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SCALAR PARAMETERS IN EFFECTIVE GAUGE THEORIES

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We apply Weinberg's method of effective gauge theories to the study of light scalar masses and coupling constants, in a simplified model with two widely separated mass scales. No improvement of the unnaturalness in the Higgs sector is seen.

1. Introduction

The unification of the SU(3) and SU(2) $\times$ U(1) gauge groups of the observed strong and electroweak interactions in a simple gauge group G [1] inevitably leads to the existence of two enormously different mass scales [2], one associated with the breaking of the unification gauge group G down to SU(3) $\times$ SU(2) $\times$ U(1) and the other associated with the breaking SU(3) $\times$ SU(2) $\times$ U(1) $\rightarrow$ SU(3) $\times$ U$(1)_{em}$. A theory with two so disparate mass scales poses well-known problems. The central problem is whether it is possible and natural to arrange the parameters of the theory so that the desired mass hierarchy can be achieved at the tree level and then stay stable against radiative corrections. This is known as the gauge hierarchy problem [3]. A large mass hierarchy can be arranged, but only at the price of fine-tuning the Higgs and Yukawa couplings of the theory. The deep question of how such a mass hierarchy arises naturally remains unanswered, although some attempts have been made to bypass it by completely discarding the Higgs fields [4], but with no apparent success.

The next more realistic question to ask is, if, given a large mass hierarchy at the tree level, one can maintain it in the presence of radiative corrections. Once this is done, a consistent separation of light and heavy particles is possible. Without a stable gauge hierarchy, no decoupling of heavy particles and no definition of an effective light theory at low energies is meaningful. The decoupling of heavy

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particles in spontaneously broken gauge theories is supported by many explicit one-loop calculations [5]. On the other hand, different methods to define an effective low energy theory have been employed by different people [6].

In the present work, we consider an SU(2) gauge model with a triplet and a complex doublet of Higgs fields and, in analogy with more realistic models, such as SU(5), we assume it spontaneously broken according to the pattern

$$\mathrm{SU}(2) \underset{M_{\mathrm{SH}}}{\overset{V}{\rightarrow}} \mathrm{U}(1) \underset{M_{\mathrm{L}}}{\overset{v}{\rightarrow}} \text{No symmetry},$$

with $v(M_{\mathrm{L}}) \ll V(M_{\mathrm{SH}})$. We adopt Weinberg's method [7] to construct an effective theory by integrating out the heavy degrees of freedom. First, we obtain the effective parameters of the light theory by performing a one-loop functional integral over heavy fields at renormalization scales of order M_{SH} (the mass scale of the strong breaking). In defining the heavy fields, we have assumed the existence of a gauge hierarchy at the tree level. Second, we integrate the two-loop renormalization group equations (RGE) for the parameters of the light theory, in which no heavy fields appear, from low energy scales up to M_{SH}, where we use the expressions obtained at the first step as initial conditions. In this way we can see in a transparent fashion that the gauge hierarchy problem remains and is a problem of initial conditions.

Recently, Kazama et al. [8] argued that one can define a stable gauge hierarchy if one organizes the perturbation theory properly. In fact, they take as parameters the vacuum expectation values of the theory V and v and they demand $v \ll V$. The mass parameters that appear in the lagrangian do not enter into the Green functions of the theory, but they are connected with the vacuum expectation values through the minimization conditions. In this way, a stable hierarchy is possible ($v \ll V$), but the unnatural adjustment, which has to be done in each order of perturbation theory, has been carried over to the minimization conditions. To us, this merely disguises the question of the gauge hierarchy, burying the fine tuning into the minimization conditions.

The paper is organized as follows. In sect. 2 we review the essential points of Weinberg's approach of constructing a light effective gauge theory, by integrating out the heavy fields. In sect. 3 we describe the model which we are working with and derive the effective parameters to be used as boundary conditions when solving the renormalization group equations of the effective theory. In sect. 4 we discuss the effective theory and the minimization condition of its effective potential up to the two-loop level. In sect. 5 we derive the renormalization group equations for the effective theory and give their solutions. Finally, in sect. 6 we draw our conclusions.

2. Effective gauge theories

Consider a field theory with two sets of fields, χ of small or zero mass and ϕ of much larger mass. All the connected Green functions with χ external legs can be obtained from the generating functional

$$e^{iW(J)} = \int [d\chi][d\phi] \exp\left\{ iS(\chi, \phi) + i\int d^D x\, J \cdot \chi \right\}. \tag{1}$$

$S(\chi, \phi)$ is the action and J a c-number source. Integrating out the ϕ fields, we are left with an alternative expression for the connected functional:

$$e^{iW(J)} = \int [d\chi] \exp\left\{ i\tilde{S}(\chi) + i\int d^D x\, J \cdot \chi \right\}. \tag{2}$$

The new action of the effective theory is defined by

$$\tilde{S}(\chi) \equiv -i \ln\left\{ \int [d\phi] \exp[iS(\chi, \phi)] \right\} \tag{3}$$

and can be expanded in ϕ loops.

Although, in principle, all Green functions of the light field χ can be extracted from either expression, $\tilde{S}(\chi)$ is non-local and contains an infinite number of non-renormalizable terms. Hence, in practice, we would be able to compute with it only for energies for which it could be replaced by a limiting local and renormalizable lagrangian. The non-renormalizable couplings have dimensions of inverse mass to some power, this mass being naturally the scale of the ϕ fields. At energies E well below the ϕ mass scale, the non-renormalizable terms are suppressed by powers of E/M_ϕ and the effective lagrangian becomes renormalizable at low energies. The graphs participating in the loop expansion of $\tilde{S}(\chi)$ have only ϕ internal lines and hence we can deal with renormalized ϕ graphs in the external χ field. The local limit can then be taken and we end up with a local and renormalizable effective theory at low energies. The resulting theory involves only light particles.

Calculations performed with the effective theory incorporate the memory of the full theory via the definition of the effective parameters. The expressions that one obtains in perturbation theory are reliable at energies close to the large mass. They can be put in touch with the light theory computations through the renormalization group*.

For a spontaneously broken gauge theory, care must be taken whether this separation of fields can be done in a way that will leave us with an effective theory that possesses the desired local gauge invariance. Following Weinberg [7] we employ a gauge-fixing term of the form

$$-\frac{1}{2\xi} \int d^D x \sum_{\phi_i} \left[\partial_\mu \phi_i^\mu + g c_{ij\alpha} \phi_\mu^j \chi_\alpha^\mu + i g \xi (V, \tau_i \Phi) \right]^2$$

* The renormalization group equations for the light effective parameters, say to nth order in the loop expansion, must be integrated up to the heavy scale, where their expressions to $(n-1)$th order in terms of the parameters of the full theory are employed as boundary conditions.

suited to give an effective theory which is locally gauge invariant under the unbroken subgroup of χ's. The summation is only over heavy fields and V is the expectation value of the heavy Higgs field Φ. It is easy to see that, in the Landau gauge, such a prescription will lead to the usual Fadeev-Popov-de Witt (FPdW) determinant.

3. The model

As a model, where we explicitly carry out the above stated program, we have considered an SU(2) gauge theory with a triplet and a complex doublet of scalar fields. Needless to say that our results are expected to hold, with no qualitative changes, in more realistic models.

The triplet Higgs develops an expectation value which breaks the SU(2) symmetry down to U(1). Subsequently, the U(1) breaks down to no symmetry via the doublet. The fields that developed a mass, at the tree level, in the first breaking, will be designated as the heavy fields. Of course, the important issue here is whether the light fields, i.e., the fields that got mass in the second breaking or no mass at all, will remain light.

Our lagrangian is

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4}\mathrm{Tr}\left\{\left(\partial_\mu W_\nu - \partial_\nu W_\mu + g[W_\mu, W_\nu]\right)^2\right\} + \tfrac{1}{2}\mathrm{Tr}\left\{\left(\partial_\mu \Phi - \sqrt{\tfrac{1}{2}}\, gi[W_\mu, \Phi]\right)^2\right\} \\ &+ \left|\left(\partial_\mu - \sqrt{\tfrac{1}{2}}\, giW_\mu\right)H\right|^2 + \tfrac{1}{2}\mu^2 \mathrm{Tr}(\Phi^2) + \tfrac{1}{2}\nu^2 |H|^2 \\ &- \tfrac{1}{4}f(\mathrm{Tr}(\Phi^2))^2 - \tfrac{1}{4}\lambda|H|^4 - \tfrac{1}{2}\alpha|H|^2\mathrm{Tr}(\Phi^2).\end{aligned} \tag{4}$$

The $SU(2) \to U(1)$ breaking is achieved by the development of an expectation value by the triplet Φ. The lagrangian is supplemented by the appropriate gauge fixing term and the Fadeev-Popov ghost term (see appendix A and fig. 1). The adjoint Higgs is chosen to get an expectation value in the τ_3 direction

$$\langle\Phi\rangle = \sqrt{\tfrac{1}{2}}\, V\tau_3, \qquad \text{or} \qquad \langle\Phi_\alpha\rangle = V\delta_{\alpha 3}. \tag{5}$$

As a result of the breaking in the τ_3 direction, the W_μ^1 and W_μ^2 gauge fields acquire masses $M_V^2 = g^2V^2$ at the tree level. The surviving scalar particle corresponding to the field ϕ_3 gets a mass $M_\Phi^2 = 2fV^2$. These will be defined as the heavy particles of the theory.

The U(1) symmetric vacuum which the theory has chosen after the spontaneous breaking of the SU(2) symmetry is only approximate. Subsequently, the U(1) symmetry will be broken via the doublet. Nevertheless, we choose to define first a U(1) symmetric effective theory, irrespectively of any breaking of the remnant symmetry, by integrating out the heavy degrees of freedom, as explained in the

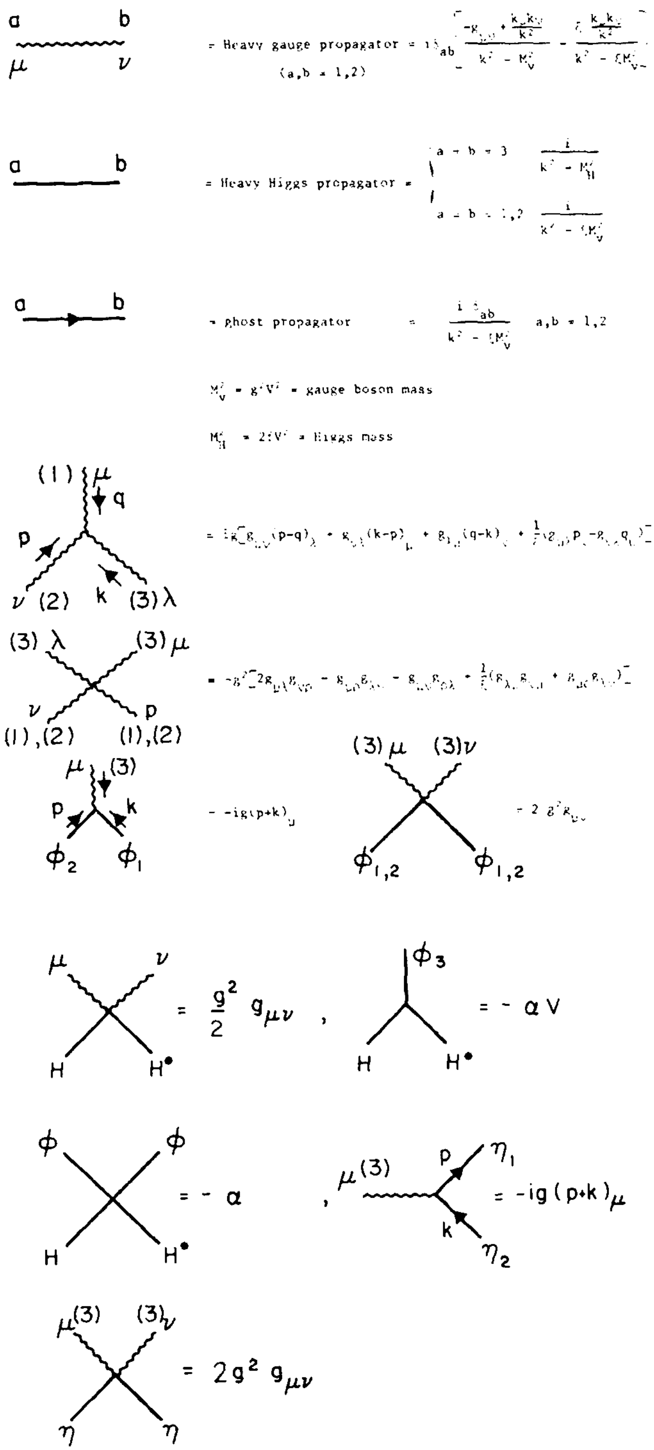


Fig. 1. Feynman rules used to calculate effective coupling and mass parameters (see figs. 2, 3, 4).

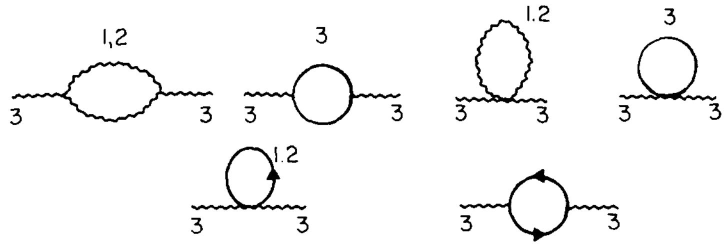


Fig. 2. Graphs involving only heavy loops leading to renormalization of the light gauge boson (photon). We only depict the non-vanishing graphs in the Landau gauge. Wavy internal lines may also designate heavy Goldstones.

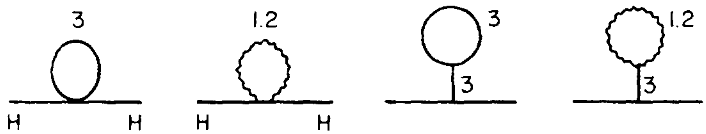


Fig. 3. Non-vanishing graphs (Landau gauge) renormalizing the light Higgs and the mass parameter. The internal wavy lines are as in fig. 2.

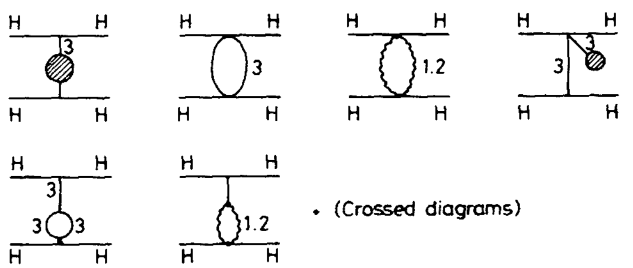


Fig. 4. Graphs leading to renormalization of the light quartic coupling. Internal gauge boson lines are as in figs. 2 and 3.

previous section. The coefficients in the light fields in the effective lagrangian, after we take the local limit, will be light particle irreducible diagrams, with only heavy fields propagating in the internal lines.

The diagrams, with heavy internal lines, contributing in the Landau gauge ($\xi = 0$) to the light gauge and scalar propagators are shown in figs. 2 and 3, respectively. The graphs of fig. 2 provide us with the renormalization of the gauge coupling, due to heavy loops. The graphs of fig. 3 provide us with the corrections of the light mass parameter ν^2, induced by the heavy fields. The graphs, which give the effective quartic coupling of the U(1) theory are depicted in fig. 4. For the effective couplings one finds*

$$g_{\text{eff}}^{-2}(\mu) = g^{-2}(\mu) - \frac{1}{24\pi^2}\left(1 - \tfrac{21}{2}\ln\left(\frac{g^2V^2}{\mu^2}\right)\right), \tag{6}$$

$$\lambda_{\text{eff}}(\mu) = \lambda(\mu) - \frac{\alpha^2}{f} + \frac{1}{8\pi^2}\left\{3g^4\left(\frac{\alpha}{f} - \tfrac{1}{2}\right)^2\ln\left(\frac{g^2V^2}{\mu^2}\right)\right.$$
$$\left. + 2\alpha\ln\left(\frac{2fV^2}{\mu^2}\right) + 2g^4\left(\frac{\alpha}{f} - \tfrac{1}{2}\right)^2\right\}. \tag{7}$$

The effective mass ν_{eff}^2 is given by

$$\nu_{\text{eff}}^2(\mu) = \nu^2 - \alpha V^2 + \frac{V^2}{8\pi^2}\left\{3g^4\left(\frac{\alpha}{f} - \tfrac{1}{2}\right)\ln\left(\frac{g^2V^2}{\mu^2}\right)\right.$$
$$\left. + 2\alpha f\ln\left(\frac{2fV^2}{\mu^2}\right) - g^4\left(\frac{\alpha}{f} - \tfrac{1}{2}\right) - 2\alpha f\right\}. \tag{8}$$

Note that $v_{\text{eff}}^2 \equiv \nu_{\text{eff}}^2/\lambda_{\text{eff}}$ would be the vacuum expectation value of the light sector, when U(1) breaks, with the contributions of heavies taken into account. At the tree level this becomes $(\nu^2 - \alpha V^2)/(\lambda - \alpha^2/f)$, which is indeed the tree level vacuum expectation value obtained by minimizing the potential.

At this point, we should comment on ν_{eff}^2. It is clear that at the tree level one needs $\alpha \sim O(v^2/V^2)$ in order to achieve a large mass separation (fine tuning). What is now disturbing is that, at the one-loop level, the condition for keeping ν_{eff}^2 light has to be changed [readjustment, see eq. (8)]. Instead of ν_{eff}^2, we could, of course, use the parameter $v_{\text{eff}}^2(\equiv \nu_{\text{eff}}^2/\lambda_{\text{eff}})$. In this way the relation $\nu_{\text{eff}}^2 = \lambda_{\text{eff}} v_{\text{eff}}^2$ simply determines ν^2 in terms of v_{eff}^2, V^2 but ν_{eff}^2 never appears in the Green functions. Taking $v_{\text{eff}} \ll V$, the light sector remains light, without further fine tuning. This is the line of reasoning followed by Kazawa et al. [8] and the relation $\nu_{\text{eff}}^2 = \lambda_{\text{eff}} v_{\text{eff}}^2$, which determines ν^2 in terms of v_{eff}^2, V^2 and the couplings, is nothing else but their formulae (III.10, 11)**, solved for ν^2, when only the contribution of the heavies is taken into account. Although there is nothing wrong with this reasoning, this does not give an answer to the gauge hierarchy problem, as stated in its initial form. The whole gauge hierarchy issue is still unresolved, because in order to take $v_{\text{eff}} \ll V$, one has to satisfy $\nu_{\text{eff}}^2(V) = \lambda_{\text{eff}} v_{\text{eff}}^2$, without further fine tuning. A consistent treatment of the theory has to take the minimization conditions into account.

* Dimensional regularization and minimal subtraction are used throughout the calculations.

** See the first paper in ref. [8].

4. The effective theory

At energies much smaller than the scale of the SU(2) breaking, the physics is described by the effective U(1) symmetric lagrangian*

$$\mathcal{L} = -\tfrac{1}{4}\left(\partial_\mu W_\nu^3 - \partial_\nu W_\mu^3\right)^2 + |\left(\partial_\mu - ig^*\tau_3 W_\mu^3\right)H|^2 + \tfrac{1}{2}\nu^{*2}|H|^2 - \tfrac{1}{4}\lambda^*|H|^4. \quad (9)$$

The U(1) symmetry will be broken to no symmetry if the doublet Higgs fields develop a non-zero vacuum expectation value. This is best studied, as is well known, by calculating the effective potential and looking for an asymmetric minimum**. The physical spectrum of the U(1) theory contains two Goldstone bosons, a massive vector and a massive scalar, with masses $\frac{1}{2}g^{*2}v^2$ and $\lambda^* v^2$, respectively, at the tree level where $\langle 0|H|0\rangle \equiv (v/\sqrt{2})\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $v^2 \equiv \nu^{*2}/\lambda^*$. It is easy to observe that there is a global O(4) invariance in the Higgs sector, which facilitates the computation of the effective potential***. The calculation is performed in the Landau gauge and MS renormalization scheme is being used. The vacuum graphs that contribute up to the two-loop approximation are shown in fig. 5. The relevant Feynman rules are listed in fig. 6. (We use the shift of the fields $\varphi_i \to \varphi_i + \tilde{\varphi}_i$ [9].) The potential is best expressible in terms of the "vector" and "scalar" masses

$$\begin{aligned} M_V^2 &= \tfrac{1}{2}g^{*2}|H|^2, \\ M_1^2 &= -\tfrac{1}{2}\nu^{*2} + \tfrac{3}{2}\lambda^*|H|^2, \\ M_2^2 &= -\tfrac{1}{2}\nu^{*2} + \tfrac{1}{2}\lambda^*|H|^2. \end{aligned} \quad (10)$$

The evaluation of the one-loop graphs gives the one-loop potential

$$\begin{aligned} V_{(H)}^{\text{1-loop}} = &-\tfrac{1}{2}\nu^{*2}|H|^2 + \tfrac{1}{4}\lambda^*|H|^4 - \frac{5}{128\pi^2}M_V^4 - \frac{9}{128\pi^2}M_2^4 \\ &- \frac{3}{128\pi^2}M_1^4 + \frac{3}{64\pi^2}M_V^4 \ln\left(\frac{M_V^2}{\mu^2}\right) \\ &+ \frac{3M_2^4}{64\pi^2}\ln\left(\frac{M_2^2}{\mu^2}\right) + \frac{M_1^4}{64\pi^2}\ln\left(\frac{M_1^2}{\mu^2}\right). \end{aligned} \quad (11)$$

* All asterisks indicate effective quantities.

** We are justified in looking separately at the light particle theory since V receives only small corrections from the light particles, having arranged a large gauge hierarchy.

*** In fact, we can put

$$H = \sqrt{\tfrac{1}{2}}\begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 - i\varphi_4 \end{pmatrix}$$

and then we have invariance under the O(4) symmetry $\varphi_i \to O_{ij}\varphi_j$, O being an orthogonal 4×4 matrix.

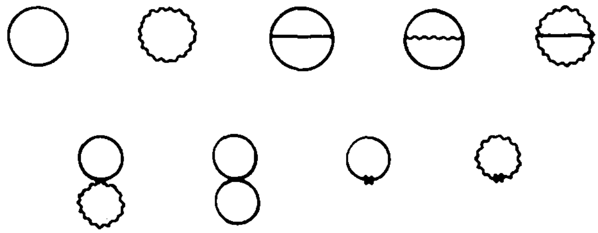
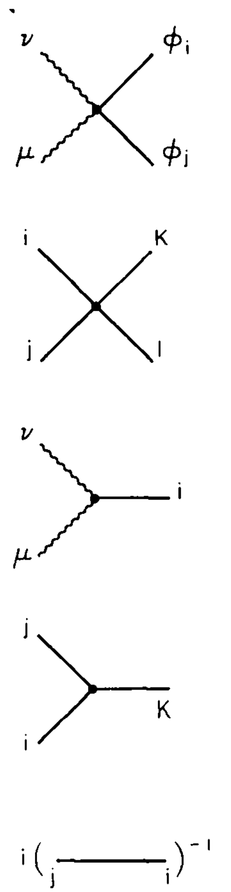


Fig. 5. The one- and two-loop contributions to the effective potential. Crosses denote counterterms. The rules for evaluating the graphs are given in fig. 6.



$$= \frac{g^2}{2} g_{\mu\nu}\delta_{ij}$$

$$= -\frac{\lambda}{2}(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

$$= \frac{g^2}{2} g_{\mu\nu} \bar{\phi}_i$$

$$= -\frac{\lambda}{2}[\delta_{ij}\bar{\Phi}_k + \delta_{jk}\bar{\phi}_i + \delta_{ki}\bar{\Phi}_j]$$

$$= [-\Box + \frac{\nu^2}{2} - \frac{\lambda}{4}\bar{\Phi}^2]\delta_{ij} - \frac{\lambda}{2}\bar{\phi}_i\bar{\Phi}_j$$

Fig. 6. Feynman rules used to calculate the graphs of fig. 5.

The minimization condition is, at scales $\mu^2 = \frac{1}{2}g^{*2}v^2$,

$$\nu^{*2} = v^2\left\{\lambda^* - \frac{3\lambda^{*2}}{32\pi^2}\left(1 - \ln\left(\frac{2\lambda^*}{g^{*2}}\right)\right) - \frac{g^{*4}}{64\pi^2}\right\}. \tag{12}$$

Adding the two-loop corrections yields the two-loop effective potential

$$\begin{aligned}
V_{(H)}^{2\text{-loop}} = {} & V_{(H)}^{1\text{-loop}} + \frac{\lambda^*}{8(16\pi^2)^2} \\
& \times\left[\tfrac{63}{2}M_1^4 - 12M_1^2M_2^2 - \tfrac{15}{2}M_2^4 + (-27M_1^4 + 21M_1^2M_2^2)\ln\left(\frac{M_1^2}{\mu^2}\right)\right. \\
& + (-3M_2^4 - 15M_1^2M_2^2)\ln\left(\frac{M_2^2}{\mu^2}\right) + (\tfrac{15}{2}M_1^4 - 6M_1^2M_2^2)\ln^2\left(\frac{M_1^2}{\mu^2}\right) \\
& \left. + (\tfrac{9}{2}M_2^4 + 3M_1^2M_2^2)\ln^2\left(\frac{M_2^2}{\mu^2}\right) + 3M_1^2M_2^2\ln\left(\frac{M_1^2}{\mu^2}\right)\ln\left(\frac{M_2^2}{\mu^2}\right)\right] \\
& - \frac{g^{*2}}{4(16\pi^2)^2}\left[\tfrac{7}{4}M_1^4 + \tfrac{45}{4}M_2^4 - \tfrac{161}{12}M_V^4 + 6M_1^2M_2^2 + \tfrac{3}{4}M_1^2M_V^2 + \tfrac{27}{4}M_2^2M_V^2\right. \\
& + (\tfrac{9}{2}M_1^2M_V^2 - \tfrac{5}{2}M_1^4 - M_1^2M_2^2)\ln\left(\frac{M_1^2}{\mu^2}\right) + (3M_2^2M_V^2 - \tfrac{19}{2}M_2^4 - M_1^2M_2^2) \\
& \times\ln\left(\frac{M_2^2}{\mu^2}\right) + (\tfrac{35}{3}M_V^4 - 3M_1^2M_V^2 + \tfrac{1}{2}M_1^4 + \tfrac{1}{2}M_2^4 - M_1^2M_2^2 - 6M_2^2M_V^2) \\
& \times\ln\left(\frac{M_V^2}{\mu^2}\right) + (-\tfrac{3}{2}M_1^2M_V^2 + \tfrac{1}{2}M_1^2M_V^2 + \tfrac{1}{2}M_1^4)\ln^2\left(\frac{M_1^2}{\mu^2}\right) \\
& + (-\tfrac{3}{4}M_2^2M_V^2 + \tfrac{1}{2}M_1^2M_2^2 + 2M_2^4)\ln^2\left(\frac{M_2^2}{\mu^2}\right) \\
& + (-\tfrac{11}{4}M_V^4 - \tfrac{1}{4}M_1^4 - \tfrac{1}{4}M_2^4 + \tfrac{3}{4}M_1^2M_2^2 + \tfrac{3}{2}M_2^2M_V^2 + \tfrac{1}{2}M_1^2M_V^2)\ln^2\left(\frac{M_V^2}{\mu^2}\right) \\
& + (-\tfrac{3}{2}M_1^2M_V^2 - \tfrac{1}{2}M_1^2M_2^2 + \tfrac{1}{2}M_1^4)\ln\left(\frac{M_1^2}{\mu^2}\right)\ln\left(\frac{M_V^2}{\mu^2}\right) - \tfrac{1}{2}M_1^2M_2^2\ln\left(\frac{M_1^2}{\mu^2}\right) \\
& \left. \times\ln\left(\frac{M_2^2}{\mu^2}\right) + (-3M_2^2M_V^2 - \tfrac{1}{2}M_1^2M_2^2 + \tfrac{1}{2}M_2^4)\ln\left(\frac{M_2^2}{\mu^2}\right)\ln\left(\frac{M_V^2}{\mu^2}\right)\right] \\
& + U(M_1^2, M_2^2, M_V^2).
\end{aligned}$$

The function $U(M_1^2, M_2^2, M_V^2)$ can be expressed in terms of logarithms and Spence functions. It is given in appendix B. The separate contributions of the diagrams are also listed in the appendix B. The minimization condition is now

$$\nu^{*2} = v^2 \left\{ \lambda^* - \frac{3\lambda^{*2}}{32\pi^2}\left(1 - \ln\left(\frac{2\lambda^*}{g^{*2}}\right)\right) - \frac{g^{*4}}{64\pi^2} + \lambda^{*3} B\left(\frac{2\lambda^*}{g^{*2}}\right)\right\}. \tag{14}$$

B is a complicated function of (λ^*/g^{*2}), involving logarithms and Spence functions. No qualitative difference arises at the two-loop level with respect to the position of the vacuum and, in order to have a small v^2, one surely needs small ν^{*2}. Therefore, the study of the ν^{*2} via the RGE, at the two-loop order, is needed in order to reach a definite conclusion.

5. The renormalization group

As we have already seen, the low-energy computations performed with the effective theory can incorporate the heavy particle effects through the boundary conditions, near the heavy mass scale. The two energy regions can be bridged via the RG. In order to match the one-loop boundary conditions, we must use the two-loop RGE. In the dimensionally regularized theory, the mass term is replaced by $\hat{\nu}^2\mu^2$ and the evolution of the now dimensionless $\hat{\nu}^2$ is studied like the evolution of any coupling. For the calculation of the β function we need the wave function renormalization constants for the "photon" and the Higgs fields. The divergent part of the potential we already have. The relevant graphs are shown in fig. 7.

The resulting two-loop RGE are

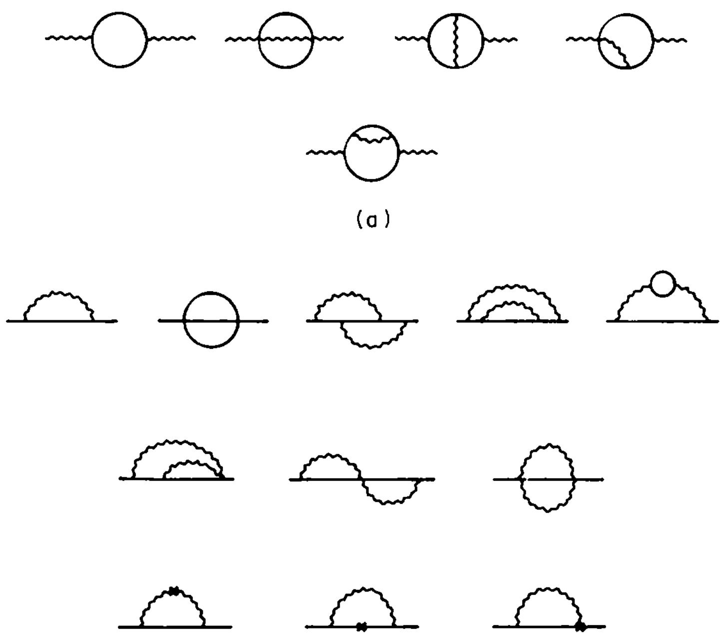
$$\frac{\mathrm{d}g^{*2}}{\mathrm{d}\ln\mu^2} = \frac{1}{(16\pi^2)}\frac{g^{*4}}{6} + \frac{1}{(16\pi^2)^2}\frac{g^{*6}}{2}, \tag{15}$$

$$\frac{\mathrm{d}\lambda^*}{\mathrm{d}\ln\mu^2} = \frac{1}{(16\pi^2)}\left(\tfrac{3}{4}g^{*4} - \tfrac{3}{2}\lambda^* g^{*2} + 3\lambda^{*2}\right) + \frac{1}{(16\pi^2)^2}\left(-\tfrac{59}{24}g^{*6} + \tfrac{229}{48}\lambda^* g^{*4} + \tfrac{9}{2}\lambda^{*2}g^{*2} - \tfrac{39}{4}\lambda^{*3}\right), \tag{16}$$

$$\frac{\mathrm{d}\ln\nu^{*2}}{\mathrm{d}\ln\mu^2} = \frac{1}{(16\pi^2)}\left(\tfrac{3}{2}\lambda^* - \tfrac{3}{4}g^{*2}\right) + \frac{1}{(16\pi^2)^2}\left(\tfrac{157}{96}g^{*4} + 3\lambda^* g^{*2} - \tfrac{15}{8}\lambda^{*2}\right). \tag{17}$$

The gauge coupling constant renormalization group equation can be integrated exactly to give

$$g^{*-2}(\mu) = g^{*-2}(\mu_0) - \frac{1}{(96\pi^2)}\ln\left(\frac{\mu^2}{\mu_0^2}\right) + \frac{3}{(16\pi^2)}\ln\left\{1 - \frac{g^{*2}(\mu_0)}{(96\pi^2)}\ln\left(\frac{\mu^2}{\mu_0^2}\right)\right\}. \tag{18}$$



(a)

(b)

Fig. 7. (a) The non-vanishing graphs (Landau gauge) needed to determine the wave function renormalization constant for the "photon" field. Diagrams involving one-loop and one-counterterm add up to zero as a result of the Ward identity and are not shown. (b) Non-vanishing graphs (Landau gauge) used to determine the Higgs wave function renormalization constant. Crosses denote counterterms.

The one-loop equations for λ^* and ν^{*2} are also exactly integrable and yield

$$\frac{\lambda^*(\mu)}{g^{*2}(\mu)} = \tfrac{5}{18} + \frac{\sqrt{56}}{18}\tan\left\{\sqrt{56}\ln\frac{g^{*2}(\mu)}{g^{*2}(\mu_0)} + \tan^{-1}\left(\frac{18}{\sqrt{56}}\frac{\lambda^*(\mu_0)}{g^{*2}(\mu_0)} - \frac{5}{\sqrt{56}}\right)\right\}, \quad (19)$$

$$\frac{\nu^{*2}(\mu)}{\nu^{*2}(\mu_0)} = \left(\frac{g^{*2}(\mu)}{g^{*2}(\mu_0)}\right)^{-2} \times\left\{\frac{\cos\left[\sqrt{56}\ln\left(g^{*2}(\mu)/g^{*2}(\mu_0)\right) + \tan^{-1}\left(\left(18/\sqrt{56}\right)\lambda^*(\mu_0)/g^{*2}(\mu_0) - 5/\sqrt{56}\right)\right]}{\cos\left[\tan^{-1}\left(\left(18/\sqrt{56}\right)\lambda^*(\mu_0)/g^{*2}(\mu_0) - 5/\sqrt{56}\right)\right]}\right\}^{-1/2}. \quad (20)$$

In order to integrate the two-loop equation (17) we have to make some approximations and it is reasonable to consider logarithms of the couplings virtually constant*. In this approximation one finds

$$\left(\frac{\nu^{*2}(\mu)}{\nu^{*2}(\mu_0)}\right)_{\text{2-loop}} \simeq \left(\frac{\nu^{*2}(\mu)}{\nu^{*2}(\mu_0)}\right)_{\text{1-loop}} \left\{1 + \frac{A(\mu_0)}{(96\pi^2)} g^{*2}(\mu) g^{*2}(\mu_0) \ln\left(\frac{\mu^2}{\mu_0^2}\right)\right\}, \quad (21)$$

where

$$A(\mu_0) = \frac{1}{(16\pi^2)}\left[18\frac{\lambda^*(\mu_0)}{g^{*2}(\mu_0)} - \frac{45}{4}\left(\frac{\lambda^*(\mu_0)}{g^{*2}(\mu_0)}\right)^2 + \frac{157}{16}\right]. \quad (22)$$

No significant change from the one-loop result is found, apart from mild corrections and conclusively, the two-loop RGE do not change the situation. The term on the right-hand side in (21), multiplying $(\nu^{*2}(\mu)/\nu^{*2}(\mu_0))_{\text{one-loop}}$, cannot become vanishingly small and, hence, even at the two-loop level, for having $\nu^{*2}(\mu)$ small, it is demanded that $\nu^{*2}(\mu_0)$ be small too.

The conclusion of this section is that the RGE for the "mass" do not improve the gauge hierarchy problem. Hence, a fine tuning along with order-by-order readjustment is needed if one uses the "masses" as arbitrary parameters, at least in the context of the perturbation theory.

6. Conclusions

We have studied an SU(2) gauge theory, which suffers two stages of spontaneous symmetry breaking: one strong at energies M_{SH}, with a scalar triplet, and another weak at $M_L \ll M_{SH}$, with a scalar doublet. We designate as heavy fields those acquiring masses via the first breaking; the rest we call light. After integrating out, at the first stage of the breaking, the heavy degrees of freedom, we have an effective U(1) symmetric theory, whose parameters at scales $\sim M_{SH}$ are connected to the parameters of the initial SU(2) theory.

In order to have an effective U(1) theory at low energies and light particles to be really light, one needs to show that the subsequent breaking U(1) → no symmetry is caused by a vacuum expectation value $v \sim M_L \ll M_{SH}$. We investigated the breaking of the remnant U(1) symmetry, by calculating the effective potential at two-loop approximation. The couplings and masses appearing in it are those of the effective theory. Their values at energies $E \sim M_L \ll M_{SH}$ are given by solving the RGE, given that at M_{SH} they are related to the couplings and masses of the full SU(2) theory (initial conditions). In order to have a very small vacuum expectation value for the "lights", one needs fine tuning as well as unnatural adjustment. Therefore, the way out of this difficulty does not seem to lie within perturbation theory, at least without new dynamical input.

* From the one-loop calculation we see that the logarithms of the couplings are very slow functions of the scale μ^2.

Taking the vacuum expectation value as parameters to replace the pair of "masses" and demanding $v \ll V$, the gauge hierarchy issue is rephrased, but not resolved. The lights remain indeed light, as $v \ll V$, but the mass parameters are connected to v, V through the minimization conditions. However, these conditions contain unnatural adjustments to each order.

The conclusions of this work can be extrapolated to more realistic models, as the simple SU(2) model we deal with carries all the qualitative features of such grand unifying models.

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Appendix A

In what follows, we explain the notation and conventions used in the main text. The τ matrices are chosen to obey the convention

$$\mathrm{Tr}(\tau^a \tau^b) = 2\delta^{ab}$$

and satisfy the commutation relations

$$[\tau^a, \tau^b] = 2i\varepsilon^{abc}\tau^c .$$

The gauge and Higgs fields written in the adjoint representation are

$$W_\mu = \sqrt{\tfrac{1}{2}}\,\tau^a W_\mu^a, \qquad \Phi = \sqrt{\tfrac{1}{2}}\,\tau^a \Phi^a .$$

The fundamental Higgs is a complex doublet

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \sqrt{\tfrac{1}{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 - i\varphi_4 \end{pmatrix} .$$

The gauge-fixing term for the SU(2) → U(1) breaking is chosen

$$I_{\mathrm{GF}} = -\frac{1}{2\xi}\int \mathrm{d}^4x \sum_{a=1,2} \left(\partial^\mu W_\mu^a + g\varepsilon^{ab3} W_\mu^b W^{\mu 3} + ig\xi V \varepsilon^{ab3}\Phi^3\right).$$

The ghost lagrangian is

$$\mathcal{L}_{\mathrm{FPdW}} = \bar{\eta} G^{-1}\eta ,$$

the ghost propagator being

$$G^{-1} = -\begin{Bmatrix} \left(\Box + g^2\left(W_2^2 - W_3^2\right) + g^2\xi V\Phi_3\right) & \left(-2gW_\mu^3\partial^\mu + g\partial^\mu W_\mu^3 - g^2 W_\mu^1 W^{\mu 2}\right) \\ \left(2gW_\mu^3\partial^\mu + g\partial^\mu W_\mu^3 - g^2 W_\mu^1 W^{\mu 2}\right) & \left(\Box + g^2\left(W_1^2 - W_3^2\right) + g^2\xi V\Phi_3\right) \end{Bmatrix}.$$

The Feynman rules of the heavy sector are listed in fig. 1.

Appendix B

The function $U(M_1^2, M_2^2, M_3^2)$ appearing in the potential is

$$\begin{aligned}
U(M_1^2, M_2^2, M_V^2) = {} & \frac{3\lambda^*}{8(16\pi^2)^2} M_1^2\left(M_1^2 - M_2^2\right)\left[-3.513 + J_0\left(\frac{M_2^2}{M_1^2}\right)\right] \\
& - \frac{g^{*2}}{4(16\pi^2)^2}\left\{\left[M_1^2\left(\tfrac{1}{2}M_1^2 - \tfrac{11}{4}M_V^2\right) - \frac{M_1^2}{4M_V^2}\left(M_1^2 - M_V^2\right)^2\right] J_0\left(\frac{M_V^2}{M_1^2}\right)\right. \\
& + \tfrac{1}{2}\left[-\tfrac{1}{2}\left(M_1^2 - M_2^2\right)^2 + M_V^2\left(M_1^2 + M_2^2 - \frac{M_V^2}{2}\right)\right] \\
& \times\left(J_0\left(\frac{M_1^2}{M_V^2}\right) + J_0\left(\frac{M_2^2}{M_V^2}\right)\right) \\
& + M_V^2(2M_2^2 - \tfrac{1}{2}M_V^2) J_0\left(\frac{M_2^2}{M_V^2}\right) \\
& + \tfrac{1}{6}\pi^2\left[M_1^2\left(\tfrac{1}{4}M_V^2 - \tfrac{1}{2}M_1^2\right) - \frac{M_1^2}{4M_V^2}\left(M_1^2 - M_2^2\right)^2\right] \\
& + \left(M_1^2 - M_V^2\right)^2 \frac{M_1^2}{2M_V^2} L\left(1 - \frac{M_V^2}{M_1^2}\right) \\
& + \tfrac{1}{2}\left(M_1^2 - M_2^2\right)^2 L\left(1 - \frac{M_1^2}{M_V^2}\right) \\
& + \left(M_1^2 + M_2^2 - \tfrac{1}{2}M_V^2\right)\left(M_1^2 L\left(1 - \frac{M_2^2}{M_1^2}\right)\right. \\
& \left.\left. + M_2^2 L\left(1 - \frac{M_1^2}{M_2^2}\right)\right)\right\},
\end{aligned}$$

where

$$L(x) \equiv -\int_0^x \frac{\mathrm{d}t}{t} \ln(1-t)$$

is the Spence function or dilogarithm, and

$$J_0(x) \equiv \int_0^1 \mathrm{d}t \ln\left(1 - \frac{x}{t(1-t)}\right)$$

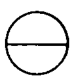
$$= -\tfrac{1}{2}\ln^2 x + \theta(1-4x)\left\{-(\ln x)\sqrt{1-4x}\ln\left(\frac{1+\sqrt{1-4x}}{1-\sqrt{1-4x}}\right)\right.$$

$$\left. -\sqrt{1-4x}\left[L\left(\frac{2}{1+\sqrt{1-4x}}\right) - L\left(\frac{2}{1-\sqrt{1-4x}}\right)\right]\right\}$$

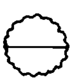
$$+\theta(4x-1)\left(-2\sqrt{4x-1}\right) f\left[2\tan^{-1}\left(\frac{1}{\sqrt{4x-1}}\right)\right],$$

with $f(x) \equiv \mathrm{Im}\, L(\mathrm{e}^{ix})$ (Clausen's function).

The separate contributions of the various graphs are

 $= \tfrac{3}{8} i\lambda^*(M_1^2 - M_2^2)[J(M_2, M_2, M_1) + J(M_1, M_1, M_1)]$,

 $= \tfrac{3}{16} i\lambda^*[5K^2(M_2) + 2K(M_1)K(M_2) + K^2(M_1)]$,

 $= \tfrac{1}{4} ig^{*2} M_V^2[(D-2)J(M_V, M_1, M_V) + X(M_V, M_1, M_V)]$,

 $= \tfrac{1}{8} ig^{*2}(D-1)[3K(M_V)K(M_2) + K(M_V)K(M_1)]$,

 $= -\tfrac{1}{4} ig^{*2}[2Y(M_2, M_2, M_V) + Y(M_1, M_2, M_V) + Y(M_2, M_1, M_V)]$,

where

$$K(M) \equiv \int \frac{\mathrm{d}^D p}{(2\pi)^D}[p^2 - M^2]^{-1} = \frac{-i}{(4\pi)^{D/2}}(M^2)^{D/2-1}\Gamma(1 - \tfrac{1}{2}D),$$

$$J(A,B,C) \equiv \int \frac{\mathrm{d}^D p\, \mathrm{d}^D q}{(2\pi)^{2D}} \left[p^2 - A^2\right]^{-1} \left[(p+q)^2 - B^2\right]^{-1} \left[q^2 - C^2\right]^{-1}$$

$$= -\frac{1}{(4\pi)^D} \frac{\Gamma(\varepsilon)\Gamma(2\varepsilon)\Gamma(1+\varepsilon)}{\Gamma(2+2\varepsilon)}$$

$$\times \Bigg\{ (A^2+B^2+C^2)$$

$$+\varepsilon\left[5(A^2+B^2+C^2) - 2\left(A^2 \ln\frac{A^2}{\mu^2} + B^2 \ln\frac{B^2}{\mu^2} + C^2 \ln\frac{C^2}{\mu^2}\right)\right]$$

$$+\varepsilon^2\Bigg[13(A^2+B^2+C^2) - 10\left(A^2 \ln\frac{A^2}{\mu^2} + B^2 \ln\frac{B^2}{\mu^2} + C^2 \ln\frac{C^2}{\mu^2}\right)$$

$$+2\left(A^2 \ln^2\frac{A^2}{\mu^2} + B^2 \ln^2\frac{B^2}{\mu^2} + C^2 \ln^2\frac{C^2}{\mu^2}\right) + 2S(A^2, B^2, C^2)\Bigg]\Bigg\}.$$

$D = 4 - 2\varepsilon$ is the number of dimensions and S is a symmetric function of A^2, B^2, C^2:

$$S(A^2, B^2, C^2) \equiv A^2 \int_0^1 \mathrm{d}x\, L\left(1 - \frac{B^2}{A^2}\frac{1}{x} - \frac{C^2}{A^2}\frac{1}{1-x}\right)$$

$$+B^2 L\left(1 - \frac{C^2}{B^2}\right) + C^2 L\left(1 - \frac{B^2}{C^2}\right).$$

Finally,

$$Y(A,B,C) \equiv \int \frac{\mathrm{d}^D p\, \mathrm{d}^D q}{(2\pi)^{2D}} \left(p^2 - \frac{(p\cdot q)^2}{q^2}\right) \left[p^2 - A^2\right]^{-1} \left[(p+q)^2 - B^2\right]^{-1} \left[q^2 - C^2\right]^{-1}$$

$$= -\frac{1}{4C^2}(A^4 + B^4 + C^4 - 2A^2B^2 - 2B^2C^2 - 2A^2C^2) J(A,B,C)$$

$$+\tfrac{1}{4}\left(1 + \frac{B^2 - A^2}{C^2}\right) K(B)K(C) - \tfrac{1}{4}K(A)K(B)$$

$$+\tfrac{1}{4}\left(1 + \frac{A^2 - B^2}{C^2}\right) K(A)K(C) + \frac{(B^2 - A^2)^2}{4C^2} J(A,B,0),$$

$$X(A,B,C) \equiv \int \frac{\mathrm{d}^D p\,\mathrm{d}^D q}{(2\pi)^{2D}} \frac{(p\cdot q)^2}{p^2 q^2} \left[p^2 - A^2\right]^{-1} \left[(p+q)^2 - B^2\right]^{-1} \left[q^2 - C^2\right]^{-1}$$

$$= J(A,B,C) - \frac{1}{A^2}\left[Y(A,B,C) - Y(0,B,C)\right].$$

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