ON THE RELATION BETWEEN THE GAUGE-COVARIANT FORMULATION OF STRING FIELD THEORIES

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By using the language of differential forms, which illuminates the structure of the gauge symmetries of string theories in a compact notational framework, we clarify the relation between the Neveu–West–Nicolai–Schwarz formulation of the gauge-covariant string field theories and that of Banks and Peskin

1. The rapidly growing interest in the recently constructed supersymmetric string theories [1,2] as unified theories of all known interactions makes more urgent the answers to many fundamental theoretical questions, such as renormalizability or finiteness, nature of the vacuum, symmetry-breaking mechanisms, consistent compactification of extra dimensions, to mention a few of them [3]. Although partial answers to some of these questions have been given in a non-covariant formulation at the first- or the second-quantized level [3], it is expected that deeper understanding will be obtained when one has gauge- as well as Lorentz-covariant formulation of string field theories. Indeed several attempts in such a direction have recently been started [4–9].

In this letter we present a geometric interpretation of the Neveu–West–Nicolai–Schwarz (NWNS) string field theory for the free open bosonic string in $D = 26$ dimensions. Other cases will be discussed elsewhere. We will see that the geometric interpretation illuminates the structure of the huge set of gauge invariances of the string field theory, which is a consequence of the reparametrization invariance of the string. This reformulation has a further advantage of making the relation between NWNS theories and that of Banks and Peskin (BP) [6] transparent. As an application, we shall point out that BP-type theories with a finite number of auxiliary fields (minimum four) exist.

2. The basic tool for the study of the string gauge symmetries is the Virasoro algebra, which is the quantum version of the Lie algebra of the reparametrization group [3]

$$[L_n, L_m] = V^k_{n,m} L_k + \delta_{n,-m} C(n)\,, \qquad k, n, m \in \mathbf{Z}\,, \tag{2.1}$$

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where

$$V^k_{n,m} = (n-m)\delta_{n+m,k} ,$$

and

$$C(n) = \tfrac{1}{12}Dn(n^2-1) , \quad n \in \mathbf{Z} .$$

The dimension D of the space–time in which the string lives is taken to be arbitrary for the moment. (In what follows, we shall deal with the case with an infinite number of fields. Finite-number versions will be obtained simply as special cases, and so will become clear later.)

Let M be the space of functions $x^\mu(s)$ defined on a two-dimensional surface Σ ($s \in \Sigma$). M is an infinite-dimensional space and will be called the string space. String fields are defined as function(als) on M and form a Hilbert space H_M. The inner product in H_M is defined by an appropriate path integral, $A, B \in H_M$,

$$(A,B) \equiv \int \mathcal{D}x(s)\, A^+(x(s))B(x(s)) . \tag{2.2}$$

H_M is the Fock space of the states of a string. The action of an infinitesimal reparametrization transformation on Σ induces a transformation on M and hence on H_M. This can be represented by

$$\delta_n \psi = L_n \psi , \quad \psi \in H_M . \tag{2.3}$$

Consider now the tangent and the cotangent spaces, T_M and T^*_M, respectively, on M. The above transformation on M naturally defines a corresponding basis $\{e^n\}$, $n = 1, 2, ...$, say, on T^*_M. The dual basis $\{e_{-m}\}$ on T_M is defined as usual:

$$(e^n, e_{-m}) = \delta^n_m , \quad n, m \in \mathbf{Z}_+ . \tag{2.4}$$

This "inner product" is between T_M and T^*_M and should not be confused with the one on H_M.

Now, tensor fields on M can be introduced just as in the case of the finite dimensional manifold [10]. It will be useful to consider tensors for which all the covariant and the contravariant indices are separately fully antisymmetrized. Then it is natural to call such a tensor of $\binom{a}{b}$ type an $\binom{a}{b}$ form, and write it as

$$\omega = \omega^{m_1 \dots m_a}_{n_1 \dots n_b} e_{m_1} \wedge \dots \wedge e_{m_a} \otimes e^{n_1} \wedge \dots \wedge e^{n_b}$$
$$n_i \in \mathbf{Z}_+ , \quad m_j \in \mathbf{Z}_- . \tag{2.5}$$

The inner product of an $\binom{a}{b}$ form ω and of a $\binom{b}{a}$ form ρ consists of (2.2) and (2.4) and takes the form

$$(\omega, \rho) \equiv \int \mathcal{D}x(s)\, \omega^{m_1 \dots m_a}_{n_1 \dots n_b}(x(s))\rho^{n_1 \dots n_b}_{m_1 \dots m_a}(x(s)) . \tag{2.6}$$

The string fields Ψ, $\Phi^{(m)}$ and $\zeta^{(m,n)}$ of the NWNS formulation become forms in an obvious way

$$\Psi \colon \binom{0}{0} \text{ form} ,$$
$$\Phi = \Phi^{-m} e_{-m} \colon \binom{1}{0} \text{ form} ,$$
$$\zeta = \zeta^{-m}_n e_{-m} \otimes e^n \colon \binom{1}{1} \text{ form} . \tag{2.7}$$

Next we come to the important notions of the exterior derivative d and the divergence δ [10].

We define the exterior derivative of a $\binom{0}{0}$ form Ψ as

$$d\Psi = L_n \Psi e^n , \quad n \in \mathbf{Z}^+ \tag{2.8}$$

and demanding the antiderivation property

$$d(\omega \wedge \nu) = d\omega \wedge \nu + (-1)^{\deg \omega}\, \omega \wedge d\nu , \tag{2.9}$$

[where for an $\binom{a}{b}$ form ω, $\deg \omega = b$], plus the cohomology property

$$d^2 = 0$$

on Ψ, we are led to choose

$$de^n = -\tfrac{1}{2} V^n_{km} e^k \wedge e^m , \quad n, k, m \in \mathbf{Z}^+ . \tag{2.10}$$

Demanding $d^2 = 0$ on a $\binom{1}{0}$ form we may choose

$$de_{-n} = V^m_{n,-k} e_{-m} \otimes e^k . \tag{2.11}$$

One may also choose $-V^m_{n,k}$ (indices $\mathbf{Z}_+$) in place of $V^m_{n,-k}$ to get $d^2 = 0$. This choice, however, is not useful in what follows. Note also that $V^m_{n,-k} = W^n_{n,k}$ of BP [6]. With the definitions (2.8), (2.10) and (2.11) we can build the action of d naturally on any $\binom{a}{b}$ form ω [6]:

$$d\omega = (L_{n_1} \omega^{m_1 \dots m_a}_{n_2 \dots n_{b+1}} + a V^{m_1}_{p,-n_1} \omega^{p m_2 \dots m_a}_{n_2 \dots n_{b+1}} - \tfrac{1}{2} b V^p_{n_1 n_2} \omega^{m_1 \dots m_a}_{p n_3 \dots n_{b+1}}) e^{n_1} \wedge \dots \wedge e^{n_{b+1}} \otimes e_{m_1} \wedge \dots \wedge e_{m_a} ,$$
$$m_i \in \mathbf{Z}^- , \quad n_j \in \mathbf{Z}^+ . \tag{2.12}$$

The next step in our construction is to define the dual operator δ, the divergence. This is defined to be the conjugate of d with respect to the inner product (2.6)

$$(\mathrm{d}\omega, \nu) \equiv (\omega, \delta\nu) . \tag{2.13}$$

From (2.6), (2.12) and (2.13), we see that d maps $\binom{a}{b}$ forms to $\binom{a}{b+1}$ forms and δ, $\binom{a}{b}$ forms to $\binom{a-1}{b}$ forms. From (2.12) and (2.13) we find the action of δ on any $\binom{a}{b}$ form [6]:

$$\delta\omega = \{L_{-p}\omega^{pm_1\ldots m_{a-1}}_{n_1\ldots n_b} + bV^q_{n_1,-p}\omega^{pm_1\ldots m_{a-1}}_{n_1\ldots n_b} - \tfrac{1}{2}(a-1)V^{m_1}_{k,l}\omega^{klm_2\ldots m_{a-1}}_{n_1\ldots n_b}\} e^{n_1} \wedge \ldots \wedge e^{n_b} \otimes e_{m_1} \wedge \ldots \wedge e_{m_a} ,$$
$$m_i \in \mathbf{Z}^- , \quad n_j \in \mathbf{Z}^+ . \tag{2.14}$$

We present two examples of the action of δ, namely on $\binom{1}{0}$ and $\binom{1}{1}$ forms, which we shall need later. Let $\omega = \omega^{-m}e_{-m}$ and $J = J_n^{-m}e^n$, e be two such forms. Applying (2.14) we find:

$$\delta\omega = L_{-m}\omega^{-m} , \tag{2.15a}$$

and

$$\delta J = (L_{-m}J_n^{-m} + V^p_{n,-m}J_p^{-m})e^n . \tag{2.15b}$$

Another crucial operation we shall need is the star operation which turns $\binom{a}{b}$ forms into $\binom{b}{a}$ forms and it is defined on forms by transforming all the basis elements as follows [‡1]

$$*e_{-n} \equiv n_{nm}e^m , \quad *e^n \equiv n^{nm}e_{-m} \tag{2.16a, b}$$

where

$$n_{nm} = m\delta_{n,m} , \quad n^{nm} = (1/m)\delta_{n,m} .$$

We can check that the following properties hold:

$$** = 1 , \quad (*\omega, *\nu) = (\omega, \nu) , \tag{2.16c, d}$$

for any $\binom{a}{b}$, $\binom{b}{a}$ forms ω and ν.

Finally, we need the generalized kinetic operator

$$\mathsf{K} = 2(L_0 - 1 + \hat{N}) ,$$

where on any $\binom{a}{b}$ form

$$\omega = \omega^{m_1\ldots m_a}_{n_1\ldots n_b} e_{m_1} \wedge \ldots \wedge e_{m_a} \otimes e^{n_1} \wedge \ldots \wedge e^{n_b} ,$$
$$m_i \in \mathbf{Z}^- , \quad n_j \in \mathbf{Z}^+ ,$$
$$\hat{N}\omega = \omega^{m_1\ldots m_a}_{n_1\ldots n_b} \sum_{ij} (n_j + |m_i|) \times e_{m_1} \wedge \ldots \wedge e_{m_a} \otimes e^{n_1} \wedge \ldots \wedge e^{n_b} . \tag{2.17}$$

Later we shall use the following properties of d, δ, K, $*$ operations which can be easily checked: using the properties of the structure constants V^l_{mn} of the Virasoro algebra,

$$[\mathrm{d}, \delta]\Phi = \mathsf{K}{*}\Phi \quad (\Leftrightarrow D = 26!) \tag{2.18a}$$

$$[\mathrm{d}, \mathsf{K}]\Psi = 0 , \quad [\delta, \mathsf{K}]\zeta = 0 , \tag{2.18b, c}$$

$$[\mathrm{d}, \mathsf{K}]\Phi = 0 , \quad \mathsf{K}{*}\Phi = {*}\mathsf{K}\Phi , \tag{2.18d, e}$$

where Ψ, Φ, ζ are $\binom{0}{0}$, $\binom{1}{0}$ and $\binom{1}{1}$ forms, respectively.

3. The NWS action of the gauge covariant formulation for the open bosonic string is of the form [9]

$$A_{\mathrm{NWS}} = \tfrac{1}{2}(\Psi, (L_0 - 1)\Psi) + \sum_{n=1}^{\infty} (L_n\Psi + n\Phi^{(n)}, \Phi^{(n)}) + \sum_{n,m=1}^{\infty} (L_n\Phi^{(m)} + (2n+m)\Phi^{(n+m)}, \zeta^{(n,m)}) - \frac{1}{2}\sum_{n,m=1}^{\infty} (\zeta^{(n,m)}, (L_0 - 1 + n + m)\zeta^{(n,m)}) . \tag{3.1}$$

This action is invariant under the gauge transformations

$$\delta_\Lambda\Psi = \sum_{n=1}^{\infty} L_{-n}\Lambda_n , \quad \delta_\Lambda\Phi^{(n)} = -(L_0 - 1 + n)\Lambda_n . \tag{3.2a, b}$$

$$\delta_\Lambda\zeta^{(n,m)} = -L_m\Lambda_n - (2m+n)\Lambda_{n+m} ,$$
$$n, m = 1, 2, \ldots , \tag{3.2c}$$

only if the identity [9]

‡1 The $*$ operation here should not be confused with the Hodge, $*$ operation [10]. It is similar to the $\uparrow$, $\downarrow$ operations of Peskin and Banks [6].

$$-\sum_{m=1}^{n-1}(2n-m)(n+m)+\tfrac{1}{12}Dn(n^2-1)$$

$$-2n(n-1)=0 \qquad (3.3)$$

is true $\forall n = 2, 3, ...,$ and this happens only when $D = 26$. It is possible to choose a specific gauge such that

$$\Phi^{(n)} = \zeta^{(n,m)} = 0\,, \quad n, m = 1, 2, \ldots\,, \qquad (3.4)$$

and the equations of motion for the action (3.1) become

$$(L_0 - 1)\Psi = 0\,, \quad L_n\Psi = 0\,, \quad n = 1, 2, \ldots\,. \qquad (3.5)$$

We proceed now to the geometrical formulation of the NWS string field theory. First we identify the fields Ψ, $\Phi^{(n)}$, $\zeta^{(n,m)}$ and the gauge functions Λ_n with the forms

Ψ: $\binom{0}{0}$ form ,

$\Phi^{(n)} \to \Phi = \Phi^{-n}e_n$: $\binom{1}{0}$ form ,

$\zeta^{(n,m)} \to \zeta = \zeta_m^{-n}e^m \otimes e_{-n}$: $\binom{1}{1}$ form ,

$\Lambda_n \to \Lambda = \Lambda^{-n}e_{-n}$: $\binom{1}{0}$ form .

Then we observe that the different parts of the action (3.1) can be written as:

$$\tfrac{1}{2}(\Psi, (L_0 - 1)\Psi) = \tfrac{1}{4}(\Psi, \mathsf{K}\Psi)\,, \qquad (3.6a)$$

$$\frac{1}{2}\sum_{n,m=1}^{\infty}(\zeta^{(n,m)}, (L_0 - 1 + n + m)\zeta^{(m,n)})$$

$$= \tfrac{1}{4}(\zeta, \mathsf{K}\zeta)\,, \qquad (3.6b)$$

$$\sum_{n=1}^{\infty} n(\Phi^{(n)}, \Phi^{(n)}) = (\Phi, *\Phi)\,, \qquad (3.6c)$$

$$\sum_{n,m=1}^{\infty}(L_n\Phi^{(m)} + (2n+m)\Phi^{(n+m)}, \zeta^{(n,m)})$$

$$= \sum_n \Big(\Phi^{-n}, \sum_m L_{-m}\zeta_n^{-m} + \sum_{p+q=n}(2p+q)\zeta_q^{-p}\Big)$$

$$\overset{(2.15\text{b})}{=} (\Phi, \delta\zeta)\,. \qquad (3.6d)$$

So we obtain:

$$A_{\mathrm{NWS}} = \tfrac{1}{4}(\Psi, \mathsf{K}\Psi) - \tfrac{1}{4}(\zeta, \mathsf{K}\zeta) + (\Phi, \mathrm{d}\Psi + \delta\zeta + *\Phi)\,. \qquad (3.7)$$

The gauge transformations (3.2) become

$$\delta_\Lambda\Psi = \delta\Lambda\,, \quad \delta_\Lambda\Phi = -\tfrac{1}{2}\mathsf{K}\Lambda\,, \quad \delta_\Lambda\zeta = -\mathrm{d}\Lambda\,. \qquad (3.8\text{a, b, c})$$

The variation of the action under (3.8) is

$$\delta_\Lambda A_{\mathrm{NWS}} = \tfrac{1}{2}(\Psi, [\mathsf{K}\delta - \delta\mathsf{K} + \mathsf{K}\mathrm{d} - \mathrm{d}\mathsf{K}]\Lambda) + (\Phi, [\mathrm{d}\delta - \delta\mathrm{d} - *\mathsf{K}]\Lambda)\,.$$

The vanishing of $\delta_\Lambda A$ is guaranteed by the relations (2.18). We see that gauge invariance in covariant gauges follows if and only if $D = 26$. We now briefly discuss the formulations with a finite number of auxiliary fields and their mutual relations. First we easily observe that, for a given $n(\geqslant 2)$, we can consider Φ, ζ and Λ such that

$$\Phi^{-k} = \zeta_l^{-k} = \Lambda^{-k} = 0\,, \quad \forall k > n\,.$$

Then we get an action, call it $A^{(n)}$, of NWNS [9,8] with a finite number of auxiliary fields. Now given $A^{(n)}$, we can go to $A^{(n-1)}$ by the following procedure: the fields not in $A^{(n-1)}$ are Φ^{-n}, ζ_e^{-n} ($e = 1, 2, ..., n$) and ζ_n^{-1} ($e = 1, 2, ..., n-1$). It is easy to see that the ζ_n^{-1} are Lagrange multiplier fields and, upon integration, produce δ functions

$$\sum_{l=1}^{n-1}\delta(L_l\Phi^{-n} - \tfrac{1}{2}\mathsf{K}\zeta_l^{-n})\,. \qquad (3.9)$$

Since K is an invertible operator by using (3.8b), we can gauge Φ^{-n} to zero, i.e., we can insert $\delta(\Phi^{-n})$ with an appropriate ghost term. Then (3.9) becomes a δ function constraint on the ζ^{-n}. Upon doing this, we find that the ζ_l^{-n} completely decouple from the fields in $A^{(n-1)}$ and these can also be integrated. In this way we arrive at $A^{(n-1)}$ plus a ghost term. The relation between the theories based on $A^{(n)}$ and $A^{(n-1)}$, displayed above, suggests that they may well lead to different interacting theories.

4. The relation between the NWNS formulation and the BP one [6] becomes obvious if we write down the BP action in the form

$$\mathcal{A}_{\rm PB} = \tfrac{1}{4}(\Psi, \mathrm{K}\Psi) - \tfrac{1}{4}(\zeta, \mathrm{K}\zeta)$$
$$- \tfrac{1}{4}(\mathrm{d}\Psi + \delta\zeta, *\mathrm{d}\Psi + *\delta\zeta)\,, \qquad (4.1)$$

where we identified the Φ and Λ fields of BP as

$$\Phi = \Psi\,, \quad \Lambda = \zeta\,.$$

Comparing (3.9) and (4.1), one immediately recognizes that integration over Φ in $\mathcal{A}_{\rm NWNS}$ gives $\mathcal{A}_{\rm BP}$. Indeed, by completing the square

$$\mathcal{A}_{\rm NWS} = \mathcal{A}_{\rm BP} + (G, *G)\,, \qquad (4.2)$$

where

$$G = *\Phi + \tfrac{1}{2}(\mathrm{d}\Psi + \delta\zeta)\,, \qquad (4.3)$$

it is readily checked that G is gauge invariant. Gauge invariance of $\mathcal{A}_{\rm BP}$ under (3.8a) and (3.8c) is thus guaranteed. The difference between the two approaches lies simply in the use of the first-order (NWNS) or the second-order (BP) formalism.

One can now easily obtain a BP-type action $\mathcal{A}^{(n)}_{\rm BP}$ with a finite number of auxiliary fields, starting from $\mathcal{A}^{(n)}_{\rm NWNS}$. Especially for $N = 2$, $\mathcal{A}^{(2)}_{\rm BP}$ contains only four auxiliary fields. It should be remarked, however, that one cannot go from $\mathcal{A}^{(n)}_{\rm BP}$ to $\mathcal{A}^{(n-1)}_{\rm BP}$, in contrast to the case of $\mathcal{A}_{\rm NWNS}$. The reason is two-fold: first, gauge transformation of ζ_e^{-n} involves non-invertible operators. Thus one cannot gauge away ζ_e^{-n}. Further, none of the ζ_n^{-1} are Lagrange multiplier fields any more. Integration over them produces non-local expressions. The gauge fixing of Φ^{-n} in NWNS formulation does not correspond to a simple procedure in the BP version.

It should also be mentioned that there exists another action which is local and invariant under the set of gauge transformations (3.8). Namely, one can add to $\mathcal{A}_{\rm NWNS}$ a term $(\mathrm{d}\zeta, *\mathrm{d}\zeta)$, which after fixing the gauge appropriately gives the same equations of motion [3, 5]. It is possible to extend the geometric interpretation to the closed as well as to the supersymmetric string theories. We hope that it will be of some help for the search of the covariant interacting string field theories.

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