

PRIMORDIAL INFLATION IN SIMPLE SUPERGRAVITY

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Received 22 December 1982

We present a model for cosmological inflation at the Planck time in the context of simple ($N = 1$) supergravity. The model uses only one gauge singlet chiral superfield and requires non-renormalizable terms at least up to sixth order in the superpotential. Inflation itself imposes no serious constraints on any of the parameters in the model. In addition this model is capable of producing scale independent density fluctuations $\delta\rho/\rho \sim O(10^{-4})$.

It is well known by now that a large number of cosmological peculiarities such as the high degree of isotropy and flatness, large-scale homogeneity, great longevity and apparent lack of magnetic monopoles, could all be understood as natural provided there existed some epoch in which the universe expanded exponentially [1]. The period of exponential growth or inflation is generally associated with a supercooled phase transition at [1–4] or before [5] the Grand Unified Theory (GUT) era. During this transition the universe becomes dominated by the vacuum energy density due to a scalar field and passes from a radiation dominated Friedmann–Robertson–Walker (FRW) state to an approximate De Sitter state [6]. The return to a FRW universe had been one of the main sore spots in Guth's original scenario [1]. This problem, however, was resolved [2,3] by considering an inflationary period which begins after the transition has taken place and the scalar field has begun its roll-over to a new global minimum.

Conventional scenarios [1–4] consider the phase transition driving inflation to be associated with the breaking of a GUT such as $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$ and the scalar field responsible for this breaking to be in the adjoint representation. Revised scenarios [2–4] all employ the Coleman–

Weinberg mechanism [7] for the symmetry breaking, i.e. through one-loop corrections to the scalar potential. These models all require, however, unnatural fine-tunings of parameters. In addition, because the amount of inflation produced is very sensitive to the ϕ^4 coupling which in turn depends on the gauge coupling g , it has been shown [8] that sufficient inflation only occurs when $g^2 \lesssim 10^{-2}$. Other effects such as scalar field fluctuations all support the same conclusion [8–10]: standard $SU(5)$ using the Coleman–Weinberg mechanism does not provide one with a suitable inflationary scenario.

There is yet another problem with standard $SU(5)$ and inflation, namely the magnitude of density perturbations produced by the phase transition. Although phase transitions driven by scalar fields yield a scale independent spectrum for density fluctuations [11] several groups [10,12] have all concluded that the magnitude of these perturbations are a factor of $O(10^5)$ too large and would totally disrupt the observed isotropy of the cosmic background radiation. It is interesting to note that this problem also arises because of the ϕ^4 coupling's relation to the unadjustable gauge coupling.

These problems have led several of us to consider [5,8,13,14] supersymmetric versions of the infla-

tionary scenario. In supersymmetry, the ϕ^4 coupling can be made independent of the gauge coupling. Although this resolves technically many of these problems, inflation at the GUT scale still requires the unnatural fine-tunings of several parameters. It has been shown [5] that by pushing the scale at which inflation occurs above the GUT scale and closer to the Planck scale, the fine-tuning problems are removed. New problems associated with first order gravitational (FOG) effects arise as the inflationary scale nears the Planck scale. We propose here a simple toy model involving a single scalar gauge singlet field in the context of simple supergravity [15]. We derive general conditions under which inflation is possible in such a model and conditions in which density perturbations are also at an acceptable level. We assume that simple supergravity provides a reasonable model for gravitational interactions, at and below the Planck scale, and that all FOG effects are accounted for by including non-renormalizable terms in the superpotential [16].

We begin, therefore by writing down the most general form for the superpotential in one field ϕ (which we take to be real).

$$f = \mu^3 \left[\sum_{n=0}^m \frac{\lambda_n}{n+1} \left(\frac{\phi}{M}\right)^{n+1} + \lambda' \right], \quad (1)$$

where m is arbitrary and may be infinite, and $M = M_P / (8\pi)^{1/2} = 2.4 \times 10^{18}$ GeV. The couplings λ_i are all dimensionless, and μ is an as yet unspecified mass parameter.

The effective potential in $N = 1$ supergravity is given by [15]

$$V(\phi) = \exp(|\phi|^2/M^2) [|f_\phi|^2 - (3/M^2)|f|^2], \quad (2)$$

where the generalized derivative f_ϕ is

$$\begin{aligned} f_\phi &= \partial f / \partial \phi + \phi^* f / M^2 \\ &= \frac{\mu^3}{M} \left[\lambda_0 + (\lambda_1 + \lambda') \frac{\phi}{M} \right. \\ &\quad \left. + \sum_{n=0} \left(\frac{\lambda_n}{n+1} + \lambda_{n+2} \right) \left(\frac{\phi}{M}\right)^{n+2} \right]. \end{aligned} \quad (3)$$

From this point on, we will use units in which $M = 1$. In order to have a flat potential at the origin [$\partial V / \partial \phi(0) = 0$] one must eliminate the linear term in the

potential V by either setting $\lambda_1 = \lambda'$ or $\lambda_0 = 0$. To simplify the following analyses we wish to have as few parameters in the low order terms in V as possible; therefore we will choose to take $\lambda_1 = \lambda' = 0$. The effective potential then takes the following form

$$\begin{aligned} V(\phi)/\mu^6 &= \exp(\phi^2) [\lambda_0^2 + \lambda_0(2\lambda_2 - \lambda_0)\phi^2 + 2\lambda_0\lambda_3\phi^3 \\ &\quad + (\lambda_0^2 + \lambda_2^2 + \frac{2}{3}\lambda_0\lambda_2 + 2\lambda_0\lambda_4)\phi^4 + \dots]. \end{aligned} \quad (4)$$

Because sufficient inflation will require $\phi \ll 1$ and because we are only interested in the dynamics of the inflationary epoch, we can expand $\exp(\phi^2)$ and we find that

$$V(\phi)/\mu^6 = \delta + \gamma\phi^2 - \beta\phi^3 + \alpha\phi^4 + \dots, \quad (5)$$

where

$$\alpha = \frac{1}{2}\lambda_0^2 + \lambda_2^2 + \frac{8}{3}\lambda_0\lambda_2 + 2\lambda_0\lambda_4, \quad (6)$$

$$\beta = -2\lambda_0\lambda_3, \quad \gamma = 2\lambda_0\lambda_2, \quad \delta = \lambda_0^2. \quad (7,8,9)$$

This will be the basic form for the potential we consider for inflation.

Because inflation is occurring at the Planck scale, we will work in the context of exact supersymmetry. Effects due to the breaking of supersymmetry will only be valid at scales $\sim m_s \lesssim (m_w m_P)^{1/2}$. In order to ensure that supersymmetry remains unbroken, we must require that the generalized derivative of the superpotential vanish at the minimum of V . We choose this minimum to occur at $\langle \phi \rangle = v = 1$. Thus we require

$$f_\phi(1) = \mu^3 \left(\sum_{n=0} \frac{n+2}{n+1} \lambda_n \right) = 0. \quad (10)$$

In addition to preserving supersymmetry, we also will require that at the minimum of V , the cosmological constant also vanish. While in principle, the cosmological constant may have further contributions due to other phase transition (e.g., the GUT phase transition), these contributions will all be $\ll 1$. Therefore, for all intents and purposes we must have at least a negligible cosmological constant at this scale. Together with condition (10) it is easily seen by (2) that $V(1)$ will vanish if

$$\frac{\partial f}{\partial \phi}(1) = \mu^3 \sum_{n=0} \lambda_n = f(1) = \mu^3 \sum_{n=0} \frac{\lambda_n}{n+1} = 0. \quad (11)$$

We now make the following observation: In any $N = 1$ supergravity model, if there exists a point p at

which supersymmetry is unbroken and has a vanishing cosmological constant, then:

(i) That point will be an extremum.

(ii) The curvature at p is positive-semi-definite.

This can be seen by computing the derivatives $\partial V/\partial\phi_i(p)$ and $\partial^2 V/(\partial\phi_i\partial\phi_k^*)(p)$ (in general the superpotential may be a function of an arbitrary number of fields ϕ_i). The term $\partial V/\partial\phi_i$ is always proportional to a linear combination of $V, f_{\phi_i}^*$ and f^* which at p are all zero. The second derivative

$$\frac{\partial^2 V}{\partial\phi_i\partial\phi_k^*}(p) = \sum_j \frac{\partial^2 f}{\partial\phi_i\partial\phi_j}(p) \frac{\partial^2 f^*}{\partial\phi_j^*\partial\phi_k^*}(p), \quad (12)$$

and clearly has only non-negative eigenvalues. Thus the theorem is proved ^{#1}. In this case of a single field ϕ the second derivative is just $\partial^2 V/(\partial\phi\partial\phi^*) = |\partial^2 f/\partial\phi^2|^2$. From the point of view of inflation, in order to ensure that the point $p = 1$ is a minimum we must simply require

$$\frac{\partial^2 f}{\partial\phi^2} = \mu^3 \sum_{n=1} n\lambda_n \neq 0. \quad (13)$$

Eqs. (11) and (13) are therefore sufficient to have a minimum at $\phi = 1$ with $V(1) = 0$.

Let us now examine what values of the couplings are necessary in order to achieve sufficient inflation to solve the cosmological peculiarities mentioned above. To begin with, we must require that the field ϕ be constrained near the origin long enough so that the temperature will have fallen to $T_H = H/2\pi$ where in these units the Hubble parameter is

$$H^2 = \frac{1}{3}\delta = \frac{1}{3}\lambda_0^2\mu^6. \quad (14)$$

Otherwise, thermal fluctuation could carry the field and the amount of inflation ensuing could not be determined. To achieve this, we have the necessary condition ^{#2} that the universe as a whole does not make the tunnelling transition too soon. The probability for tunnelling is given by

$$P \sim \gamma^2 \exp(-B), \quad (15)$$

^{#1} This may be thought of as the generalization of the positivity of the effective potential in global supersymmetry [17].

^{#2} This condition may not be sufficient, e.g. if there is some other smaller action; however, it is at least necessary [18].

where the euclidean gravitation action B is

$$B = -4\pi \int d^4x \sqrt{g}(R - 2\Lambda), \quad (16)$$

where g is the determinant of the De Sitter metric, R is the scalar curvature ($R = 4\Lambda$ in a De Sitter space) and $\Lambda = \delta$ is the cosmological constant before the phase transition. The action for tunnelling from the local minimum to the local maximum is then [4] [for $(V_1 - V_0) \ll V_0$]

$$B = 24\pi^2(1/V_0 - 1/V_1) \approx 24\pi^2(V_1 - V_0)/V_0^2, \quad (17)$$

where $V_0 = \delta$ is the value of V at the local minimum and V_1 is the value of V at the local maximum. We now require $B \gg 1$ [19].

To find the value V_1 we first calculate the value of ϕ at the local maximum,

$$\begin{aligned} \phi_1 &= 3\beta/8\alpha - (9\beta^2 - 32\alpha\gamma)^{1/2}/8\alpha \\ &\approx 2\gamma/3\beta = -2\lambda_2/3\lambda_3 \quad (\text{for } 32\alpha\gamma/9\beta^2 \ll 1), \end{aligned} \quad (18)$$

we then have

$$V_1 - V_0 \approx (8\lambda_0\lambda_2^3/27\lambda_3^2)\mu^6 \quad (19)$$

and the action is just

$$B = \frac{64}{9}\pi^2\lambda_2^3/\lambda_0^3\lambda_3^2\mu^6. \quad (20)$$

The final constraint comes from requiring that the roll-over timescale $\tau \approx 3H/2\gamma\mu^6$, i.e., the time in which it takes for the field to move from the local maximum to the "global" minimum ^{#3}, to be large

$$\begin{aligned} H\tau &= 3H^2/2\gamma\mu^6 = \lambda_0/4\lambda_2 > 65, \\ \lambda_0 &> 260\lambda_2. \end{aligned} \quad (21)$$

This constraint (21) together with $B \gg 1$ implies

$$-\lambda_3\mu^3 \ll 2 \times 10^{-3} \quad (\lambda_3 < 0). \quad (22)$$

Finally, our approximation (18) requires

$$\lambda_0\mu^3 \lesssim 3 \times 10^{-3}. \quad (23)$$

Therefore if we set $\lambda_0\mu^3 \sim \lambda_3\mu^3 \sim 3 \times 10^{-3}$ and $\lambda_2\mu^3 \sim 10^{-5}$, this model will give a sufficient amount of inflation. To satisfy conditions (10) and (11) at

^{#3} This model presumably possesses many other minima at $\phi \gg 1$, but these will be separated by enormous barriers and are not to be feared.

least ϕ^5 and ϕ^6 terms must be considered with their couplings fixed by (11) and (13).

We note at this point that we have not discussed any finite temperature corrections to the potential (4). We now show that they are negligible. One can estimate finite temperature corrections to V by

$$\delta V = (\partial^2 f / \partial \phi^2)^2 \frac{1}{8} T^2 \approx \frac{1}{2} \mu^6 \lambda_2^2 T^2 \phi^2 \quad (24)$$

for $\phi \ll T$ and $\phi \ll 1$. But because $B \gg 1$, the universe will quickly cool down to T_H so that

$$\delta V \approx \frac{1}{2} \mu^6 \lambda_2^2 T_H^2 \phi^2 = \mu^{12} (\lambda_0^2 \lambda_2^2 / 24\pi^2) \phi^2, \quad (25)$$

and hence negligible to the ϕ^2 term already present in V .

We now briefly discuss the magnitude of density perturbations that arise during inflation [10,12,13]. For a potential containing a cubic term the magnitude of fluctuations as they enter the horizon is [13]

$$\delta\rho/\rho \simeq [\mu^3 / (2\pi^3)^{1/2}] (\beta/H) \ln^2(Hk^{-1}), \quad (26)$$

where k is the wave number of the fluctuation $\delta\phi$ leading to the density fluctuation $\delta\rho$. In terms of the potential (4) we have

$$\begin{aligned} \delta\rho/\rho &\simeq (6/\pi^3)^{1/2} \lambda_3 \mu^3 \ln^2[(\lambda_0 \mu^3 / \sqrt{3}) k^{-1}] \\ &\sim O(10^3) \lambda_3 \mu^3. \end{aligned} \quad (27)$$

Thus we need couplings somewhat lower than those sufficient for inflation in order to have $\delta\rho/\rho \sim O(10^{-4})$. Couplings of the order $\lambda_0 \sim \lambda_3 \sim 10^{-1}$ and $\lambda_2 \sim 10^{-3}$ and $\mu \sim 10^{-2}$ will provide both enough inflation and density perturbation of an acceptable magnitude.

Finally we note once again that primordial inflation does not solve the monopole problem [20]. Instead a natural explanation of monopole suppression might be due to a delayed breaking on SU(5). It is well known that if the SU(5) phase transition takes place at $T_c \lesssim 10^{10}$ GeV the number of monopoles will be at an acceptable level [21]. That this scale coincides with the strong coupling scale $\Lambda_{SU(5)}$ of SU(5) and the supersymmetry breaking scale m_s , we feel, is no accident. In a forthcoming paper [22], we describe in detail such a supersymmetric SU(5) model coupled to $N = 1$ supergravity.

In summary, we have displayed a simple toy model for primordial inflation in the context of simple supergravity. This model does not suffer from previous problems inherent in non-supersymmetric inflation,

e.g. a large ϕ^4 coupling and technically unnatural fine-tunings. It also takes completely into account all FOG effects which were a worry to globally supersymmetric inflationary models in which the inflation scale was at or close to the Planck scale. While we do not expect that this is the final solution to inflation, it seems to point in the direction that gravitational interactions are somehow related to our present universe, maybe not such a far-fetched idea.

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