Radiative Electroweak symmetry breaking in the MSSM and Low Energy Thresholds

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Abstract

We study Radiative Electroweak Symmetry Breaking in the Minimal Supersymmetric Standard Model (MSSM). We employ the 2-loop Renormalization Group equations for running masses and couplings taking into account sparticle threshold effects. The decoupling of each particle below its threshold is realized by a step function in all one-loop Renormalization Group equations (RGE). This program requires the calculation of all wavefunction, vertex and mass renormalizations for all particles involved. Adapting our numerical routines to take care of the succesive decoupling of each particle below its threshold, we compute the mass spectrum of sparticles and Higgses consistent with the existing experimental constraints. The effect of the threshold corrections is in general of the same order of magnitude as the two-loop contributions with the exception of the heavy Higgses and those neutralino and chargino states that are nearly Higgsinos for large values of the parameter μ.

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1. Introduction

The low energy values of the three gauge coupling constants known to the present experimental accuracy rule out the simplest versions of the Grand Unified Theories. In contrast, supersymmetric unification, in the framework of Supersymmetric Grand Unified Theories[1] [2], is in excellent agreement[3] with a unification energy scale M_{GUT} within the proton decay lower bounds. Moreover, softly broken supersymmetry, possibly resulting from an underlying Superstring framework, could lead to $SU(2)_L \times U(1)_Y$ gauge symmetry breaking through radiative corrections for a certain range of values of the existing free parameters[4]. In such a scenario the elegant ideas of Supersymmetry, Unification and Radiative Symmetry Breaking are realized within the same framework. The Minimal Supersymmetric extension of the Standard Model (MSSM)[1] incorporates all the above. Due to its minimal content and the radiatively induced symmetry breaking it is the most predictive of analogous theories.

As in the numerous[5][6] existing analyses of radiative symmetry breaking in the MSSM, in the present article we employ the Renormalization Group. The Higgs boson running mass-squared matrix, although positive definite at large energy scales of the order of M_{GUT}, yields a negative eigenvalue at low energies causing the spontaneous breakdown of the electroweak symmetry. The "running" of mass parameters from large to low scales is equivalent to computing leading logarithmic radiative corrections. Although this scenario depends on the values of few (3 or 4) free parameters, one could interpret it as leading to the prediction of M_Z in terms of M_{GUT}, or the Planck mass, and the top quark Yukawa coupling. Another way to interpret the predictions of this model is to consider M_Z determined in terms of the supersymmetry breaking scale. The analysis of the results helps us find out to which extent the low energy data can constrain the type and scale of supersymmetry breaking.

The purpose of the present article is to include in the above stated scenario the so-called low-energy "threshold effects". Since we have employed the $\overline{DR}$ scheme in writting down the one-loop Renormalization Group equations, which is by definition mass-indepedent, we could "run" them from M_{GUT} down to M_Z without taking notice of the numerous sparticle thresholds existing in the neighborhood of the supersymmetry breaking scale near and above M_Z. This approach of working in the "full" theory consisting of particles with masses varying over 1-2 orders of magnitude has to overcome the technical problems of the determination of the pole masses. Our approach, also shared by other analyses, is to introduce a succession of effective theories defined as the theories resulting after we functionally integrate out all heavy degrees of freedom at each particle threshold. Above and below each physical threshold we write down the Renormalization Group equations in the $\overline{DR}$ scheme only with the degrees of freedom that are light in each case. This is realized by the use of a theta function at each physical threshold. The integration of the Renormalization Group equations in the "step approximation" keeps the logarithms $\ln(\frac{m}{\mu})$ and neglects constant terms. The physical masses are determined by the condition $m(m_{phys}) = m_{phys}$ which coincides with the pole condition if we keep leading logarithms and neglect constant terms. The great advantage of this approach is that the last step of determining the physical mass presents no extra technical problem and it is trivially incorporated in the integration of the Renormalization Group equations.

2. The softly broken Minimal Supersymmetric Standard Model

The superpotential of the minimal supersymmetric extension of the standard model, or just MSSM, is

$$\mathcal{W} = Y_e L^j E^c H_1^i \epsilon_{ij} + Y_d Q^{ja} D_a^c H_1^i \epsilon_{ij} + Y_u Q^{ja} U_a^c H_2^i \epsilon_{ij} + \mu H_1^i H_2^j \epsilon_{ij} \tag{1}$$

($\epsilon_{12} = +1$) in terms of the quark $Q(3,2,1/6)$, $D^c(\overline{3},1,1/3)$, $U^c(\overline{3},1,-2/3)$, lepton $L(1,2,-1/2)$, $E^c(1,1,1)$ and Higgs $H_1(1,2,-1/2)$, $H_2(1,2,1/2)$ chiral superfields. We have suppressed family indices. The second Higgs doublet H_2 is necessary in order to give mass to the up quarks since the conjugate of H_1 cannot be used due to the analyticity of the superpotential. It is also required in order to cancel the new anomalies generated by the fermions in H_1. Note that the superpotential (1) is not the most general $SU(3)_C \times SU(2)_L \times U(1)_Y$ -invariant superpotential that can be written in terms of the given chiral superfields since terms like $D^c D^c U^c$,QLD^c,...etc not containing any ordinary standard model interaction could be present. The superpotential (1) could be arrived at by a straightforword supersymmetrization of the standard three Yukawa interaction terms. It possesses an anomalous R-parity broken by supersymmetry-breaking gaugino masses down to a discrete R-parity which ascribes -1 to matter and +1 to Higgses. There is also an unwanted continuous PQ-type symmetry leading to an observable electroweak axion which is broken by the last term in (1). This term introduces a scale μ which has to be of the order of the soft supersymmetry breaking scale in order to achieve electroweak breaking at the observed M_Z value. Although it appears unnatural that the scale of breaking of a PQ symmetry should be related to the supersymmetry breaking, there exist schemes based on an enlarged framework (extra fields or non-minimal supergravitational couplings) that lead to dynamical explanation of the order of magnitude of the scale μ[7].

The fact that supersymmetry is not observed at low energies requires the introduction of extra supersymmetry breaking interactions. This is achieved by adding to the Lagrangian density, defined by the given $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry and $\mathcal{W}$, extra interaction terms that respect the gauge symmetry but break supersymmetry. This breaking however should be such that no quadratic divergences appear and the technical "solution" to the hierarchy problem is not spoiled. Such terms are generally termed "soft". The most general supersymmetry breaking interaction Lagrangian resulting from spontaneously broken Supergravity in the flat limit ($M_P \to \infty$) contains just four types of soft terms , i.e gaugino masses, $\Phi^*\Phi$-scalar masses, $\Phi\Phi\Phi$-scalar cubic superpotential interactions and $\Phi\Phi$-scalar quadratic superpotential interactions. For the MSSM this amounts to

$$\begin{aligned}
\mathcal{L}_{SB} &= -\frac{1}{2}\sum_A M_A \bar{\lambda}_A \lambda_A - m_{H_1}^2 |H_1|^2 - m_{H_2}^2 |H_2|^2 - m_{\tilde{Q}}^2 |\tilde{Q}|^2 - m_{\tilde{D}}^2 |\tilde{D}^c|^2 - m_{\tilde{U}}^2 |\tilde{U}^c|^2 \\
&- m_{\tilde{L}}^2 |\tilde{L}|^2 - m_{\tilde{E}}^2 |\tilde{E}^c|^2 - (Y_e A_e \tilde{L}^j \tilde{E}^c H_1^i \epsilon_{ij} + Y_d A_d \tilde{Q}^{ja} \tilde{D}_a^c H_1^i \epsilon_{ij} \\
&+ Y_u A_u \tilde{Q}^{ja} \tilde{U}_a^c H_2^i \epsilon_{ij} + h.c) - (B\mu H_1^i H_2^j \epsilon_{ij} + h.c) \qquad (2)
\end{aligned}$$

Again we have suppressed family indices. We denote with H_1,H_2 the ordinary Higgs boson doublets and with $\tilde{Q}$,$\tilde{D}^c$,$\tilde{U}^c$,$\tilde{L}$,$\tilde{E}^c$ the squark and slepton scalar fields .The gauginos λ_A are considered as four component Majorana spinors. Apart from the three gaugino masses, B and the two soft Higgs masses, there are also $5N_G$ masses and $3N_G$ dimensionfull cubic couplings in the simplest case that we retain only family diagonal couplings.Thus, totally $6 + 8N_G$ new parameters. Note that these new parameters are dimensionfull and that without a simplifying principle they could in general represent different scales.

A dramatic simplification of the structure of the supersymmetry breaking interactions is provided either by Grand Unification assumptions or by Superstrings. For example, $SU(5)$ unification implies at tree level $m_{\tilde{Q}} = m_{\tilde{U}} = m_{\tilde{E}}$, $m_{\tilde{L}} = m_{\tilde{D}}$, $M_1 = M_2 = M_3$ and $A_d = A_e$. $SO(10)$ unification implies further equality of all sparticle masses, equality of Higgs masses and equality of the three types of cubic couplings. The simplest possible choice at tree level is to take all sparticle and Higgs masses equal to a common mass parameter m_o, all gaugino masses equal to some parameter $m_{1/2}$ and all cubic couplings flavour blind and equal to A_o. This situation is common in the effective Supergravity theories resulting from Superstrings but there exist more complicated alternatives. For example Superstrings with massless string modes of different modular weights lead to different sparticle masses at tree level[8]. The equality of gaugino masses can also be circumvented in an effective supergravity theory with a suitable non-minimal gauge kinetic term[9]. Note however that such non-minimal alternatives like flavour dependent sparticle masses are constrained by limits on FCNC processes. In what follows we shall consider this simplest case of four parameters m_o, $m_{1/2}$, A_o and B_o.

3. Radiative corrections and symmetry breaking

The scalar potential of the model is a sum of three terms, being the contribution of F-terms,

$$\begin{aligned} V_F &= |\sum (Y_d \tilde{Q}^{ja} \tilde{D}^c_a + Y_e \tilde{L}^j \tilde{E}^c)\epsilon_{ij} + \mu H_2^j \epsilon_{ij}|^2 + \sum |Y_e H_1^j \tilde{E}^c \epsilon_{ij}|^2 \\ &+ |\sum (Y_u \tilde{Q}^{ja} \tilde{U}^c_a)\epsilon_{ij} - \mu H_1^j \epsilon_{ij}|^2 + \sum |Y_e H_1^i \tilde{L}^j \epsilon_{ij}|^2 \\ &+ \sum |Y_d H_1^j \epsilon_{ji} \tilde{D}^c_a + Y_u H_2^j \epsilon_{ji} \tilde{U}^c_a|^2 + \sum |Y_d H_1^i \epsilon_{ij} \tilde{Q}^{ja}|^2 + \sum |Y_u H_2^i \tilde{Q}^{ja} \epsilon_{ij}|^2 \end{aligned} \tag{3}$$

where the sums are over the omitted family indices, the contribution of the D-terms

$$\begin{aligned} V_D &= \frac{1}{2} g'^2 \left[-\frac{1}{2}|H_1|^2 + \frac{1}{2}|H_2|^2 + \sum (-\frac{1}{2}|\tilde{L}|^2 + |\tilde{E}^c|^2 - \frac{2}{3}|\tilde{U}^c|^2 + \frac{1}{3}|\tilde{D}^c|^2 + \frac{1}{6}|\tilde{Q}|^2) \right]^2 \\ &+ \frac{1}{2} g_3^2 \left[\sum (\tilde{Q}^\dagger \frac{\lambda^A}{2} \tilde{Q} - \tilde{D}^c \frac{\lambda^A}{2} \tilde{D}^{c\dagger} - \tilde{U}^c \frac{\lambda^A}{2} \tilde{U}^{c\dagger}) \right]^2 \\ &+ \frac{1}{8} g^2 \left[|H_1|^4 + |H_2|^4 + (\sum |\tilde{Q}|^2)^2 + (\sum |\tilde{L}|^2)^2 - 2|H_1|^2|H_2|^2 \right. \\ &- 2|H_1|^2 \sum |\tilde{Q}|^2 - 2|H_2|^2 \sum |\tilde{Q}|^2 - 2|H_1|^2 \sum |\tilde{L}|^2 - 2|H_2|^2 \sum |\tilde{L}|^2 \\ &- 2 \sum |\tilde{Q}|^2 \sum |\tilde{L}|^2 + 4|H_1^\dagger H_2|^2 + 4 \sum \tilde{Q}_i^\dagger \tilde{Q}_j \sum \tilde{L}_j^\dagger \tilde{L}_i + 4 \sum |H_1^\dagger \tilde{L}|^2 \\ &+ \left. 4 \sum |H_2^\dagger \tilde{L}|^2 + 4 \sum |H_1^\dagger \tilde{Q}|^2 + 4 \sum |H_2^\dagger \tilde{Q}|^2 \right] \end{aligned} \tag{4}$$

and finally the scalar part of $-\mathcal{L}_{SB}$ shown in (2). The tree level scalar potential leads to electroweak breaking as long as

$$m_1^2 m_2^2 - \mu^2 B^2 < 0$$

This is clear from

$$\begin{aligned} V &= m_1^2|H_1|^2 + m_2^2|H_2|^2 + (B\mu H_1^i H_2^j \epsilon_{ij} + h.c) \\ &+ \frac{1}{8}g'^2(|H_1|^2 - |H_2|^2)^2 + \frac{1}{8}g^2(|H_1|^4 + |H_2|^4 + 4|H_1^\dagger H_2|^2 - 2|H_1|^2|H_2|^2) + ... \end{aligned} \quad (5)$$

written in terms of

$$m_{1,2}^2 \equiv m_{H_{1,2}}^2 + \mu^2. \quad (6)$$

We have replaced the appearing parameters with their running values $m_1^2(Q)$, $m_2^2(Q)$,... as defined by the Renormalization Group. The results based on the tree level study of the potential cannot be always trusted since they are sensitive to the choice of the renormalization scale. Adding the one-loop radiative corrections obtained in the $\overline{DR}$ scheme,

$$\Delta V_1 = \frac{1}{64\pi^2} Str\{\mathcal{M}^4(ln(\mathcal{M}^2/Q^2) - 3/2)\} \quad (7)$$

we end up with an Effective Potential that upon minimization supports a vacuum with spontaneously broken electroweak symmetry[6][10]. A reasonable approximation to (7) would be to allow only for the dominant top-stop loops. Note that although the Renormalization Group improved tree level potential depends on the scale Q this is not the case for the full 1-loop Effective Potential which is Q-independent up to, irrelevant for minimization, Q-dependent but field-independent terms.

Minimization of the 1-loop Effective Potential gives two conditions

$$\frac{1}{2}M_Z^2 = \frac{\overline{m}_1^2 - \overline{m}_2^2 tan^2\beta}{tan^2\beta - 1} \quad (8)$$

and

$$sin2\beta = -\frac{2B\mu}{\overline{m}_1^2 + \overline{m}_2^2} \quad (9)$$

The angle β is defined as $\beta = tan^{-1}(\upsilon_2/\upsilon_1)$ in terms of the Higgs v.e.v.'s $\upsilon_{1,2} =< H^o_{1,2} >$. The masses appearing in (8) and (9) are defined as

$$\overline{m}^2_{1,2} \equiv m^2_{1,2} + \frac{\partial(\Delta V_1)}{\partial \upsilon^2_{1,2}} \tag{10}$$

All parameters are Q-dependent. At $Q = M_Z$

$$M_Z^2 = \frac{1}{2}(g^2 + g'^2)(\upsilon_1^2 + \upsilon_2^2) + ... \tag{11}$$

M_Z denotes the Z - boson pole mass $M_Z = 91.187 GeV$, and the ellipses are higher order corrections. Note also that in our convention μB has the same sign with $\overline{m}_1^2 + \overline{m}_2^2$.

We shall assume that at a very high energy scale M_{GUT} the soft supersymmetry breaking is represented by four parameters m_o, $m_{1/2}$, A_o and B of which we shall consider as input parameters only the first three and treat $B(M_Z)$ as determined through equation (9). Actually we can treat $\beta(M_Z)$ as input parameter and both $B(M_Z), \mu(M_Z)$ are determined by solving the minimization conditions (8) and (9), with the sign of μ left undetermined. The top-quark mass[11], or equivalently the top-quark Yukawa coupling, although localized in a small range of values should also be considered as an input parameter since the sparticle spectrum and the occurrance of symmetry breaking itself is sensitive to its value.Thus, the input parameters are m_o, $m_{1/2}$, A_o, $\beta(M_Z)$ and $m_t(M_Z)$.

Since radiative corrections are generally expected to be small with the exception of the contributions from the top-stop system, a reasonable approximation of (7) is

$$\Delta V_1 = \frac{3}{32\pi^2} \sum_{i=+,-} m_i^4 [ln(m_i^2/Q^2) - 3/2] - \frac{3}{16\pi^2} m_t^4 [ln(m_t^2/Q^2) - 3/2] \tag{12}$$

where we have kept only the $\tilde{t}$,$\tilde{t^c}$ and t contributions[6][10][12]. The field-dependent "masses" appearing in (12) are

$$m_t \equiv Y_t H_2^o \quad , \quad m_\pm^2 = \frac{1}{2}\{m_{LL}^2 + m_{RR}^2 \pm [(m_{LL}^2 - m_{RR}^2)^2 + 4m_{RL}^4]^{\frac{1}{2}}\} \tag{13}$$

where

$$
\begin{aligned}
m^2_{LL} &\equiv m^2_{\tilde{t}} + m^2_t + (\frac{1}{12}g'^2 - \frac{1}{4}g^2)(|H^o_2|^2 - |H^o_1|^2) \\
m^2_{RR} &\equiv m^2_{\tilde{t}^c} + m^2_t - \frac{1}{3}g'^2(|H^o_2|^2 - |H^o_1|^2) \\
m^2_{LR} &\equiv -Y_t(H^o_2 A_t + \mu H^{o*}_1) \qquad (14) \\
m^2_{RL} &\equiv {m^2_{LR}}^* \qquad (15)
\end{aligned}
$$

Note that the use of the t-$\tilde{t}$ contribution can be misleading in some cases due to large cancellations occuring with terms that are not included [Arnowitt and Nath in ref.6].A complete analysis requires that the contributions of all sectors to the effective potential are duly taken into account.

4. The Renormalization Group and Threshold effects

Consider the Renormalization Group equation for a soft mass parameter derived in the $\overline{DR}$ scheme[16]

$$\frac{dm}{dlnQ^2} = \frac{b}{16\pi^2}m \qquad (16)$$

This equation should be integrated from a superlarge scale $Q = M_{GUT}$, where we impose a boundary condition $m(M_{GUT}) = m_o$, down to any desirable value of Q. As we come down from M_{GUT} as long as we are at scales larger than the heaviest particle in the spectrum we include in b contributions from all the particles in the MSSM. When we cross the heaviest particle threshold we switch and compute b in a new theory, an effective field theory[13] with the heaviest particle integrated out. Coming further down in energy we encounter the next particle threshold at which point we switch again to a new effective field theory with the two heaviest particles integrated out. It is clear how we procced from then on.

The change in the running mass parameter m at a particle threshold M in the above scheme comes out to be for $m < M$

$$\frac{\Delta m}{m} \simeq \frac{(b_+ - b_-)}{16\pi^2} ln(\frac{M^2}{m^2}) \tag{17}$$

where b_+ and b_- are the Renormalization Group coefficients computed in the effective theories above and below the threshold respectively. Comparing (17) with the exact result obtained from the 2-point function associated with m we find that there is a finite non-logarithmic part that is missed by our approximation[14]. The further M and m are apart the better the approximation becomes.Of course, the great advantage of the approximation lies in the fact that it is done entirely at the level of the Renormalization Group without the need to calculate the finite parts of n-point functions.

The Renormalization Group equation (16) referring to a particular running mass $m(Q)$ is integrated stepwise in the above stated manner down to the physical mass corresponding to $m(Q)$. The physical mass is determined by the condition

$$m(m_{phys}) = m_{phys} \tag{18}$$

Note that in the $\overline{DR}$ scheme the inverse two-point function corresponding to the running mass $m(Q)$ will be at 1-loop of the general form

$$\Gamma^{(2)}(Q^2, m(Q)) = Q^2(c_1 + c_2 ln(\frac{m^2(Q)}{Q^2})) + m^2(Q)(c_1' + c_2' ln(\frac{m^2(Q)}{Q^2})) \tag{19}$$

Imposing the condition (18) we see that the right hand side of (19) gives $(c_1 + c_1')m_{phys}^2$. Thus, condition (18) coincides with the true (pole) condition for the physical mass only when the constant non-logarithmic contributions can be neglected.

In what follows we shall present the 1-loop β-functions of gauge and Yukawa couplings as well as those for the soft masses, cubic parameters A and quadratic parameters B and μ[15]. Note that the threshold corrections introduced in our approximation by the theta-functions at 1-loop are expected to be comparable to the standard 2-loop RG corrections. In our numerical analysis that will follow we shall employ the 2-loop RG equations which have not been presented here due to their complicated form but can be found elsewhere[17]. In our notation, for a physical mass M,

$$\theta_M \equiv \theta(Q^2 - M^2) \tag{20}$$

Also t stands for $t = lnQ^2$ and $\beta_\lambda \equiv \frac{d\lambda}{dt}$ for each parameter λ. Note also that we assume diagonal couplings in family space.

The β-functions for the three gauge couplings are

$$\frac{dg_i}{dt} \equiv \beta(g_i) = \frac{b_i}{2(4\pi)^2} T_i \, {g_i}^3 \quad , \quad i = 1, 2, 3 \tag{21}$$

The coefficients b_i are $\frac{33}{5}$,1,-3 respectively.

Keeping the Yukawa couplings $Y_{t,b,\tau}$ of the third generation fermions, the corresponding β functions are,

$$\frac{dY_\tau}{dt} \equiv \beta(Y_\tau) = \frac{Y_\tau}{(4\pi)^2}\{-\frac{3}{2}T_{\tau 2}{g_2}^2 - \frac{9}{10}T_{\tau 1}{g_1}^2 + 2T_{\tau\tau}{Y_\tau}^2 + \frac{3}{2}{Y_b}^2\} \tag{22}$$

$$\frac{dY_b}{dt} \equiv \beta(Y_b) = \frac{Y_b}{(4\pi)^2}\{-\frac{8}{3}T_{b3}{g_3}^2 - \frac{3}{2}T_{b2}{g_2}^2 - \frac{7}{30}T_{b1}{g_1}^2 + \frac{1}{2}T_{bt}{Y_t}^2 + 3T_{bb}{Y_b}^2 + \frac{1}{2}{Y_\tau}^2\} \tag{23}$$

$$\frac{dY_t}{dt} \equiv \beta(Y_t) = \frac{Y_t}{(4\pi)^2}\{-\frac{8}{3}T_{t3}{g_3}^2 - \frac{3}{2}T_{t2}{g_2}^2 - \frac{13}{30}T_{t1}{g_1}^2 + 3T_{tt}{Y_t}^2 + \frac{1}{2}T_{tb}{Y_b}^2\} \tag{24}$$

The threshold coefficients T_i, $T_{\tau i}, etc$ appearing in the expressions above are shown in Table I. We denote by $\tilde{G}$,$\tilde{W}$, $\tilde{B}$ the $SU(3)$,$SU(2)$ and $U(1)$ gauge fermions respectively.

The β-functions for the cubic couplings are

$$\begin{aligned}\frac{dA_\tau}{dt} = & \frac{1}{(4\pi)^2}\{-3{g_2}^2 M_2 \theta_{\tilde{W}\tilde{H}_1} - \frac{3}{5}{g_1}^2 M_1 (2 + \theta_{\tilde{H}_1})\theta_{\tilde{B}} \\ & + 3{Y_b}^2 A_b \theta_{\tilde{D}\tilde{Q}} + 4{Y_\tau}^2 A_\tau + A_\tau[Z_{\tau 1}{g_1}^2 + Z_{\tau 2}{g_2}^2 + Z_{\tau\tau}{Y_\tau}^2]\}\end{aligned} \tag{25}$$

$$\frac{dA_b}{dt} = \frac{1}{(4\pi)^2}\{-\frac{16}{3}{g_3}^2 M_3\theta_{\tilde{G}} - 3{g_2}^2 M_2\theta_{\tilde{W}\tilde{H}_1} - \frac{1}{30}{g_1}^2 M_1(-4+18\theta_{\tilde{H}_1})\theta_{\tilde{B}}$$

$$+ \ {Y_\tau}^2 A_\tau\theta_{\tilde{E}\tilde{L}} + {Y_t}^2 A_t\theta_{H_2\tilde{U}} + 6{Y_b}^2 A_b$$

$$+ \ A_b[Z_{b3}{g_3}^2 + Z_{b2}{g_2}^2 + Z_{b1}{g_1}^2 + Z_{bt}Y_t^2 + Z_{bb}{Y_b}^2]\} \quad (26)$$

$$\frac{dA_t}{dt} = \frac{1}{(4\pi)^2}\{-\frac{16}{3}{g_3}^2 M_3\theta_{\tilde{G}} - 3{g_2}^2 M_2\theta_{\tilde{W}\tilde{H}_2} - \frac{1}{15}{g_1}^2 M_1(4+9\theta_{\tilde{H}_2})\theta_{\tilde{B}}$$

$$+ \ 6Y_t^2 A_t\theta_{H_1\tilde{U}} + {Y_b}^2 A_b\theta_{H_1\tilde{D}}$$

$$+ \ A_t[Z_{t3}{g_3}^2 + Z_{t2}{g_2}^2 + Z_{t1}{g_1}^2 + Z_{tt}Y_t^2 + Z_{tb}{Y_b}^2]\} \quad (27)$$

In our notation $\theta_{ab} \equiv \theta_a\theta_b$ and $\theta_{abc} \equiv \theta_a\theta_b\theta_c$. The coefficients Z_{qi} are shown in Table II.

Next we procceed to the RG equations for the scalar masses. The RG equations for the sparticle masses refer to the third generation. For the other two generations the Yukawa couplings could be set to zero due to their smallness.

$$\frac{dm_{\tilde{Q}}^2}{dt} = \frac{1}{(4\pi)^2}\{-[\frac{8}{3}{g_3}^2(\theta_{\tilde{Q}} - \theta_{\tilde{G}}) + \frac{3}{2}{g_2}^2(\theta_{\tilde{Q}} - \theta_{\tilde{W}}) + \frac{1}{30}{g_1}^2(\theta_{\tilde{Q}} - \theta_{\tilde{B}})]\, m_{\tilde{Q}}^2$$

$$- \ \frac{16}{3}{g_3}^2 M_3^2\theta_{\tilde{G}} - 3{g_2}^2 M_2^2\theta_{\tilde{W}} - \frac{1}{15}{g_1}^2 M_1^2\theta_{\tilde{B}} + \frac{1}{10}{g_1}^2 S$$

$$+ \ {Y_t}^2[m_{\tilde{Q}}^2\theta_{\tilde{H}_2} + m_{\tilde{U}}^2\theta_{\tilde{U}} + m_2^2\theta_{H_2} + A_t^2\theta_{H_2\tilde{U}} + \mu^2(\theta_{H_1\tilde{U}} - 2\theta_{\tilde{H}_2})]\}$$

$$+ \ {Y_b}^2[m_{\tilde{Q}}^2\theta_{\tilde{H}_1} + m_{\tilde{D}}^2\theta_{\tilde{D}} + m_1^2\theta_{H_1} + A_b^2\theta_{H_1\tilde{D}} + \mu^2(\theta_{H_2\tilde{D}} - 2\theta_{\tilde{H}_1})]\} \quad (28)$$

$$
\begin{aligned}
\frac{dm_{\tilde{U}}^2}{dt} &= \frac{1}{(4\pi)^2}\{-[\frac{8}{3}{g_3}^2(\theta_{\tilde{U}}-\theta_{\tilde{G}})+\frac{8}{15}{g_1}^2(\theta_{\tilde{U}}-\theta_{\tilde{B}})]\,m_{\tilde{U}}^2 \\
&- \frac{16}{3}{g_3}^2M_3^2\theta_{\tilde{G}}-\frac{16}{15}{g_1}^2M_1^2\theta_{\tilde{B}}-\frac{2}{5}{g_1}^2S \\
&+ 2{Y_t}^2[m_{\tilde{U}}^2\theta_{\tilde{H}_2}+m_{\tilde{Q}}^2\theta_{\tilde{Q}}+m_2^2\theta_{H_2}+A_t^2\theta_{H_2\tilde{Q}}+\mu^2(\theta_{H_1\tilde{Q}}-2\theta_{\tilde{H}_2})]\} \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
\frac{dm_{\tilde{D}}^2}{dt} &= \frac{1}{(4\pi)^2}\{-[\frac{8}{3}{g_3}^2(\theta_{\tilde{D}}-\theta_{\tilde{G}})+\frac{2}{15}{g_1}^2(\theta_{\tilde{D}}-\theta_{\tilde{B}})]\,m_{\tilde{D}}^2 \\
&- \frac{16}{3}{g_3}^2M_3^2\theta_{\tilde{G}}-\frac{4}{15}{g_1}^2M_1^2\theta_{\tilde{B}}+\frac{1}{5}{g_1}^2S \\
&+ 2{Y_b}^2[m_{\tilde{D}}^2\theta_{\tilde{H}_1}+m_{\tilde{Q}}^2\theta_{\tilde{Q}}+m_1^2\theta_{H_1}+A_b^2\theta_{H_1\tilde{Q}}+\mu^2(\theta_{H_2\tilde{Q}}-2\theta_{\tilde{H}_1})]\} \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
\frac{dm_{\tilde{L}}^2}{dt} &= \frac{1}{(4\pi)^2}\{-[\frac{3}{2}{g_2}^2(\theta_{\tilde{L}}-\theta_{\tilde{W}})+\frac{3}{10}{g_1}^2(\theta_{\tilde{L}}-\theta_{\tilde{B}})]\,m_{\tilde{L}}^2 \\
&- 3{g_2}^2M_2^2\theta_{\tilde{W}}-\frac{3}{5}{g_1}^2M_1^2\theta_{\tilde{B}}-\frac{3}{10}{g_1}^2S \\
&+ {Y_\tau}^2[m_{\tilde{L}}^2\theta_{\tilde{H}_1}+m_{\tilde{E}}^2\theta_{\tilde{E}}+m_1^2\theta_{H_1}+A_\tau^2\theta_{H_1\tilde{E}}+\mu^2(\theta_{H_2\tilde{E}}-2\theta_{\tilde{H}_1})]\} \qquad (31)
\end{aligned}
$$

$$
\begin{aligned}
\frac{dm_{\tilde{E}}^2}{dt} &= \frac{1}{(4\pi)^2}\{-[\frac{6}{5}{g_1}^2(\theta_{\tilde{E}}-\theta_{\tilde{B}})]\,m_{\tilde{E}}^2 \\
&- \frac{12}{5}{g_1}^2M_1^2\theta_{\tilde{B}}+\frac{3}{5}{g_1}^2S
\end{aligned}
$$

$$+ \quad 2{Y_\tau}^2[m_{\tilde{E}}^2\theta_{\tilde{H}_1} + m_{\tilde{L}}^2\theta_{\tilde{L}} + m_1^2\theta_{H_1} + A_\tau^2\theta_{H_1\tilde{L}} + \mu^2(\theta_{H_2\tilde{L}} - 2\theta_{\tilde{H}_1})]\} \tag{32}$$

$$\frac{dm_1^2}{dt} = \frac{1}{(4\pi)^2}\{-[\frac{3}{2}{g_2}^2(\theta_{H_1} - \theta_{\tilde{H}_1\tilde{W}}) + \frac{3}{10}{g_1}^2(\theta_{H_1} - \theta_{\tilde{H}_1\tilde{B}})]\, m_1^2$$

$$- \quad 3{g_2}^2(M_2^2 + \mu^2)\theta_{\tilde{H}_1\tilde{W}} - \frac{3}{5}{g_1}^2(M_1^2 + \mu^2)\theta_{\tilde{H}_1\tilde{B}} - \frac{3}{10}{g_1}^2 S$$

$$+ \quad {Y_\tau}^2[m_1^2 + m_{\tilde{L}}^2\theta_{\tilde{L}} + m_{\tilde{E}}^2\theta_{\tilde{E}} + A_\tau^2\theta_{\tilde{L}\tilde{E}}]$$

$$+ \quad 3{Y_b}^2[m_1^2 + m_{\tilde{Q}}^2\theta_{\tilde{Q}} + m_{\tilde{D}}^2\theta_{\tilde{D}} + A_b^2\theta_{\tilde{Q}\tilde{D}}] + 3{Y_t}^2\mu^2\theta_{\tilde{Q}\tilde{U}}\} \tag{33}$$

$$\frac{dm_2^2}{dt} = \frac{1}{(4\pi)^2}\{-[\frac{3}{2}{g_2}^2(\theta_{H_2} - \theta_{\tilde{H}_2\tilde{W}}) + \frac{3}{10}{g_1}^2(\theta_{H_2} - \theta_{\tilde{H}_2\tilde{B}})]\, m_2^2$$

$$- \quad 3{g_2}^2(M_2^2 + \mu^2)\theta_{\tilde{H}_2\tilde{W}} - \frac{3}{5}{g_1}^2(M_1^2 + \mu^2)\theta_{\tilde{H}_2\tilde{B}} + \frac{3}{10}{g_1}^2 S$$

$$+ \quad 3{Y_t}^2[m_2^2 + m_{\tilde{Q}}^2\theta_{\tilde{Q}} + m_{\tilde{U}}^2\theta_{\tilde{U}} + A_t^2\theta_{\tilde{Q}\tilde{U}}]$$

$$+ \quad 3{Y_b}^2\mu^2\theta_{\tilde{Q}\tilde{D}} + {Y_\tau}^2\mu^2\theta_{\tilde{E}\tilde{L}}\} \tag{34}$$

The quantity S appearing in the equations above is defined as

$$S \equiv \; Tr\; \{\frac{Y}{2}\; \theta_m\; m^2\} \tag{35}$$

In the absence of the threshold effects this quantity is multiplicatively renormalized. Therefore if it vanishes at the unification scale, due to appropriate boundary conditions, it vanishes everywhere and its effect can be ommited altogether from the RGE's. However in our case this does not any longer hold owing to its explicit threshold dependence and S starts becoming nonvanishing as soon as we pass the heaviest of the thresholds. For the Higgs and Higgsino mixing parameters $m_3^2 \equiv B\mu$ and μ respectively we have,

$$\begin{aligned}
\frac{dm_3^2}{dt} &= \frac{1}{(4\pi)^2}\{[-\frac{3}{4}{g_2}^2(\theta_{H_1}+\theta_{H_2}+2\theta_{H_1H_2}-\theta_{\tilde{H}_1\tilde{W}}-\theta_{\tilde{H}_2\tilde{W}}) \\
&- \frac{3}{20}{g_1}^2(\theta_{H_1}+\theta_{H_2}+2\theta_{H_1H_2}-\theta_{\tilde{H}_1\tilde{B}}-\theta_{\tilde{H}_2\tilde{B}}) \\
&+ \frac{3}{2}{Y_t}^2+\frac{3}{2}{Y_b}^2+\frac{1}{2}{Y_\tau}^2]\ m_3^2 \\
&+ \mu\,[-3{g_2}^2M_2\theta_{\tilde{H}_1\tilde{H}_2\tilde{W}}-\frac{3}{5}{g_1}^2M_1\theta_{\tilde{H}_1\tilde{H}_2\tilde{B}} \\
&+ 3A_t{Y_t}^2\theta_{\tilde{Q}\tilde{U}}+3A_b{Y_b}^2\theta_{\tilde{Q}\tilde{D}}+A_\tau{Y_\tau}^2\theta_{\tilde{L}\tilde{E}}]\} \qquad (36)
\end{aligned}$$

$$\begin{aligned}
\frac{d\mu}{dt} &= \frac{1}{(4\pi)^2}\{\frac{3}{8}{g_2}^2(\theta_{\tilde{H}_1}+\theta_{\tilde{H}_2}-8\theta_{\tilde{H}_1\tilde{H}_2}+\theta_{H_1\tilde{W}}+\theta_{H_2\tilde{W}}) \\
&+ \frac{3}{40}{g_1}^2(\theta_{\tilde{H}_1}+\theta_{\tilde{H}_2}-8\theta_{\tilde{H}_1\tilde{H}_2}+\theta_{H_1\tilde{B}}+\theta_{H_2\tilde{B}}) \\
&+ \frac{3}{4}{Y_b}^2(\theta_{\tilde{Q}}+\theta_{\tilde{D}})+\frac{3}{4}{Y_t}^2(\theta_{\tilde{Q}}+\theta_{\tilde{U}})+\frac{1}{4}{Y_\tau}^2(\theta_{\tilde{L}}+\theta_{\tilde{E}})\}\,\mu \qquad (37)
\end{aligned}$$

Finally the beta functions for the three gaugino masses are

$$\frac{dM_i}{dt} = S_i\ \frac{b_i}{(4\pi)^2}\ g_i^2\ M_i\ ,\ i=1,2,3 \qquad (38)$$

where b_i are the beta function coefficients of the gauge couplings given earlier and S_i are threshold function coefficients given by,

$$S_3 = -3\ \theta_{\tilde{G}}-\frac{1}{6}\sum_{i=1}^{N_g}\ (2\theta_{\tilde{Q}_i}+\theta_{\tilde{U}_i}+\theta_{\tilde{D}_i}) \qquad (39)$$

$$S_2 \;=\; -6\;\theta_{\tilde{W}} - \frac{1}{2}\sum_{i=1}^{N_g}\,(3\theta_{\tilde{Q}_i} + \theta_{\tilde{L}_i}) - \frac{1}{2}(\theta_{H_1\tilde{H}_1} + \theta_{H_2\tilde{H}_2}) \tag{40}$$

$$S_1 \;=\; \frac{1}{11}\,[\sum_{i=1}^{N_g}\,(\frac{1}{6}\theta_{\tilde{Q}_i} + \frac{4}{3}\theta_{\tilde{U}_i} + \frac{1}{3}\theta_{\tilde{D}_i} + \frac{1}{2}\theta_{\tilde{L}_i} + \theta_{\tilde{E}_i}) + \frac{1}{2}(\theta_{H_1\tilde{H}_1} + \theta_{H_2\tilde{H}_2})] \tag{41}$$

The dimensionful parameters, masses and cubic couplings, are meant to freeze out when the energy crosses below the mass scale associated with the heaviest particle participating. This can be implemented by multiplying the corresponding quantity by the relevant theta function. Thus for instance A_t freezes out below the thresholds of either $\tilde{t}$, or $\tilde{t}^c$, or H_2, whichever is the heaviest, and the associated theta functions should multiply the r.h.s of eq. (27). For simplicity of our notation we do not indicate that explicitly in the RGE's displayed in eqs. (25)-(38).

5. Formulation of the problem and numerical analysis

The problem at hand consists in finding the physical masses of the presently unobserved particles, i.e. squarks, sleptons, Higgses, Higgsinos and gauginos, as well as their physical couplings to other observed particles. This will be achieved by integrating the Renormalization Group equations from a superheavy scale M_{GUT}, taken to be in the neighbourhood of $10^{16} GeV$, down to a scale Q_o in the stepwise manner stated. If the equation at hand is the Renormalization Group equation for a particular running mass $m(Q)$, then Q_o is the corresponding physical mass determined by the condition $m(Q_o) = Q_o$. If the equation at hand is the Renormalization Group equation for a gauge or Yukawa coupling the integration will be continued down to $Q_o = M_Z$. Acceptable solutions should satisfy the constraints (8) and (9) at M_Z, i.e. describe a low energy theory with broken electroweak symmetry at the right value of $M_Z \simeq 91.187 GeV$.

The boundary condition at high energy will be chosen as simple as possible, postponing for elsewhere the study of more complicated alternatives. Thus at the (unification) point M_{GUT}, taken to be 10^{16} GeV, we shall take

$$\begin{aligned} m_{\tilde{Q}}(M_{GUT}) &= m_{\tilde{D}^c}(M_{GUT}) = m_{\tilde{U}^c}(M_{GUT}) = m_{\tilde{L}}(M_{GUT}) = m_{\tilde{E}^c}(M_{GUT}) \\ &= m_{H_1}(M_{GUT}) = m_{H_2}(M_{GUT}) \equiv m_o \end{aligned} \tag{42}$$

and

$$M_1(M_{GUT}) = M_2(M_{GUT}) = M_3(M_{GUT}) \equiv m_{1/2} \tag{43}$$

In addition we take equal cubic couplings at M_{GUT}, i.e.

$$A_e(M_{GUT}) = A_d(M_{GUT}) = A_u(M_{GUT}) \equiv A_o \tag{44}$$

All our boundary conditions are family blind. We shall also denote with B_o and μ_o the boundary values at M_{GUT} of the parameters $B(Q)$ and $\mu(Q)$. This five parameters m_o, $m_{1/2}$, A_o, B_o and μ_o are not all free due to conditions (8) and (9) which could be viewed as determining B and as trading μ for $\beta \equiv tan^{-1}(\upsilon_2/\upsilon_1)$. Thus, m_o, $m_{1/2}$, A_o, $\beta(M_Z)$ as well as the sign of $\mu(M_Z)$ can be our free parameters.

Our set of constraints includes the low energy experimental gauge coupling values which we have taken to be $\alpha_3(M_Z)_{\overline{MS}} = 0.117 \pm 0.010$, $\alpha_{em}(M_Z)^{-1}{}_{\overline{MS}} = 127.9 \pm 0.1$ and $(sin^2\theta_W)\ |_{\overline{MS}} = 0.2316 - .88\ 10^{-7}(M_t{}^2 - 160^2)GeV^{-2}$[21]. The $\overline{MS}$ values for the couplings are related to their $\overline{DR}$[1]ones through the relations $g_{\overline{MS}} = g_{\overline{DR}}(1 - Cg^2/96\pi^2)$, where $C = 0, 2, 3$ respectively for the three gauge groups. The unification scale M_{GUT} is determined from the intersection of $\alpha_{1\overline{DR}}$ and $\alpha_{2\overline{DR}}$ gauge couplings and is found to be in the vicinity of 10^{16} GeV.This value of M_{GUT} is not easily reconcilable with the low energy value of α_3 quoted above and the universal boundary condition, as given in

[1]Note that at the 2-loop order the $\overline{DR}$ scheme needs to be modified so that no contribution to the scalar masses due to the"ϵ-scalars"[18] shows up.

eqs.(42-44) if the effective SUSY breaking scale M_S is below 1 Tev[22] If we treat the low energy value of α_3 as an output we find it to be $\geq$0.125, i.e slightly larger than the most favourite experimental value quoted previously with a tendency to decrease as M_S gets larger than $\geq$1 Tev.Our interest in this paper is mainly focused on the mass spectrum and on the effect of the mass thresholds to it.The subtle issue of the gauge coupling unification in conjuction with the small value of α_3 shall be addressed to in a forthcoming publication. In the course of our numerical computations we allow for switching off the effect of the the thresholds from masses and cubic couplings involved. In this way we can compare the predictions for couplings and pole masses in the two cases i) With all thresholds present in both couplings and dimensionful parameters and ii) thresholds appearing only in gauge and Yukawa couplings. The latter case has already been considered by several groups. Although the difference is expected to be small only case (i) represents a consistent prediction of the spectrum, together with α_{GUT} and M_{GUT}, in the framework of the leading logarithmic approximation.

For the b quark and τ lepton our inputs are $M_b^{pole} = 5$ GeV and $M_\tau^{pole} = 1.777$ GeV which are related to their running $\overline{MS}$ masses by

$$m_b(M_b) = \frac{M_b}{(1 + \frac{4\alpha_3}{3\pi} + 12.4(\frac{\alpha_3}{\pi})^2)} \tag{45}$$

$$m_\tau(M_\tau) = M_\tau \tag{46}$$

These are evolved up to M_Z according to the $SU_c(3) \times U_{em}(1)$ RGE's and are then converted to $\overline{DR}$ values.From these the $\overline{DR}$ values for the Yukawa couplings $Y_b(M_Z)$, $Y_\tau(M_Z)$ are determined. The numerical procedure for the determination of M_{GUT} respecting the experimental inputs for α_{em},$sin^2\theta_W$ and $M_{b,\tau}^{pole}$ needs several iterations to reach convergence.

As for the top Yukawa coupling our input is the $\overline{DR}$ running mass $m_t(M_Z)$ related to the physical top mass M_t^{pole} by

$$m_t(M_t) = \frac{M_t}{(1 + \frac{5\alpha_3}{3\pi} + \cdots)} \tag{47}$$

The recent evidence[11] for the top-quark mass has motivated values for M_t in the neighborhood of 176 ± 8 GeV.

In our numerical procedure we follow a two loop renormalization group analysis for all parameters involved, i.e couplings and dimensionful parameters, v.e.v's included. We start with the $\overline{MS}$ values for the gauge couplings at M_Z giving a trial input value for the strong coupling constant α_3 in the vicinity of .120, which are then converted to their $\overline{DR}$ values. These are run down to M_b, M_τ with the $SU_c(3) \times U_{em}(1)$ RGE' s to know the running bottom and tau masses as given before. From these by use of the RGE's for the running masses m_b, m_τ we run upwards to know their $\overline{MS}$ values at M_Z which are subsequently converted to their corresponding $\overline{DR}$ values. This provides us with the bottom and tau Yukawa couplings at the scale M_Z. The top Yukawa coupling is known from the input running top quark mass $m_t(M_Z)$ as said earlier. The evolution of all couplings from M_Z running upwards to high energies determines the unification scale M_{GUT} and the value of the unification coupling α_{GUT} by

$$\alpha_{1\overline{DR}}(M_{GUT}) = \alpha_{2\overline{DR}}(M_{GUT}) = \alpha_{GUT}. \tag{48}$$

Running down from M_{GUT} to M_Z the trial input value for α_3 has now changed. This procedure is iterated several times until convergence is reached. In each iteration the values of B, μ, which as stated previously are not inputs in this approach, are determined by minimizing the scalar potential. For their determination at the scale M_Z we take into account the one loop corrected scalar potential. This procedure modifies the tree level values $B(M_Z)$, $\mu(M_Z)$. It is well known that the value of μ affects the predictions for the physical masses especially those of the neutralinos and charginos. In approaches in which the effect of the thresholds is ignored in the RGE's the determination of B, μ is greatly facilitated by the near decoupling of these parameters from the rest of the RGE's. However with the effects of the thresholds taken into account such a decoupling no longer holds since the thresholds themselves depend on B, μ, or equivalently on μ, m_3^2.

In solving the system of the 33 RGE's involved, among these those for the v.e.v's, we have used special $FORTRAN$ routines of the $IMSL$ library available to us which use the $Runge - Kutta - Verner$ sixth order method and are capable of handling stiff systems of nonlinear differential equations with high accuracy.

Throughout our analysis we avoid considering values for $\tan\beta$ for which the couplings are driven to large values outside of the perturbative regime.

The experimental lower bounds for the masses of new particles extracted from accelerator data are as follows[2]. The four neutralino mass eigenstates have to be heavier than 20,45,70 and 108 GeV while the two chargino states have lower mass limits of 43 and 99 GeV. Charged sleptons have to be heavier than 45 GeV, while sneutrinos have to be heavier than 41.8 GeV. There is also a 150 GeV lower bound on the mass of squarks and gluinos. Charged Higgses should be heavier than 41 GeV while the CP-odd neutral Higgs should be heavier than 22 GeV. The lightest of the two neutral CP-even Higgs eigenstates should have a mass larger than 44 if the CP-odd Higgs is lighter than M_Z or 60 GeV if the opposite is true respectively.

The interesting part of the output of the numerical integration of the RG equations consists of the mass spectrum of the new particles (gaugino-Higgsinos, squarks and sleptons) as well as the Higgs masses. The neutralino mass eigenstates can be read off from reference [12]. The two stop mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$ correspond to the mass eigenvalues $m_{\pm}^2$ shown in(13).The Higgs mass eigenvalues at tree level are $m_A^2 = m_1^2 + m_2^2$ for the neutral pseudoscalar Higgs A, and

$$m_{H,h}^2 = \frac{1}{2}[{M_Z}^2 + m_A^2 \pm \sqrt{(m_A^2 + M_Z^2)^2 - 4M_Z^2 m_A^2 cos^2(2\beta)}] \tag{49}$$

[2]We have also constrained the output to cases of neutral and colourless LSP.

for the two neutral scalar Higgses h and H. The charged Higgs has a tree level mass $M_{H^+}^2 = m_A^2 + M_W^2$. As is well known, the 1-loop radiative corrections, mostly due to the large value of the top Yukawa coupling, are important for the Higgs masses. Following an approximation that has been tested in the literature[10], we have computed the Higgs mass eigenvalues based on the 1-loop effective potential (12), where the dominant third generation contribution has been kept.

The radiative corrections to the pseudoscalar and charged Higgses are known to have an explicit dependence on the scale Q which cancels against the implicit Q dependence of the mixing mass $m_3^2(Q)$ and $\sin 2\beta(Q)$. Following other authors[10] we choose as appropriate scale for the evaluation of their masses a scale Q_0 for which the effect of the radiative corrections is vanishingly small. Then Q_0 turns out to be in the vicinity of the heavier of the stops and in this regime the stops have not been decoupled yet. It is therefore permissible to consider their loop effects to the effective potential as given in the references cited above. Keeping only the dominant third generation corrections to the Higgs masses the aforementioned cancellation among the Q dependent pieces is rather incomplete since the evolution of $m_3^2(Q)$ depends on the gaugino masses as well (see *eq.(36)*). Therefore there is a residual scale dependence, induced by the gauginos, whose effect on $m_3^2(Q)$ is not guaranteed to be small especially for large values of the soft mass $m_{1/2}$.

In our approach we have verified that the radiatively corrected Higgs masses for the heavy Higgses as these are calculated at the scale M_Z and the corresponding masses at Q_0 are very close to each other even for large values of the soft mass $m_{1/2}$. This is due to the appearance of the thresholds within the RGE of $m_3^2(Q)$ which properly takes care of the gaugino decoupling. This would not have been the case in schemes in which such a decoupling is not present. In those cases it is required that either the physical Higgs masses are evaluated as poles of the one loop propagators or the effect of the gaugino fields is duly taken into account in the effective potential approach, for the effect of the radiative corrections to be numerically insensitive to the choice of the scale. This subtle issue is under investigation and the results of this analysis will appear in a forthcoming publication[20].

6. Conclusions

We have displayed some of our results in tables III to VIII. In all tables we have taken $\mu(M_Z)$ positive. The negative $\mu(M_Z)$ values lead to qualitatively similar results. In the first four tables (III - VI) for a fixed value of $m_t(M_Z) = 175$ GeV, we present acceptable spectra that have been obtained for different values of the four parameter m_o, A_o, $m_{1/2}$ and $tan\beta(M_Z)$. In the table III, for a characteristic set of values $A_o = 400$ GeV, $m_o = 300$ GeV and $m_{1/2} = 200$ GeV we have varied $tan\beta$ between 2 and 25. For values of $tan\beta$ larger than about 60 no electroweak breaking occurs in this case. Note the well known[19] approximate equality between the masses of one of the neutralinos and one of the charginos. The lightest Higgs turns out to be heavier than the Z - boson and increases with increasing the value of $tan\beta$. Althought not displayed, for negative μ its mass drops below M_Z for small values of the angle $tan\beta \simeq 2$. In table IV, for fixed characteristic values of $tan\beta = 10$, $A_o = 400$ GeV and $m_o = 300$GeV,we vary $m_{1/2}$ between 75 GeV and 700 GeV. The sensitivity of the whole spectrum on $m_{1/2}$ is apparent. In table V, for fixed $tan\beta = 10, m_o = 300$ GeV and $m_{1/2} = 200$ GeV, we vary A_o between 0 and 800 GeV. In this case, as in all other acceptable cases, the LSP is a neutralino with a mass roughly independent of A_o. We have also included in this table negative values of A_o in the range -800 GeV to 0. Note that in both cases the gluino mass is roughly stable. In table VI we keep $tan\beta = 10$, $A_o = 400$GeV and $m_{1/2} = 200$GeV fixed and vary m_o between 100 and 800 GeV. In all cases shown the LSP is a neutralino. In table VII we have presented spectra obtained, for fixed m_o, $m_{1/2}$, A_o and $tan\beta$ values, when we vary the top quark mass. In all cases displayed in the tables III - VIII we have the approximate equality $m_{\chi_2^o} \simeq m_{\chi_2^c}$[19] due to the fact that $\mu(M_Z)$ turns out to be significantly heavier than M_Z.

As is apparent from the results displayed in tables IV to VI the value of the strong coupling constant has the tendency to decrease with increasing the supersymmetry breaking scale. In the table IV for instance keeping fixed the values of m_0, A_0, $\alpha_3(M_Z)$ takes values between $\simeq .131$ and $\simeq .127$ if $m_{1/2}$ varies from 75 to $700 GeV$. The situation is similar for the other two cases of tables V and VI where two of the soft SUSY breaking parameters are kept fixed and the third varies from low to large values. One notices however that the decrease in $\alpha_3(M_Z)$ in these cases is rather slower as compared to that of table IV. Thus $\alpha_3(M_Z)$ decreases faster in the direction of increasing the soft gaugino mass $m_{1/2}$.

Finally in table VIII for a characteristic choice of the parameter values we compare outputs for three dinstict cases. One loop predictions (case [a]), two loop predictions with thresholds only in couplings (case [b]), and the complete two loop case where thresholds appear in the RGE's of both couplings and dimensionful quantities (case [c]). Comparing cases [a] and [b] we see that as expected the value of the strong coupling is increased from its one loop value from $\simeq .118$ to .133. The bulk of this increase, as is well known, is due to the two loop effects. At the same time the value of the unification scale is increased too from $2.1881 \quad 10^{16}$ to $2.8876 \quad 10^{16} GeV$. Two loop effects delay the merging of the α_1 and α_2 gauge couplings resulting in larger values of α_3. As can be seen from the comparison of the two outputs, two loops and threshold effects in couplings have a small effect on the mass spectrum. Differences are small of the order of 2% or smaller in all sectors except the neutralino and chargino states $\tilde{\chi}^o_{1,2}$, $\tilde{\chi}^c_2$, whose masses are exactly

M_1, M_2 in the absence of electroweak symmetry breaking effects. In these states we have differences of the order of 10% or so originating mainly from the different evolutions of the soft gaugino masses $M_{1,2}$ and the $\alpha_{1,2}$ gauge couplings in the 1 - loop and 2 - loop cases.

Switching on the the mass and cubic coupling thresholds produces a minor effect, as can be seen by comparison of cases [b] and [c], except in the case of the heavy Higgses and those neutralino and chargino states that are nearly Higgsinos, i.e. of mass $\simeq \mu$, for large values of the parameter μ. These states are labelled as $\tilde{\chi}^o_{3,4}$, $\tilde{\chi}^c_1$ In those cases relatively large differences are observed. For the neutralinos and charginos the large differences are attributed to the value of the parameter $\mu(M_Z)$ which turns out to be larger, by about 10%, in case [c] as compared to that of case [b]. As far as the heavy Higgses are concerned from the discussion in the previous section it is evident that this discrepancy is due mainly to the evolution of m_3^2, whose value affects substantially the masses of the pseudoscalar and charged Higgses, and in particular on its dependence on the gaugino masses. In order to determine the Higgs masses we need, among other things, the value of m_3^2 at a scale Q_o, which minimizes the loop corrections due to the quarks and squarks of the third family, knowing its value at M_Z. However its evolution in the two approaches, with and without thresholds in masses and cubic couplings, is quite different. In the complete 2 - loop case, which takes into account all threshold effects, we have properly taken care of the decoupling of all particles as we move below their thresholds, gauginos included, unlike the case where the threshold effects are totally ignored. This results in rather large logarithmic corrections and hence to the relatively large differences ($\simeq 6\%$) occuring in this cases. For a proper treatment of the radiative corrections in the second scheme, where threshold effects are not present in the RGE's, explicit one loop calculations of the gaugino contributions to the two point Green's functions of the Higgses should be carried out. It is only in this case where comparison of the predictions for the heavy Higgses spectrum of the two approaches at hand can be made. At any rate the rather large differences seen in some particular cases point to the fact that a more refined analysis of the radiative effects to the Higgs sector is needed which also takes into account of the contributions of the gauginos and not just those of the heavy quarks. We have undertaken such a calculation and the results will appear in a future publication[20].

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Table Captions

Table I: Threshold coefficients appearing in the renormalization group equations of the gauge and Yukawa couplings. Above all thresholds these become equal to unity.

Table II: Threshold coefficients appearing in the renormalization group equations of the trilinear scalar couplings. Above all thresholds these are vanishing.

Table III: MSSM predictions for $m_t = 175\,GeV$, $A_o = 400\,GeV, m_o = 300$, $m_{1/2} = 200 GeV$ and for values of $\tan\beta$ ranging from 2 to 25. Only the $\mu > 0$ case is displayed. The $\overline{MS}$ values of α_{em}, $sin^2\theta_W$, α_3 and the unification scale are also shown. α_{GUT} is the $\overline{DR}$ value of the unification coupling. M_t is the physical top quark mass, $\tilde{g}$ denotes the gluino and $\tilde{\chi}_i^o$, $\tilde{\chi}_i^c$ are neutralino and chargino states. $(\tilde{t}_{1,2}, \tilde{b}_{1,2})$ and $(\tilde{\tau}_{1,2}, \tilde{\nu}_\tau)$ are squarks and sleptons of the third generation while $\tilde{u}_i$,$\tilde{u}_i^c$, $\tilde{d}_i$,$\tilde{d}_i^c$ and $\tilde{e}_i$,$\tilde{e}_i^c$, $\tilde{\nu}_i$ denote squarks and sleptons of the first two generations. h_o,H_o, A, $H^\pm$ are the CP even, CP odd and the charged Higgses respectively.

Table IV: Same as in table III for $m_t = 175\,GeV$, $A_o = 400\,GeV$, $m_o = 300, \tan\beta = 10$ and for values of $m_{1/2}$ ranging from $75 GeV$ to $700 GeV$ $(\mu > 0)$.

Table V: Same as in table III for $m_t = 175\,GeV$, $m_o = 300\,GeV$, $m_{1/2} = 200 GeV$ and for values of A_o 0, ± 300, ± 800 GeV $(\mu > 0)$.

Table VI: Same as in table III for $m_t = 175\,GeV$, $A_o = 400\,GeV, m_{1/2} = 200\,GeV, \tan\beta = 10$ and values of m_o from $100 GeV$ to $800 GeV$ $(\mu > 0)$.

Table VII: Mass spectrum of the MSSM for input values $m_o = 400\, GeV, m_{1/2} = 300 GeV, A_o = 200\, GeV, \tan\beta = 20$ and values for the running top quark mass equal to $175, 180, 185, 190$ and $195\, GeV$ ($\mu > 0$) respectively.Couplings and masses shown are as in table III.

Table VIII: MSSM mass spectrum for the inputs shown in the first row ($\mu > 0$). We compare 1 - loop (case [a]), 2 - loop with thresholds in couplings (case [b]) and complete 2 - loop predictions (case [c]) with thresholds in both couplings and dimensionful parameters.

TABLE I

$$T_1 = \frac{1}{33}[20 + \theta_{\tilde{H}_1} + \theta_{\tilde{H}_2} + \frac{1}{2}(\theta_{H_1} + \theta_{H_2}) + \sum_{i=1}^{3}(\frac{1}{2}\theta_{\tilde{L}_i} + \theta_{\tilde{E}_i} + \frac{1}{6}\theta_{\tilde{Q}_i} + \frac{4}{3}\theta_{\tilde{U}_i} + \frac{1}{3}\theta_{\tilde{D}_i})]$$

$$T_2 = -\frac{10}{3} + \frac{4}{3}\theta_{\tilde{W}} + \frac{1}{3}(\theta_{\tilde{H}_1} + \theta_{\tilde{H}_2}) + \frac{1}{6}(\theta_{H_1} + \theta_{H_2}) + \frac{1}{6}\sum_{i=1}^{3}(3\theta_{\tilde{Q}_i} + \theta_{\tilde{L}_i})$$

$$T_3 = \frac{7}{3} - \frac{2}{3}\theta_{\tilde{G}} - \frac{1}{18}\sum_{i=1}^{3}(2\theta_{\tilde{Q}_i} + \theta_{\tilde{D}_i} + \theta_{\tilde{U}_i})$$

$$T_{\tau 2} = \frac{1}{4}[-1 + 4\theta_{H_1} - 2\theta_{\tilde{H}_1\tilde{W}} - \theta_{\tilde{L}\tilde{W}} + 4\theta_{\tilde{H}_1\tilde{L}\tilde{W}}]$$

$$T_{\tau 1} = \frac{1}{12}[11 - 4\theta_{\tilde{B}\tilde{E}} + 8\theta_{\tilde{B}\tilde{E}\tilde{H}_1} - 2\theta_{\tilde{B}\tilde{H}_1} + 4\theta_{H_1} - \theta_{\tilde{B}\tilde{L}} - 4\theta_{\tilde{B}\tilde{L}\tilde{H}_1}]$$

$$T_{\tau\tau} = \frac{1}{8}[2 + \theta_{\tilde{H}_1\tilde{E}} + 3\theta_{H_1} + 2\theta_{\tilde{L}\tilde{H}_1}]$$

$$T_{b3} = \frac{1}{4}[6 - \theta_{\tilde{G}\tilde{D}} - \theta_{\tilde{G}\tilde{Q}}]$$

$$T_{b2} = \frac{1}{4}[-1 + 4\theta_{H_1} - 2\theta_{\tilde{H}_1\tilde{W}} - \theta_{\tilde{Q}\tilde{W}} + 4\theta_{\tilde{H}_1\tilde{Q}\tilde{W}}]$$

$$T_{b1} = \frac{1}{28}[-21 - 4\theta_{\tilde{B}\tilde{D}} - 18\theta_{\tilde{B}\tilde{H}_1} + 24\theta_{\tilde{H}_1\tilde{D}\tilde{B}} + 36\theta_{H_1} - \theta_{\tilde{B}\tilde{Q}} + 12\theta_{\tilde{H}_1\tilde{Q}\tilde{B}}]$$

$$T_{bt} = \frac{1}{2}[\theta_{H_2} + \theta_{\tilde{H}_2\tilde{U}}]$$

$$T_{bb} = \frac{1}{12}[6 + \theta_{\tilde{D}\tilde{H}_1} + 3\theta_{H_1} + 2\theta_{\tilde{Q}\tilde{H}_1}]$$

$$T_{t3} = \frac{1}{4}[6 - \theta_{\tilde{G}\tilde{Q}} - \theta_{\tilde{G}\tilde{U}}]$$

$$T_{t2} = \frac{1}{4}[-1 + 4\theta_{H_2} - 2\theta_{\tilde{H}_2\tilde{W}} - \theta_{\tilde{Q}\tilde{W}} + 4\theta_{\tilde{H}_2\tilde{Q}\tilde{W}}]$$

$$T_{t1} = \frac{1}{52}[15 - 18\theta_{\tilde{H}_2\tilde{B}} + 36\theta_{H_2} - \theta_{\tilde{B}\tilde{Q}} - 12\theta_{\tilde{B}\tilde{Q}\tilde{H}_2} - 16\theta_{\tilde{B}\tilde{U}} + 48\theta_{\tilde{B}\tilde{U}\tilde{H}_2}]$$

$$T_{tt} = \frac{1}{12}[6 + 3\theta_{H_2} + 2\theta_{\tilde{H}_2\tilde{Q}} + \theta_{\tilde{H}_2\tilde{U}}]$$

$$T_{tb} = \frac{1}{2}[\theta_{H_1} + \theta_{\tilde{H}_1\tilde{D}}]$$

TABLE II

$$Z_{\tau 1} = \frac{3}{40}[11 + 10\theta_{\tilde{B}} - 8\theta_{\tilde{E}} - 4\theta_{\tilde{B}\tilde{E}} + 8\theta_{\tilde{B}\tilde{E}\tilde{H}_1} +$$

$$2\theta_{H_1} - 8\theta_{H_1\tilde{E}} - 2\theta_{\tilde{L}} - \theta_{\tilde{B}\tilde{L}} - 8\theta_{\tilde{E}\tilde{L}} - 4\theta_{\tilde{B}\tilde{H}_1\tilde{L}} + 4\theta_{H_1\tilde{L}}]$$

$$Z_{\tau 2} = \frac{1}{8}[-3 + 6\theta_{H_1} - 6\theta_{\tilde{L}} - 12\theta_{H_1\tilde{L}} + 6\theta_{\tilde{W}} - 3\theta_{\tilde{L}\tilde{W}} + 12\theta_{\tilde{W}\tilde{H}_1\tilde{L}}]$$

$$Z_{\tau\tau} = \frac{1}{4}[-16 + +6\theta_{\tilde{H}_1} - \theta_{\tilde{H}_1\tilde{E}} - 3\theta_{H_1} + 4\theta_{H_1\tilde{E}} + 4\theta_{\tilde{E}\tilde{L}} - 2\theta_{\tilde{H}_1\tilde{L}} + 8\theta_{H_1\tilde{L}}]$$

$$Z_{b3} = \frac{2}{3}[6 - 2\theta_{\tilde{D}} + 4\theta_{\tilde{G}} - \theta_{\tilde{D}\tilde{G}} - 2\theta_{\tilde{Q}} - 4\theta_{\tilde{D}\tilde{Q}} - \theta_{\tilde{G}\tilde{Q}}$$

$$Z_{b2} = \frac{1}{8}[-3 + 6\theta_{H_1} - 6\theta_{\tilde{Q}} - 12\theta_{H_1\tilde{Q}} + 6\theta_{\tilde{W}} - 3\theta_{\tilde{Q}\tilde{W}} + 12\theta_{\tilde{W}\tilde{H}_1\tilde{Q}}]$$

$$Z_{b1} = \frac{1}{120}[-21 + 10\theta_{\tilde{B}} - 8\theta_{\tilde{D}} - 4\theta_{\tilde{B}\tilde{D}} + 24\theta_{\tilde{B}\tilde{D}\tilde{H}_1} + 18\theta_{H_1} - 24\theta_{H_1\tilde{D}}$$

$$-2\theta_{\tilde{Q}} - \theta_{\tilde{Q}\tilde{B}} + 8\theta_{\tilde{Q}\tilde{D}} + 12\theta_{\tilde{B}\tilde{Q}\tilde{H}_1} - 12\theta_{H_1\tilde{Q}}]$$

$$Z_{bb} = \frac{1}{4}[-24 + 6\theta_{\tilde{H}_1} - \theta_{\tilde{H}_1\tilde{D}} - 3\theta_{H_1} + 4\theta_{H_1\tilde{D}} + 12\theta_{\tilde{Q}\tilde{D}} - 2\theta_{\tilde{Q}\tilde{H}_1} + 8\theta_{H_1\tilde{Q}}]$$

$$Z_{bt} = \frac{1}{4}[2\theta_{\tilde{H}_2} - \theta_{H_2} - \theta_{\tilde{U}\tilde{H}_2}]$$

$$Z_{t3} = \frac{2}{3}[6 - 2\theta_{\tilde{Q}} + 4\theta_{\tilde{G}} - \theta_{\tilde{Q}\tilde{G}} - 2\theta_{\tilde{U}} - 4\theta_{\tilde{U}\tilde{Q}} - \theta_{\tilde{G}\tilde{U}}]$$

$$Z_{t2} = \frac{1}{8}[-3 + 6\theta_{H_2} - 6\theta_{\tilde{Q}} - 12\theta_{H_2\tilde{Q}} + 6\theta_{\tilde{W}} - 3\theta_{\tilde{Q}\tilde{W}} + 12\theta_{\tilde{W}\tilde{H}_2\tilde{Q}}]$$

$$Z_{t1} = \frac{1}{120}[15 + 34\theta_{\tilde{B}} + 18\theta_{H_2} - 2\theta_{\tilde{Q}} - \theta_{\tilde{B}\tilde{Q}} - 12\theta_{\tilde{B}\tilde{Q}\tilde{H}_2} + 12\theta_{H_2\tilde{Q}}$$

$$-32\theta_{\tilde{U}} - 16\theta_{\tilde{B}\tilde{U}} + 48\theta_{\tilde{B}\tilde{U}\tilde{H}_2} - 48\theta_{H_2\tilde{U}} - 16\theta_{\tilde{U}\tilde{Q}}]$$

$$Z_{tb} = \frac{1}{4}[2\theta_{\tilde{H}_1} - \theta_{H_1} - \theta_{\tilde{D}\tilde{H}_1}]$$

$$Z_{tt} = \frac{1}{4}[-24\theta_{H_1\tilde{U}} + 6\theta_{\tilde{H}_2} - \theta_{\tilde{H}_2\tilde{U}} - 3\theta_{H_2} + 4\theta_{H_2\tilde{U}} + 12\theta_{\tilde{Q}\tilde{U}} - 2\theta_{\tilde{Q}\tilde{H}_2} + 8\theta_{H_2\tilde{Q}}]$$

TABLE III

$m_t = 175$, $A_o = 400$, $m_o = 300$, $m_{1/2} = 200$, $\mu(M_Z) > 0$					
$tan\beta$	25	20	15	10	2
M_{GUT} $(10^{16} GeV)$	2.632	2.633	2.633	2.632	2.515
α_{GUT}	.04161	.04161	.04161	.04161	.04134
α_{em}^{-1}	127.9	127.9	127.9	127.9	127.9
$sin^2\theta_W$	.2311	.2311	.2311	.2311	.2311
α_3	.13128	.13129	.13129	.13126	.12909
M_t	177.0	177.0	177.0	177.0	176.8
$\tilde{g}$	495.6	495.8	495.9	495.9	492.4
$\tilde{\chi}_1^o$	77.1	77.0	76.8	76.4	74.7
$\tilde{\chi}_2^o$	139.1	138.9	138.5	137.6	136.0
$\tilde{\chi}_3^o$	350.5	352.3	354.7	359.1	487.8
$\tilde{\chi}_4^o$	-337.2	-338.6	-340.3	-343.2	-467.7
$\tilde{\chi}_1^c$	352.3	354.0	356.0	359.9	484.5
$\tilde{\chi}_2^c$	138.8	138.6	138.1	137.0	134.7
$\tilde{t}_1$,$\tilde{t}_2$	527.3,315.4	531.9,315.2	535.9,314.4	539.4,312.3	543.9,285.3
$\tilde{b}_1$,$\tilde{b}_2$	506.2,441.0	514.2,451.4	520.9,459.4	525.8,465.0	525.9,462.4
$\tilde{\tau}_1$,$\tilde{\tau}_2$	328.0,266.5	330.4,281.0	331.7,292.9	331.9,302.1	329.4,309.2
$\tilde{\nu}_\tau$	306.4	311.5	315.6	318.6	323.4
$\tilde{u}_{1,2}$,$\tilde{u}_{1,2}^c$	537.8,528.9	537.8,528.9	537.8,528.9	537.7,528.8	535.4,525.8
$\tilde{d}_{1,2}$,$\tilde{d}_{1,2}^c$	543.3,529.6	543.3,529.6	543.3,529.6	543.2,529.5	538.6,525.8
$\tilde{e}_{1,2}$,$\tilde{e}_{1,2}^c$	330.1,311.7	330.1,311.7	330.0,311.7	330.0,311.6	328.8,310.3
$\tilde{\nu}_{1,2}$	321.0	321.0	320.9	321.0	323.5
A	630.7	612.0	588.1	560.1	669.0
h_o,H_o	114.1,630.6	114.2,611.9	114.2,588.0	113.8,560.2	92.1,672.8
$H^\pm$	635.4	616.9	593.1	565.4	673.5

TABLE IV					
	$m_t = 175$, $A_o = 400$, $m_o = 300$, $tan\beta = 10$, $\mu(M_Z) > 0$				
$m_{1/2}$	700	500	300	100	75
M_{GUT} $(10^{16} GeV)$	1.705	1.915	2.290	3.145	3.182
α_{GUT}	.04030	.04066	.04120	.04215	.04222
α_{em}^{-1}	127.9	127.9	127.9	127.9	127.9
$sin^2\theta_W$	.2311	.2311	.2311	.2311	.2311
α_3	.12669	.12795	.12983	.13228	.13110
M_t	177.0	177.0	177.0	177.0	177.0
$\tilde{g}$	1552.7	1140.1	714.6	267.8	207.1
$\tilde{\chi}_1^o$	291.5	204.3	118.6	34.7	24.4
$\tilde{\chi}_2^o$	527.9	371.3	215.1	62.2	44.5
$\tilde{\chi}_3^o$	752.0	612.0	450.5	259.5	234.7
$\tilde{\chi}_4^o$	-733.9	-595.8	-434.8	-243.8	-219.5
$\tilde{\chi}_1^c$	751.4	611.6	450.6	261.9	237.9
$\tilde{\chi}_2^c$	527.7	371.1	214.7	60.7	42.5
$\tilde{t}_1$,$\tilde{t}_2$	1347.6,1077.8	1022.5,780.2	698.2,470.9	388.3,167.5	353.0,144.7
$\tilde{b}_1$,$\tilde{b}_2$	1380.8,1311.8	1039.9,975.9	695.6,634.7	370.2,305.3	337.8,270.6
$\tilde{\tau}_1$,$\tilde{\tau}_2$	548.5,390.1	446.6,346.7	362.5,313.7	311.9,294.8	308.8,293.8
$\tilde{\nu}_\tau$	542.2	438.4	351.2	296.9	293.6
$\tilde{u}_{1,2}$,$\tilde{u}_{1,2}^c$	1444.7,1394.9	1083.2,1049.7	718.1,701.0	372.1,371.1	337.6,338.4
$\tilde{d}_{1,2}$,$\tilde{d}_{1,2}^c$	1446.6,1389.7	1085.9,1046.7	722.1,700.3	380.0,373.4	346.3,341.1
$\tilde{e}_{1,2}$,$\tilde{e}_{1,2}^c$	550.0,399.3	447.3,355.5	361.8,322.8	309.1,304.7	305.9,303.7
$\tilde{\nu}_{1,2}$	544.9	440.9	353.7	299.3	295.9
A	1341.8	1036.5	718.7	414.6	383.9
h_o,H_o	119.4,1341.9	118.1,1036.6	115.9,718.8	108.7,414.7	105.9,384.0
$H^\pm$	1344.0	1039.3	722.8	421.9	391.8

TABLE V					
$m_t = 175$, $tan\beta = 10$, $m_o = 300$, $m_{1/2} = 200$, $\mu(M_Z) > 0$					
$Ao =$	800	-800	300	-300	0
M_{GUT} ($10^{16} GeV$)	2.574	2.657	2.645	2.690	2.676
α_{GUT}	.04155	.04165	.04163	.04169	.04167
α_{em}^{-1}	127.9	127.9	127.9	127.9	127.9
$sin^2\theta_W$	.2311	.2311	.2311	.2311	.2311
α_3	.13031	.13183	.13148	.13235	.13206
M_t	177.0	177.0	177.0	177.0	177.0
$\tilde{g}$	493.0	501.1	496.7	500.3	498.6
$\tilde{\chi}_1^o$	77.3	75.9	76.0	73.7	74.6
$\tilde{\chi}_2^o$	141.8	134.0	136.1	127.1	130.6
$\tilde{\chi}_3^o$	457.4	305.2	338.5	271.3	291.2
$\tilde{\chi}_4^o$	-447.5	-282.1	-320.5	-240.5	-265.7
$\tilde{\chi}_1^c$	458.4	305.6	339.2	271.4	291.5
$\tilde{\chi}_2^c$	141.6	132.7	135.3	124.6	128.9
$\tilde{t}_1$,$\tilde{t}_2$	522.6,182.5	514.9,413.8	541.6,332.8	538.0,405.5	543.4,378.5
$\tilde{b}_1$,$\tilde{b}_2$	520.8,433.0	529.8,484.2	526.8,470.9	530.5,490.2	529.2,484.1
$\tilde{\tau}_1$,$\tilde{\tau}_2$	331.9,294.0	328.0,302.5	331.7,303.5	330.1,306.8	330.9,306.3
$\tilde{\nu}_\tau$	316.2	317.7	319.0	319.5	319.6
$\tilde{u}_{1,2}$,$\tilde{u}_{1,2}^c$	536.1,527.2	540.9,532.0	538.2,529.2	540.4,531.6	539.4,530.5
$\tilde{d}_{1,2}$,$\tilde{d}_{1,2}^c$	541.6,527.8	546.3,532.6	543.6,529.9	545.8,532.2	544.8,531.2
$\tilde{e}_{1,2}$,$\tilde{e}_{1,2}^c$	330.1,311.5	330.4,311.6	330.0,311.7	330.1,311.7	330.0,311.7
$\tilde{\nu}_{1,2}$	321.1	321.4	321.0	321.1	321.0
A	661.3	449.1	537.2	447.5	480.5
h_o,H_o	119.6,660.8	105.7,449.5	112.7,537.4	108.2,447.8	110.1,480.8
$H^\pm$	665.8	455.7	542.7	454.1	486.6

TABLE VI

$m_t = 175$, $tan\beta = 10$, $Ao = 400$, $m_{1/2} = 200$, $\mu(M_Z) > 0$

m_o	800	600	400	200	100
M_{GUT} ($10^{16} GeV$)	2.687	2.681	2.660	2.581	2.482
α_{GUT}	.04125	.04138	.04153	.04169	.04176
α_{em}^{-1}	127.9	127.9	127.9	127.9	127.9
$sin^2\theta_W$	.2311	.2311	.2311	.2311	.2311
α_3	.13076	.13087	.13110	.13142	.13146
M_t	177.0	177.0	177.0	177.0	177.0
$\tilde{g}$	510.9	504.7	498.4	494.2	494.3
$\tilde{\chi}_1^o$	77.4	77.1	76.6	76.2	76.0
$\tilde{\chi}_2^o$	137.2	137.7	137.7	137.6	137.5
$\tilde{\chi}_3^o$	340.1	351.6	357.3	360.6	361.5
$\tilde{\chi}_4^o$	-322.1	-334.9	-341.2	-344.9	-345.9
$\tilde{\chi}_1^c$	340.6	352.2	358.0	361.4	362.3
$\tilde{\chi}_2^c$	136.4	137.0	137.0	137.0	137.0
$\tilde{t}_1$,$\tilde{t}_2$	762.2,522.9	652.9,432.2	569.7,349.3	517.5,281.9	504.0,260.9
$\tilde{b}_1$,$\tilde{b}_2$	879.9,734.4	719.6,611.1	581.4,507.1	482.8,431.9	454.6,410.1
$\tilde{\tau}_1$,$\tilde{\tau}_2$	805.2,793.5	611.8,596.2	422.6,399.7	247.5,205.7	178.6,113.9
$\tilde{\nu}_\tau$	800.1	604.9	412.3	229.2	152.3
$\tilde{u}_{1,2}$,$\tilde{u}_{1,2}^c$	890.2,887.3	730.7,725.6	593.2,585.5	494.8,484.6	466.5,455.2
$\tilde{d}_{1,2}$,$\tilde{d}_{1,2}^c$	893.5,888.2	734.6,726.3	598.1,586.2	500.7,485.2	472.8,455.8
$\tilde{e}_{1,2}$,$\tilde{e}_{1,2}^c$	807.6,803.0	612.8,604.9	421.9,408.4	244.0,217.6	173.0,132.2
$\tilde{\nu}_{1,2}$	804.1	608.1	414.9	231.6	155.0
A	910.8	756.5	618.1	514.5	485.0
h_o,H_o	112.3,911.0	112.4,756.7	113.2,618.2	114.4,514.6	114.8,485.0
$H^\pm$	914.1	760.4	622.9	520.3	491.1

TABLE VII					
$tan\beta = 20$, $Ao = 200$, $m_o = 400$, $m_{1/2} = 300$, $\mu(M_Z) > 0$					
m_t	175	180	185	190	195
M_{GUT} ($10^{16}GeV$)	2.337	2.414	2.495	2.581	2.670
α_{GUT}	.04117	.04118	.04119	.04120	.04120
α_{em}^{-1}	127.9	127.9	127.9	127.9	127.9
$sin^2\theta_W$	.23110	.23094	.23078	.23061	.23044
α_3	.13005	.13058	.13114	.13171	.13226
M_t	177.0	181.7	186.4	191.1	195.8
$\tilde{g}$	717.3	719.0	720.7	722.5	724.4
$\tilde{\chi}_1^o$	118.9	119.0	119.1	119.2	119.3
$\tilde{\chi}_2^o$	213.2	215.0	216.4	217.5	218.5
$\tilde{\chi}_3^o$	402.0	422.0	441.3	459.9	477.9
$\tilde{\chi}_4^o$	-384.0	-406.2	-427.2	-447.2	-466.3
$\tilde{\chi}_1^c$	402.6	422.7	442.1	460.8	478.8
$\tilde{\chi}_2^c$	212.8	214.7	216.1	217.3	218.3
$\tilde{t}_1$,$\tilde{t}_2$	720.3,526.9	717.7,522.9	715.2,519.5	712.9,516.8	710.7,515.1
$\tilde{b}_1$,$\tilde{b}_2$	727.0,665.1	728.9,663.1	730.8,661.3	732.7,659.8	734.6,658.7
$\tilde{\tau}_1$,$\tilde{\tau}_2$	444.4,392.8	444.9,392.2	445.5,391.6	446.0,391.1	446.5,390.5
$\tilde{\nu}_\tau$	432.2	432.3	432.4	432.5	432.5
$\tilde{u}_{1,2}$,$\tilde{u}_{1,2}^c$	761.3,745.7	762.7,747.2	764.3,748.7	765.9,750.3	767.5,752.0
$\tilde{d}_{1,2}$,$\tilde{d}_{1,2}^c$	765.1,745.2	766.6,746.6	768.1,748.2	769.7,749.8	771.3,751.5
$\tilde{e}_{1,2}$,$\tilde{e}_{1,2}^c$	447.3,417.0	447.3,417.0	447.4,417.0	447.4,417.1	447.5,417.1
$\tilde{\nu}_{1,2}$	440.7	440.8	440.8	440.9	441.0
A	795.6	820.6	843.0	862.8	879.8
h_o,H_o	115.0,795.6	116.9,820.6	118.8,843.0	120.7,862.7	122.6,879.7
$H^{\pm}$	799.3	824.1	846.4	866.1	883.1

TABLE VIII			
$m_t = 175$, $tan\beta = 10$, $A_o = 250$, $m_o = 200$, $m_{1/2} = 150$, $\mu(M_Z) > 0$			
	Case [a] 1-loop	Case [b] 2-loop	Case [c] Complete 2-loop
M_{GUT} ($10^{16} GeV$)	2.1881	2.8876	2.8766
α_{GUT}	.04127	.04201	.04202
α_{em}^{-1}	127.9	127.9	127.9
$sin^2\theta_W$	.23105	.23110	.23110
α_3	.11767	.13284	.13289
M_t	181.0	177.0	177.0
$\tilde{g}$	398.4	381.4	382.6
$\tilde{\chi}_1^o$	59.2	55.0	54.4
$\tilde{\chi}_2^o$	109.0	98.3	96.7
$\tilde{\chi}_3^o$	302.8	304.1	279.6
$\tilde{\chi}_4^o$	-284.1	-287.7	-260.1
$\tilde{\chi}_1^c$	304.0	305.5	280.9
$\tilde{\chi}_2^c$	108.0	97.4	95.3
$\tilde{t}_1$,$\tilde{t}_2$	443.6,247.0	442.0,235.2	440.2,234.7
$\tilde{b}_1$,$\tilde{b}_2$	401.7,357.2	395.4,352.8	394.9,352.6
$\tilde{\tau}_1$,$\tilde{\tau}_2$	235.3,203.6	232.2,201.6	231.3,202.7
$\tilde{\nu}_\tau$	216.6	212.6	212.5
$\tilde{u}_{1,2}$,$\tilde{u}_{1,2}^c$	410.5,401.4	400.1,394.1	400.1,394.1
$\tilde{d}_{1,2}$,$\tilde{d}_{1,2}^c$	417.8,402.4	407.5,395.7	407.5,395.7
$\tilde{e}_{1,2}$,$\tilde{e}_{1,2}^c$	231.6,212.6	227.7,211.6	227.5,211.8
$\tilde{\nu}_{1,2}$	218.1	214.2	214.1
A	412.1	421.0	394.5
h_o,H_o	113.0,412.2	110.7,421.1	110.5,394.7
$H^\pm$	419.5	428.1	402.1