Some selection rules for non-renormalizable chiral couplings in 4D fermionic superstring models

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We present a simple method for deriving some general selection rules for nonrenormalizable chiral terms based entirely on the field charges under the U(1) currents associated with the $N=2$ world sheet supersymmetry algebra. We explicitly derive these rules for terms up to ninth order.

Superstring theory is our best candidate for a unified theory of all known interactions including gravity. In the last few years various approaches for constructing superstring models have been presented [1,2]. In the fermionic formulation of the heterotic string in four dimensions models with realistic features have been constructed [3,4]. In a recent paper [5], rules for the calculation of non-renormalizable superpotential terms have been presented and these terms for a specific model [3] have been explicitly derived [6,7] up to the $N=5$ order. Although the method of calculation is straightforward, the number of terms increases rapidly with the order. Since in these models some very high VEV's are invariably involved, one needs to consider NR terms of quite high order ($N\sim 10$) before one decides for the phenomenological viability of the model. In this paper we present a very simple method for derivation of general selection rules referring to arbitrarily high order of NR terms. These rules are based on the $N=2$ superconformal U(1) charges of the states and they can be readily applied to any $N=1$ supersymmetric model. Actually, in a recent paper [8] similar arguments were used in the case of the SU(5)×U(1) model in order to consider possible higher order contributions to mass matrices.

In the free fermionic formulation of the heterotic string extra world-sheet fermions are introduced in order to cancel the conformal anomaly in four dimensions [2]. The left movers are the spacetime fields X^μ, ψ^μ and the 18 real free fermions x^I, y^I, $\omega^I (I=1, ..., 6)$, transforming according to the adjoint of $SU(2)^6$, and the right movers $\bar{X}^\mu$ and 44 real free fermions $\bar{\phi}^\alpha$ ($\alpha=1, ..., 44$), in the usual literature notation. In terms of these fields the supercurrent is

$$T_F=\psi^\mu \partial X_\mu + i \sum_I x^I y^I \omega^I . \quad (1)$$

A specific model is defined by a set of boundary conditions for all world-sheet fermions. Depending on the boundary conditions some real fermions can be either complexified in left–left or right–right pairs or form nontrivial real fermion left–right pairs corresponding to an Ising model. In the usual construction we can choose a basis vector

$$S=\{\psi^\mu, x^1, ..., x^6\} ,$$

where the fields included are periodic and the rest antiperiodic. The left-moving fields $x^1, ..., x^6$ can be bosonized [5]

$$\frac{1}{\sqrt{2}}(x^1+ix^2)=\exp(iS_{12}) ,$$

$$\frac{1}{\sqrt{2}}(x^1-ix^2)=\exp(-iS_{12}) ,$$

$$\frac{1}{\sqrt{2}}(x^3+ix^4)=\exp(iS_{34}) , \quad (2)$$

$$\frac{1}{\sqrt{2}}(x^3-\mathrm{i}x^4)=\exp(-\mathrm{i}S_{34}),$$

$$\frac{1}{\sqrt{2}}(x^5+\mathrm{i}x^6)=\exp(\mathrm{i}S_{56}),$$

$$\frac{1}{\sqrt{2}}(x^5-\mathrm{i}x^6)=\exp(-\mathrm{i}S_{56}). \quad (2\text{ cont'd})$$

Some of the remaining left movers can be bosonized as well.

According to refs. [9,5] $N=1$ spacetime SUSY implies $N=2$ superconformal invariance as well as the existence of an extra $\mathrm{U}_J(1)$ world-sheet current. According to ref. [5] this current is expressed in terms of S_{12}, S_{34}, S_{56} as

$$J(z)=\mathrm{i}\partial_z(S_{12}+S_{34}+S_{56}), \quad (3)$$

and the $\mathrm{U}_J(1)$ symmetry is extended to $\mathrm{U}(1)^3$ with the three U(1)'s generated by S_{12}, S_{34}, S_{56}.

In a particular $N=1$ supersymmetric model, the coupling of a specific superpotential term involving the chiral superfields $\Phi_i (i=1, ..., N)$

$$\int \mathrm{d}^2\theta\, \Phi_1...\Phi_N \quad (4)$$

is proportional to the correlator

$$\langle V_1^{\mathrm{F}} V_2^{\mathrm{F}} V_3^{\mathrm{B}}...V_N^{\mathrm{B}}\rangle \quad (5)$$

where $V_i^{\mathrm{F}}(V_i^{\mathrm{B}})$ is the fermionic (bosonic) part of the vertex operator corresponding to the superfield Φ_i. Physical states are divided in two classes according to the sector they arise from: (i) the NS sector (untwisted) and (ii) the R sector (twisted). According to ref. [5], the part of the vertex operators involving the bosonized fields (2) is

$$V^{\mathrm{B}}_{(-1)} \sim \exp(\alpha S_{12})\exp(\beta S_{34})\exp(\gamma S_{56}), \quad (6\mathrm{a})$$

$$V^{\mathrm{F}}_{(-1/2)} \sim \exp[(\alpha-\tfrac{1}{2})S_{12}]\exp[(\beta-\tfrac{1}{2})S_{34}] \times\exp[(\gamma-\tfrac{1}{2})S_{56}], \quad (6\mathrm{a})$$

with

$$(\alpha,\beta,\gamma)\in\{(1,0,0), (0,1,0), (0,0,1)\} \quad (7\mathrm{a})$$

for NS fields, and

$$(\alpha,\beta,\gamma)\in\{(0,\tfrac{1}{2},\tfrac{1}{2}), (\tfrac{1}{2},0,\tfrac{1}{2}), (\tfrac{1}{2},\tfrac{1}{2},0)\} \quad (7\mathrm{b})$$

for R fields, with the subscript (-1) or $(-\tfrac{1}{2})$ referring to the ghost number.

Of course, in order for the correlator (5) to be modular invariant the total ghost number should add up to -2 which implies that the vertex operators $V_4^{\mathrm{B}}...V_N^{\mathrm{B}}$ have to be picture-changed in the zero picture according to

$$V^{\mathrm{B}}_{(0)}(z)=\lim_{w\to z} e^{c}(w)T_{\mathrm{F}}(w)V^{\mathrm{B}}_{(-1)}(z). \quad (8)$$

Following ref. [5] the contributing part of the supercurrent is

$$T'_{\mathrm{F}}=\exp(-\mathrm{i}S_{12})\tau_{12}+\exp(-\mathrm{i}S_{34})\tau_{34} +\exp(-\mathrm{i}S_{56})\tau_{56}, \quad (9)$$

with

$$\tau_{ij}=\frac{\mathrm{i}}{\sqrt{2}}(y^i\omega^i+\mathrm{i}y^j\omega^j). \quad (10)$$

Using (6) and (8), we find that the charge of a picture-changed bosonic NS field is always (0, 0, 0) while for a R boson two contributions arise with charges $\{(0,-\tfrac{1}{2},+\tfrac{1}{2}), (0,+\tfrac{1}{2},-\tfrac{1}{2})\}$, $\{(+\tfrac{1}{2},0,-\tfrac{1}{2}), (-\tfrac{1}{2},0,+\tfrac{1}{2})\}$, $\{(+\tfrac{1}{2},-\tfrac{1}{2},0), (-\tfrac{1}{2},+\tfrac{1}{2},0)\}$ for each of the cases (7b). We note that we can introduce a simple vector notation for the state charges. We define $\boldsymbol{\delta}_i$ $(i=1,2,3)$

$$\boldsymbol{\delta}_1=(1,0,0), \quad \boldsymbol{\delta}_2=(0,1,0), \quad \boldsymbol{\delta}_3=(0,0,1) \quad (11)$$

and assign each vertex operator a subscript $i\in\{1, 2, 3\}$ corresponding to the position of 1 in (7a) for a NS field or the position of 0 in (7b) for a R field, as shown in table 1. This number to which we will refer as "category" together with the field type (R/NS) completely determines the field charges under S_{12}, S_{34}, S_{56} as shown in table 1. The charges of the various types of vertex operators in this notation are shown

Table 1
Charges under S_{12}, S_{34}, S_{56} for various types of vertex operators in vector notation.

Field type	Charge (α,β,γ)
$(\mathrm{NS})^{\mathrm{B}}_{i(-1)}$	$\boldsymbol{\delta}_i$
$(\mathrm{NS})^{\mathrm{F}}_{i(-1/2)}$	$\tfrac{1}{2}(2\boldsymbol{\delta}_i-\mathbf{1})$
$(\mathrm{R})^{\mathrm{B}}_{i(-1)}$	$\tfrac{1}{2}(\mathbf{1}-\boldsymbol{\delta}_i)$
$(\mathrm{R})^{\mathrm{F}}_{i(-1/2)}$	$-\tfrac{1}{2}\boldsymbol{\delta}_i$
$(\mathrm{NS})^{\mathrm{B}}_{i(0)}$	$\mathbf{0}$
$(\mathrm{R})^{\mathrm{B}}_{i(0)}$	$\tfrac{1}{2}(\mathbf{1}-\boldsymbol{\delta}_i-2\boldsymbol{\delta}_{i'}),\ i'\neq i$

in table 2, where we have used the notation i' for a variable with values different than i, which actually accounts for the contribution of the i' term of the supercurrent in (9).

Let us now start our analysis by examining the correlator

$$\langle V^{\mathrm{F}}_{i1\,(-1/2)} V^{\mathrm{F}}_{i2\,(-1/2)} V^{\mathrm{B}}_{i3\,(-1)} V^{\mathrm{B}}_{i4\,(0)} \dots V^{\mathrm{B}}_{iN\,(0)} \rangle \tag{12}$$

corresponding to the superpotential coupling (4), using the following facts:

(a) Due to the well known conformal field theory result the total charge of the vertex operators in (12), under each of the three U(1)'s, should add up to zero for the correlator to be nonvanishing.

(b) Due to the $N=1$ supersymmetry the correlator is independent of which two fields we choose to be fermions [#1].

(c) Due to conformal invariance the correlator is independent of which particular bosons we choose to picture-change [#1].

Furthermore in our notation one can immediately verify the following identity:

$$\boldsymbol{\delta}_i + \boldsymbol{\delta}_j + \boldsymbol{\delta}_k = \mathbf{1} \leftrightarrow i \neq j \neq k\,. \tag{13}$$

We shall start with the $N=3$ superpotential couplings and see what rules we get when using (a), (b), (c). We have the following possible types of couplings [#2]:

(i) $\mathrm{R}^{\mathrm{F}}_i \mathrm{R}^{\mathrm{F}}_j \mathrm{R}^{\mathrm{B}}_k$: Vanishing of the total charge (a) requires $\boldsymbol{\delta}_i+\boldsymbol{\delta}_j+\boldsymbol{\delta}_k=\mathbf{1}$ which according to (13) implies $i \neq j \neq k$.

(ii) $\mathrm{R}^{\mathrm{F}}_i \mathrm{R}^{\mathrm{F}}_j \mathrm{NS}^{\mathrm{B}}_k$: Vanishing of the total charge (a) requires $-\frac{1}{2}\boldsymbol{\delta}_i-\frac{1}{2}\boldsymbol{\delta}_j+\boldsymbol{\delta}_k=\mathbf{0}$ which implies $i=j=k$.

(iii) $\mathrm{NS}^{\mathrm{F}}_i \mathrm{NS}^{\mathrm{F}}_j \mathrm{R}^{\mathrm{B}}_k$: Vanishing of the total charge implies $-\boldsymbol{\delta}_i-\boldsymbol{\delta}_j+\frac{1}{2}\boldsymbol{\delta}_k=\mathbf{1}$. This is clearly impossible and thus couplings of this explicitly vanish.

(iv) $\mathrm{NS}^{\mathrm{F}}_i \mathrm{NS}^{\mathrm{F}}_j \mathrm{NS}^{\mathrm{B}}_k$: We have again $\boldsymbol{\delta}_i+\boldsymbol{\delta}_j+\boldsymbol{\delta}_k=\mathbf{1}$ which according to (13) implies $i \neq j \neq k$.

Similarly we have for the $N=4$ NR terms [#2]:

(i) $\mathrm{R}^{\mathrm{F}}_i \mathrm{R}^{\mathrm{F}}_j \mathrm{R}^{\mathrm{B}}_k \mathrm{R}^{\mathrm{B}}_m$: We obtain $\boldsymbol{\delta}_i+\boldsymbol{\delta}_j+\boldsymbol{\delta}_k+\boldsymbol{\delta}_m+2\boldsymbol{\delta}_{m'}=\mathbf{2}$ which using (13) implies $i=j\neq k=m\neq m'$.

(ii) $\mathrm{R}^{\mathrm{F}}_i \mathrm{R}^{\mathrm{F}}_j \mathrm{R}^{\mathrm{B}}_k \mathrm{NS}^{\mathrm{B}}_m$: Since the charge of the NS field in the zero picture is zero, we immediately get the tree level result $i\neq j\neq k$. This implies that m should be equal to one of $\{i, j, k\}$. Assuming $m=k$ we consider the equivalent, according to (c), coupling $\mathrm{R}^{\mathrm{F}}_i \mathrm{R}^{\mathrm{F}}_j \mathrm{NS}^{\mathrm{B}}_m \mathrm{R}^{\mathrm{B}}_k$ and get $\boldsymbol{\delta}_i+\boldsymbol{\delta}_j+\boldsymbol{\delta}_k+2\boldsymbol{\delta}_{k'}-2\boldsymbol{\delta}_m=\mathbf{1}$. Using (13) and the previous results we find $\boldsymbol{\delta}_{k'}=\boldsymbol{\delta}_k$. This is not possible because by definition $k'\neq k$, and thus this coupling vanishes.

(iii) $\mathrm{R}^{\mathrm{F}}_i \mathrm{R}^{\mathrm{F}}_j \mathrm{NS}^{\mathrm{B}}_k \mathrm{NS}^{\mathrm{B}}_m$: From the tree level results, we obtain $i=j=k$, but if we consider $\mathrm{R}^{\mathrm{F}}_i \mathrm{R}^{\mathrm{F}}_j \mathrm{NS}^{\mathrm{B}}_m \mathrm{NS}^{\mathrm{B}}_k$ we get $i=j=m$. On the other hand from $\mathrm{NS}^{\mathrm{F}}_m \mathrm{NS}^{\mathrm{F}}_k \mathrm{R}^{\mathrm{B}}_i \mathrm{R}^{\mathrm{B}}_j$ we get $\boldsymbol{\delta}_m+\boldsymbol{\delta}_k-\frac{1}{2}\boldsymbol{\delta}_i-\frac{1}{2}\boldsymbol{\delta}_j-\boldsymbol{\delta}_{j'}=\mathbf{0}$ which is not compatible with the previous two. Thus this coupling also vanishes.

(iv) $\mathrm{R}^{\mathrm{F}}_i \mathrm{NS}^{\mathrm{F}}_j \mathrm{NS}^{\mathrm{B}}_k \mathrm{NS}^{\mathrm{B}}_m$: From the tree level result (iii) we find that this coupling also vanishes.

(v) $\mathrm{NS}^{\mathrm{F}}_i \mathrm{NS}^{\mathrm{F}}_j \mathrm{NS}^{\mathrm{B}}_k \mathrm{NS}^{\mathrm{B}}_m$: Using the tree level results (iv) for cyclic reorderings we get the incompatible relations $i\neq j\neq k$, $m\neq i\neq j$, $k\neq m\neq i$. Thus this coupling also vanishes.

One can now repeat the above steps and derive constraints for higher order terms.

Alternatively, we can prove some general theorems which will allow us to construct the selection rules by inspection.

Theorem 1. A coupling of the type $(\mathrm{R})_{i_1}\dots(\mathrm{R})_{i_N}$ with $i_1=i_2=\dots=i_m\neq\{i_{m+1}, \dots, i_N\}$, $N\geqslant 3$, is nonvanishing only if $m=N \bmod 2$, $m\neq N$.

Theorem 2. A coupling of the type $(\mathrm{R})_{i_1}\dots(\mathrm{R})_{i_N}$ $(\mathrm{NS})_j$, $N\geqslant 3$ with $i_1=i_2=\dots=i_m=j\neq\{i_{m+1}, \dots, i_N\}$ vanishes for $m\geqslant N-2$.

[#1] Possible field reorderings could give rise to a phase factor [10] but we are only interested in cases where the correlator (12) vanishes.

[#2] The $N=3$ as well as some of the $N=4$ and $N=5$ rules have also been derived in ref. [5] using a different method.

Table 2
Category assignment for various types of fields according to S_{12}, S_{34}, S_{56} charges.

Field type	Fermionic charge	Bosonic charge	Category
R	$(-\frac{1}{2}, 0, 0)$	$(0, \frac{1}{2}, \frac{1}{2})$	1
	$(0, -\frac{1}{2}, 0)$	$(\frac{1}{2}, 0, \frac{1}{2})$	2
	$(0, 0, -\frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2}, 0)$	3
NS	$(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})$	$(1, 0, 0)$	1
	$(-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$	$(0, 1, 0)$	2
	$(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$	$(0, 0, 1)$	3

Theorem 3. Any pure NS coupling vanishes unless $N=3$ [#3].

Theorem 4. Any coupling with n R and $(N-n)$ NS fields, $N>3$, vanishes for $n=1, 2, 3$.

Proof of Theorem 1. The charge conservation (a) for this type of coupling implies

$$\sum_{J=1}^{N} \boldsymbol{\delta}_{i(J)} + 2\sum_{J=4}^{N} \boldsymbol{\delta}_{i'(J)} = (N-2)\mathbf{1} . \tag{14}$$

Assuming $i_1 = i_2 = ... = i_m = i \neq \{i_{m+1}, ..., i_N\}$, this becomes

$$m\boldsymbol{\delta}_i + \sum_{J=m+1}^{N} \boldsymbol{\delta}_{i(J)} + 2\sum_{J=4}^{N} \boldsymbol{\delta}_{i'(J)} = (N-2)\mathbf{1} . \tag{15}$$

Looking at the ith component we have $m+2(...) = N-2 \Rightarrow m=N \bmod 2$. On the other hand, if $m=N$, (14) requires

$$N\boldsymbol{\delta}_i + 2(\boldsymbol{\delta}_{i'(4)} + ... + \boldsymbol{\delta}_{i'(N)}) = (N-2)\mathbf{1} , \tag{16}$$

with $\{i'_{(4)}, i'_{(5)}, ..., i'_{(N)}\} \neq i$ which fails to be satisfied in the ith component.

Proof of Theorem 2. Since a picture-changed NS field gets zero charge, one constraint for the coupling $(\mathrm{R})_{i_1}...(\mathrm{R})_{i_N}(\mathrm{NS})_j$ immediately follows from the $(\mathrm{R})_{i_1}...(\mathrm{R})_{i_N}$ coupling and requires $m=N \bmod 2$, $m\neq N$, according to Theorem 1, and thus $m \leqslant N-2$. If $m=N-2$ we consider the equivalent coupling $(\mathrm{R})_{i_{N-1}}(\mathrm{R})_{i_N}(\mathrm{NS})_j(\mathrm{R})_{i_1}...(\mathrm{R})_{i_{N-2}}$. From charge conservation we get

$$\begin{aligned} &\boldsymbol{\delta}_{i(N)} + \boldsymbol{\delta}_{i(N-1)} + (N-2)\boldsymbol{\delta}_i + 2(\boldsymbol{\delta}_{i'(1)} + ... + \boldsymbol{\delta}_{i'(N-2)}) \\ &\quad = (N-2)\mathbf{1} + 2\boldsymbol{\delta}_j , \end{aligned} \tag{17}$$

with $\{i_{(N)}, i_{(N-1)}, i'_{(J)}\} \neq i = i_1 = ... = i_{(N-2)} = j$, which fails to be satisfied in the ith component and thus the coupling vanishes.

Proof of Theorem 3. A pure NS coupling is of the form $(\mathrm{NS})_{i_1}...(\mathrm{NS})_{i_N}$. Charge conservation requires $\boldsymbol{\delta}_{i_1} + \boldsymbol{\delta}_{i_2} + \boldsymbol{\delta}_{i_3} = \mathbf{1}$ which implies $i_1 \neq i_2 \neq i_3$ according to (13). Thus the $N=3$ case is possible if all fields belong to different categories. If $N>3$, we consider the cyclic reorderings $(\mathrm{NS})_{i_4}(\mathrm{NS})_{i_1}(\mathrm{NS})_{i_2}(\mathrm{NS})_{i_3}...(\mathrm{NS})_{i_N}$ and $(\mathrm{NS})_{i_3}(\mathrm{NS})_{i_4}(\mathrm{NS})_{i_1}(\mathrm{NS})_{i_2}...(\mathrm{NS})_{i_N}$. They require $i_4 \neq i_1 \neq i_2$ and $i_3 \neq i_4 \neq i_1$ respectively. These three constraints are incompatible and thus the coupling vanishes for all $N>3$.

[#3] This has also been proved in ref. [5].

Proof of Theorem 4. For the $n=1$ case we consider a coupling of the form $\mathrm{R}_{i_1}(\mathrm{NS})_{i_2}...(\mathrm{NS})_{i_N}$. Vanishing of the total charge requires $-\frac{1}{2}\boldsymbol{\delta}_{i_1} + \boldsymbol{\delta}_{i_2} + \boldsymbol{\delta}_{i_3} = \frac{1}{2}\cdot\mathbf{1}$ as in the $N=3$ case. This is clearly impossible. For the $n=2$ case we have a coupling of the form $\mathrm{R}_{i_1}\mathrm{R}_{i_2}(\mathrm{NS})_{i_3}...(\mathrm{NS})_{i_N}$ requiring $-\frac{1}{2}\boldsymbol{\delta}_{i_1} - \frac{1}{2}\boldsymbol{\delta}_{i_2} + \boldsymbol{\delta}_{i_3} = \mathbf{0}$ which implies $i_1 = i_2 = i_3$. Since $N>3$, we can then interchange $i_3 \leftrightarrow i_4$ and get $i_1 = i_2 = i_4$. But if we then consider $(\mathrm{NS})_{i_3}(\mathrm{NS})_{i_4}\mathrm{R}_{i_1}\mathrm{R}_{i_2}(\mathrm{NS})_{i_5}...(\mathrm{NS})_{i_N}$, we get

$$2\boldsymbol{\delta}_{i_3} + 2\boldsymbol{\delta}_{i_4} - \boldsymbol{\delta}_{i_2} - \boldsymbol{\delta}_{i_1} - 2\boldsymbol{\delta}_{i_{2'}} = \mathbf{0} . \tag{18}$$

This, for $i_1 = i_2 = i_3 = i_4$, implies $\boldsymbol{\delta}_{i_2} = \boldsymbol{\delta}_{i_{2'}}$ and thus fails to be satisfied since $i_2 \neq i_2'$ by definition. For the $n=3$ case Theorem 1 requires one R field to each category. On the other hand Theorem 2 requires that no NS field belongs to the category with the maximum number of elements (one, in this case). Thus the $n=3$ couplings are also impossible.

One can now proceed and obtain some constraints by considering Theorem 1 for a pure R coupling and Theorem 1 in conjuction with Theorems 2 and 3 for a mixed coupling.

As corollaries one can easily show:

(1) Any coupling of the form $(\mathrm{R})_{i_1}(\mathrm{R})_{i_2}(\mathrm{R})_{i_3}...(\mathrm{R})_{i_M}(\mathrm{NS})_{j_1}...(\mathrm{NS})_{j_K}$ with $i_2 = i_2 = ... = i_m = i$, $m = M-2$, $M \geqslant 3$ vanishes if for some I ($I=1, ..., K$), $j_I = i$.

(2) Any coupling with four R fields of the type $(\mathrm{R})_{i_1}(\mathrm{R})_{i_2}(\mathrm{R})_{i_3}(\mathrm{R})_{i_4}(\mathrm{NS})_{i_5}...(\mathrm{NS})_{i_N}$ is nonvanishing only if $i_1 = i_2 \neq i_3 = i_4 \neq i_5 = ... = i_N$.

In order to prove Corollary 1, we first apply Theorem 1, neglecting the picture-changed (NS) fields which give zero charge, and obtain that the maximum allowed number of equal index R fields is $m = M-2$. Then we apply Theorem 2 for each of the (NS) fields consecutively and obtain $j_I \neq i$, $I=1, ..., K$. Corollary 2 follows from the $\mathrm{R}_i\mathrm{R}_j\mathrm{R}_k\mathrm{R}_m$ result requiring $i=j\neq k=m$ in conjuction with Corollary 1.

We can now consider specific types of higher order couplings. For $N=5$ Theorem 1 gives that $\mathrm{R}_i\mathrm{R}_j\mathrm{R}_k$

R_mR_n is only possible if $i=j=k\neq m\neq n$, and the coupling $R_iR_jR_kR_mNS_n$ is possible for $i=j\neq k=m\neq n$ according to Corollary 1. All other types of $N=5$ couplings vanish according to Theorems 3, 4. At this point we note that we can introduce an easier notation for the allowed couplings, showing the number of equal index fields of each type. In this notation we write the allowed (5R) couplings as $[3_R, 1_R, 1_R]$ and the allowed (4R)(1NS) couplings as $[2_R, 2_R, 1_{NS}]$. Using the above notation we present explicit results following from the use of Theorems 1, 2, 3 and 4 for terms up to $N=9$, in table 3. Some comments are now in order about how one is to read these results. Table 3 contains the field partitions in the three categories which lead to nonvanishing couplings for the various types of couplings. All partitions not included in table 3 lead to explicitly vanishing couplings. Furthermore partitions indicated are orderless. That means that if $[n_1, n_2, n_3]$ is allowed, then $[n_1, n_3, n_2]$, $[n_2, n_1, n_3]$, $[n_2, n_3, n_1]$, $[n_3, n_1, n_2]$, $[n_3, n_2, n_1]$ are allowed too. We must also note that in case the number of NS fields indicated is less than the total number of NS fields, the rest of the NS fields can be freely distributed to the three categories except the one which indicates 0_{NS}. For example, in the box corresponding to the $N=9$ coupling with 6 R and 3 NS fields we write $[4_R+0_{NS}, 2_R, 0_R]$ indicating that the couplings $[4_R, 2_R, 3_{NS}]$, $[4_R, 2_R+1_{NS}, 2_{NS}]$, $[4_R, 2_R+2_{NS}, 1_{NS}]$, $[4_R, 2_R+3_{NS}, 0]$ are allowed.

Let us now summarize our results. For $N=1$ supersymmetric models built in the free fermionic formulation one has to choose six of the left-moving real fermions to be bosonized forming three U(1)'s. Physical states have specific charges under these U(1)'s and can be divided in three categories according to these charges. We find that some general selection rules associated with type {R, NS} and the category {1, 2, 3} of the involved fields can be derived. More specifically, for any $N\geqslant 3$ superpotential coupling

(i) A pure NS coupling exists only at $N=3$ with each field of different category.

(ii) A pure R coupling of order N partitioned as $[n_1, n_2, n_3]$ to the three categories vanishes unless if $n_i = N \bmod 2$ and $n_i\neq N$.

(iii) Any coupling with n R and $(N-n)$ NS fields, $N>3$, vanishes if $n\leqslant 3$.

(iv) A coupling of order $N>3$ involving some R and 1 NS field is nonvanishing only if the pure R part is nonvanishing according to (ii) and the NS field does not belong to the category with the maximum allowed number of elements.

(v) Any coupling with 4 R and $(N-4)$ NS fields vanishes unless if the associated fields are divided as $[2_R, 2_R, (N-4)_{NS}]$ in the three categories.

(vi) For the remaining coupling types, involving

Table 3
Field partitions to the three categories which lead to nonvanishing superpotential couplings up to the ninth order.

3	$[1_R, 1_R, 1_R]$	$[2_R+1_{NS}, 0, 0]$	–	$[1_{NS}, 1_{NS}, 1_{NS}]$	·	·
4	$[2_R, 2_R, 0]$	–	–	–	–	·
5	$[3_R, 1_R, 1_R]$	$[2_R, 2_R, 1_{NS}]$	–	–	–	–
6	$[4_R, 2_R, 0]$ $[2_R, 2_R, 2_R]$	$[3_R, 1_R+1_{NS}, 1_R]$	$[2_R, 2_R, 2_{NS}]$	–	–	–
7	$[5_R, 1_R, 1_R]$ $[3_R, 3_R, 1_R]$	$[4_R, 2_R, 1_{NS}]$ $[4_R, 2_R+1_{NS}, 0]$ $[2_R, 2_R, 2_R+1_{NS}]$	$[3_R, 1_R+2_{NS}, 1_R]$ $[3_R, 1_R+1_{NS}, 1_R+1_{NS}]$	$[2_R, 2_R, 3_{NS}]$	–	–
8	$[6_R, 2_R, 0]$ $[4_R, 4_R, 0]$ $[4_R, 2_R, 2_R]$	$[5_R, 1_R+1_{NS}, 1_R]$ $[3_R, 3_R, 1_R+1_{NS}]$ $[3_R, 3_R+1_{NS}, 1_R]$	$[4_R, 2_R, 2_{NS}]$ $[4_R, 2_R+1_{NS}, 1_{NS}]$ $[4_R, 2_R+2_{NS}, 0]$ $[2_R, 2_R, 2_R]$	$[3_R, 1_R, 1_R+3_{NS}]$ $[3_R, 1_R+1_{NS}, 1_R+2_{NS}]$	$[2_R, 2_R, 4_{NS}]$	–
9	$[7_R, 1_R, 1_R]$ $[5_R, 3_R, 1_R]$ $[3_R, 3_R, 3_R]$	$[6_R, 2_R, 1_{NS}]$ $[6_R, 2_R+1_{NS}, 0]$ $[4_R, 4_R, 0_R]$ $[4_R, 2_R, 2_R]$	$[5_R, 1_R+1_{NS}, 1_R+1_{NS}]$ $[5_R, 1_R, 1_R+2_{NS}]$ $[3_R, 3_R, 1_R]$	$[4_R+0_{NS}, 2_R, 0_R]$ $[2_R, 2_R, 2_R]$	$[3_R+0_{NS}, 1_R, 1_R]$ $[2_R, 2_R, 2_R]$	$[2_R, 2_R, 5_{NS}]$

both R and more than 1 NS field, rules arising from the combination of (ii) and (iv) can be constructed.

Explicitly, the rules for al possible coupling types up to $N=9$ are presented in table 3.

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