

**RADIATIVE GUT SYMMETRY BREAKING IN A  $R$ -SYMMETRIC  
FLIPPED  $SU(5)$  MODEL**

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**Abstract**

We study the generation of the GUT scale through radiative corrections in the context of a  $R$ -symmetric “flipped”  $SU(5) \times U(1)_X$  model. A negative mass squared term for the GUT Higgs fields develops due to radiative effects along a flat direction at a superheavy energy scale. The  $R$ -symmetry is essential in maintaining triplet-doublet splitting and  $F$ -flatness in the presence of non-renormalizable terms. The model displays radiative electroweak symmetry breaking and satisfies all relevant phenomenological constraints.

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## INTRODUCTION

The Standard Model (SM) and its  $N = 1$  supersymmetric extension [1], the Minimal Supersymmetric Standard Model (MSSM), can be naturally embedded in a Grand Unified Theory [2] (GUT) with interesting phenomenological and cosmological consequences. GUTs can successfully predict the electroweak mixing angle  $\sin^2 \theta_W$ , fermion mass relations as well as provide a mechanism for the creation of the baryon asymmetry of the Universe [3]. However, in the framework of quantum field theory no severe restrictions exist on the gauge group or the field content of a GUT apart from the requirement that it should incorporate the Standard Model. Many possibilities are allowed including minimal  $SU(5)$  and its extensions [4], varieties of  $SO(10)$  [5] and  $E(6)$  [6] models e.t.c. A GUT can be in principle accommodated in the framework of superstring theory [7]. The assumption that the GUT is the low energy field theory limit of a four dimensional heterotic superstring compactification imposes serious restrictions on the spectrum. For gauge groups realized at level  $k = 1$  of the World Sheet Affine Algebra, only the chiral multiplets in the vector and antisymmetric tensor representations of  $SU(n)$  groups and the vector and spinor representations of  $SO(n)$  groups are massless. The absence of adjoint scalars severely restricts the possibilities of breaking to the MSSM through the Higgs mechanism and diminishes the number of candidate GUT models. Apart from these restrictions superstrings offer a new possibility. The GUT gauge group does not have to be simple in order to guarantee unification \* . Semi-simple or product groups are equally acceptable since string theory takes over the task of gauge coupling unification. Among the few GUT examples embeddable in superstrings is the so-called “flipped”  $SU(5) \times U(1)$  [8] model which has been explicitly constructed and studied in the framework of four dimensional free fermionic superstrings [9] [10] [11]. When a GUT is considered in the context of string theory the GUT scale [12] is distinct from the string scale. Despite the fact that this is not necessarily a problem for product gauge groups it has been termed as

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\*The term GUT now refers to a gauge group that only partially contains the SM gauge group

the *mismatch problem*.

Can the GUT scale be generated by radiative corrections in an analogous fashion to the generation of the electroweak breaking scale through dimensional transmutation in the MSSM ? The point of view realized in the present article is that the symmetry breaking scales associated with the effective field theories of a GUT or the Standard Model are generated through dimensional transmutation [13] [14] while the Planck scale  $M_P$  and the supersymmetry (SUSY) breaking scale  $m_s$  are “fundamental” and, presumably, accounted for by strings or non-perturbative physics. Although the idea of generating the GUT scale through radiative corrections is not new, its existing realizations [15] are not satisfactory for various reasons. These are, non-embeddability in strings, “baroque” field content or lack of symmetries that could guarantee flatness in the presence of non-renormalizable corrections. In a recent paper H. Goldberg [16] considers a gauge singlet superfield  $S$  coupled to a pair of adjoint fields in a supersymmetric  $SU(5)$  model. In this model, the soft breaking mass term of the gauge singlet  $S$  becomes negative and develops a vacuum expectation value (v.e.v)  $\langle S \rangle$  which ultimately defines the GUT scale.

In the present article we are going to study the generation of the GUT scale through radiative corrections in a prototype  $k = 1$  string embeddable GUT. These corrections, controlled by the supersymmetry breaking scale  $m_s$ , can give rise to a logarithmically distant from  $m_s$  scale  $M_X$  lying close to the scale at which the soft SUSY-breaking squared masses of the GUT Higgs scalars become negative. Such a mechanism requires, of course, the existence of a D- and F-flat direction for the relevant fields in the supersymmetric limit. As a prototype GUT we shall employ a version of the “flipped”  $SU(5) \times U(1)_X$  model that satisfies the  $k = 1$  string embeddability criteria, possesses a discrete  $R$ -symmetry that guarantees triplet-doublet splitting and flatness along the direction responsible for the generation of  $M_X$ , displays radiative electroweak symmetry breaking and satisfies all phenomenological low energy constraints.

## 1. A $R$ -SYMMETRIC VERSION OF $SU(5) \times U(1)_X$ .

In the present article we shall work with a simple version of the flipped  $SU(5)$  model possessing an almost minimal field content which, however, is sufficient to illustrate the mechanism under investigation for generating the GUT scale. The minimal chiral superfield content of the flipped  $SU(5) \times U(1)_X$  model consists of the *matter superfields*, in three family replicas

$$F_i(\mathbf{10}, 1/2) , \quad f_i^c(\bar{\mathbf{5}}, -3/2) , \quad l_i^c(\mathbf{1}, 5/2) , \quad (1)$$

the *GUT Higgses*

$$H(\mathbf{10}, 1/2) , \quad \bar{H}(\bar{\mathbf{10}}, -1/2) , \quad (2)$$

and the *electroweak Higgses*

$$h(\mathbf{5}, -1) , \quad \bar{h}(\bar{\mathbf{5}}, 1) . \quad (3)$$

We shall also introduce four additional gauge singlet superfields, three  $N_i$ 's and one  $\phi$  , and an additional pair of tenplets

$$H'(\mathbf{10}, 1/2) , \quad \bar{H}'(\bar{\mathbf{10}}, -1/2) . \quad (4)$$

All these fields are massless at the Planck scale. The masslessness of most of them will be protected by additional symmetries that will be shortly imposed. Nevertheless, experience from the string model [10] [11] itself has shown that tree-level  $O(M_P)$  mass terms allowed by symmetries are not always present. Adopting this point of view we shall assume that  $\phi$  and the additional pair of tenplets  $H'$  ,  $\bar{H}'$  have an intermediate scale mass despite the fact that  $O(M_P)$  masses for them are allowed by the symmetries.

The  $SU(5) \times U(1)_X$  gauge symmetry breaking at a superheavy scale requires the existence of a F-flat direction for  $H$  ,  $\bar{H}$  . In order to achieve F-flatness one could impose

discrete symmetries. However, with conventional discrete symmetries one can hardly protect F-flatness from non-renormalizable terms. For example, the frequently imposed discrete symmetry  $H \rightarrow -H$  does not forbid the dangerous superpotential term  $(H\overline{H})^2$  which lifts the F-flatness and forbids a v.e.v.  $\sim 10^{16} GeV$ . Moreover, such a symmetry may generate a serious domain wall problem in the early Universe. This problem becomes more severe if the phase transition associated with the superheavy scale takes place at temperatures  $\sim m_s$  as is expected to be the case in the context of superstring embeddable models. In contrast,  $R$ -symmetries are capable of forbidding dangerous non-renormalizable terms to all orders. For example, if we impose a  $R$ -symmetry under which  $H$  and  $\overline{H}$  transform trivially, all terms of the type  $(H\overline{H})^n$  are forbidden. Such a discrete symmetry is not broken by a large v.e.v. of  $H$ ,  $\overline{H}$  and the domain wall problem has a good chance to be avoided. If the discrete symmetry is eventually spontaneously broken, the domain wall problem could still be avoided provided the discrete  $R$ -symmetry carries colour anomalies i.e. does not leave invariant the effective instanton vertex for QCD. In this case the degeneracy between vacua separated by domain walls is lifted by QCD instanton effects at temperatures of the order of the QCD scale (100 MeV). The resulting pressure on the walls causes their collapse soon after the QCD phase transition [17]. This mechanism assumes that the domain wall system does not dominate the energy density of the Universe before its collapse. If the anomalous discrete  $R$ -symmetry breaks at the electroweak scale, this condition is readily satisfied [18]. In what follows we shall construct a version of the flipped  $SU(5)$  model possessing such an anomalous discrete  $R$ -symmetry which is broken at the electroweak scale.

Consider a  $\mathcal{Z}_3$   $R$ -symmetry under which the fields transform as

$$\{F, f^c, l^c, H, \overline{H}\} \rightarrow \{F, f^c, l^c, H, \overline{H}\} \quad (5)$$

$$\{h, \overline{h}, N\} \rightarrow e^{\frac{2\pi i}{3}} \{h, \overline{h}, N\} \quad (6)$$

$$\{\phi, H', \overline{H}'\} \rightarrow e^{-\frac{2\pi i}{3}} \{\phi, H', \overline{H}'\} \quad (7)$$

and the *superpotential* as

$$\mathcal{W} \rightarrow e^{\frac{2\pi i}{3}} \mathcal{W}. \quad (8)$$

The above transformations actually refer to the bosonic components of the corresponding superfields. The fermionic components transform with an additional factor of  $e^{\frac{2\pi i}{3}}$  relative to the bosonic ones. Consequently the effective instanton QCD vertex is multiplied by a factor  $e^{\frac{2\pi i}{3}}$  under a discrete  $R$ -symmetry transformation which means that the discrete  $R$ -symmetry carries QCD anomalies.

In addition to the  $\mathcal{Z}_3$   $R$ -symmetry we also impose a  $\mathcal{Z}_2$  *matter parity* under which the only fields transforming non-trivially are the matter fields and the three singlets  $N$

$$\{F, f^c, l^c, N, H', \overline{H}'\} \rightarrow -\{F, f^c, l^c, N, H', \overline{H}'\}. \quad (9)$$

The matter parity singles out  $\phi$  among the four singlets as the only one allowed to acquire an electroweak scale v.e.v., and generate the  $\mu$ -term. It also forbids  $\phi$  to participate in the see-saw mechanism. Moreover, the  $\mathcal{Z}_2$  matter parity generates a cold dark matter candidate, the lightest supersymmetric particle, which is necessary given that neutrinos are superlight ( $m_\nu \sim M_W^3/M_X^2$ ).

The renormalizable part of the superpotential respecting the symmetry  $SU(5) \times U(1)_X \times \mathcal{Z}_2 \times \mathcal{Z}_3$  is

$$\mathcal{W} \sim FFh + Ff^c\overline{h} + f^cl^ch + HHh + \overline{H}\overline{H}\overline{h} + F\overline{H}N + \phi h\overline{h} + N^2\phi + F\overline{H}'\phi + H'\overline{H}' + \phi^2. \quad (10)$$

Assuming that the extra pair of tenplets  $H', \overline{H}'$  as well as the singlet  $\phi$  remain massless at the Planck scale, they obtain intermediate scale masses  $\sim M_X^2/M_P$  through the non-renormalizable terms  $(H\overline{H})H'\overline{H}', H\overline{H}\phi^2$ . Note that, in general, supergravity corrections generate a SUSY-breaking,  $R$ -symmetry-breaking tadpole [19]

$$m_s^2 M_P (\phi + \phi^*). \quad (11)$$

As a result, a v.e.v for  $\phi$  is induced

$$\langle \phi \rangle \sim M_P (m_s/m_\phi)^2, \quad (12)$$

where  $m_\phi$  is the mass of  $\phi$ . Demanding that  $\langle \phi \rangle \sim m_s$ , we obtain

$$m_\phi^2 \sim m_s M_P. \quad (13)$$

This takes care of the  $\mu$ - problem. It should be noted that due to the imposed symmetries the presence of non-renormalizable terms does not affect neither the triplet-doublet splitting nor the F-flatness. Also the model possesses a mechanism to generate a  $\mu$ -term, provided the  $R$ -symmetry breaking is  $\sim m_s$ . These are virtues in themselves which are worth emphasizing independently of the GUT scale generation realized by the model.

## 2. RADIATIVE CORRECTIONS AND THE RGEs.

As already emphasized the superpotential  $\mathcal{W}$  allows a large v.e.v. along the  $F$ -flat and  $D$ -flat direction  $|H| = |\overline{H}|$  for the SM-singlets  $N_H^c, \overline{N}_{\overline{H}}^c$  in  $H, \overline{H}$ , respectively whose value  $V_X$  is not determined at the tree-level. Such a v.e.v. breaks the  $SU(5) \times U(1)_X$  gauge symmetry down to the SM gauge group  $SU(3)_c \times SU(2)_L \times U(1)_Y$ . The colour triplets  $D_H^c(\mathbf{3}, \mathbf{1}, 1/3), \overline{D}_{\overline{H}}^c(\overline{\mathbf{3}}, \mathbf{1}, -1/3)$  that survive the Higgs phenomenon pair up with the colour triplets  $d_h(\overline{\mathbf{3}}, \mathbf{1}, -1/3), \overline{d}_{\overline{h}}(\mathbf{3}, \mathbf{1}, 1/3)$  in the Higgs pentaplets to form states with masses  $\sim M_X \sim V_X$  through the superpotential couplings  $HHh$  and  $\overline{H}\overline{H}\overline{h}$ . The model exhibits triplet-doublet splitting which, however, is to a large extent a consequence of the imposed discrete symmetries.

The GUT scale  $M_X$  is defined as the scale at which the breaking  $SU(5) \times U(1)_X \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$  occurs<sup>†</sup> or, equivalently, the scale at which the gauge couplings  $\alpha_3$  and  $\alpha_2$  meet, i.e.

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<sup>†</sup> The relation imposed by the  $E_6$ -normalization of  $U(1)_X$  in flipped  $SU(5)$  is

$$25\alpha_1^{-1}(M_X) = \alpha_5^{-1}(M_X) + 24\alpha_X^{-1}(M_X)$$

$$\alpha_3(M_X) = \alpha_2(M_X) = \alpha_5(M_X) \equiv \alpha_G . \quad (14)$$

One of the aims of the present article is to show how radiative corrections determine the value  $M_X$  of this scale. Above  $M_X$  the  $SU(5)$  and  $U(1)_X$  gauge couplings  $\alpha_5$  and  $\alpha_X$  converge and eventually meet at some scale  $M_s$  which will be taken to be the *string unification scale*  $M_s \sim 5 \times 10^{17} GeV$

$$\alpha_5(M_s) = \alpha_X(M_s) \equiv \alpha_{SU} . \quad (15)$$

The Renormalization Group Equations (RGEs) for the gauge couplings, apart from the standard minimal set of fields, involve the pair of extra tenplets  $H'$ ,  $\overline{H}'$  as well. For their masses  $\sim M_X^2/M_P$  we shall adopt a phenomenological attitude and adjust them in the range  $10^{11} - 10^{12} GeV$  in order to obtain an optimal fit of the low energy data.

Let us now consider the RGEs for the gauge couplings. The leading logarithmic radiative corrections to the various parameters of the model are represented by the scale dependence of the running parameters that satisfy the RGEs. We shall assume that the only appreciable couplings in the superpotential are the following :

$$\begin{aligned} \mathcal{W}_1 = & \frac{1}{8} \lambda \epsilon^{AB\Gamma\Delta E} H_{AB} H_{\Gamma\Delta} h_E + \frac{1}{8} \overline{\lambda} \epsilon_{AB\Gamma\Delta E} \overline{H}^{AB} \overline{H}^{\Gamma\Delta} \overline{h}^E + Y_t^{ij} F_{AB}^i f_j^c \overline{h}^B \\ & + \frac{1}{2} Y_N^i F_{AB}^i \overline{H}^{AB} N_i \quad (i = 1, \dots, 3) . \end{aligned} \quad (16)$$

The relevant soft-SUSY breaking terms corresponding to  $\mathcal{W}_1$  are

$$\begin{aligned} -\mathcal{L}_{soft} = & m_{\mathcal{H}}^2 |\mathcal{H}|^2 + m_{\overline{\mathcal{H}}}^2 |\overline{\mathcal{H}}|^2 + m_h^2 |h|^2 + m_{\overline{h}}^2 |\overline{h}|^2 + m_{\mathcal{F}_i}^2 |\mathcal{F}_i|^2 + m_{f_i^c}^2 |f_i^c|^2 \\ & + m_{\mathcal{N}_i}^2 |\mathcal{N}_i|^2 + \left( \frac{1}{8} A_\lambda \lambda \mathcal{H}^2 h + \frac{1}{8} \overline{A_\lambda} \overline{\lambda} \overline{\mathcal{H}}^2 \overline{h} + A_t^{ij} Y_t^{ij} \mathcal{F}_i f_j^c \overline{h} + \frac{1}{2} A_N^i Y_N^i \mathcal{F}_i \overline{\mathcal{H}} \mathcal{N}_i \right. \\ & \left. + \frac{1}{2} M_5 \lambda_5 \lambda_5 + \frac{1}{2} M_1 \lambda_1 \lambda_1 + h.c. \right) \end{aligned} \quad (17)$$



where  $\mathcal{H}, \overline{\mathcal{H}}, h, \overline{h}, \mathcal{F}, f^c, \mathcal{N}$  are the scalar components of the superfields  $H, \overline{H}, h, \overline{h}, F, f^c, N$ , respectively and  $\lambda_5, \lambda_1$  the gauginos of  $SU(5)$  and  $U(1)_X$ , respectively.

As we shall explain shortly, we are interested in the evolution of the soft SUSY-breaking masses from  $M_s$  to  $M_X$  and more specifically in the RGEs <sup>‡</sup> for  $m_{\mathcal{H}}^2$  and  $m_{\overline{\mathcal{H}}}^2$  :

$$Q \frac{dm_{\mathcal{H}}^2}{dQ} = \frac{1}{8\pi^2} \{ 3\lambda^2 [m_h^2 + 2m_{\mathcal{H}}^2 + A_\lambda^2] - \frac{72}{5} M_5^2 g_5^2 - \frac{1}{10} M_1^2 g_X^2 + \frac{1}{4} g_X^2 \mathcal{S} \} \quad (18)$$

$$Q \frac{dm_{\overline{\mathcal{H}}}^2}{dQ} = \frac{1}{8\pi^2} \{ 3\overline{\lambda}^2 [m_{\overline{h}}^2 + 2m_{\overline{\mathcal{H}}}^2 + \overline{A}_\lambda^2] + \sum_{i=1}^3 Y_N^{i2} [m_{\mathcal{F}_i}^2 + m_{\overline{\mathcal{H}}}^2 + m_{\mathcal{N}_i}^2 + A_{\mathcal{N}_i}^2] - \frac{72}{5} M_5^2 g_5^2 - \frac{1}{10} M_1^2 g_X^2 - \frac{1}{4} g_X^2 \mathcal{S} \} , \quad (19)$$

where

$$\mathcal{S} = (m_{\mathcal{H}}^2 - m_{\overline{\mathcal{H}}}^2) - (m_h^2 - m_{\overline{h}}^2) - \frac{3}{2} \sum_{i=1}^3 m_{f_i^c}^2 + \sum_{i=1}^3 m_{\mathcal{F}_i}^2 . \quad (20)$$

Consider the flat direction that allows for a non-zero v.e.v. in the supersymmetric limit. The soft SUSY-breaking mass terms induce a small deviation from flatness which, with the leading logarithmic corrections present in the 1-loop effective potential for  $\mathcal{H}, \overline{\mathcal{H}}$  taken into account, are, in principle, able to generate a minimum at a scale  $V_X \sim M_X$  with depth  $\sim m_s^2 V_X^2$ . . If this is the case, the GUT scale can be thought of as generated through radiative corrections. In order to investigate this phenomenon it would be sufficient to consider the tree-level potential, given essentially by the soft mass terms only, and study the renormalization group evolution of its parameters. Let us start at high energies with positive soft masses-squared for the relevant fields  $\mathcal{H}, \overline{\mathcal{H}}$ , and assume that at some lower

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<sup>‡</sup>We consider the third generation of the Yukawa couplings  $Y_t^{ij}$  ( $Y_t^{33} \equiv Y_t$ ). The quantity  $\mathcal{S}$  plays no role in the evolution and can be ignored.

energy  $Q_0$  a reversal of sign occurs for them<sup>§</sup>, i.e.

$$m_{\mathcal{H}}^2(Q_0) + m_{\overline{\mathcal{H}}}^2(Q_0) = 0 . \quad (21)$$

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<sup>§</sup> Consider a toy model with gauge group  $U(1)$  involving the superfields  $\phi(1)$ ,  $\overline{\phi}(-1)$ ,  $f(-\frac{1}{2})$ ,  $\overline{f}(\frac{1}{2})$ . We impose a  $\mathcal{Z}_2$  symmetry under which  $f \rightarrow -f$  and a  $\mathcal{Z}_4$  R-symmetry under which  $(f, \overline{f}) \rightarrow i(f, \overline{f})$ , with the superpotential  $\mathcal{W} \rightarrow -\mathcal{W}$ . Then, the renormalizable part of  $\mathcal{W}$  is

$$\mathcal{W} = \frac{1}{2}\lambda f f \phi + \frac{1}{2}\overline{\lambda} \overline{f} \overline{f} \overline{\phi} .$$

The potential of the model in the supersymmetric limit is given by

$$\begin{aligned} V_1 = & |\lambda|^2 |f|^2 \left( \frac{1}{4} |f|^2 + |\phi|^2 \right) + |\overline{\lambda}|^2 |\overline{f}|^2 \left( \frac{1}{4} |\overline{f}|^2 + |\overline{\phi}|^2 \right) \\ & + \frac{1}{2} g^2 (|\phi|^2 - |\overline{\phi}|^2 - \frac{1}{2} (|f|^2 - |\overline{f}|^2))^2 , \end{aligned}$$

and possesses the exact D- and F- flat direction  $|\phi| = |\overline{\phi}|$  with  $f = \overline{f} = 0$ . Adding to  $V_1$  the soft SUSY-breaking terms

$$\begin{aligned} V_2 = & m_f^2 |f|^2 + m_{\overline{f}}^2 |\overline{f}|^2 + m_\phi^2 |\phi|^2 + m_{\overline{\phi}}^2 |\overline{\phi}|^2 \\ & + \left( \frac{1}{2} A_\lambda \lambda f f \phi + \frac{1}{2} A_{\overline{\lambda}} \overline{\lambda} \overline{f} \overline{f} \overline{\phi} + \frac{1}{2} M_1 \lambda_1 \lambda_1 + h.c. \right) , \end{aligned}$$

the flat direction is lifted. However,  $f = \overline{f} = 0$  is still a minimum of  $V = V_1 + V_2$  for fixed  $\phi, \overline{\phi}$  along the D-flat direction  $|\phi| = |\overline{\phi}|$ , provided  $|\phi| \gg m_s$ . Setting  $f = \overline{f} = 0$  and  $|\phi| = |\overline{\phi}| \gg m_s$  the potential  $V$  reduces to

$$V = (m_\phi^2 + m_{\overline{\phi}}^2) |\phi|^2 .$$

A non-trivial minimum will occur at a scale  $V_X \sim Q_0 \gg m_s$  if the quantity  $m_\phi^2(Q) + m_{\overline{\phi}}^2(Q)$  flips its sign at  $Q = Q_0$ , i.e.

$$m_\phi^2(Q_0) + m_{\overline{\phi}}^2(Q_0) = 0 .$$

Then, this reversal of sign signals the development of a symmetry breaking minimum along the flat direction with v.e.v.  $V_X \sim Q_0$ .

The energy range in which  $Q$  takes values is divided into three regions:

**a)**  $M_X < Q < M_s$ : The gauge couplings run according to the following RGEs \*\*

$$Q \frac{d\alpha_i}{dQ} = \frac{\alpha_i^2}{2\pi} \left( b_i + \frac{1}{4\pi} \sum_{j=5,X} b_{ij} \alpha_j \right) \quad (22)$$

$$b_5 = -2$$

$$b_X = 8 \quad (23)$$

and

$$b_{ij} = \begin{pmatrix} \frac{776}{5} & \frac{23}{5} \\ \frac{336}{5} & \frac{83}{10} \end{pmatrix}, (i, j = 5, X).$$

**b)**  $M_{10} < Q < M_X$  : In this energy region the RGEs receive contributions from the spectrum of MSSM and from  $D_{H'}^c(\bar{3}, 1, 1/3)$ ,  $\bar{D}_{H'}^c(3, 1, -1/3)$ ,  $Q_{H'}(3, 2, 1/6)$ ,  $\bar{Q}_{H'}(\bar{3}, 2, -1/6)$  contained in  $H'$ ,  $\bar{H}'$ . The 2-loop beta functions of the gauge couplings are given by  $b_1 = 36/5$ ,  $b_2 = 4$ ,  $b_3 = 0$  and

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A more accurate determination of the position  $V_X$  of the minimum can be obtained numerically by solving the RGEs for the soft SUSY-breaking terms and looking for a minimum at  $|\phi| = V_X$  of the “effective potential” :

$$(m_\phi^2(|\phi|) + m_\phi^2(|\phi|)) |\phi|^2.$$

\*\*We ignore the Yukawa coupling contribution to the 2-loop expression for the beta function of the gauge couplings.

$$b_{ij} = \begin{pmatrix} 48 & 15 & 7 \\ 40 & 46 & 2 \\ \frac{104}{5} & 6 & \frac{202}{25} \end{pmatrix}, \quad (i, j = 3, 2, 1).$$

in a notation analogous to the one employed previously. Here we evolve the Yukawa couplings as well as the soft SUSY-breaking masses by making use of their one-loop beta functions.

c)  $M_Z < Q < M_{10}$  :

In this range there are contributions to the RGEs only from the MSSM spectrum. For the evolution of all couplings and masses the 2-loop approximation was made.

In the following section we are going to combine the above RGEs in order to achieve gauge coupling unification at  $M_s \sim 5 \times 10^{17} GeV$  and generation of the symmetry breaking scale  $M_X$  in a way consistent with the low energy data.

### 3. NUMERICAL ANALYSIS AND CONCLUSIONS.

We evolve the 2-loop RGEs from  $M_Z$  up to the scale  $M_{10} = 3 \times 10^{11} GeV$ , keeping fixed the value for the strong coupling at  $\alpha_s(M_Z) = 0.120$ , for  $m_t = 174 GeV$ ,  $m_b = 4.9 GeV$ ,  $m_\tau = 1.777 GeV$ . Above  $M_{10}$  the thresholds of  $H'$ ,  $\overline{H}'$  are switched on. The unification scale is defined by the equality of the gauge couplings  $\alpha_3$  and  $\alpha_2$ . Above  $M_X$  the two gauge couplings  $\alpha_5$  and  $\alpha_X$  are evolved up to the string unification scale  $M_s$  where they become equal. There, we impose universal boundary conditions for the soft SUSY-breaking masses

$$\begin{aligned} m_{\mathcal{H}} &= m_{\overline{\mathcal{H}}} = m_h = m_{\overline{h}} = m_{\mathcal{F}_i} = m_{\overline{\mathcal{F}}_i} = m_{\mathcal{N}_i} \equiv M_0 \\ M_1 &= M_5 \equiv M_{1/2} \\ A_\lambda &= \overline{A}_\lambda = A_t = A_N^i \equiv A_0, \quad i = 1 \dots 3 \end{aligned} \quad (24)$$

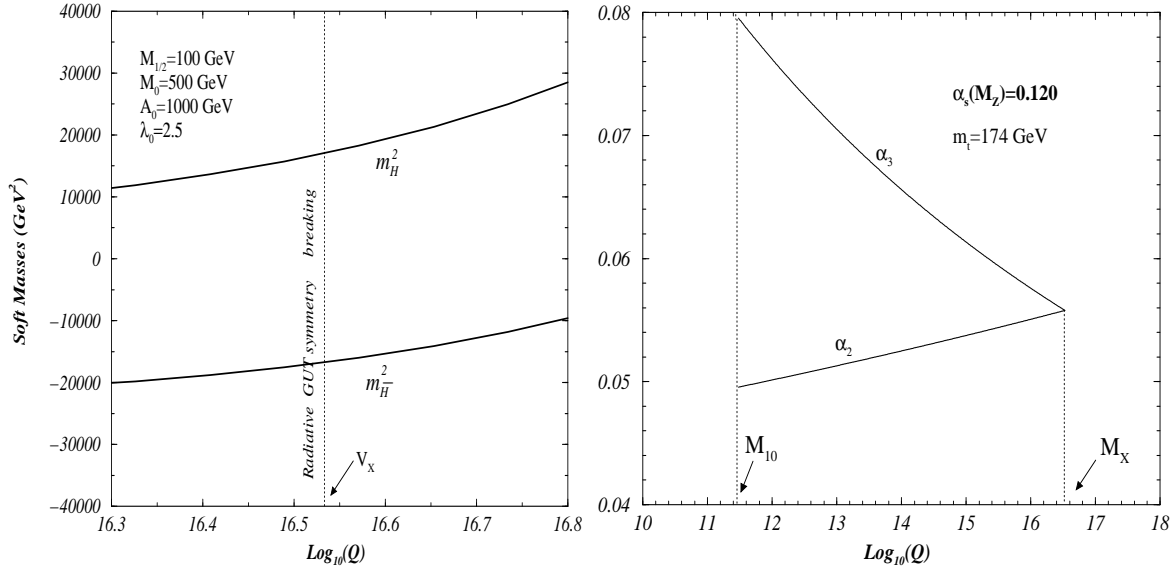
and for simplicity we take the Yukawa couplings appearing in the superpotential  $\mathcal{W}_1$  to be

$$\lambda = \overline{\lambda} = Y_N^i \equiv \lambda_0. \quad (25)$$

Starting from these boundary conditions, we come down in energy and demand that the relation

$$m_{\mathcal{H}}^2(Q_0) + m_{\overline{\mathcal{H}}}^2(Q_0) = 0 \quad (26)$$

be satisfied at a scale  $Q_0 \sim V_X \sim M_X < M_s$ . The whole procedure is carried out under the constraints of electroweak radiative symmetry breaking at  $M_Z$ , perturbativity of all couplings up to  $M_s$ , as well as the experimental constraints on the values of  $\sin^2\theta$  and the sparticle masses [20].



**Figure 1:** Evolution of the soft masses squared  $m_{\mathcal{H}}^2$  and  $m_{\overline{\mathcal{H}}}^2$  from  $M_s \sim 5 \times 10^{17} \text{ GeV}$  to lower scales. The vertical line indicates the scale  $V_X$  where the radiative GUT symmetry breaking of the flipped  $SU(5)$  occurs. This scale practically coincides with the unification point of the gauge couplings  $\alpha_2$  and  $\alpha_3$ .

The evolution of the soft masses squared  $m_{\mathcal{H}}^2$  and  $m_{\overline{\mathcal{H}}}^2$  is depicted in Figure 1. These masses although positive definite at  $M_s$  develop a negative sign at a somewhat lower scale with the masses-squared of all the other gauge non-singlets remaining positive, if we adopt suitable values for the parameters. When the one of the two squared masses (usually  $m_{\overline{\mathcal{H}}}^2$ ) becomes negative, with its absolute value being greater than the value of the other, radiative

GUT symmetry breaking occurs. From the structure of the renormalization group equations (18),(19), it is easy to deduce that the way to ensure the desired sequence of events is to keep gaugino masses  $M_{1/2}$  at their lowest possible value compatible with all the relevant phenomenological constraints and employ values of  $A_0$  and  $M_0$  considerably larger than  $M_{1/2}$ . Thus, for  $M_0 = 500$  GeV,  $A_0 = 1000$  GeV,  $M_{1/2} = 100$  GeV and  $\lambda_0 = 2.5$  the radiative GUT symmetry breaking picture outlined above is achieved.

In Figure 1 we have taken  $M_{10} = 3 \times 10^{11}$  GeV. If we increase  $M_{10}$  by one order of magnitude,  $M_X$  decreases towards a value less than  $\simeq 10^{16}$  GeV, which is dangerous for proton decay. If we decrease  $M_{10}$ , the obtained value of the weak mixing angle is quite large. The choice of the Yukawa couplings  $\lambda = \bar{\lambda} = Y_N^i = \lambda_0$  at  $M_s$ , plays a crucial role in the determination of  $M_X$  with the values of  $M_0$ ,  $M_{1/2}$  and  $A_0$  kept fixed. In the region where  $\lambda_0 \lesssim 2.0$ , no radiative GUT symmetry breaking occurs. On the other hand, if  $\lambda_0 \gtrsim 3.0$  the breaking takes place at a scale which is too close to  $M_s$  pushing among others the weak mixing angle out of the limits imposed by experiment. Although the soft SUSY-breaking parameters  $A_0$  and  $M_0$  must be both large, in order to obtain the successful symmetry breaking picture just outlined, they are still subject to the constraints of charge-color breaking minima and radiative electroweak symmetry breaking, respectively. The case where  $M_{1/2}$  is appreciable, and  $M_0$ ,  $A_0$  tend to zero, results in  $M_X \sim M_s$  which is unacceptable as already explained. In conclusion, the generation of the superheavy scale is achieved in a relatively narrow range of values of the parameters under the assumption that  $M_{1/2} \ll M_0, A_0$ .

The  $SU(5) \times U(1)_X$  model studied in the present article should be regarded as a phenomenologically viable example realizing radiative GUT scale generation even in cases where  $F$ -flatness is essentially exact. The flatness of the potential is lifted by the small SUSY-breaking scalar mass squared terms which through radiative effects flip their sign along the almost flat direction. The relevant radiatively generated scale is practically the energy at which this flipping occurs.

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