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Radiative symmetry breaking and the top quark mass in local supersymmetry

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Exact low energy expressions are derived for the top-squark and Higgs masses, taking into account radiative contributions due to a heavy top quark. Their masses are expressed as analytic functions of $m_{1/2}$, $m_{3/2}$, $m_t/\sin\beta$. Constraints from the radiative symmetry breaking mechanism are used then, to put lower bounds on the top-quark mass m_t. In particular, when $m_{3/2} \gg m_{1/2}$, we obtain the bound $m_t \geqslant 155 \sin\beta$ GeV, while in the case of $m_{3/2} \equiv 0$, we obtain $m_t \geqslant 80 \sin\beta$ GeV.

It has by now been established that the idea of Grand Unification with the minimal standard model content, is realised only when supersymmetry is present down to the TeV scale. On the other hand, local supersymmetry is necessary in order to solve the gauge hierarchy problem [1]. Therefore, spontaneously broken supergravity coupled to Grand Unified Theories, is a promising scenario for the physics beyond the standard model.

Another nice feature of supersymmetric theories, is the radiative breaking scenario of the electroweak symmetry [2,3] through the generalization of the Coleman–Weinberg mechanism [4]. The key role in this mechanism is played by the soft SUSY breaking terms and, in particular, the scalar masses which, as going from the unification scale (M_U) down to low energies, get modified due to large radiative corrections [3,5]. The $SU(2) \times U(1)$ symmetry breaking is obtained if some of the Higgs doublets develops a negative $(\text{mass})^2$. The only radiative corrections which may drive the Higgs $(\text{mass})^2$ negative, arise from the contributions of Yukawa couplings through the renormalization group running of the scalar masses. Obviously, the most important contribution of this kind arises from the top-quark–Yukawa coupling. Such contributions have already been estimated in the literature [3,5–7].

Because of their importance in the radiative breaking mechanism, and in view of the new theoretical predictions and experimental constraints for the top-mass range, we would like to present in this letter a new approach and calculate more accurately their effects. We will make a first attempt to derive analytic expressions for these negative contributions, and discuss the bounds imposed on the top–Yukawa coupling.

In order to set our formalism and the general framework, we would like to start from the well-known results on the evolution of the top–Yukawa coupling λ_{top}, due to renormalization group running from the unification point down to low energies. We will assume that λ_{top} is much bigger than all other fermion–Yukawa couplings. In this case we may ignore the contributions of the latter, thus the top–Yukawa coupling differential equation may be cast in the form

$$16\pi^2 \frac{\mathrm{d}}{\mathrm{d}t} \lambda_{\text{top}} = \lambda_{\text{top}} [6\lambda_{\text{top}}^2 - G_U(t)] . \tag{1}$$

The relevant gauge contribution G_U is given by

$$G_U = \sum_{i=1}^{3} c_U^i g_i^2(t) , \tag{2}$$

$$g_i^2(t) = \frac{g_i^2(t_0)}{1 - (b_i/8\pi^2) g_i^2(t_0)(t - t_0)} . \tag{3}$$

The g_i are the three gauge coupling constants of the Standard Model and b_i are the corresponding supersymmetric beta functions. The coefficients c_U^i are given by

$$\{c_U^i\}_{i=1,2,3} = \{\tfrac{13}{15}, 3, \tfrac{16}{3}\}. \tag{4}$$

The solution of eq. (1) is

$$\lambda_{\text{top}}(t) = \lambda_{\text{top}}(t_0)\xi(t)^6\gamma_U(t), \tag{5}$$

where [8,9]

$$\gamma_U(t) = \exp\left(-\frac{1}{16\pi^2}\int_{t_0}^{t} G_U(t)\,\mathrm{d}t\right) = \prod_{j=1}^{3}\left(1-\frac{b_{j,0}\alpha_{j,0}(t-t_0)}{2\pi}\right)^{c_U^j/2b_j}, \tag{6}$$

and

$$\xi(t) = \exp\left(\frac{1}{16\pi^2}\int_{t_0}^{t}\lambda_{\text{top}}^2(t')\,\mathrm{d}t'\right) = \left(1-\frac{6}{8\pi^2}\lambda_{\text{top}}^2(t_0)\int_{t_0}^{t}\gamma_U^2(t')\,\mathrm{d}t'\right)^{-1/12}. \tag{7}$$

Let us now turn our attention to the scalar masses. As we have already mentioned, we are interested in those low energy scalar masses which are affected by the top–Yukawa contributions. These are the top-squarks $m_{\tilde{t}_L}$, $m_{\tilde{t}_R}$, the Higgs mass $m_{\bar{h}}$ (where $\bar{h}$ is the Higgs which gives masses to the up-quarks) and the trilinear scalar coupling parameter A. Let us denote $m_{\tilde{t}_L} \equiv \tilde{m}_1$, $m_{\tilde{t}_R} \equiv \tilde{m}_2$, and $m_{\bar{h}} \equiv \tilde{m}_3$. Then, at any scale $t = \ln\mu$, we can write the following general formula for the squark and $\bar{h}$-Higgs masses which couple to the top–Yukawa coupling:

$$\tilde{m}_n^2(t) = \alpha_n m_{3/2}^2 + C_n(t)m_{1/2}^2 - n\Delta_A^2(t) - n\Delta_{\tilde{m}}^2(t). \tag{8}$$

In the above,

$$\Delta_A^2(t) + \Delta_{\tilde{m}}^2(t) = \int_{t}^{t_0}\frac{\lambda_{\text{top}}^2(t')}{8\pi^2}\left(A^2(t') + \sum_{j=1}^{3}\tilde{m}_j^2(t')\right)\mathrm{d}t', \tag{9}$$

where with $\Delta_A(t)$ we have denoted the radiative corrections due to the trilinear scalar coupling which in general depends on the scale parameter $t = \ln\mu$ [3]. $C_n(t) = C_Q, C_U, C_L$ are calculable coefficients [10] which represent gauge corrections, while $\alpha_n \equiv \alpha_Q = \alpha_U = \alpha_L = 1$ in the minimal case where the scalars are in a flat manifold [11]. Thus, eq. (8) can be transformed to an integral equation of Volterra type which can be solved exactly. We start by summing all the scalar masses containing the Yukawa corrections. Thus we get

$$\sum_{n=1}^{3}\tilde{m}_n^2(t) = \sum_{n=1}^{3}\alpha_n m_{3/2}^2 + \sum_{n=1}^{3}C_n(t)m_{1/2}^2 - \sum_{n=1}^{3}n\Delta_A^2(t) - \sum_{n=1}^{3}n\Delta_{\tilde{m}}^2(t). \tag{10}$$

Let us define

$$u(t) = \sum_{n=1}^{3}\tilde{m}_n^2(t),$$

$$u_0(t) = \sum_{n=1}^{3}\alpha_n m_{3/2}^2 + \sum_{n=1}^{3}C_n m_{1/2}^2 - \sum_{n=1}^{3}n\Delta_A^2(t),$$

$$C = -\frac{1}{8\pi^2}\sum_{n=1}^{3}n. \tag{11}$$

Using the definitions in eqs. (11) above, eq. (10) can be cast in a standard form:

$$u(t) = u_0(t) + C\int_{t}^{t_0}\mathrm{d}t'\,\lambda_{\text{top}}^2(t')u(t'). \tag{12}$$

The above equation can be solved easily. Define

$$f(t) = C\int_{t}^{t_0}\mathrm{d}t'\,\lambda_{\text{top}}^2(t')u(t'). \tag{13}$$

Differentiating eq. (13), and substituting back eq. (12), we obtain

$$\frac{\mathrm{d}f(t)}{u_0(t)+f(t)} = -C\lambda_{\text{top}}^2(t)\,\mathrm{d}t. \tag{14}$$

The solution of the above is

$$f(t) = -q(t)^{-1} C \int_{t_0}^{t} q(t') u_0(t') \lambda_{\rm top}^2(t')\, dt' , \tag{15}$$

where the *integrating factor* $q = q(t)$ is given by

$$q = \exp\left(C \int_{t_0}^{t} \lambda_{\rm top}^2(t')\, dt' \right)$$
$$= \exp[-12 \ln \xi(t)] \tag{16}$$
$$= \xi^{-12} , \tag{17}$$

where ξ is given again by the integral (7). Now, the solution for $u(t)$ can be given by substituting eq. (15) into (12). The corrections $\Delta^2(t)$, can also be expressed in terms of the function $f(t)$,

$$\Delta_{\tilde{m}}^2(t) = -\tfrac{1}{6} f(t) . \tag{18}$$

Substituting the above results in eq. (8), we find that the scalar masses are given by

$$\tilde{m}_n^2(t) = \alpha_n m_{3/2}^2 + C_n(t) m_{1/2}^2 - n\delta_A^2(t) - n\delta_m^2(t) , \tag{19}$$

where

$$\delta_m^2(t) = \xi(t)^{12} \int_{t_0}^{t} \xi(t')^{-12} [C\lambda_{\rm top}^2(t')\, dt'] \times \frac{1}{6} \left(\sum_{n=1}^{3} \alpha_n m_{3/2}^2 + \sum_{n=1}^{3} C_n(t') m_{1/2}^2 \right) , \tag{20}$$

and

$$\delta_A^2(t) = \Delta_A^2(t) - \xi(t)^{12} \int_{t_0}^{t} \xi(t')^{-12} [C\lambda_{\rm top}^2(t')\, dt'] \Delta_A^2(t') . \tag{21}$$

The above expressions can be simplified by substituting $\lambda_{\rm top}^2$ from eq. (5) into the integrals. Taking also into account that the top mass is given by the formula $m_t = \lambda_{\rm top}(v/\sqrt{2}) \sin\beta$ where $v = 246$ GeV, we may write $\delta_m^2(t)$, as follows:

$$\delta_m^2(t) = \left(\frac{m_t}{2\pi v \gamma_U \sin\beta} \right)^2 \times (3m_{3/2}^2 I + m_{1/2}^2 J) , \tag{22}$$

where I, J, are integrals containing only functions of gauge couplings, i.e.

$$I = \int_{t}^{t_0} dt' \gamma_U^2(t') , \tag{23}$$

$$J = \int_{t}^{t_0} dt' \gamma_U^2(t') C(t') , \tag{24}$$

with $C(t) \equiv \sum_{n=1}^{3} C_n(t')$ [3]. A similar form may be obtained for $\delta_A^2(t)$.

As an application, let us calculate the $(\text{mass})^2$-parameter of the $\bar{h}$-Higgs. We start with the integral (20). The integrals I, J can be easily performed numerically. Thus for $t \sim \ln m_t$, with m_t in the experimentally allowed region, we get $I \approx 112$, and $J \approx 590$.

The trilinear coupling A is given by [3] $A(t) = [A_0 m_{3/2} + C_A(t) m_{1/2}]/(1 - 12 \ln\xi)$. For our present purposes let us assume an average value for $C_A(t)$ and ignore the t-dependence. Now the integral can be performed rather easily by observing that the various t-dependent quantities can finally be expressed as functions of ξ. Indeed, first we observe that $(\xi(t') \equiv \xi')$

$$C\lambda_{\rm top}^2(t')\, dt' = d(-12 \ln\xi') . \tag{25}$$

Under the previous assumption, the integral Δ_A^2 is

$$\Delta_A^2(t) = \tfrac{1}{6} Q_0^2 \int_0^{-12\ln\xi} \frac{d(-12 \ln\xi')}{(1 - 12 \ln\xi')^2} , \tag{26}$$

with $Q_0 = (A_0 m_{3/2} + C_A m_{1/2})$. Thus, the quantity $\delta_A^2(t)$ becomes

$$\delta_A^2(t) = \tfrac{1}{6} Q_0^2 \left(\int_0^{-12\ln\xi} \frac{1}{(1+x)^2}\, dx - \xi(t)^{12} \int_0^{-12\ln\xi} \frac{x\, e^x}{1+x}\, dx \right) . \tag{27}$$

For $\mu \approx O(m_t)$ as previously, a correct bottom mass prediction requires [9,12] $\xi \approx 0.81$, thus $\delta_A^2 \approx 2.52 \times$

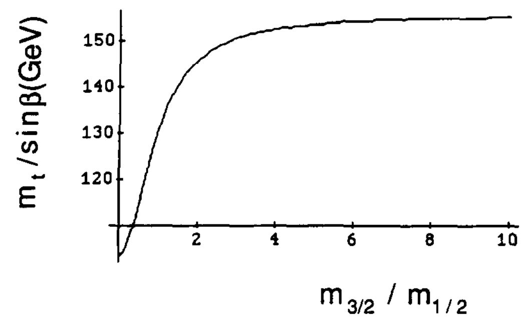

Fig. 1.

$10^{-2}Q_0^2$, which is negligible compared to the δ_m^2 contribution.

Substituting (22) back into eq. (19), and requiring $m_{\bar{h}}^2 < 0$, for any given $m_{3/2}/m_{1/2}$ ratio we can obtain a lower bound on the top-quark mass. In fig. 1 we plot the lower bound on $m_t/\sin\beta$, as a function of the ratio $r = m_{3/2}/m_{1/2}$. There is a lower value of this bound, $m_t \geqslant 80\sin\beta$ GeV, which corresponds to the case $r = 0$, while the curve exhibits an asymptotic behaviour as $r \geqslant 2$, and the bound reaches its maximum value $m_t \geqslant 155\sin\beta$ GeV, as $r \to \infty$. No upper bound in obtained for m_t from the other scalar masses.

It is a remarkable fact that the constraints on the top mass from the radiative symmetry breaking mechanism are in perfect agreement with previous analyses of predictive fermion mass textures in supersymmetric grand unified models. Indeed, the value of $\xi \approx 0.81$ dictated by the bottom mass, implies [9,12] that $\sin\beta \approx m_t/180$ GeV, which is consistent with the bounds derived in the present analysis. If we adopt this value for $\sin\beta$, we may finally express the scalar masses $\tilde{m}_n^2$ only in terms of the supersymmetric mass parameters $m_{3/2}$ and $m_{1/2}$. Indeed, substituting the numbers for the various parameters discussed above we finally get

$$\tilde{m}_n^2 \approx (\alpha_n - 0.45n)m_{3/2}^2 + (C_n - 0.9n)m_{1/2}^2, \quad (28)$$

with $\alpha_n = 1$ in the flat case and $C_{1,2,3} \approx 5.29$, 4.89, 0.50 [10] respectively. Note that, for $n = 1, 2$ one always obtains $m_{1,2}^2 \equiv m_{\tilde{t}_L,\tilde{t}_R}^2 \geqslant 0$.

In conclusion, in this letter we have derived analytic formulae for the scalar masses in supersymmetric grand unified models taking into account radiative corrections from the top–Yukawa coupling. We have found that the radiative symmetry breaking mechanism implies plausible bounds on the top mass consistent with the top mass predictions from fermion mass matrix textures discussed in the recent literature. Using the predictions of the latter, scalar masses are finally expressed only in terms of the supersymmetric mass parameters $m_{1/2}$ and $m_{3/2}$.

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