Retracing the phenomenology of the flipped SU(5)×U(1) superstring model

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We study in detail gauge symmetry breaking in the $SU(5)\times U(1)'\times U(1)^4\times SO(10)\times SO(6)$ superstring model, solving the D- and F-flatness conditions and taking into account quartic and quintic superpotential terms. We find that, to this order, the model describes two massive generations of quarks and leptons as well as a massless generation expected to receive naturally suppressed masses from higher order non-renormalizable terms. We show that D-flatness restricts the number of massless isodoublets to four. We also extract an inequality relating the top quark mass to M_W.

A presently popular model, constructed in the framework of the fermionic formulation of four dimensional superstrings [1,2], is the flipped $SU(5)\times U(1)'\times U(1)^4\times SO(10)\times SO(6)$ [3] superstring model. In this article, we study in detail the gauge symmetry breaking $SU(5)\times U(1)'\times U(1)^4\to SU(5)\times U(1)'\to SU(3)_C\times SU(2)_L\times U(1)_Y$ to the standard model solving the D- and F-flatness conditions that determine the pattern of VEV's. We calculate, with the rules exhibited in ref. [4], the quartic and quintic non-renormalizable terms in the superpotential of chiral superfields. We find that, to this order, the model possesses two massive families of quarks and leptons with a hierarchical mass structure. A third generation stays massless at this level of calculation. We show that D-flatness restricts the number of massless isodoublets to four. We derive an inequality relating the top quark mass to M_W. We also find that the singlet VEV pattern determined by the D-flatness conditions at the cubic level maintains F-flatness up to the quintic level without fixing any of the undetermined VEVs.

The model is defined by eight vectors of boundary conditions for all world sheet fermions [3]. The massless spectrum generated by this basis is listed in tables 1 and 2. The trilinear superpotential of chiral superfields is the sum of the "observable sector" superpotential

$$
\begin{aligned}
&F_1F_1h_1+F_2F_2h_2+F_4F_4h_1+\bar F_5\bar F_5\bar h_2+\frac{1}{\sqrt2}F_4\bar F_5\phi_3+F_4\bar f_5\bar h_{45}+F_3\bar f_3\bar h_3+\bar f_1 l_1^c h_1+\bar f_2 l_2^c h_2+\bar f_5 l_5^c h_2\\
&+\frac{1}{\sqrt2}f_4\bar f_5\bar\phi_2+\frac{1}{\sqrt2}\bar l_4^c l_5^c\bar\phi_2+f_4\bar l_4^c\bar h_1+h_1\bar h_2\Phi_{12}+\bar h_1h_2\bar\Phi_{12}+h_2\bar h_3\Phi_{23}+\bar h_2h_3\bar\Phi_{23}+h_3\bar h_1\Phi_{31}+\bar h_3h_1\bar\Phi_{31}\\
&+h_3\bar h_{45}\bar\phi_{45}++\bar h_3h_{45}\phi_{45}+\tfrac12 h_{45}\bar h_{45}\Phi_3+\phi_1\bar\phi_2\Phi_4+\bar\phi_1\phi_2\Phi_4+\phi_3\bar\phi_4\Phi_5+\bar\phi_3\phi_4\Phi_5+\Phi_{12}\Phi_{23}\Phi_{31}+\bar\Phi_{12}\bar\Phi_{23}\bar\Phi_{31}\\
&+\Phi_{12}\phi_+\phi_-+\bar\Phi_{12}\bar\phi_+\bar\phi_-+\tfrac12\sum_{i=1}^4\phi_i\bar\phi_i\Phi_3+\Phi_{12}\sum_{i=1}^4\phi_i^2+\bar\Phi_{12}\sum_{i=1}^4\bar\phi_i^2\\
&+\tfrac12\phi_{45}\bar\phi_{45}\Phi_3+\tfrac12\phi_+\bar\phi_+\Phi_3+\tfrac12\phi_-\bar\phi_-\Phi_3\,,
\end{aligned}
\tag{1}
$$

and the "hidden sector" superpotential [5]

Table 1
Chiral superfields in terms of their $SU(5)\times U(1)\prime\times U(1)^4$ quantum numbers.

$F_1(10, \frac{1}{2}; -\frac{1}{2}, 0, 0, 0)$	$\bar{f}_1(\bar{5}, -\frac{3}{2}; -\frac{1}{2}, 0, 0, 0)$	$l_1^c(1, \frac{5}{2}; -\frac{1}{2}, 0, 0, 0)$
$F_2(10, \frac{1}{2}; 0, -\frac{1}{2}, 0, 0)$	$\bar{f}_2(\bar{5}, -\frac{3}{2}; 0, -\frac{1}{2}, 0, 0)$	$l_2^c(1, \frac{5}{2}; 0, -\frac{1}{2}, 0, 0)$
$F_3(10, \frac{1}{2}; 0, 0, \frac{1}{2}, -\frac{1}{2})$	$\bar{f}_3(\bar{5}, -\frac{3}{2}; 0, 0, \frac{1}{2}, \frac{1}{2})$	$l_3^c(1, \frac{5}{2}; 0, 0, \frac{1}{2}, \frac{1}{2})$
$F_4(10, \frac{1}{2}; -\frac{1}{2}, 0, 0, 0)$	$f_4(5, \frac{3}{2}; \frac{1}{2}, 0, 0, 0)$	$\bar{l}_4^c(1, -\frac{5}{2}; \frac{1}{2}, 0, 0, 0)$
$\bar{F}_5(\overline{10}, -\frac{1}{2}; 0, \frac{1}{2}, 0, 0)$	$\bar{f}_5(\bar{5}, -\frac{3}{2}; 0, -\frac{1}{2}, 0, 0)$	$l_5^c(1, \frac{5}{2}; 0, -\frac{1}{2}, 0, 0)$
$h_1(5, -1; 1, 0, 0, 0)$	$h_2(5, -1; 0, 1, 0, 0)$	$h_3(5, -1; 0, 0, 1, 0)$
$\bar{h}_1(\bar{5}, 1; -1, 0, 0, 0)$	$\bar{h}_2(\bar{5}, 1; 0, -1, 0, 0)$	$\bar{h}_3(\bar{5}, 1; 0, 0, -1, 0)$
$h_{45}(5, -1; -\frac{1}{2}, -\frac{1}{2}, 0, 0)$	$\bar{h}_{45}(\bar{5}, 1; \frac{1}{2}, \frac{1}{2}, 0, 0)$	
$\phi_{45}(1, 0; \frac{1}{2}, \frac{1}{2}, 1, 0)$	$\phi_+(1, 0; \frac{1}{2}, -\frac{1}{2}, 0, 1)$	$\phi_-(1, 0; \frac{1}{2}, -\frac{1}{2}, 0, -1)$
$\bar{\phi}_{45}(1, 0; -\frac{1}{2}, -\frac{1}{2}, -1, 0)$	$\bar{\phi}_+(1, 0; -\frac{1}{2}, \frac{1}{2}, 0, ; -1)$	$\bar{\phi}_-(1, 0; -\frac{1}{2}, \frac{1}{2}, 0, 1)$
$\Phi_{23}(1, 0; 0, -1, 1, 0)$	$\Phi_{31}(1, 0; 1, 0, -1, 0)$	$\Phi_{12}(1, 0; -1, 1, 0, 0)$
$\bar{\Phi}_{23}(1, 0; 0, 1, -1, 0)$	$\bar{\Phi}_{31}(1, 0; -1, 0, 1, 0)$	$\bar{\Phi}_{12}(1, 0; 1, -1, 0, 0)$
$\phi_i(1, 0; \frac{1}{2}, -\frac{1}{2}, 0, 0), i=1, ..., 4$	$\bar{\phi}_i(1, 0; -\frac{1}{2}, \frac{1}{2}, 0, 0), i=1, ..., 4$	$\Phi_I(1, 0; 0, 0, 0, 0), I=1, ..., 5$

Table 2
Chiral superfields in terms of their $U(1)'\times SO(10)\times SO(6)\times(U(1))^4$ quantum numbers.

$\Delta_1(0; 1, 6; 0, -\frac{1}{2}, \frac{1}{2}, 0)$	$T_1(0; 10, 1; 0, -\frac{1}{2}, \frac{1}{2}, 0)$
$\Delta_2(0; 1, 6; -\frac{1}{2}, 0, \frac{1}{2}, 0)$	$T_2(0; 10, 1; -\frac{1}{2}, 0, \frac{1}{2}, 0)$
$\Delta_3(0; 1, 6; -\frac{1}{2}, -\frac{1}{2}, 0, \frac{1}{2})$	$T_3(0; 10, 1; -\frac{1}{2}, -\frac{1}{2}, 0, -\frac{1}{2})$
$\Delta_4(0; 1, 6; 0, -\frac{1}{2}, \frac{1}{2}, 0)$	$T_4(0; 10, 1; 0, \frac{1}{2}, -\frac{1}{2}, 0)$
$\Delta_5(0; 1, 6; \frac{1}{2}, 0, -\frac{1}{2}, 0)$	$T_5(0; 10, 1; -\frac{1}{2}, 0, \frac{1}{2}, 0)$
$\bar{X}_1(-\frac{5}{4}; 1, \bar{4}; -\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{2})$	$\bar{X}_2(-\frac{5}{4}; 1, \bar{4}; -\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$
$Y_1(\frac{5}{4}; 1, 4; -\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, \frac{1}{2})$	$Y_2(\frac{5}{4}; 1, 4; -\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, -\frac{1}{2})$
$Z_1(-\frac{5}{4}; 1, 4; \frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, \frac{1}{2})$	$\bar{Z}_1(\frac{5}{4}; 1, \bar{4}; -\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$
$Q_1(\frac{5}{4}; 1, 4; -\frac{1}{4}, \frac{3}{4}, -\frac{1}{4}, 0)$	$\bar{Q}_1(-\frac{5}{4}; 1, \bar{4}; -\frac{3}{4}, \frac{1}{4}, -\frac{1}{4}, 0)$
$Y'_2(\frac{5}{4}; 1, 4; -\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, -\frac{1}{2})$	$\bar{Y}_1(-\frac{5}{4}; 1, \bar{4}; \frac{1}{4}, -\frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$
$X_1(\frac{5}{4}; 1, 4; -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{2})$	$\bar{X}'_2(-\frac{5}{4}; 1, \bar{4}; -\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$

$$\Delta_1^2\bar{\Phi}_{23}+\Delta_2^2\Phi_{31}+\Delta_4^2\bar{\Phi}_{23}+\Delta_5^2\Phi_{31}+\frac{1}{\sqrt{2}}\Delta_4\Delta_5\bar{\phi}_3+T_1^2\bar{\Phi}_{23}+T_2^2\Phi_{31}+T_4^2\Phi_{23}+T_5^2\Phi_{31}$$

$$+\frac{1}{\sqrt{2}}T_4T_5\phi_2+\frac{1}{\sqrt{2}}Y_1\bar{X}_2\phi_4+\frac{1}{\sqrt{2}}Y_2\bar{X}_1\phi_1+Y_2\bar{X}_2\phi_++\tfrac{1}{2}Z_1\bar{Z}_1\Phi_3+Q_1\bar{Q}_1\bar{\Phi}_{12}+Z_1\bar{X}'_2l_5^c+Y'_2Z_1\Delta_1\,. \qquad (2)$$

The F- and D-flatness conditions relevant to the hidden sector are trivially satisfied with vanishing VEVs for all hidden sector fields Δ, T, ... and, therefore, unbroken $SO(10)\times SO(6)$ gauge symmetry. Although (2) is exhibited for completeness it will not have any effect on the question of gauge symmetry breaking in the observable sector.

The observable gauge group $SU(5)\times U(1)'\times U(1)^4$ can be broken down to $SU(5)\times U(1)'$ provided that a subset of the singlets obtain non-vanishing vacuum expectation values at some high scale. The four surplus U(1)'s are not traceless. They can, however, be transformed into four orthogonal combinations $U(1)'_1$, $U(1)'_2$, $U(1)'_3$, and $U(1)'_A$ of which only the last is traceful, namely

$$U(1)'_1=U(1)_3+2U(1)_4\,,\quad U(1)'_2=U(1)_1-3U(1)_2\,,$$

$$U(1)'_3=3U(1)_1+U(1)_2+4U(1)_3-2U(1)_4\,,\quad U(1)_A=-3U(1)_1-U(1)_2+2U(1)_3-U(1)_4\,.$$

It is clear that $Tr(U(1)'_i)=0$ while $Tr(U(1)_A)=180$. Due to the presence of the traceful $U(1)_A$, a D-term is generated in the form

$$D_A = \sum_i (Q_A)_i |\phi_i|^2 + \xi ,$$

with $\xi \equiv (90g/96\pi^2)e^{\Phi_D}$, which requires that the $U(1)_i$'s are broken through non-vanishing VEVs of the ϕ's in order to preserve supersymmetry [6]. The natural order of magnitude of these VEVs is $O(\xi^{1/2})$. The four D-flatness conditions imposed by the $U(1)^4$ breaking, assuming that all $SU(5)\times U(1)'$ breaking VEVs are zero, take the form [#1]

$$|\phi_{45}|^2 - |\bar{\phi}_{45}|^2 = \tfrac{1}{15}\xi , \tag{3a}$$

$$|\Phi_{31}|^2 - |\bar{\Phi}_{31}|^2 - |\Phi_{23}|^2 + |\bar{\Phi}_{23}|^2 = \tfrac{1}{5}\xi , \tag{3b}$$

$$|\Phi_{23}|^2 - |\bar{\Phi}_{23}|^2 - |\Phi_{12}|^2 + |\bar{\Phi}_{12}|^2 = |\bar{\phi}_+|^2 - |\phi_+|^2 - \tfrac{1}{2}\sum_{i=1}^{4} (|\phi_i|^2 - |\bar{\phi}_i|^2) , \tag{3c}$$

$$|\phi_+|^2 - |\bar{\phi}_+|^2 - |\phi_-|^2 + |\bar{\phi}_-|^2 = \tfrac{1}{15}\xi . \tag{3d}$$

The F-flatness conditions derived from the cubic superpotential (1) are

$$\sum_{i=1}^{4} \phi_i\bar{\phi}_i + \phi_{45}\bar{\phi}_{45} + \phi_+\bar{\phi}_+ + \phi_-\bar{\phi}_- = 0 , \quad \phi_1\bar{\phi}_2 + \bar{\phi}_1\phi_2 = \phi_3\bar{\phi}_4 + \bar{\phi}_3\phi_4 = 0 , \tag{4a,b}$$

$$\sum_{i=1}^{4} \phi_i^2 + \phi_+\phi_- + \Phi_{31}\Phi_{23} = 0 , \quad \Phi_{12}\Phi_{31} = \Phi_{12}\Phi_{23} = \phi_{45}\Phi_3 = 0 , \tag{4c,d}$$

$$\bar{\phi}_2\Phi_4 + \tfrac{1}{2}\bar{\phi}_1\Phi_3 + 2\Phi_{12}\phi_1 = \bar{\phi}_1\Phi_4 + \tfrac{1}{2}\bar{\phi}_2\Phi_3 + 2\Phi_{12}\phi_2 = 0 , \tag{4e}$$

$$\bar{\phi}_4\Phi_5 + \tfrac{1}{2}\bar{\phi}_3\Phi_3 + 2\Phi_{12}\phi_3 = \bar{\phi}_3\Phi_5 + \tfrac{1}{2}\bar{\phi}_4\Phi_3 + 2\Phi_{12}\phi_4 = 0 , \tag{4f}$$

$$\Phi_{12}\phi_- + \tfrac{1}{2}\Phi_3\bar{\phi}_+ = \Phi_{12}\phi_+ + \tfrac{1}{2}\Phi_3\bar{\phi}_- = 0 , \tag{4g}$$

together with the ones for the "barred" representations wherever they are different (giving totally 23 constraints). The subset (4d) of these constraints together with its "barred" counterpart gives eight candidate solutions $\Phi_{12} = \bar{\Phi}_{12} = \Phi_3 = 0$, $\Phi_{12} = \bar{\Phi}_{31} = \bar{\Phi}_{23} = \Phi_3 = 0$, $\Phi_{31} = \Phi_{23} = \bar{\Phi}_{12} = \Phi_3 = 0$, $\Phi_{31} = \Phi_{23} = \bar{\Phi}_{31} = \bar{\Phi}_{23} = \Phi_3 = 0$, ..., etc. All except the first three are rejected by the D-flatness conditions (3a) and (3b). Applying the constraints (4g) to the remaining three, we see that the first of them, namely $\Phi_{12} = \bar{\Phi}_{12} = \Phi_3 = 0$, satisfies them automatically. For the second, these constraints imply $\bar{\phi}_+ = \bar{\phi}_- = 0$ which, if we apply (4e) and (4f), leads to $\bar{\phi}_i = 0$. All that reduces the D-flatness conditions (3c) and (3d) to $|\Phi_{23}|^2 + |\bar{\Phi}_{12}|^2 = -|\phi_+|^2 - \tfrac{1}{2}\sum_{i=1}^{4}\phi_i^2$, which is unacceptable. Similarly, for the third candidate, we get eventually $\phi_+ = \phi_- = \phi_i = 0$ which reduces (3c) and (3d) into the unacceptable $|\bar{\Phi}_{23}|^2 + |\Phi_{12}|^2 = -|\bar{\phi}_+|^2 - \tfrac{1}{2}\sum_{i=1}^{4}\bar{\phi}_i^2$. Therefore, we are left with the unique candidate solution $\Phi_{12} = \bar{\Phi}_{12} = \Phi_3 = 0$ which satisfies the constraints (4d) and (4g). The constraints (4d) and (4f) are satisfied with one of the following four choices: $\Phi_4 = \Phi_5 = 0$, $\Phi_5 = \phi_1 = \bar{\phi}_1 = \phi_2 = \bar{\phi}_2 = 0$, $\Phi_4 = \phi_3 = \bar{\phi}_3 = \phi_4 = \bar{\phi}_4 = 0$ and $\phi_i = \bar{\phi}_i = 0$. The constraints (4b),

$$\phi_3\bar{\phi}_4 = -\bar{\phi}_3\phi_4 , \quad \phi_1\bar{\phi}_2 = -\bar{\phi}_1\phi_2 ,$$

determine two of the ϕ_i, $\bar{\phi}_i$'s for the first choice or the second and third, while they are trivially satisfied for the fourth. It is worth pointing out also that Φ_5 is expected to be flat to all orders in string dynamics [3]. This might favour the last two choices although the first two are not forbidden.

The left over F-flatness conditions

[#1] Both the F- and D-flatness conditions have also been derived and solved in ref. [7]. Our constraints are slightly different (eqs. (4b), (4f)) because of the extra superpotential terms $\Phi_5(\bar{\phi}_3\phi_4 + \phi_3\bar{\phi}_4)$.

$$\sum_{i=1}^{4} \phi_i\bar{\phi}_i + \phi_{45}\bar{\phi}_{45} + \phi_+\bar{\phi}_+ + \phi_-\bar{\phi}_- = 0\,, \tag{5a}$$

$$\sum_{i=1}^{4} \phi_i^2 + \phi_+\phi_- + \Phi_{23}\Phi_{31} = 0\,, \quad \sum_{i=1}^{4} \bar{\phi}_i^2 + \bar{\phi}_+\bar{\phi}_- + \bar{\Phi}_{23}\bar{\Phi}_{31} = 0\,, \tag{5b,c}$$

together with the D-flatness conditions (3c) and (3d) can be thought as a set of five non-linear equations for ϕ_+, ϕ_-, $\bar{\phi}_+$, $\bar{\phi}_-$ and the six undetermined ϕ_i, $\bar{\phi}_i$'s in terms of Φ_{23}, $\bar{\Phi}_{23}$, Φ_{31}, $\bar{\Phi}_{31}$, ϕ_{45} and $\bar{\phi}_{45}$. The last six VEVs are not all independent since they are still constrained by the remaining two D-flatness conditions (3a) and (3b). There is enough freedom to choose all but one of the undetermined ϕ_i, $\bar{\phi}_i$'s and still to have a four-parameter solution. From the $(\mathrm{U}(1)')^3\times\mathrm{U}(1)_\mathrm{A}$ assignments of $\Phi_{23}(1,3,3,3)$, $\Phi_{31}(-1,1,-1,-5)$, $\phi_{45}(1,-1,6,0)$, $\phi_i(0,2,1,-1)$, $\phi_+(2,2,-1,-2)$, $\phi_-(-2,2,3,0)$ it is evident that all the surplus U(1)'s can be broken [#2].

The singlet VEV pattern has an immediate effect on the Higgs pentaplet mass matrix determining the number of massless pentaplets. This can be read off from (1) in the form $h_i\bar{h}_iM_{ij}$ with

$$M = \begin{pmatrix} 0 & 0 & \bar{\Phi}_{31} & 0 \\ 0 & 0 & \Phi_{23} & 0 \\ \Phi_{31} & \bar{\Phi}_{23} & 0 & \bar{\phi}_{45} \\ 0 & 0 & \phi_{45} & 0 \end{pmatrix}, \tag{6}$$

and $h_i = (h_1, h_2, h_3, h_{45})$, $\bar{h}_i = (\bar{h}_1, \bar{h}_2, \bar{h}_3, \bar{h}_{45})$. If S and T are two unitary matrices with the property $(SMT^\dagger)_{ij} = m_i\delta_{ij}$ it immediately follows that $SMM^\dagger S^\dagger = TM^\dagger MT^\dagger = |m|^2$. The unitary rotations $h' = T\bar{h}$ and $h' = S^*h$ lead to $h^\perp M\bar{h} = m_i h'_i \bar{h}'_i$. Therefore, in order to diagonalize the pentaplet mass matrix, we should first determine the eigenstates m_i of $MM^\dagger$ and $M^\dagger M$ and then the matrices S, T in terms of which the eigenfields h'_i, $\bar{h}'_i$ are defined. Assuming real VEVs for simplicity, we obtain the eigenvalues

$$0,\quad 0,\quad m = (\bar{\Phi}_{31}^2 + \Phi_{23}^2 + \phi_{45}^2)^{1/2}\,, \quad m' = (\Phi_{31}^2 + \bar{\Phi}_{23}^2 + \bar{\phi}_{45}^2)^{1/2}\,. \tag{7}$$

The eigenvectors are better expressed in terms of angles,

$$\theta \equiv \arctan\left(-\frac{\phi_{45}}{\Phi_{23}}\right), \quad \theta' \equiv \arctan\left(\frac{\bar{\Phi}_{31}}{\Phi_{23}}\right), \quad \bar{\theta} \equiv \arctan\left(-\frac{\bar{\phi}_{45}}{\bar{\Phi}_{23}}\right), \quad \bar{\theta}' \equiv \arctan\left(\frac{\Phi_{31}}{\bar{\Phi}_{23}}\right). \tag{8}$$

They are

$$h_0 = -\cos\theta'\, h_1 + \sin\theta'\, h_2\,, \quad h'_0 = -N[\tan\theta\,(\sin\theta'\, h_1 + \cos\theta'\, h_2) + (\cos\theta')^{-1}h_{45}]\,,$$

$$h_m = N[\tan\theta'\, h_1 + h_2 - \tan\theta\, h_{45})\,, \quad h'_m = h_3\,, \tag{9}$$

as well as the "barred" ones $\bar{h}_0$, $\bar{h}'_0$, $\bar{h}_m$, $\bar{h}'_m$ obtained by the changes $\theta\to\bar{\theta}$, $\theta'\to\bar{\theta}'$ and $m\leftrightarrow m'$. The factors N and $\bar{N}$ stand for $N = (1+\tan^2\theta+\tan^2\theta')^{-1/2}$ and $\bar{N} = (1+\tan^2\bar{\theta}+\tan^2\bar{\theta}')^{-1/2}$.

The original pentaplets can be reexpressed in terms of the eigenfields. Neglecting the massive fields, we get

$$h_1 = -\cos\theta'\, h_0 - N\tan\theta\sin\theta'\, h'_0 + \dots\,, \quad h_2 = \sin\theta'\, h_0 - N\tan\theta\cos\theta'\, h'_0 + \dots\,,$$

$$h_{45} = -N(\cos\theta')^{-1}h'_0 + \dots\,, \quad h_3 = 0 + \dots\,, \tag{10}$$

[#2] There is actually one special solution of both F- and D-flatness constraints, $\phi = \bar{\phi}_i = \Phi_{23} = \bar{\Phi}_{23} = \phi_+ = \bar{\phi}_+ = \Phi_{12} = \bar{\Phi}_{12} = \Phi_3 = 0$, $3(|\phi_{45}|^2 - |\bar{\phi}_{45}|^2) = |\Phi_{31}|^2 - |\bar{\Phi}_{31}|^2 = 3(|\bar{\phi}_-|^2 - |\phi_-|^2) = \frac{1}{5}\xi$, $\phi_{45}\bar{\phi}_{45} + \phi_-\bar{\phi}_- = 0$, which keeps one extra U(1) factor $\mathrm{U}(1)'_1 + \mathrm{U}(1)'_2$ unbroken. See also ref. [7].

as well as the corresponding expressions for the "barred" fields. There is always the possibility of an O(2) rotation among the massless fields h_0, h'_0.

It is interesting to note that due to the D-flatness conditions (3a) and (3b), the choices $\bar{\Phi}_{31} = \Phi_{23} = \phi_{45} = 0$ or $\Phi_{31} = \bar{\Phi}_{23} = \bar{\phi}_{45} = 0$ are forbidden. Thus, we always have for the Higgs pentaplet masses $m \neq 0$ and $m' \neq 0$. We can say that the anomalous U(1) D-term through the D-flatness conditions determines the number of massless pentaplet pairs to two. Another interesting constraint is obtained from the difference $m'^2 - m^2$ which, if we subtract (3a) and (3b) becomes

$$m'^2 - m^2 = \tfrac{2}{15}\xi .$$

Substituting the definitions (8) in m and m', we obtain $m'^2 = (\bar{\phi}_{45})^2/(\bar{N}^2 \tan^2\bar{\theta})$ and $m^2 = (\phi_{45})^2/(N^2 \tan^2\theta)$, which, using (3a), gives

$$m^2 N^2 \tan^2\theta - m'^2 \bar{N}^2 \tan^2\bar{\theta} = \tfrac{1}{15}\xi .$$

Solving this system of equations for the masses, we get

$$m^2 = \frac{\xi}{15} \frac{(1 + 2\bar{N}^2 \tan^2\bar{\theta})}{(N^2 \tan^2\theta - \bar{N}^2 \tan^2\bar{\theta})}, \quad m'^2 = \frac{\xi}{15} \frac{(1 + 2N^2 \tan^2\bar{\theta})}{(N^2 \tan^2\theta - \bar{N}^2 \tan^2\bar{\theta})} .$$

These equations imply the inequalities [#3]

$$N^2 \tan^2\theta > \bar{N}^2 \tan^2\bar{\theta} , \tag{11a}$$

for $\xi > 0$, or

$$N^2 \tan^2\theta < \bar{N}^2 \tan^2\bar{\theta} , \tag{11b}$$

for $\xi < 0$. These inequalities have interesting phenomenological consequences. As will become evident later, the fields F_4, $\bar{f}_5$, l_5^c can play the role of the top family of quarks and leptons with masses "m_b"$= g\sqrt{2}\langle h_1 \rangle$, "$m_t$"$= g\sqrt{2}\langle \bar{h}_{45} \rangle$ and "m_τ"$= g\sqrt{2}\langle h_2 \rangle$ as can be read from (1). These masses, in terms of the massless doublets in (10), become $g\sqrt{2}(\cos\theta' \langle h_0 \rangle + N \tan\theta \sin\theta' \langle h'_0 \rangle)$, $g\sqrt{2}[(\bar{N}/\cos\bar{\theta}')\langle \bar{h}'_0 \rangle]$ and $g\sqrt{2}(N \times \tan\theta \cos\theta' \langle h'_0 \rangle - \sin\theta' \langle h_0 \rangle)$ correspondingly. It is not difficult to see that

$$\left(\frac{m_t}{g\sqrt{2}\langle \bar{h}'_0 \rangle}\right)^2 + \left(\frac{m_b}{g\sqrt{2}\langle h'_0 \rangle}\right)^2 + \left(\frac{m_\tau}{g\sqrt{2}\langle h'_0 \rangle}\right)^2 = \left(\frac{\langle h_0 \rangle}{\langle h'_0 \rangle}\right)^2 + \frac{\bar{N}^2}{\cos^2\bar{\theta}'} + N^2 \tan^2\theta$$
$$= \left(\frac{\langle h_0 \rangle}{\langle h'_0 \rangle}\right)^2 + 1 + N^2 \tan^2\theta - \bar{N}^2 \tan^2\bar{\theta} .$$

If $\langle h_0 \rangle = \langle \bar{h}_0 \rangle = 0$ and $M_W^2 = \frac{1}{2} g^2 (\langle h'_0 \rangle^2 + \langle \bar{h}'_0 \rangle^2)$, we get

$$m_t^2 + u^2(m_b^2 + m_\tau^2) = \frac{4u^2}{1+u^2} M_W^2 (1 + N^2 \tan^2\theta - \bar{N}^2 \tan^2\bar{\theta}) ,$$

in terms of $u \equiv \langle h'_0 \rangle / \langle \bar{h}'_0 \rangle$. Thus, making use of (11), we end up with the inequalities

$$m_t^2 + u^2(m_b^2 + m_\tau^2) > \frac{4u^2}{1+u^2} M_W^2 \quad \xi > 0 , \qquad m_t^2 + u^2(m_b^2 + m_\tau^2) < \frac{4u^2}{1+u^2} M_W^2 \quad \xi < 0 . \tag{12a,b}$$

In the general case where all doublets aquire VEVs we have

[#3] Although ξ is by our definition positive, we prefer to be more general in order to circumvent a sign dispute that has arisen in the calculation of the anomalous D-term [8].

$$m_t^2+\left(\frac{\langle \bar{h}'_0\rangle}{\langle h'_0\rangle}\right)^2(m_b^2+m_\tau^2)\gtrsim 4M_W^2\frac{\langle \bar{h}'_0\rangle^2(\langle h_0\rangle^2+\langle h'_0\rangle^2)}{\langle h'_0\rangle^2(\langle h_0\rangle^2+\langle h'_0\rangle^2+\langle \bar{h}_0\rangle^2+\langle \bar{h}'_0\rangle^2)}\,. \tag{12c}$$

The remarkable fact about these inequalities is that they are a direct consequence of th D-flatness conditions and the surplus U(1) anomaly. We shall come back to them later [#4].

Up to now we examined the $(\mathrm{U}(1))^4$ symmetry breaking setting all $\mathrm{SU}(5)\times\mathrm{U}(1)'$ breaking VEVs to zero. This is consistent with the expectation that the $\mathrm{SU}(5)\times\mathrm{U}(1)'$ breaking occurs at a scale M_X lower than the natural scale $\xi^{1/2}$ of the surplus U(1) breakdown. This is confirmed by a renormalization group analysis. Thus, the exact form of the $(\mathrm{U}(1))^4$ D-flatness conditions will contain corrections of $\mathrm{O}((M_X/M_P)^2)$ which will have a negligible effect in what has been said. The F-flatness conditions will not be modified at all apart from the contribution of the term $F_4\bar{F}_5\phi_3$ which will not have any effect if two of the appearing fields get zero VEVs. The complete set of F-flatness conditions arising from the cubic superpotential is satisfied with our solution for the singlets plus independent non-vanishing VEVs for the SU(5) decaplets that break $\mathrm{SU}(5)\times\mathrm{U}(1)'$ down to $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$. Although the two scales are completely independent in the context of the F-flatness conditions, as we remarked before, they are mixed by the $(\mathrm{U}(1))^4$ D-flatness conditions but this is of no relevance since the renormalization group supports an M_X a few orders of magnitude lower than the surplus U(1) breaking scale.

The $\mathrm{SU}(5)\times\mathrm{U}(1)'\to\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ breaking is achieved with non-vanishing VEVs for $F_i(10,\frac{1}{2})$ and $\bar{F}_5(\overline{10},-\frac{1}{2})$ fields. $\mathrm{SU}(5)\times\mathrm{U}(1)'$ D-flatness dictates that

$$\sum_{i=1}^{4}\langle N_i^c\rangle^2=\langle \bar{N}_5^c\rangle^2\,.$$

In order to keep the direction $\bar{F}_5$ flat it is necessary to take $\phi_3=0$ [#5] in view of the term $F_4\bar{F}_5\phi_3$. Since the only relevant u-quark coupling that appears at the cubic level involves F_4, it is phenomenologically appealing to take $\langle F_4\rangle=0$, otherwise the model would not account for a top quark heavier than the rest of the fermions. This choice has the extra phenomenological merit that it allows the "right-handed" neutrino to combine with ϕ_3 through a Dirac mass term $g\langle \bar{N}_5^c\rangle N_4^c\phi_3$ [3]. From now on we shall restrict ourselves to the choice $F_4=0$. Next, we can immediately observe that F_3 couples exclusively to a supermassive pentaplet while F_1, F_2 and F_4 have couplings to the massless pentaplets h_0, h'_0, $\bar{h}'_0$ and necessarily to the massless colour triplets in them. A realization of the doublet–triplet splitting mechanism [3] requires that the breaking does not occur exclusively in the direction of F_3. The simplest choice is to take $\langle F_3\rangle=0$. Assuming that $\langle F_4\rangle=\langle F_3\rangle=0$, we can parametrize the VEVs as $\langle \bar{N}_5^c\rangle=V$, $\langle N_1^c\rangle=V\sin\alpha$ and $\langle N_2^c\rangle=V\cos\alpha$. A set of new decaplets can be defined as

$$F_1=\sin\alpha\,F'_1-\cos\alpha\,F'_2\,,\quad F_2=\cos\alpha\,F'_1+\sin\alpha\,F'_2\,,$$

so that $\langle N_1^{c\prime}\rangle=V$ and $\langle N_2^{c\prime}\rangle=0$. In terms of the new fields we have

$$\begin{aligned}F_1F_1h_1+F_2F_2h_2&=(\sin^2\alpha\,h_1+\cos^2\alpha\,h_2)F'_1F'_1+(\cos^2\alpha\,h_1+\sin^2\alpha\,h_2)F'_2F'_2-2\cos\alpha\sin\alpha\,(h_1-h_2)F'_1F'_2\\&=2V(\sin^2\alpha\,D_1+\cos^2\alpha\,D_2)d_1^{c\prime}-\mathrm{V}\sin(2\alpha)\,(D_1-D_2)d_2^{c\prime}-\sin(2\alpha)\,(H_1-H_2)d_1^{c\prime}Q'_2\\&\quad+(\cos^2\alpha\,h_1+\sin^2\alpha\,h_2)F'_2F'_2\,.\end{aligned} \tag{13}$$

We have symbolized as D_1 and D_2 the coloured triplets in h_1 and h_2 and as H_1 and H_2 the Higgs isodoublets. The contents of the decaplets are d^c, Q, N^c in standard notation. In addition to the above couplings, $\bar{F}_5\bar{F}_5\bar{h}_2$ gives $2V\bar{d}_5^c\bar{D}_2$. As can be seen in (13), for a general direction $0<\alpha<\frac{1}{2}\pi$ the surviving component of F'_1, namely $d_1^{c\prime}$, gets a superheavy mass by its coupling to the combination $\sin^2\alpha\,D_1+\cos^2\alpha\,D_2$. The same is true for $d_2^{c\prime}$ which

[#4] These relations are, of course, considered at the unification scale.

[#5] This implies that one of the $\bar{\phi}_3$, ϕ_4 must be zero as well due to the condition (4b).

obtains a superheavy mass combining with $\sin(2\alpha)$ $(D_1 - D_2)$. As a result in this case there are not enough right-handed d-quarks to complete three generations. A state with the necessary quantum numbers exists, namely $\bar{D}_1$ in $\bar{h}_1$, but it is missing the appropriate couplings to play that role at least at this level of the superpotential. For the specific direction $\alpha = \frac{1}{2}\pi$, only the pair d_1^c, D_1 becomes massive, since

$$F_1 F_1 h_1 + F_2 F_2 h_2 = 2 V D_1 d_1^c + F_2 F_2 h_2 .$$

In this case, the light spectrum consists of three complete F's, namely F_4, F_3 and F_2, three complete $\bar{f}$'s out of $\bar{f}_5, \bar{f}_1, \bar{f}_2, \bar{f}_3$, of which one together with f_4 are not wanted, and three l^c's out of the l_5^c, l_1^c, l_2^c, l_3^c, of which one together with $\bar{l}_4^c$ are not wanted as well. The case $\alpha = 0$, for which

$$F_1 F_1 h_1 + F_2 F_2 h_2 = 2 V D_2 d_2^c + h_1 F_1 F_1 ,$$

has identical particle content with the previous one. It is certainly a different case, however, since the superpotential couplings for it are different. In both these cases we have the additional massless coloured triplets D_2 (D_1) together with $\bar{D}_1$ which has no chiral couplings at the cubic level. With the above analysis it is evident that the phenomenologically promising cases are either $\langle F_1 \rangle = \langle \bar{F}_5 \rangle = V$ or $\langle F_2 \rangle = \langle \bar{F}_5 \rangle = V$. There also exist at the cubic level terms $(f_4 \bar{f}_5 + \bar{l}_4^c l_5^c)\bar{\phi}_2$ that could rid the light spectrum of the unwanted states through a VEV for $\bar{\phi}_2$. Nevertheless, they involve $\bar{f}_5$ which contains the only candidate for the right-handed top quark since $\bar{f}_5$ participates in a u-quark mass term at the cubic level. Thus, we should adhere to $\bar{\phi}_2 = 0$ and keep the unwanted states temporarily with us until at some higher order a non-renormalizable term makes them massive.

The fact that the $(\mathrm{U}(1))^4$ breaking scale is quite close to M_{P} makes it necessary to investigate the effects of non-renormalizable superpotential corrections [4] to the Yukawa couplings [3,4,9]. This is certainly welcome since a phenomenological analysis of the fermion mass terms is hardly satisfactory at the cubic level and the only hopeful direction we could look into is the direction of non-renormalizable couplings. The number of non-renormalizable terms increases rapidly with order and it would be in principle a difficult task to find general solutions to the F-flatness conditions. It turns out that for the quartic and quintic terms our solution to the F-flatness conditions obtained at the cubic level suffices and F-flatness is maintained. The quartic part of the observable sector superpotential is very simple [#6]. It just consists of the two terms

$$(F_2 \bar{f}_2 \bar{\phi}_4 \bar{h}_{45} + \bar{f}_1 F_1 \phi_1 \bar{h}_{45})/M . \tag{14}$$

For $\langle F_1 \rangle$, $\langle \phi_1 \rangle \neq 0$, the doublets in $\bar{f}_1$ and $\bar{h}_{45}$ become massive. For $\langle F_1 \rangle$, $\langle \bar{\phi}_4 \rangle \neq 0$ and $\langle \phi_1 \rangle = 0$, we obtain a u-quark mass term $(\langle \bar{\phi}_4 \rangle / M) F_2 \bar{f}_2 \bar{h}_{45}$. For $\langle F_2 \rangle$, $\langle \bar{\phi}_4 \rangle \neq 0$, the doublets in $\bar{f}_2$ and $\bar{h}_{45}$ become massive. For $\langle F_2 \rangle$, $\langle \phi_1 \rangle \neq 0$ and $\langle \bar{\phi}_4 \rangle = 0$, we obtain a u-quark mass term $(\langle \phi_1 \rangle / M) F_1 \bar{f}_1 \bar{h}_{45}$.

An enormously simplifying act at the quintic level, that allows us to maintain F-flatness without modifying our singlet VEV solution obtained at the cubic level, is the absence of any terms Φ^5 involving only singlet fields. No such terms are present. In addition, no terms $h\bar{h}\Phi^3$ are present either and the pentaplets mass matrix is not modified. Terms of the type $h\bar{h}h\bar{h}\Phi$ would not, obviously, have any effect on anything. Potentially important terms, however, might be the ones of the type $F\bar{F}\Phi^3$ and $(F\bar{F})^2\Phi$ with F, $\bar{F}$ standing for 10, $\overline{10}$'s. Apart from terms involving at least two singlets with vanishing VEVs which are not of any relevance to F-flatness, the only terms of this type are $\bar{F}_5 \bar{F}_5 F_4 F_4 \Phi_{12}$ and $\bar{F}_5 \bar{F}_5 F_1 F_1 \Phi_{12}$. When the $\mathrm{SU}(5) \times \mathrm{U}(1)'$ breaking occurs in the direction $\langle \bar{F}_5 \rangle = \langle F_1 \rangle = V$, the second term introduces a $(V/M)^4$ correction to the F-flatness condition for Φ_{12}. This is a small correction to eq. (5c) that prescribes a small modification to its solution.

The remaining possible quintic terms can be classified in a class of terms that do not involve any Higgs pentaplets and a class of terms that do. The first class involves terms with at least three vanishing VEVs except the four terms

$$(\bar{F}_5 F_2 \bar{l}_4^c l_2^c \bar{\phi}_2 + \bar{F}_5 F_2 f_4 \bar{f}_2 \bar{\phi}_2 + \bar{F}_5 F_1 \bar{l}_4^c l_1^c \phi_3 + \bar{F}_5 F_1 f_4 \bar{f}_1 \phi_3)/M^2 . \tag{15}$$

[#6] The coefficient of each non-renormalizable term can be calculated [4]. For the purposes of our analysis we have just derived the non-vanishing non-renormalizable terms and assumed the coefficient to be O(1).

The second class can be further divided by considering the number of Higgs pentaplets that appear. Obviously those terms involving more than one Higgs pentaplet, matter fields and, possibly, a singlet (e.g. $\bar{f}\bar{f}h\bar{h}\Phi$) are of no relevance. That is not true for the terms that contain one Higgs pentaplet which are very important. These terms are

$$\begin{aligned}
&\bar{F}_5\bar{F}_5\bar{h}_1(\phi_1^2+\phi_4^2+\phi_+\phi_-)+\bar{F}_5\bar{F}_5\bar{h}_{45}\Phi_{23}\bar{\phi}_{45}+F_4F_4h_2(\phi_1^2+\phi_4^2+\phi_+\phi_-)+F_4F_4h_{45}\Phi_{31}\phi_{45}\\
&+F_1F_1h_2\left(\sum_{i=1}^{4}\phi_i^2+\phi_+\phi_-\right)+F_2F_2h_1(\bar{\phi}_1^2+\bar{\phi}_2^2+\bar{\phi}_3^2+\bar{\phi}_+\bar{\phi}_-)+F_1F_1h_{45}\Phi_{31}\phi_{45}+F_2F_2h_{45}\bar{\Phi}_{23}\phi_{45}\\
&+\bar{f}_1l_1^ch_2\left(\sum_{i=1}^{4}\phi_i^2+\phi_+\phi_-\right)+\bar{f}_1l_1^ch_{45}\Phi_{31}\phi_{45}+\bar{f}_2l_2^ch_1\left(\sum_{i=1}^{4}\bar{\phi}_i^2+\bar{\phi}_+\bar{\phi}_-\right)+\bar{f}_2l_2^ch_{45}\bar{\Phi}_{23}\phi_{45}+\bar{f}_5l_2^ch_2\bar{F}_5F_2\\
&+\bar{f}_5l_5^ch_1(\bar{\phi}_1^2+\bar{\phi}_4^2+\bar{\phi}_+\bar{\phi}_-)+\bar{f}_5l_5^ch_{45}\bar{\Phi}_{23}\phi_{45}+\bar{f}_2l_5^ch_2\bar{F}_5F_2+f_4\bar{l}_4^c\bar{h}_2(\bar{\phi}_1^2+\bar{\phi}_4^2+\bar{\phi}_+\bar{\phi}_-)\\
&+f_4\bar{l}_4^c\bar{h}_{45}\bar{\Phi}_{31}\bar{\phi}_{45}+F_4\bar{f}_2\bar{h}_{45}F_2\bar{F}_5\,. \qquad (16)
\end{aligned}$$

The only singlet VEV choice, in addition to our solution of the flatness conditions we made based on phenomenological arguments, is $\phi_3=\bar{\phi}_2=0$. In the view of (4b) this splits into the four possibilities $\phi_3=\bar{\phi}_2=\bar{\phi}_3=\bar{\phi}_1=0$, $\phi_3=\bar{\phi}_2=\bar{\phi}_3=\phi_2=0$, $\phi_3=\bar{\phi}_2=\phi_4=\bar{\phi}_1=0$, $\phi_3=\bar{\phi}_2=\phi_4=\phi_2=0$. Concerning the $SU(5)\times U(1)'$ breaking, we have, for the moment, the choice between $\langle F_1\rangle\neq 0$ and $\langle F_2\rangle\neq 0$. Let us start with the examination of the first case $\langle F_1\rangle=\langle\bar{F}_5\rangle=V$. In that case, the quartic terms (14) demand $\phi_1=0$ in addition to $\phi_3=\bar{\phi}_2=0$ [#7]. The relevant mass terms are

$$\begin{aligned}
&F_2F_2[h_2+h_1(\bar{\phi}_1^2+\bar{\phi}_3^2+\bar{\phi}_+\bar{\phi}_-)+h_{45}\bar{\Phi}_{23}\phi_{45}]+F_2\bar{f}_2\bar{h}_{45}\bar{\phi}_4+F_4F_4[h_1+h_2(\phi_4^2+\phi_+\phi_-)+h_{45}\Phi_{31}\phi_{45}]+F_4\bar{f}_5\bar{h}_{45}\\
&+\bar{f}_1l_1^c[h_1+h_2(\phi_2^2+\phi_4^2+\phi_+\phi_-)+h_{45}\Phi_{31}\phi_{45}]+\bar{f}_2l_2^c[h_2+h_1(\bar{\phi}_1^2+\bar{\phi}_3^4+\bar{\phi}_4^2+\bar{\phi}_+\bar{\phi}_-)+h_{45}\bar{\Phi}_{23}\phi_{45}]\\
&+\bar{f}_5l_5^c[h_2+h_1(\bar{\phi}_1^2+\bar{\phi}_4^2+\bar{\phi}_+\bar{\phi}_-)+h_{45}\bar{\Phi}_{23}\phi_{45}]+f_4\bar{l}_4^c[\bar{h}_1+\bar{h}_2(\bar{\phi}_1^2+\bar{\phi}_4^2+\bar{\phi}_+\bar{\phi}_-)+\bar{h}_{45}\bar{\Phi}_{31}\bar{\phi}_{45}]\,. \qquad (17)
\end{aligned}$$

The top generation can be made out of F_4, $\bar{f}_5$ and l_5^c. The u-quark of this generation ("t")gets a mass $g\bar{H}_{45}$ while the corresponding d-quark ("b") a mass $g[H_1+(H_2/M^2)(\phi_4^2+\phi_+\phi_-)+(H_{45}/M^2)\Phi_{31}\phi_{45}]$. The charged lepton of this family ("τ") gets a mass $g[H_2+(H_1/M^2)(\bar{\phi}_1^2+\bar{\phi}_4^2+\bar{\phi}_+\bar{\phi}_-)+(H_{45}/M^2)\bar{\Phi}_{23}\phi_{45}]$ while the corresponding left-handed neutrino, mostly ν_5, remains light and the right-handed neutrino becomes supermassive through $N_4^c\phi_3 V$. We can consider F_2, $\bar{f}_2$, l_2^c as the second generation with a u-quark mass $g\bar{H}_{45}(\bar{\phi}_4/M)$, naturally suppressed in comparison to the mass of the top generation. Nevertheless, the d-quark mass for this generation is $g[H_2+(H_1/M^2)(\bar{\phi}_1^2+\bar{\phi}_3^2+\bar{\phi}_+\bar{\phi}_-)+(H_{45}/M^2)\bar{\Phi}_{23}\phi_{45}]$ while the corresponding charged lepton mass is $g[H_2+(H_1/M^2)(\bar{\phi}_1^2+\bar{\phi}_4^2+\bar{\phi}_+\bar{\phi}_-)+(H_{45}/M^2)\bar{\Phi}_{23}\phi_{45}]$, which are not suppressed in comparison to the masses of the top generation in any obvious way. The neutrino of this generation has a large Dirac mass. This general mass pattern is not satisfactory but can be partially remedied in various ways. For instance, we could choose the parameters $\theta=\bar{\theta}=0$, i.e. $\phi_{45}=\bar{\Phi}_{31}=0$, and make h_2 superheavy [#8]. The resulting mass pattern has an improved hierarchical structure

$$\text{"}m_t\text{"}\sim g\langle\bar{H}_0'\rangle\frac{\bar{N}}{\cos\bar{\theta}'}\,,\quad \text{"}m_\tau\text{"}\sim g\langle H_0\rangle\,,\quad \text{"}m_\tau\text{"}\sim g\langle H_0\rangle O\left(\frac{\phi^2}{M^2}\right),$$

$$\text{"}m_c\text{"}\sim g\langle\bar{H}_0'\rangle\frac{\bar{N}}{\cos\bar{\theta}'}\left(\frac{\bar{\phi}_4}{M}\right),\quad \text{"}m_s\text{"}\sim g\langle H_0\rangle O\left(\frac{\phi^2}{M^2}\right),\quad \text{"}m_\mu\text{"}\sim g\langle H_0\rangle O\left(\frac{\phi^2}{M^2}\right),$$

but this is not entirely satisfactory, mainly because of "m_b" in relation to "m_t". The rest of the masses can be

[#7] Conditions (4b) now demand $\bar{\phi}_1\phi_2=\bar{\phi}_3\phi_4=0$.
[#8] This is only possible for $\xi<0$.

expected to be corrected by higher order effects. In order to have a phenomenologically acceptable m_b/m_t ratio we must have $\langle h_0\rangle \ll \langle \bar{h}'_0\rangle$, $\langle h'_0\rangle$. This reduces the inequality analogous to (12b) into

$$m_t^2+\left(\frac{\langle \bar{h}'_0\rangle}{\langle h'_0\rangle}\right)^2 m_b^2<4M_W^2\frac{\langle \bar{h}'_0\rangle^2}{\langle h'_0\rangle^2+\langle \bar{h}'_0\rangle^2},$$

which implies a relatively light top quark [#9]. This is even more clear in the case $\langle h'_0\rangle=\langle \bar{h}'_0\rangle$, when

$$m_t^2+m_b^2<2M_W^2 .$$

Another possibility in order to get a more satisfactory mass pattern is to assume $\langle h_0\rangle=0$ in which case

$$\text{“}m_b\text{''}=g\sqrt{2}\,N\tan\theta\sin\theta'\ \langle h'_0\rangle\ ,\qquad \text{“}m_\tau\text{''}=\text{“}m_s\text{''}=\text{“}m_\mu\text{''}=g\sqrt{2}\,N\tan\theta\cos\theta'\ \langle h'_0\rangle\ ,$$

and the rest as before. The second equality is certainly wrong but if we ignore it and believe only in the expressions for the heaviest quarks, we obtain from (12)

$$m_t^2+u^2m_b^2>\frac{4u^2}{1+u^2}M_W^2\quad \xi>0\ ,\qquad m_t^2+u^2m_b^2<\frac{4u^2}{1+u^2}M_W^2\quad \xi<0\ .$$

A heavy top quark can be obtained for $\xi>0$. For instance, in the case $\theta'\sim\pi/2-\epsilon$, $m_b\sim g\tan\theta\cos\theta'\ \langle h'_0\rangle$ and we can have a heavy top quark (see footnote 9)

$$m_t^2>2M_W^2$$

for $u=1$.

Let us complete our analysis considering the case $\langle F_2\rangle=\langle \bar{F}_5\rangle=V$. In this case, the quartic terms (14) dictate $\bar{\phi}_4=0$ in addition to $\phi_3=\bar{\phi}_2=0$ (see footnote 7). The relevant mass terms are

$$\begin{aligned}&F_1F_1\left[h_1+h_2\left(\sum_{i=1}^{4}\phi_i^2+\phi_+\phi_-\right)+h_{45}\Phi_{31}\phi_{45}\right]+F_1\bar{f}_1\bar{h}_{45}\phi_1+F_4F_4[h_1+h_2(\phi_4^2+\phi_1^2+\phi_+\phi_-)+h_{45}\Phi_{31}\phi_{45}]\\&\quad+F_4\bar{f}_5\bar{h}_{45}+\bar{f}_1l_1^c[h_1+h_2(\phi_1^2+\phi_2^2+\phi_4^2+\phi_+\phi_-)+h_{45}\Phi_{31}\phi_{45}]+\bar{f}_5l_2^ch_2V^2\\&\quad+\bar{f}_2l_2^c[h_2+h_1(\bar{\phi}_1^2+\bar{\phi}_3^2+\bar{\phi}_+\bar{\phi}_-)+h_{45}\bar{\Phi}_{23}\phi_{45}]+F_4\bar{f}_2\bar{h}_{45}V^2+\bar{f}_5l_5^c[h_2+h_1(\bar{\phi}_1^2+\bar{\phi}_+\bar{\phi}_{p-})+h_{45}\bar{\Phi}_{23}\phi_{45}]\\&\quad+f_4\bar{l}_4^c[\bar{h}_1+\bar{h}_2(\bar{\phi}_1^2+\bar{\phi}_+\bar{\phi}_-)+\bar{h}_{45}\bar{\Phi}_{31}\bar{\phi}_{45}]\ .\end{aligned}\tag{18}$$

Again, the top generation F_4, $\bar{f}_5$, l_5^c has a “t”-quark mass $g\langle \bar{H}_{45}\rangle$, a “b”-quark mass $g[H_1+(H_2/M^2)(\phi_1^2+\phi_4^2+\phi_+\phi_-)+(H_{45}/M^2)\Phi_{31}\phi_{45}]$ and a “τ”-lepton mass $g[H_2+(H_1/M^2)(\bar{\phi}_1^2+\bar{\phi}_+\bar{\phi}_-)+(H_{45}/M^2)\bar{\Phi}_{23}\phi_{45}]$ and a light neutrino as in the previous case. A second generation can be made out of F_1, $\bar{f}_1$ and l_1^c with u-quark mass $g\langle \bar{H}_{45}\rangle\phi_1/M$, naturally suppressed in comparison of the “top” mass. The d-quark and charged lepton masses for this generation are both equal to $g[H_1+(H_2/M^2)(\phi_1^2+\phi_2^2+\phi_4^2+\phi_+\phi_-)+(H_{45}/M^2)\Phi_{31}\phi_{45}]$. The equality of d-quark masses for the two generations in this case cannot be circumvented even if we assume a special isodoublet VEV direction for the $SU(2)_L\times U(1)_Y$ breaking. Thus, this case should be declared as phenomenologically unsatisfactory. We can conclude that the direction $\langle F_1\rangle=V$ seems to be phenomenologically more promising. We expect that higher order non-renormalizable terms will introduce naturally suppressed masses for the yet decoupled lightest generation F_3, $\bar{f}_3$, l_3^c. In addition one has to show that higher order terms will provide masses for the unwanted states f_4, $\bar{l}_4^c$ together with a combination of $\bar{f}_i$, l_i^c. In order to be able to answer the question of the phenomenological viability of the model conclusively, higher order superpotential terms have to be calculated. Nevertheless, so far the hierarchical structure of the fermion masses is a unique and very appealing feature of the model that sets in automatically in contrast to older field theoretic models.

In conclusion, it is worthwhile pointing out some interesting features of the above analysis. We show that the

[#9] This relation is of course considered at the unification scale.

D-flatness conditions determined the number of massless pentaplets of the model. We also extracted the inequality (see footnote 9)

$$m_{\mathrm{t}}^2+\left(\frac{\langle \bar{h}_0' \rangle}{\langle h_0' \rangle}\right)^2 (m_{\mathrm{b}}^2+m_\tau^2) \gtrless 4M_{\mathrm{W}}^2 \frac{\langle \bar{h}_0' \rangle^2(\langle h_0 \rangle^2+\langle h_0' \rangle^2)}{\langle h_0' \rangle^2(\langle h_0 \rangle^2+\langle h_0' \rangle^2+\langle \bar{h}_0 \rangle^2+\langle \bar{h}_0' \rangle^2)},$$

for $\xi \gtrless 0$, that relates the heaviness or lightness of the top-generation charged fermion masses to M_{W} and to the $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y$ breaking VEV ratios. In addition, we saw that the unique $\mathrm{SU}(5) \times \mathrm{U}(1)' \times \mathrm{U}(1)^4$ breaking VEV pattern determined at the cubic level satisfies F-flatness even when quartic and quintic non-renormalizable terms are taken into account.

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