Phenomenological constraints imposed by the hidden sector in the flipped SU(5)×U(1) superstring model

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We calculate the trilinear superpotential of the hidden sector of the three generation flipped SU(5)×U(1)×U(1)^4× SO(10)×SU(4) superstring model. We perform a renormalization group analysis of the model taking into account the hidden sector. We find that, in all relevant cases, fractionally charged tetraplets of the hidden SO(6) gauge group are confined at a high scale. Nevertheless, their contribution to the observable U(1) gauge coupling evolution results in a drastic reduction of the available freedom in the values of $a_3(m_w)$, $\sin^2\theta_w$ and M_X that allow superunification.

The most attractive feature of modern string theory [1] is that it provides a framework for a unified theory for all interactions including gravitation. In earlier approaches some of the spatial dimensions of the string ere compactified on a Calabi–Yau manifold [2] or on an orbifold [3]. In more recent approaches, four-dimensional strings [4,5] are constructed directly without any intermediate higher dimensional state. In a particular approach, free fermionic internal degrees of freedom on the world-sheet are used to cancel the superconformal anomaly in four dimensions. Models [6,7] build along this line have $N=1$ supersymmetry and chiral fermions and do not possess any Higgs fields in the adjoint representation of the gauge group, as is conventionally required for the gauge symmetry breaking down to the standard model, provided we do not go beyond level $k=1$ Kac–Moody algebras [8].

A currently popular model [7] constructed in the framework of the fermionic formulation of the four-dimensional superstrings is the "flipped" SU(5)× U(1)′ model [9]. The complete gauge group of the model is SU(5)×U(1)′×U(1)^4×SO(10)× SU(4). The standard gauge group of electroweak and strong interactions is embedded in SU(5)×U(1)′. The ordinary quark and lepton matter fields transform also non-trivially under the extra U(1)^4 gauge factor which turns out to be broken at a superheavy scale. This part of the gauge group, together with the fields that transform under it, constitute the "observable sector" of the model. The remaining gauge group, namely SO(10)×SU(4), defines the "hidden sector", although the term "hidden" is not strictly correct since there exist fields that transform under it and simultaneously have non-zero electric charge. In fact, these states carry fractional electric charges of $\pm\frac{1}{2}$.

Light fractionally charged particles are not easily accommodated in the present low energy phenomenology. It is interesting to point out, however, that such states are not particular to the flipped SU(5)×U(1) superstring model but they are rather generic [10]. In several models, arising either from Calabi–Yau compactifications or from four dimensional fermionic constructions, one ends up with massless fractionally charged states in the spectrum of the theory. A detailed discussion, valid for any heterotic string theory, showing at which price one may avoid the appearance of these states, was given recently in ref. [11]. In particular it has been proved that a modular invariant string theory without fractional charges can exist only if the SU(2) and SU(3) levels k_2 and k_3 of the Kac–Moody algebra, to which the gauge group of the theory extends, and the U(1) normalization factor k_1 of the standard model embedding in string theory, satisfy the constraint $k_1+9k_2+12k_3=0 \bmod 36$. When other considerations, like the weak mixing angle etc., are taken into

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account, one is forced to SU(5) unification which requires adjoint Higgses and, therefore, inevitably dictates higher level Kac–Moody algebras. Although this option is challenging, model builders up to now have considered only level-one realistic models.

In this article, we examine in detail the "hidden sector" of the revamped flipped $SU(5)\times U(1)$ superstring model [7]. In particular, we have computed the superpotential couplings of hidden matter-fields and the resulting massless spectrum under a given vacuum choice. We have performed a renormalization group analysis of all relevant parameters taking into account the hidden sector as well. We address the question of the survival of the problematic fractionally charged states at low energies and the feasibility of the confinement mechanism via SU(4) hidden interactions proposed in ref. [7].

The construction of the model is based on a basis of eight vectors of boundary conditions for all world-sheet fermions. These basis vectors generate three chiral families of massless matter superfields each forming a complete **16** of SO(10) with the following $SU(5)\times U(1)'$ quantum numbers:

$$F(\mathbf{10},\tfrac{1}{2})+\bar{f}(\bar{\mathbf{5}},-\tfrac{3}{2})+\ell^c(\mathbf{1},\tfrac{5}{2}) .$$

The families differ in their $U(1)^4$ quantum number content. There is also a number of massless Higgs superfields in the **10**, $\overline{\mathbf{10}}$, **5** and $\bar{\mathbf{5}}$ representations of SU(5) which are used to break the symmetry to the standard model and provide the weak isodoublets necessary for the subsequent breaking to $SU(3)_c\times U(1)_{em}$.

The massless spectrum contains also a large number of $SU(5)\times U(1)'$ singlets with various charge assignments under the four U(1) factors. The latter are found to have $\mathrm{Tr}\,U(1)_i\neq 0$. However, without loss of generality, one can define four orthogonal combinations [7] in such a way that only one, namely $U(1)_A=-3U(1)_1-U(1)_2+2U(1)_3-U(1)_4$, remains anomalous [$\mathrm{Tr}\,U(1)_A=180$]. This is broken by the Dine–Seiberg–Witten mechanism [12] in which the anomalous D-term generated by a vacuum expectation value of the dilaton is canceled by vacuum expectation values that break some of the non-anomalous gauge symmetries so that supersymmetry is preserved. The cancelation conditions together with the F-flatness conditions, constrain the pattern of possible vacuum expectation values. There exist, however, more F-flat directions than constraints and the model possesses a number of degenerate $SU(5)\times U(1)'$-invariant vacua. Not all of these possible vacua are phenomenologically acceptable. since a complete analysis of the possible vacuum expectation value patterns has not been done yet, we shall limit ourselves to the specific phenomenologically acceptable choice of $\bar{\Phi}_{23}$, Φ_{23}, Φ_{31}, φ_{45}, $\bar{\varphi}_{45}$, φ_+ (and possibly some of Φ_{31}, φ_i, $\bar{\varphi}_i$, $\bar{\varphi}_+$, φ_-, $\bar{\varphi}_-$ subject to the F-flatness constraints) non-vanishing vacuum expectation values made in ref. [7].

The $SU(5)\times U(1)'$ breaking to the standard model is achieved by giving a non-vanishing vacuum expectation value to a combination of the $F_i(\mathbf{10},\tfrac{1}{2})$ matter fields $\langle F\rangle=\sum_{i=1,2,3}a_i\langle F_i\rangle$ together with an $\langle\bar{F}_5\rangle$ vacuum expectation value. D-flatness is enforced by the equality $\langle F\rangle=\langle\bar{F}_5\rangle$. The above set of VEVs leads to $n_2=4$ massless SU(2)-isodoublets and $n_3=2$ massless but innocent SU(3)-triplets after the $SU(5)\times U(1)'$ breakdown.

The fields discussed so far, are singlets under $SO(10)\times SO(6)$ hidden gauge interactions. However, in addition to the above observable fields the massless spectrum contains states that transform non-trivially under the hidden gauge group. These additional states can be divided in two classes. The first class consists of $SU(5)\times U(1)'$-invariant fields that couple to the observable sector only through $U(1)^4$ gauge interactions. In contrast, the fields that belong to the second class, in addition to their $SO(10)\times SO(6)\times U(1)^4$ quantum numbers, they also carry $U(1)'$ charges.

The first class of hidden states comes from the sectors b_i+2a and $b_i+2a+\zeta$, where $i=1, ..., 5$, in the customary fermionic model-building notation [4,6,7]. These fields, listed in terms of their $SO(10)\times SO(6)\times U(1)^4$ transformation properties, are

$$\Delta_1=(\mathbf{1},\mathbf{6})(0,-\tfrac{1}{2},\tfrac{1}{2},0) ,$$

$$\Delta_2=(\mathbf{1},\mathbf{6})(-\tfrac{1}{2},0,\tfrac{1}{2},0) ,$$

$$\Delta_3=(\mathbf{1},\mathbf{6})(-\tfrac{1}{2},-\tfrac{1}{2},0,\tfrac{1}{2}) ,$$

$$\Delta_4=(\mathbf{1},\mathbf{6})(0,-\tfrac{1}{2},\tfrac{1}{2},0) ,$$

$$\Delta_5=(\mathbf{1},\mathbf{6})(\tfrac{1}{2},0,-\tfrac{1}{2},0) , \qquad (1a)$$

$T_1 = (\mathbf{10}, \mathbf{1})(0, -\frac{1}{2}, \frac{1}{2}, 0)$,

$T_2 = (\mathbf{10}, \mathbf{1})(-\frac{1}{2}, 0, \frac{1}{2}, 0)$,

$T_3 = (\mathbf{10}, \mathbf{1})(-\frac{1}{2}, -\frac{1}{2}, 0, -\frac{1}{2})$,

$T_4 = (\mathbf{10}, \mathbf{1})(0, \frac{1}{2}, -\frac{1}{2}, 0)$,

$T_5 = (\mathbf{10}, \mathbf{1})(-\frac{1}{2}, 0, \frac{1}{2}, 0)$. (1b)

We now proceed to the calculation of the trilinear superpotential involving the fields Δ_i, T_i. Adopting the normalization $\langle \sigma_\pm \sigma_\pm \rangle = 1$ and $\langle \sigma_+ \sigma_- \rangle = 0$, the only non-vanishing three-point functions are [7]

$$\langle \sigma_+ \sigma_- \mathrm{f} \rangle = \langle \sigma_+ \sigma_- \bar{\mathrm{f}} \rangle = \frac{1}{\sqrt{2}}, \quad (2)$$

with $\sigma_\pm$ being the two twist-fields of conformal dimensions $(\frac{1}{16}, \frac{1}{16})$ corresponding to the two different fermion number projections $(-1)^{\mathrm{f}\bar{\mathrm{f}}} = \pm 1$ in the Ramond sector for a given real two-dimensional fermion f(left) and $\bar{\mathrm{f}}$(right). The only non-vanishing trilinear superpotential terms are [#1]

$$W = \Delta_1^2 \bar{\Phi}_{23} + \Delta_2^2 \Phi_{31} + \Delta_4^2 \bar{\Phi}_{23} + \Delta_5^2 \bar{\Phi}_{31} + \frac{1}{\sqrt{2}} \Delta_4 \Delta_5 \bar{\varphi}_3$$
$$+ T_1^2 \bar{\Phi}_{23} + T_2^2 \Phi_{31} + T_4^2 \Phi_{23} + T_5^2 \Phi_{31} + \frac{1}{\sqrt{2}} T_4 T_5 \bar{\varphi}_2 . \quad (3)$$

For the singlet fields Φ we follow the notation of ref. [7] that corresponds to the following $U(1)^4$ assignments:

$\Phi_{23}(0, -1, 1, 0)$, $\Phi_{31}(1, 0, -1, 0)$,

$\bar{\Phi}_{23}(0, 1, -1, 0)$, $\bar{\Phi}_{31}(-1, 0, 1, 0)$,

$\varphi_2(\frac{1}{2}, -\frac{1}{2}, 0, 0)$, $\bar{\varphi}_3(-\frac{1}{2}, \frac{1}{2}, 0, 0)$. (4)

Adopting the specific vacuum choice of ref. [7] we see that the terms in (3) provide tree level masses for all but Δ_3 and T_3 fields listed in (1). The latter could in principle acquire masses from higher order terms. In any case the existence of a mass term for T_3 is not important for our discussion since it is an $SU(4) \times SU(5) \times U(1)'$ singlet. In contrast, Δ_3 plays an essential role in our results since it affects the SU(4) confinement scale and, therefore, it is important to know if it remains massless or not. However, a search over all possible fourth and fifth order non-renormalizable mass terms has shown that fourth order terms are not allowed by gauge symmetry, whilst the few allowed fifth order terms vanish due to string discrete symmetry arguments state in relation (2) [#2].

The hidden chiral superfields belonging to the second class, transform as the spinorial representation of SO(6), or equivalently as a 4 of $\bar{4}$ of SU(4). They are SU(5)-singlets but carry non-zero observable $U(1)'$ charge (corresponding to fractional electric charge $\pm\frac{1}{2}$). These are the problematic exotic states that are inevitable in any level-one construction [11]. These states arise from various different sectors, namely $b_1 \pm a$ $(+\zeta)$, and $b_1 + b_4 + b_5 \pm a$ $(+\zeta)$, $b_4 \pm a$ $(+\zeta)$, $b_2 + b_3 + b_5 \pm a$ $(+\zeta)$ and $b_1 + b_2 + b_4 \pm a$ $(+\zeta)$. They are, in terms of their $U(1)' \times SO(10) \times SO(6) \times U(1)^4$ transformation properties,

$\bar{X}_1 = (-\frac{5}{4})(1, \bar{4})(-\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{2})$,

$\bar{X}_2 = (-\frac{5}{4})(1, \bar{4})(-\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$,

$Y_1 = (\frac{5}{4})(1, 4)(-\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, \frac{1}{2})$,

$Y_2 = (+\frac{5}{4})(1, 4)(-\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, -\frac{1}{2})$,

$Z_1 = (-\frac{5}{4})(1, 4)(\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, \frac{1}{2})$,

$\bar{Z}_1 = (\frac{5}{4})(1, \bar{4})(-\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$,

$X_1 = (\frac{5}{4})(1, 4)(\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{2})$,

$\bar{X}_2' = (-\frac{5}{4})(1, \bar{4})(-\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$,

$\bar{Q}_1 = (-\frac{5}{4})(1, \bar{4})(-\frac{3}{4}, \frac{1}{4}, -\frac{1}{4}, 0)$,

$Q_1 = (\frac{5}{4})(1, 4)(-\frac{1}{4}, \frac{3}{4}, -\frac{1}{4}, 0)$,

$Y_2' = (\frac{5}{4})(1, 4)(-\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, -\frac{1}{2})$,

$\bar{Y}_1 = (-\frac{5}{4})(1, \bar{4})(\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$. (5)

The electric charge for the above twelve states is $Q^\pm = \frac{2}{5}(\pm\frac{5}{4}) + 0 = \pm\frac{1}{2}$. The trilinear superpotential that involves these states can be calculated as in the case of (3). It turns out to be [#3]

$$W' = \frac{1}{\sqrt{2}} (Y_1 \bar{X}_2 \varphi_4 + \bar{X}_1 Y_2 \varphi_1) + \bar{X}_2 Y_2 \varphi_+$$
$$+ Z_1 \bar{Z}_1 \Phi_3 + Q_1 \bar{Q}_1 \bar{\Phi}_{12} + Z_1 \bar{X}_2' \varrho_2^{\mathrm{s}} + Y_2' Z_1 \Delta_1 . \quad (6)$$

[#1] We have omitted the overall normalization factor proportional to the gauge coupling since it is not important to our analysis.

[#2] Possible higher order terms would give masses lower than the SU(4) confinement scale in all acceptable cases.

[#3] $\varphi_{1,4} = (\mathbf{1}, 0; \frac{1}{2}, -\frac{1}{2}, 0, 0)$ and Φ_3 is a total singlet.

ℓ_2^c is the second-generation "right-handed" lepton superfield with $SU(5)\times U(1)'\times U(1)^4$ assignment $(\mathbf{1}, \frac{5}{2}; 0, -\frac{1}{2}, 0, 0)$. The first five of the above superpotential couplings are potential mass terms for some (at most 8) of the appearing fractionally charged states depending on the vacuum expectation value assignments of the singlets Φ. The sixth term is the unique direct Yukawa coupling between observable and hidden sectors. This exotic coupling would imply the decay of sleptons into fractionally charged particles if these particles survived light enough. Shortly we shall provide quantitative support in favour of the conjectured mechanism of confinement of the fractionally charged states by strong SU(4) interactions. Therefore, this coupling will not be of any consequence, at least in our framework.

Since some of the singlets in (6) will be forced to have vanishing VEVs (e.g. $\bar{\Phi}_{12}$), either from phenomenology or F-flatness constraints [7], the trilinear superpotential W' can provide masses only for a few of the tetraplets (four or probably less, for the vacuum choice of ref. [7]). Therefore possible non-renormalizable mass terms will be of crucial importance for our analysis. Search up to fifth order has given the following allowed couplings (in $M_{\rm Pl}$ units) #4,5,6

$$W_{\rm n.r.} = X_1\bar{X}'_2(\varphi_+\bar{\varphi}_2+\varphi_2\bar{\varphi}_-)+\bar{Y}_1 Y'_2(\varphi_+\bar{\varphi}_3+\varphi_3\bar{\varphi}_-)$$
$$+\, Y'_2\bar{X}_2(F_2\bar{F}_5\varphi_+)+Y'_2\bar{X}_1(F_2\bar{F}_5\varphi_1)$$
$$+\, X_1\bar{X}_1\bar{F}_5F_1\varphi_3. \tag{7}$$

The existence of these terms can reduce the number of light fractionally charged states providing superheavy masses for some more tetraplet pairs or intermediate masses for others.

It has been argued [7] that the SU(4) hidden interactions become strong at some scale Λ below the unification scale resulting into confinement of all SU(4)-non-singlet states and in particular the massless fractionally charged tetraplet states. If this is the case, all fractionally charged tetraplets will form SU(4)-singlet bound states of natural masses of order Λ. For simple group theoretical reasons, these states will involve an even number of tetraplets and consequently will possess integer electric charge. In order to give a quantitative answer to the question of confinement of all the fractionally charged states as well as their effect on the various parameters of the theory (i.e. M_X, $M_{\rm SU}$, α_3, $\sin^2\theta_w$) we must consider the coupled $SU(3)\times SU(2)\times U(1)\times SU(4)$ renormalization group equations and compute all phenomenologically relevant parameters together with the SU(4) confinement scale Λ.

Let us first consider the coupled renormalization group equations in the two loop approximation. They can be written in the form ($\alpha_i = g_i^2/4\pi$ and $t=\ln\mu$, as usually)

$$\frac{d\alpha_i}{dt} = \frac{\alpha_i^2}{2\pi}\left(b_i + \frac{1}{4\pi}\sum b_{ij}\alpha_j\right), \tag{8}$$

where the beta function coefficients for $SU(5)\times U(1)'\times SU(4)$ above M_X are

$$b_i = -\begin{pmatrix}0\\15\\12\end{pmatrix} + \begin{pmatrix}2\\2\\0\end{pmatrix}n_G + \begin{pmatrix}\frac{1}{2}\\\frac{1}{2}\\0\end{pmatrix}n_5 + \begin{pmatrix}\frac{1}{4}\\\frac{3}{2}\\0\end{pmatrix}n_{10}$$
$$+\begin{pmatrix}\frac{5}{8}\\0\\\frac{1}{2}\end{pmatrix}n_4 + \begin{pmatrix}0\\0\\1\end{pmatrix}n_6,$$

$$b_{ij} = \begin{pmatrix}0 & 0 & 0\\0 & -150 & 0\\0 & 0 & -96\end{pmatrix} + \begin{pmatrix}\frac{13}{5} & \frac{72}{5} & 0\\\frac{3}{5} & \frac{232}{5} & 0\\0 & 0 & 0\end{pmatrix}n_G$$
$$+\begin{pmatrix}\frac{1}{5} & \frac{24}{5} & 0\\\frac{1}{5} & \frac{49}{5} & 0\\0 & 0 & 0\end{pmatrix}n_5 + \begin{pmatrix}\frac{1}{40} & \frac{18}{5} & 0\\\frac{3}{20} & \frac{183}{5} & 0\\0 & 0 & 0\end{pmatrix}n_{10}$$
$$+\begin{pmatrix}\frac{25}{64} & 0 & \frac{75}{16}\\0 & 0 & 0\\\frac{5}{16} & 0 & \frac{31}{4}\end{pmatrix}n_4 + \begin{pmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 18\end{pmatrix}n_6, \tag{9a}$$

where $i, j = (1, 5, 4)$, while below M_X for $SU(3)_C\times SU(2)_L\times U(1)_Y\times SU(4)$

#4 Since the selection rules for these terms have not been rigorously derived yet, we simply list the gauge invariant terms which are not explicitly vanishing from the rules stated in (2).

#5 We have not included non-renormalizable terms that constitute corrections to non-vanishing lower order ones.

#6 The $U(1)^4$ assignments for the singlets are [7]: $\varphi_i = (\frac{1}{2}, -\frac{1}{2}, 0, 0)$, $i=1, ..., 4$, $\varphi_- = (\frac{1}{2}, -\frac{1}{2}, 0, -1)$, $\varphi_+ = (\frac{1}{2}, -\frac{1}{2}, 0, 1)$, $\Phi_I = (0, 0, 0, 0)$, $I = 1, ..., 5$.

$$b_i = \begin{pmatrix} 0 \\ -6 \\ -9 \\ -12 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \\ 2 \\ 0 \end{pmatrix} n_G + \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} n_2 + \begin{pmatrix} \frac{1}{5} \\ 0 \\ \frac{1}{2} \\ 0 \end{pmatrix} n_3$$

$$+ \begin{pmatrix} \frac{1}{10} \\ \frac{3}{2} \\ 1 \\ 0 \end{pmatrix} n_{32} + \begin{pmatrix} \frac{3}{5} \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} n_4 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} n_6 ,$$

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -24 & 0 & 0 \\ 0 & 0 & -54 & 0 \\ 0 & 0 & 0 & -96 \end{pmatrix}$$

$$+ \begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} & 0 \\ \frac{2}{5} & 14 & 8 & 0 \\ \frac{11}{15} & 3 & \frac{68}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} n_G + \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 & 0 \\ \frac{3}{10} & \frac{7}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} n_2$$

$$+ \begin{pmatrix} \frac{4}{75} & 0 & \frac{16}{15} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{2}{15} & 0 & \frac{17}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} n_3 + \begin{pmatrix} \frac{1}{150} & \frac{3}{10} & \frac{8}{15} & 0 \\ \frac{1}{10} & \frac{21}{2} & 8 & 0 \\ \frac{1}{15} & 3 & \frac{34}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} n_{32}$$

$$+ \begin{pmatrix} \frac{9}{25} & 0 & 0 & \frac{9}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{3}{10} & 0 & 0 & \frac{31}{4} \end{pmatrix} n_4 + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 18 \end{pmatrix} n_6 , \qquad (9b)$$

where $i, j = (1, 2, 3, 4)$. In the above n_G is the number of generations and n_{10}, n_5, n_2, n_3, n_{32} are the number of Higgs decuplets, pentaplets, doublets, triplets and doublet-triplet representations respectively. n_4 and n_6 are the fourplets and sextets of (1) and (4). In our analysis we will take $n_G=3$, $n_{10}=2$, $n_5=10$, $n_3=4$, $n_{32}=0$ and $n_6=1$ which result from the specific VEVs choice of ref. [7]. According to our previous discussion in what follows, we will keep an open mind and perform our analysis for variable number $n_4 \leqslant 6$. We have omitted the contribution of Yukawa couplings. This amounts to neglecting the terms $[4(\lambda_t+\lambda_b)$, $6(\lambda_t+\lambda_b)+\lambda_\tau, \frac{1}{5}(26\lambda_t+14\lambda_b+18\lambda_\tau)]$ in (9b). These terms have a relative weight $\frac{1}{6}$ to the gauge coupling terms. Being a small correction to a small correction in the two-loop term, they could be ignored (including them results in a slight shift of M_X). The contribution of the massless hidden sector fields, namely tetraplets and sextets, is taken into account from the superunification scale M_{SU} up to the scale Λ at which SU(4) interactions become strong. At Λ these states decouple from the renormalization group, forming massive bound states with natural masses of order Λ. We define Λ as the scale at which $\alpha_4(\Lambda)=0.2$.

The renormalization group equations are integrated using as low energy conditions the phenomenologically acceptable values of $\alpha_3(m_W)$ and $\sin^2\theta_w$

$$0.222 < \sin^2\theta_w < 0.234 ,$$

$$0.107 < \alpha_3(m_w) < 0.138 . \qquad (10)$$

The grand unification scale M_X is defined as the scale at which the $SU(3)_C$ and the $SU(2)_L$ gauge couplings are equal, i.e.

$$\alpha_3(M_X) = \alpha_2(M_X) = \alpha_5(M_X) \qquad (11)$$

while for the $U(1)_Y$ gauge coupling at M_X we have

$$\frac{25}{\alpha_Y} = \frac{1}{\alpha_5} + \frac{24}{\alpha_1} . \qquad (12)$$

The superunification scale M_{SU} is defined as the scale at which the $U(1)'$ and the SU(5) couplings are equal:

$$\alpha_1(M_{SU}) = \alpha_5(M_{SU}) = \alpha_U . \qquad (13)$$

Since M_{SU} is expected to be at the string scale, where the couplings of all non-abelian gauge group factors realized at the same level of the Kac–Moody algebra are equal [13], we should also demand $\alpha_4(M_{SU}) = \alpha_U$. It is evident that for $\alpha_Y(M_X) > \alpha_3(M_X)$ the condition for superunification cannot be met.

We start our analysis first by determining the scale at which SU(4) becomes strong. For some mean value for the superunification scale as well as the unified gauge coupling, i.e.

$$M_{SU} \sim 1.2\times 10^{16} , \quad \alpha_U = 0.057$$

we have plotted in fig. 1 the evolution of the α_4 coupling for $n_6=1$, $n_4=0, 2, 4, ..., 12$. As one should expect, α_4 becomes strong at low energies at a scale that decreases with an increasing number of massless tetraplets.

We proceed now in a detailed analysis for the gauge couplings and the unification and superunification scales by integrating the complete system of RG equations. Our results for M_X and M_{SU} are exhibited in figs. 2–4. In fig. 2 we have plotted the values for

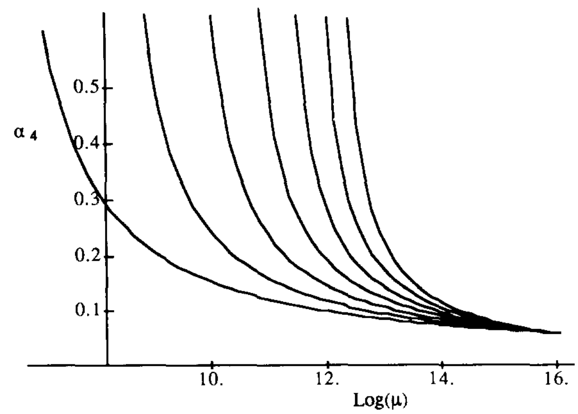

Fig. 1. Evolution of the SU(4) gauge coupling for $M_{SU} \sim 1.2\times 10^{16}$ and $\alpha_U = 0.057$.

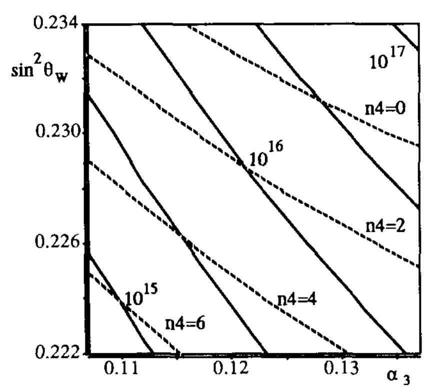

Fig. 2. Contour plots for the unification scale for $n_6=1$, $n_4=0$, 2, 4, 6 for the allowed regions of the $\sin^2\theta_w(m_w)$ and $\alpha_3(m_w)$.

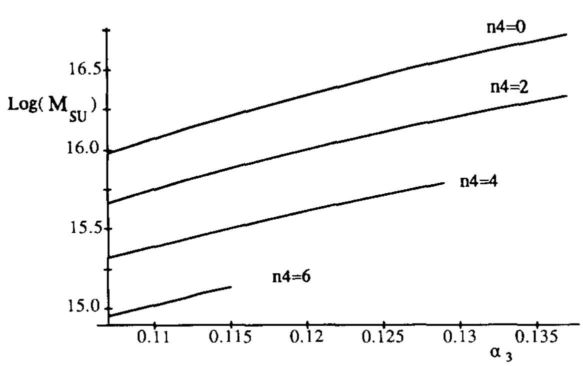

Fig. 3. Plots of M_{SU} versus $\alpha_3(m_w)$ for $n_6=1$ and $n_4=0$, 2, 4, 6.

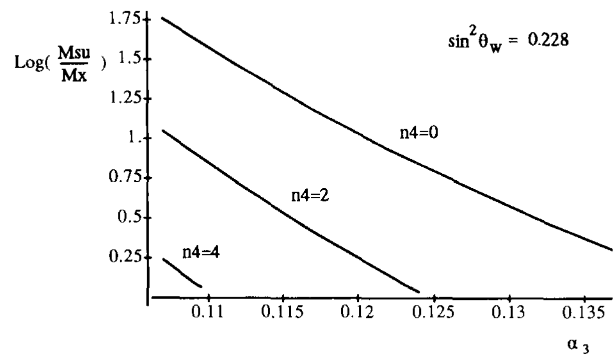

Fig. 4. Plots of the ratio $\log(M_{SU}/m_X)$ for $n_6=1$, $n_4=0$, 2, 4 and $\sin^2\theta_w(m_w)=0.228$.

the unification scale for phenomenologically acceptable values of $\alpha_3(m_w)$ and $\sin^2\theta_w(m_w)$. The solid lines correspond to the curves M_X=constant. As expected, the unification scale M_X is not sensitive to the number of massless tetraplets since they are $SU(3)\times SU(2)$ singlets. On the contrary the existence of the superunification point depends crucially on their number since they carry $U(1)'$ charges. The dotted lines correspond to the curves $M_{SU}=M_X$ for the cases $n_4=0$, 2, 4, 6. For each particular value of n_4 the superunification point above the corresponding dotted line does not exist since $\alpha_Y(M_X) > \alpha_5(M_X)$. As previously explained we have taken $n_6=1$, as the most probable case. Of course higher n_6 values would be much more restrictive.

We observe therefore, that the presence of the massless fractionally charged states in the spectrum, lowers drastically the allowed unification scale values and moreover restricts the allowed regions of the $\sin^2\theta_w(m_w)$ and $\alpha_3(m_w)$ low energy parameters.

The dependence of M_{SU} from $\alpha_3(m_w)$ is shown in fig. 3. It is worth noticing that for the special case of our analysis where $n_2+2n_{10}=6+n_3$, M_{SU} is not sensitive to $\sin^2\theta_w(m_w)$ [14].

Finally, in fig. 4 we present the ratio $\log(M_{SU}/M_X)$ for a mean value of $\sin^2\theta_w(m_w)=0.228$. This figure exhibits the behavior of M_{SU} relative to M_X for various values of n_4. It is clear that the existence of the fractionally charged tetraplets lowers the above ratio by shifting M_{SU} closer to M_X.

Let us now state our conclusions briefly summarizing our analysis.

We have considered the three family flipped

SU(5)×U(1)′ superstring model for vacuum choice within the framework proposed in ref. [7].

We have computed the trilinear Yukawa couplings, as well as the possible quartic and quintic ones relevant for mass-terms, involving the hidden sector fields. Trilinear level Yukawa couplings can provide masses for no more than four fractionally charged tetraplets for the specific VEVs choice. For other choices of VEVs, the number of massive tetraplets could increase up to eight. Mass terms arising from nonrenormalizable interactions could possibly provide superheavy or intermediate masses for another two or three pairs.

We performed a two loop renormalization group analysis of the complete system of the gauge couplings. The proposed mechanism of confinement of tetraplet states that carry fractional electric charge is generally supported by our results. However, the existence of these states lowers M_{SU} and restricts the acceptable range for M_X, $\alpha_3(m_w)$ and $\sin^2\theta_w(m_w)$ to lower values. In any case, as can be seen from fig. 4, the number of light tetraplets should not be more than four, since the scale M_X is pushed beyond the range accepted from proton decay bounds.

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