PHENOMENOLOGY OF THE HIERARCHICAL LEPTON MASS SPECTRUM IN THE FLIPPED SU(5)×U(1) STRING MODEL

G.K. LEONTARIS [a,b] and D.V. NANOPOULOS [a,c,1]

[a] CERN, CH-1211 Geneva 23, Switzerland
[b] Physics Department, University of Ioannina, GR-451 10 Ioannina, Greece
[c] Departement de Physique Théorique, Université de Genève, 24 Quai Ernest Ansermet, CH-1211 Geneva 4, Switzerland

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A detailed phenomenological analysis of the lepton mass matrices and their implications in the low energy theory are discussed, within the recently proposed SU(5)×U(1) string model. The unification scale is highly constrained while the Yukawa couplings lie in a natural region. The flavour changing decays $\mu\to e\gamma$, $\mu\to 3e$, $\mu\to e$ are highly suppressed while the depletion in the flux of muon neutrinos reported by the Kamiokande is explained through $\nu_\mu\leftrightarrow\nu_\tau$ oscillations.

Nowadays, superstring theories seem to be the most promising candidates for a unified theory of strong, electroweak and gravitational forces. In order, however, for the program of string unification to be accepted, one should arrive at a theory of four spacetime dimensions and $N=1$ supersymmetry. First attempts to obtain a realistic low energy theory involved the compactification on a Calabi–Yau manifold and later on orbifold compactifications [1].

A simpler approach to the problem, however, is to construct string theories directly in four spacetime dimensions [2–4]. In this latter case, one may have enough freedom to choose the gauge group but these theories are characterized by the absence of the Higgs fields in the adjoint or higher real representations [4,5]. Thus, one cannot in general use these theories to obtain a conventional grand unified scheme for the strong and electroweak interactions.

However, recently there has appeared a new remarkable approach [6] where one can obtain a new GUT group – the "flipped" SU(5)×U(1) – which does not need adjoint or any real higher Higgs representations to break the gauge symmetry down to $SU(3)_C\times SU(2)_L\times U(1)_Y$ [#1]. Indeed this model is quite attractive from many points of view. Firstly, the fermion fields as well as the fields needed to break the group SU(5)×U(1) down to the standard group, belong only to the **5**, $\bar{\mathbf{5}}$, **1**, **10** and $\overline{\mathbf{10}}$ representations of SU(5). Secondly, the doublet–triplet mass splitting is solved by an elegant missing partner mechanism [6,8]. In fact one can use the ten-dimensional Higgs representations of SU(5) to give large masses to the triplet components of **5** and $\bar{\mathbf{5}}$ Higgses, while keeping their doublet components massless, at the first stage of symmetry breaking. The latter are the well-known standard Higgs doublets which will realise the second stage of symmetry breaking from $SU(3)_C\times SU(2)_L\times U(1)_Y$ down to $SU(3)_C\times U(1)_{EM}$. Let us first start with the particle content of this GUT model [6]. There are three generations of matter fields with the following SU(5)×U(1) transformation properties:

$$F_i=(10,\tfrac{1}{2}),\quad \bar{f}_i=(5,-\tfrac{3}{2}),\quad \ell_i^c=(1,\tfrac{5}{2}),\quad i=1,2,3 \tag{1}$$

where for generation

[1] Permanent address: Physics Department, University of Wisconsin, Madison, WI 53706, USA.
[#1] There have been also other approaches through orbifolds. For such models, see ref. [7].

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$$F=\begin{pmatrix} 0 & d^c & -d^c & d & u \\ -d^c & 0 & d^c & d & u \\ d^c & -d^c & 0 & d & u \\ -d & -d & -d & 0 & \nu^c \\ -u & -u & -u & -\nu^c & 0 \end{pmatrix}, \quad \bar{f}=\begin{pmatrix} u^c \\ u^c \\ u^c \\ \nu \\ e \end{pmatrix}, \quad \ell^c = e^c . \tag{2}$$

The ten-dimensional Higgses needed to break $SU(5)\times U(1)$ down to the standard group are

$$H=(\mathbf{10},\tfrac{1}{2}), \quad \bar{H}=(\overline{\mathbf{10}},-\tfrac{1}{2}), \tag{3}$$

while the five-dimensional fields which break the standard group down to $SU(3)_C\times U(1)_{EM}$ are

$$h=(\mathbf{5},-1), \quad \bar{h}=(\bar{\mathbf{5}},1). \tag{4}$$

There are finally four singlet fields Φ_m. Then the most general renormalizable superpotential, invariant under $SU(5)\times U(1)\times Z_2$ ($H_1 \leftrightarrow -H_1$) reads

$$W=\lambda_1^{ij}F_iF_jh+\lambda_2^{ij}F_i\bar{f}_j\bar{h}+\lambda_3^{ij}\bar{f}_i\ell_j^c h+\lambda_4 HHh+\lambda_5\bar{H}\bar{H}\bar{h}+\lambda_6^{im}F_i\bar{H}\Phi_m+\lambda_7^m h\bar{h}\Phi_m+\lambda_8^{mnp}\Phi_m\Phi_n\Phi_p . \tag{5}$$

The advantages already mentioned in the introduction, are now obvious. The model is pretty economical, the triplets get large masses $\langle 0|\nu_H^c, \bar{\nu}_H^c|0\rangle \sim O(M_{GUT})$, the problematic mass relations for $m_{d,e}$ are discarded while the singlets provide a natural (see-saw type) neutrino mass matrix [6].

The corresponding four-dimensional string model has been constructed in refs. [8,9]. In fact it is generated by a basis of eight elements, five of them yielding an $SO(10)\times SO(6)^3$ observable gauge group together with 3×2 copies of chiral massless fields in $(16,4,1,1)+(16,\bar{4},1,1)$ representations and an E_8 hidden gauge group. The remaining basis elements break $SO(10)$ down to $SU(5)$ and $SO(6)^3$ down to $U(1)^3$. Thus the final observable GUT group is $SU(5)\times U(1)\times U(1)^3$, while there is more freedom in choosing the "hidden group". In the improved construction of the model [9] the fields are the following: Three families of quarks and leptons which transform under $U(1)^3$ as

$$M_1=(\tfrac{1}{2},0,0), \quad M_2=(0,\tfrac{1}{2},0), \quad M_3=(0,0,-\tfrac{1}{2}), \tag{6}$$

where each M_i decomposes according to the 16 spinorial representation of $SO(10)$ under $SU(5)\times U(1)$

$$M_i=F_i+\bar{f}_i+\ell_i^c . \tag{7}$$

The Higgs sector contains the fields

$$\begin{aligned} &H_1=(\mathbf{10},\tfrac{1}{2})_{(1/2,0,0)}, \quad \bar{H}_1=(\overline{\mathbf{10}},-\tfrac{1}{2})_{(-1/2,0,0)}, \quad H_2=(\mathbf{10},\tfrac{1}{2})_{(0,1/2,0)}, \quad \bar{H}_2=(\overline{\mathbf{10}},-\tfrac{1}{2})_{(0,-1/2,0)}, \\ &h_1=(\mathbf{5},-1)_{(-1,0,0)}, \quad h_2=(\mathbf{5},-1)_{(0,-1,0)}, \quad h_3=(\mathbf{5},-1)_{(0,0,1)}, \\ &\bar{h}_1=(\bar{\mathbf{5}},1)_{(1,0,0)}, \quad \bar{h}_2=(\bar{\mathbf{5}},1)_{(0,1,0)}, \quad \bar{h}_3=(\bar{\mathbf{5}},1)_{(0,0,-1)}, \\ &h_{12}=(\mathbf{5},-1)_{(1/2,1/2,0)}, \quad \bar{h}_{12}=(\mathbf{5},1)_{(-1/2,-1/2,0)} . \end{aligned} \tag{8}$$

There are also three pairs of singlets,

$$\Phi_{12}=(1,0)_{(1,-1,0)}, \quad \Phi_{23}=(1,0)_{(0,1,1)}, \quad \Phi_{13}=(1,0)_{(1,0,1)}, \tag{9}$$

and their complex conjugates, $\bar{\Phi}_{ij}$.

In this work we will extend the phenomenological analysis of the model in ref. [9]. Concentrating on the fermion mass matrices which are constructed from all the allowed renormalizable and nonrenormalizable terms of the field content of the model, we will try to specify the acceptable range of the Yukawa couplings in terms of the fermion masses and show that they lie in the expected region [10]. Then we will see that the GUT scale is fixed in a well defined region, close to the unification scale M_{SU} defined in ref. [11]. Further, phenomenological consequences of the model on neutrino oscillations, etc., will be discussed.

We consider firstly the charged lepton mass matrix. This matrix arises from the superpotential terms [9]

$$W \ni \lambda_{34}\bar{f}_3 \ell^c_3 h_1 (H_1\bar{H}_2)^2 \Phi_{23}/M^5_{SU} + \lambda_{33}\bar{f}_2 \ell^c_2 h_1 (H_1\bar{H}_2)^2/M^4_{SU} + \lambda_{32}\bar{f}_2 \ell^c_1 h_1 (H_1\bar{H}_2)/M^2_{SU}$$
$$+\lambda_{31}(\bar{f}_1 \ell^c_2 h_1 H_1 \bar{H}_2)/M^2_{SU} + \lambda_3 \bar{f}_1 \ell^c_1 h_1 , \tag{10}$$

where M_{SU} is the superunification scale where all the couplings are equal. It will become obvious below, that one should identify the fermions belonging to $M_{3,2,1}$ generations of this model [9] with the first, second and third generation of the standard classification of the particles, i.e., with e, ν_e, u, d, etc.

Furthermore for convenience we will denote the ratios $\langle H_i \rangle / M_{SU}$, $i=1$, 2 and $\langle \Phi_{23} \rangle / M_{SU}$ as η and η' respectively. Then the charged lepton mass matrix is written

$$m_\ell = \upsilon_1 \begin{pmatrix} \lambda_{34}\eta^4\eta' & 0 & 0 \\ 0 & \lambda_{33}\eta^4 & \lambda_{32}\eta^2 \\ 0 & \lambda_{31}\eta^2 & \lambda_3 \end{pmatrix} , \tag{11}$$

where $\upsilon_1 = \langle h_1 \rangle$.

We notice, however, here that the symmetry $M_3 \leftrightarrow -M_3$ which remains in the low energy theory [9,12], leaves the first generation unmixed with the other two. A mechanism for generating these terms [12] would use the VEV for the s-neutrinos $\tilde{\nu}$, $\tilde{\nu}^c$. Nevertheless in the case of the lepton mass matrix this can happen only in the two-loop level (see fig. 1) thus the matrix (11) will not alter significantly. Thus the first generation remains uncoupled. Then

$$m_e \approx \lambda_{34}\upsilon_1 \eta^4 \eta' . \tag{12}$$

For the m_μ and m_τ masses we need to diagonalize the 2×2 matrix

$$m_{\mu\tau} = \begin{pmatrix} \lambda_{33}\eta^4 & \lambda_{32}\eta^2 \\ \lambda_{31}\eta^2 & \lambda_3 \end{pmatrix} . \tag{13}$$

Denoting

$$S_L = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} . \tag{14}$$

the diagonalizing matrix, the eigenvalues of the $(mm^\dagger)$ matrix are

$$m^2_\pm = \upsilon^2_1/2\{\lambda^2_3 + 2\lambda^2_{31}\eta^4 + \lambda^2_{33}\eta^8 \pm (\lambda_3 + \lambda_{33}\eta^4)[(\lambda_3 - \lambda_{33}\eta^4)^2 + 4\lambda^2_{31}\eta^4]^{1/2}\} , \tag{15}$$

while the mixing angle is given by

$$\tan 2\theta \approx \lambda_{31}\eta^2/(\lambda_3 - \lambda_{33}\eta^4) . \tag{16}$$

Since $\eta = (M_{GUT}/M_{SU}) \leqslant O(10^{-1})$ [11], it is natural to assume that $\lambda_{33}\eta^4$, $\lambda_{31}\eta^4 < \lambda_3$. Then

$$m_\mu = m_- \approx \upsilon_1\eta^4[\lambda^4_{31}/\lambda^2_3 + \lambda_{33}(1 - 2\lambda^2_{31}/\lambda_3\lambda_{33})^{1/2}], \quad m_\tau = m_+ \approx \lambda_3\upsilon_1[1 + \eta^4\lambda^2_{31}/\lambda^2_3]^{1/2} . \tag{17}$$

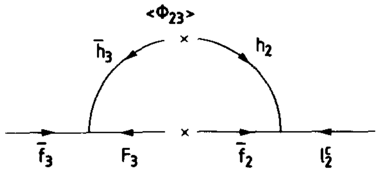

Fig. 1. Radiative corrections to the lepton masses.

From the above it can be seen that in order to have $\eta \leqslant 10^{-1}$ we must demand $\lambda_{31} > \lambda_3$, thus

$$1 < \lambda_{31}/\lambda_3 < (M_{SU}/M_{GUT})^4 . \tag{18}$$

Then, simplifying the formulae for m_τ, m_μ we have

$$m_\mu \sim v_1 \eta^4 \lambda_{31}^2/\lambda_3, \quad m_\tau \sim \lambda_3 v_1 . \tag{19}$$

Thus, from (12) and (19) the Yukawa couplings can be expressed in terms of the lepton masses and the unification scale

$$\lambda_3/\lambda_{31} \approx (m_\tau/m_\mu)^{1/2}(M_{GUT}/M_{SU})^2 \approx (2\eta)^2 , \tag{20a}$$

$$\lambda_{34}/\lambda_{31} \approx m_c/(m_\tau m_\mu)^{1/2} \, (M_{SU}/M_{GUT})^2 M_{SU}/\langle \Phi_{23} \rangle . \tag{20b}$$

Before we proceed we would like to make a comment here. It is not as yet clear what the VEV of the singlet field Φ_{23} should be. One possibility would be that the corresponding U(1) symmetry should break at the Planck scale, thus the VEV of Φ_{23} should be of the same order. The other alternative is that the singlet Φ_{23} acquires a VEV at the M_{GUT} scale, thus

$$\langle \Phi_{23} \rangle \sim \langle H_i \rangle \sim O(M_{GUT}) .$$

Both cases lead to acceptable results in this model. We choose here, however, the second alternative and comment on the other one when necessary.

The ratio $\eta = M_{GUT}/M_{SU}$ is constrained in ref. [11] to be between 10^{-1} and 10^{-3}, but here, as we will see, the constraints are much more severe, and they lead M_{GUT} very close to the upper bound, i.e., $\eta \sim O(10^{-1})$.

Indeed let us suppose that $\eta \sim 10^{-1}$. Then from (20) we get the simple relations between the Yukawa couplings

$$\lambda_3 \sim \tfrac{1}{25}\lambda_{31}, \quad \lambda_{34} \sim 1.2\lambda_{31} , \tag{21}$$

while from the mass formula for the m_τ lepton

$$v_1 \sim m_\tau/\lambda_3 \sim 25 m_\tau/\lambda_{31} . \tag{22}$$

Supposing now that $\lambda_{31} \approx \lambda_{34}/1.2 \leqslant O(1)/1.2$, we see that $\langle h_1 \rangle \leqslant 50$ GeV. This is actually what one desires, since from other phenomenological explorations [11] one expects that

$$\langle \bar{h}_{12} \rangle / \langle h_1 \rangle = v_{12}/v_1 \approx 5 . \tag{23}$$

Thus from the mass formula for the W-bosons the only natural situation is that $v_1 \approx 48$ GeV and $\bar{v}_{12} \approx 240$ GeV, in agreement with (23).

In fig. 2 we can see the variation of the ratios λ_3/λ_{31} and $\lambda_{34}/\lambda_{31}$ as a function of $x = 1/\eta = M_{SU}/M_{GUT}$. In fig. 3 the logarithms of the above quantities appear together with the function $f(v_1) = \log(\lambda_{34} v_1/50\text{ GeV})$. The above analysis leads us to accept the HIGUT scenario proposed in ref. [11]. From the above we can also put a stringent limit to the mixing angle θ which will be useful later on, when we will discuss the neutrino mass matrix. From the formula (16) we have $\tan 2\theta \geqslant \eta^2 \lambda_{31}/\lambda_3 \approx 1/4$ and thus

$$\cos^2 2\theta \geqslant 0.95 . \tag{24}$$

Next we discuss the neutrino mass matrix. For the same reasons discussed concerning the charged lepton mass matrix (see also ref. [12]) the first generation remains essentially uncoupled. Thus we can treat separately the two-generation mixing. The possible nonrenormalizable terms are [9]

$$W \ni [\lambda_{611}(F_1\bar{H}_1)^2 + \lambda_{612}(F_1\bar{H}_1)(F_2\bar{H}_2) + \lambda_{621}(F_1\bar{H}_2)(F_2\bar{H}_1) + \lambda_{622}(F_2\bar{H}_2)^2]/M_{SU} + \lambda_2(F_1\bar{f}_2\bar{h}_{12} + F_2\bar{f}_1\bar{h}_{12}) .$$

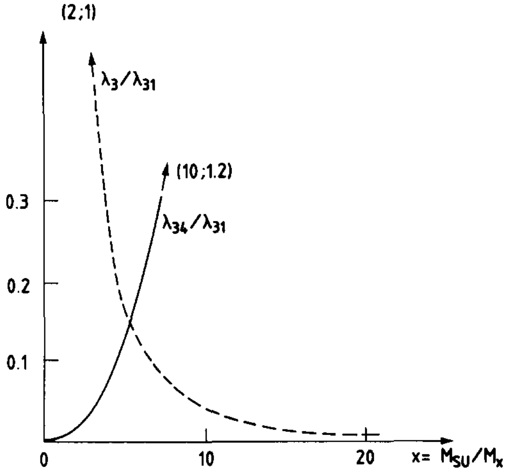

Fig. 2. Variation of the lepton Yukawa couplings as a function of $x=M_{SU}/M_{GUT}$.

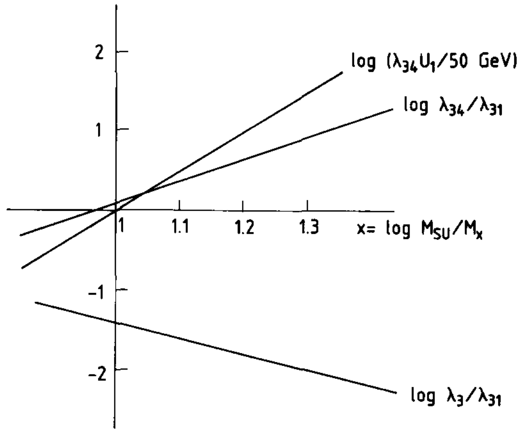

Fig. 3. Variation of the function $\log(\lambda_{34}v_1/50\ \text{GeV})$ in terms of $\log x$.

They give the 4×4 mass matrix

$$\begin{array}{c@{}c} & \begin{array}{cccc} \nu_2 & \nu_3 & N_2^c & N_3^c \end{array} \\ \begin{array}{c} \nu_2 \\ \nu_3 \\ N_2^c \\ N_3^c \end{array} & \begin{pmatrix} 0 & 0 & 0 & \lambda_2\bar{v}_{12} \\ 0 & 0 & \lambda_2\bar{v}_{12} & 0 \\ 0 & \lambda_2\bar{v}_{12} & \lambda_{611}V\eta & (\lambda_{612}+\lambda_{621})V\eta \\ \lambda_2\bar{v}_{12} & 0 & (\lambda_{612}+\lambda_{621})V\eta & \lambda_{622}V\eta \end{pmatrix} \end{array}, \tag{25}$$

where $V_1=\langle H_1\rangle\approx V_2=\langle H_2\rangle\approx V$ and $\eta=V/M_{SU}$. We write formally

$$m_{\nu N^c}=\begin{pmatrix} m_\nu & m_D \\ m_D^T & M_N \end{pmatrix}, \tag{26}$$

where

$$m_\nu=0,\quad m_D=m_D^T=\begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix},\quad M_N=\begin{pmatrix} M & M' \\ M' & M \end{pmatrix},$$

in a self-explanatory notation. Treating the above matrix perturbatively, according to standard techniques (see for example ref. [12]) we get

$$m_\nu^{\text{eff}}=m_\nu-m_D^T M_N^{-1} m_D\approx\frac{1}{a^2-1}\frac{m^2}{M}\begin{pmatrix} a & -1 \\ -1 & a \end{pmatrix}, \tag{27a}$$

where

$$a=M'/M\approx(\lambda_{612}+\lambda_{621})/\lambda_{611}\,. \tag{27b}$$

The eigenmasses are

$$m_{\nu_\tau}=m^2/(M'-M),\quad m_{\nu_\mu}=m^2/(M'+M)\,. \tag{28}$$

The diagonalizing matrix is

$$S_\nu=(1/\sqrt{2})\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}. \tag{29}$$

It is known that the difference in mass eigenstates and the mixing in the neutrino sector can lead to neutrino oscillations. In our case the $\nu_e \to \nu_\mu(\nu_\tau)$ oscillations are unobservable, since ν_e is essentially uncoupled. However $\nu_\mu \leftrightarrow \nu_\tau$ oscillations are possible. The oscillation probability is given by [13,14].

$$|\langle \nu_\beta(t)|\nu_\gamma(0)\rangle|^2 = \delta_{\mu\tau} - 4\sum_{j<k} U^*_{\mu k} U_{\tau k} U^*_{\tau j} U_{\mu j} \sin^2 \Delta_{kj} = 2\cos^2 2\theta \sin^2 \Delta_{\tau\mu} . \tag{30}$$

In the formula (30) the matrix U is given by

$$U = S_L S_V^{11} = (1/\sqrt{2}) \begin{pmatrix} 1 & \sim 0 & \sim 0 \\ \sim 0 & \cos\theta + \sin\theta & \sin\theta - \cos\theta \\ \sim 0 & -\sin\theta + \cos\theta & \cos\theta + \sin\theta \end{pmatrix}, \tag{31}$$

and

$$\Delta_{\mu\tau} \approx \tfrac{1}{2}\left(\sqrt{p_\nu^2 + m_{\nu_\tau}^2} - \sqrt{p_\nu^2 + m_{\nu_\mu}^2}\right) \approx \tfrac{1}{2}\delta m^2/2p_\nu L, \quad p_\nu \gg m_i .$$

In our case,

$$\delta m^2 = 4m^4 MM'/(M'-M)^2(M'+M)^2 .$$

For the purpose of application let us examine two limiting cases. Thus if we assume that $\lambda_{611} \sim \lambda_{612} \sim \lambda_{621} \sim \lambda_{622}$ then $M' \sim 2M$ and $\delta m^2 \sim \frac{8}{9} m^4/M^2$ and the neutrino oscillations are far away from the experimental bounds $[\delta m^2 \leqslant 4\ (\mathrm{eV})^2]$ if of course the λ_{6ij} are not very small. [For $V = 10^{16}$ GeV they should be $(\eta\lambda_{6ij}) \gg 10^{-4}$.]

There is, however, another limiting case when $M' \sim M$ or $\lambda_{612} + \lambda_{621} \sim \lambda_{611} \sim \lambda_{622}$. In that case the experimental bound for $V \sim 10^{16}$–10^{17} GeV leads to

$$\lambda_{612} + \lambda_{621} - \lambda_{611} \geqslant 10^{-3} .$$

In both cases one deduces that the neutrino masses should be less than 1 eV. On the other hand the m_{ν_e} mass is given by $\lambda v_{12}^2 V_{23} V_1^2 / M_{SU}^4$, implying that $m_{\nu_e} < 10^{-7}$ eV.

The neutrino mass spectrum and mixing comes in close agreement with the requirement of recent observations of deficits of cosmic ray muon neutrino interactions in underground detectors [15].

Indeed, let us take a specific value for the charged lepton mixing angle. As is suggested from our previous analysis tan 2θ should be taken close to $\frac{1}{4}$. Then the matrix U in (31) becomes

$$U \approx \begin{pmatrix} 1 & \sim 0 & \sim 0 \\ \sim 0 & 0.78 & 0.62 \\ \sim 0 & -0.62 & 0.78 \end{pmatrix}. \tag{31a}$$

Thus in our model, the matrix U is in agreement with the corresponding matrix obtained from phenomenological explorations in ref. [16].

The previous analysis can be extended similarly in the quark sector. However, renormalization corrections [9,11] seem to be more important here, thus there are more parameters involved in the quark mass matrices and the analysis is more complicated. However, a rough estimate would lead to the following observations:

The parameter η which expresses the ratio M_{GUT}/M_{SU} is fixed again around the value 10^{-1}, as previously from the analysis of the lepton masses.

This value can be bigger, $(3.0\text{–}4.0) \times 10^{-1}$, if we accept that the top quark mass is around 60–100 GeV, since this is the most preferable range for m_t as we know from recent experimental facts. In any case, comparing with the results of ref. [11] the most preferable value for the M_{GUT} scale seems to be of $O(10^{17})$ GeV while M_{SU} is of $O(10^{18})$ GeV. Further comparison shows that the electroweak angle is pushed to its higher acceptable values, 0.230–0.234, while $\alpha_3(m_W) \sim 0.14$–0.15.

Finally, we summarize our main results.

We have made a detailed study on the lepton mass matrices which are constructed from all the possible renormalizable and nonrenormalizable terms allowed in the model. The analysis shows that the model is consistent with the low energy theory and phenomenological consequences under very natural assumptions for the parameters involved. These assumptions are the following:

The grand unification scale should be about an order of magnitude smaller than the superunification scale.

Thus we conclude that the HIGUT scenario of ref. [11] is the most preferable. In this scenario we further find that the Yukawa couplings lie in a natural region, as they are expected from previous arguments [10].

Further phenomenological consequences of the model have been examined. Thus for example the neutrino mass matrix shows us that the neutrinos should be very light (<1 eV) in order to have consistency with $\nu_\mu \to \nu_\tau$ neutrino oscillation experiments.

Furthermore the mixing pattern of our neutrino mass matrix has a very specific form and we saw that it can interpret naturally the underground deficit of low energy ν_μ events.

Finally, there is a generic feature of this model that the first generation of leptons remains essentially uncoupled. Thus $\nu_e \leftrightarrow \nu_\mu$ or ν_τ oscillations as well as $\mu \to e\gamma$, $\mu \to 3e$ and $\mu \to e$ processes are highly suppressed in the model. Although disappointing for the experiments searching for the above decays, this fact is, however, for the time being, consistent with nature.

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