, like those in fig. 1.

In order to calculate the contribution of the diagram in fig. 1, we firstly derive the relevant soft trilinear couplings generating the squark matrices.

The scalar potential in the model is given by

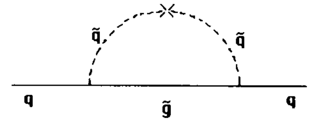

Fig. 1. Radiative corrections to quark masses.

$$V=\sum_i \left|\frac{\partial w}{\partial \phi_i}\right|^2+m_{3/2}^2\,|\phi_i|^2+(A+3B)m_{3/2}(w+w^*)\,, \tag{5}$$

where w is the trilinear superpotential in (3), $m_{3/2}$ is the gravitino mass and $A+3B=\bar{A}$ is a parameter of order one.

The $\tilde{u}$ mass matrix squared is found to be

$$\begin{pmatrix} m_{3/2}^2+|\lambda_1\bar{v}|^2 & \lambda_1\lambda_5^*v^*\mu^*+\bar{A}m_{3/2}\lambda_1\bar{v} \\ \lambda_1^*\lambda_5\mu v+\bar{A}m_{3/2}\lambda_1\bar{v} & m_{3/2}^2+|\lambda_1\bar{v}|^2 \end{pmatrix}, \tag{5'}$$

while the $\tilde{d}$ mass matrix squared has a similar form,

$$\begin{pmatrix} m_{3/2}^2+|\lambda_1\bar{v}|^2 & \lambda_1\lambda_5^*\bar{v}^*\mu^*+\bar{A}m_{3/2}\lambda_1 v \\ \lambda_1^*\lambda_5\mu\bar{v}+\bar{A}m_{3/2}\lambda_1 v & m_{3/2}^2+|\lambda_1 v|^2 \end{pmatrix}. \tag{5''}$$

We are ready now to calculate the radative corrections to the up and down quark mass matrices. From the diagram of fig. 1 the up quark mass matrix gets a contribution of the order

$$m_u\approx\frac{\alpha_3}{4\pi}\frac{T}{m_{\tilde{g}}}\left[m_{\tilde{g}}^2\left(\frac{-1}{m_{\tilde{g}}^2-m_{\tilde{u}}^2}+\frac{m_{\tilde{g}}^2}{m_{\tilde{g}}^2-m_{\tilde{u}}^2}\ln\frac{m_{\tilde{g}}^2}{m_{\tilde{u}}^2}\right)\right]$$
$$=\frac{\alpha_3}{4\pi}\frac{T}{m_{\tilde{g}}}f(x)\,, \tag{6}$$

where $T\approx\lambda_1 v\lambda_5\mu+\lambda_1\bar{v}\bar{A}m_{3/2}\approx\lambda_1\bar{v}\bar{A}m_{3/2}$ is the soft trilinear coupling (we have assumed that $v\mu\ll\bar{v}m_{3/2}$), and

$$f(x)=\frac{1}{(1-x)^2}\,(x-1-\ln x) \tag{7}$$

with x the ratio $x=m_{\tilde{u}}^2/m_{\tilde{g}}^2$. Although details for the supersymmetric masses are not known, a rough estimation of the above contribution is possible. Indeed, by inspection of the above formulae, we notice that the contribution to the up quark mass matrix is sensitive to the ratio $m_{\tilde{u}}^2/m_{\tilde{g}}^2$ through the function $f(x)$. For example, for $m_{\tilde{u}}=m_{\tilde{g}}$, $f(x)=\frac{1}{2}$, but as the ratio $x=m_{\tilde{u}}^2/m_{\tilde{g}}^2$ decreases the contribution increases. In order to have an estimation, we make the following natural assumptions. Firstly we assume that $m_{\tilde{g}}\approx m_{3/2}>\mu$ while from the $m_{\tilde{\mu}}$ matrix $m_{\tilde{u}}^2\approx \mathrm{O}(m_{3/2}^2-m_{3/2}\bar{A}\lambda_1\bar{v})\approx m_{3/2}^2-m_{3/2}\bar{A}m_t$, or [taking $\bar{A}\approx\mathrm{O}(1)$]

$$m_{3/2}^2\approx\tfrac{1}{2}\left(m_t+\sqrt{m_t^2+m_{\tilde{u}}^2}\right)\,. \tag{8}$$

Setting now a certain value for $m_{\tilde{u}}$ we can find the contribution to m_u as a function of m_t. For example in our case ($m_{\tilde{g}}>m_{\tilde{u}}$), the lower bound for $m_{\tilde{u}}$ is around 70 GeV [15]. In fig. 2 we plot m_q^{rad} as a function of m_t for $m_{\tilde{u}}=70$, 90, and 110 GeV. The top quark mass is taken to vary in the range 60–180 GeV. From these plots we notice that corrections of the order of the charm quark mass can be generated radiatively for relatively large top quark mass. For example, for $m_t\approx125$ we find that $m_{3/2}\approx160$ GeV and $m_u\approx1.4$ GeV which is indeed of the order of the charm quark mass. For higher $m_{\tilde{u}}$ values, m_t should also be higher in order to give sufficient mass corrections. Similar manipulations can be done for the down quark mass matrix. In this case we find

$$m_d\approx\frac{\alpha_3}{4\pi}\frac{\lambda_1\bar{v}\lambda_5\mu}{m_{\tilde{g}}}f(y)\,, \tag{9}$$

where now $y=m_{\tilde{d}}^2/m_{3/2}^2$. Bearing in mind that μ is of the order of the electroweak scale we find that the contribution is now of the order of a few hundred MeV. This is also sufficient to generate the down quark masses of the first and second generation. We notice therefore that the top mass and the Higgs mixing mass parameter μ, play an essential role in our model.

In an analogous manner, we can calculate the radiative corrections to the leptonic sector. Since now we have wino or zino instead of gluino exchange diagrams, the corresponding contribution is smaller. Again assuming only that the third generation receives a mass at the tree level, for natural choices of the mass parameters involved, we can obtain the rest of the lepton mass spectrum.

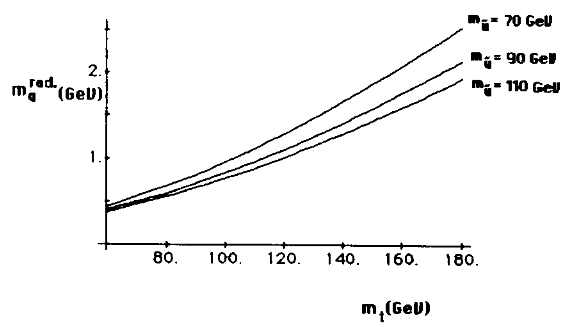

Fig. 2. Plot of m_q^{rad} as a function of m_t for $m_{\tilde{u}}=70$, 90 and 110 GeV.

In this letter, we have made qualitative predictions for the fermion mass matrices in a class of supersymmetric models where only the third generation of fermions receives masses at the tree level. Under certain assumptions, it is found that the one loop gaugino exchange graphs can generate the masses of the two lighter fermion generations. It is of course encouraging that one can find sensible values for the involved mass parameters to produce the quark and lepton masses. Nevertheless a big question remains. Do all these parameters really conspire in a miraculous way to give the desired mass ratios of the two lighter generations? Although attractive assumptions to answer this question may work [10], a definite answer to this question would need more information on the gaugino mixing and a further understanding of the underlying symmetries.

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