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String scale unification in an $SU(6) \times SU(2)$ GUT

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Abstract

We construct and analyze an $SU(6) \times SU(2)$ GUT. The model is $k=1$ string embedable in the sense that we employ only chiral representations allowed at the $k=1$ level of the associated Kač-Moody Algebra. Both cases $SU(6) \times SU(2)_L$ and $SU(6) \times SU(2)_R$ are realized. The model is characterized by the $SU(6) \times SU(2) \to SU(4) \times SU(2) \times SU(2)$ breaking scale M_X, and the $SU(4) \times SU(2) \times SU(2) \to SU(3)_C \times SU(2)_L \times U(1)_Y$ breaking scale M_R. The spectrum bellow M_R includes an extra pair of charge-1/3 colour-triplets of mass $M_I \le M_R$ that does not couple to matter fields and, possibly, an extra pair of isodoublets. Above M_X the $SU(6)$ and $SU(2)$ gauge couplings always unify at a scale which can be taken to be the string unification scale $M_s \sim 5 \times 10^{17}$ GeV. The model has Yukawa coupling unification since quarks and leptons obtain their masses from a single Yukawa coupling. Neutrinos obtain acceptably small masses through a see-saw mechanism. Coloured triplets that couple to matter fields are naturally split from the coexisting isodoublets without the need of any numerical fine tuning. © 1997 Elsevier Science B.V.

1. Introduction

Superstring Theory [1] is at present our best candidate for a unified theory of all particle interactions being a consistent theoretical framework that incorporates quantum gravity and supersymmetric gauge theories [2]. Unification takes place at energies close to the Planck scale while at lower energies gauge interactions are described in terms of an effective field theory whose spectrum consists of the massless string modes. From the low energy point of view the Standard Model and its $N=1$ supersymmetric extension, the Minimal Supersymmetric Standard Model (MSSM), can be naturally embedded in a Grand Unified Theory (GUT) with interesting phenomenological and cosmological consequences. GUTs [3] can successfully predict $\sin^2\theta_W$, fermion mass relations, charge quantization as well as provide a mechanism to explain the baryon asymmetry of the Universe [4]. However, in the context of quantum field theory no severe restrictions exist on the gauge group or the matter content of a GUT apart from the requirement that it should accommodate the MSSM. Thus a lot of possibilities arise including minimal SU(5) and its extensions [5], varieties of SO(10) [6], E(6) models [7] etc.

Accommodating a GUT in the framework of Superstring Theory [1], or more precisely assuming that

[1] The term GUT is used here in a loose sense for a gauge model with gauge group $G \supset SU(3) \times SU(2) \times U(1)$ which accommodates the MSSM particles and displays partial unification and/or correlation of the standard model group gauge factors.

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the GUT is the low energy effective field theory of a four dimensional heterotic superstring compactification, imposes serious restrictions on the GUT spectrum. For gauge groups realized at level $k=1$ of the World-Sheet Affine Algebra, only the chiral multiplets in the vector and antisymmetric representations of $SU(n)$ groups and the vector and spinor of $SO(n)$ groups are massless. The absence of adjoint scalars limits the possibilities of breaking to the MSSM through the Higgs mechanism and diminishes the number of candidate GUT models. Apart from this serious restriction, superstrings offer a new possibility. The GUT gauge group does not have to be simple in order to guarantee unification. Semi-simple or product groups are equally acceptable since string theory takes over the job of gauge coupling unification.

Historically the first GUTs that were studied as candidate low energy superstring effective models were the maximal $E(6)$ subgroups [8] as they arise naturally in the context of Calabi-Yau compactifications [9–11]. A typical example of such a model is the $SU(3)\times SU(3)\times SU(3)$ GUT [12] while other possibilities as $SU(6)\times U(1)$ have also been considered [13]. Soon it was realized that $SO(10)$ subgroups were also possible in the context of Orbifolds [14,11] or in the *free-fermionic formulation* [15]. Two typical GUT examples, namely the "flipped-SU(5)" [16] and the supersymmetric Pati-Salam model [17], were explicitly constructed in the framework of the Fermionic Superstrings [18,19]. Standard Model-like alternatives have also been explored [20].

Recently, a new the possibility has been explored, namely GUTs realized at the $k\geq 2$ level of heterotic superstring compactifications [21]. Some generic features of these models have been presented in [22,23]. Higher level models allow the presence of adjoint chiral superfields and thus the number of the candidate GUT gauge groups is enhanced. Although some explicit superstring models have been presented [24], there are still some problems to overcome [22,23,25]. Such is the absence of certain couplings. For instance, the remnants of the GUT breaking adjoint Higgs, an isotriplet and a colour-octet, remain light to all orders in perturbation theory [22,25].

Another issue raised in the framework of the Superstring embedding of the MSSM is that of the "unification mismatch". Assuming MSSM to hold, low energy data indicate gauge coupling unification at a scale $M_X\sim 10^{16}$ GeV [26] more than one order of magnitude below the string scale $M_s\sim 5\times 10^{17}$ GeV which is the typical string unification scale. The prospects to fill this gap by threshold corrections due to the infinite tower of massive string modes [27,28] seem rather restricted [29] as explicit calculations in superstring models show that such thresholds are very small [30]. This could be taken as an indication for the existence of *intermediate scales* in which extra matter states become massive and their thresholds increase the coupling unification scale up to M_s [31,32,25]. One is thus motivated to incorporate this additional feature in a candidate GUT model.

In the current state of elaboration of String Theory, the string vacuum is not uniquely determined and the various gauge groups appear on equal footing. It is hence interesting, from the low energy point of view, to classify all gauge models satisfying the known string constraints and see whether they can meet the criteria imposed by low energy data and at the same time look for signatures that distinguish among them.

As is well known, all quark and lepton fields can be accommodated in the **27** representation of $E(6)$ together with a pair of isodoublets suitable to serve as electroweak Higgses and a pair of charge $\pm\frac{1}{3}$ colour-triplets. Among the maximal subgroups of $E(6)$ we find $SU(6)\times SU(2)$, a regular subgroup which apart from [33] has not received much attention in the literature. Decomposing the **27** under $SU(6)\times SU(2)$ we get [34]

$$\mathbf{27}=(\bar{\mathbf{6}},\mathbf{2})+(\mathbf{15},\mathbf{1})$$

where **15** is the two-index antisymmetric representation of $SU(6)$. In what follows we shall construct a $k=1$ string embedable $SU(6)\times SU(2)$ GUT using chiral superfields in the **(6,2)** and **(15,1)** representations only. Two distinct ways of embedding the standard model group in $SU(6)\times SU(2)$ arise depending on whether we identify the electroweak $SU(2)_L$ with the GUT $SU(2)$ factor or with the $SU(4)\times SU(2)$ subgroup of $SU(6)$. In the latter case we have an $SU(6)\times SU(2)_R$ model while in the former an $SU(6)\times SU(2)_L$ model. Both cases $SU(6)\times SU(2)_L$ and $SU(6)\times SU(2)_R$ are realizable. Apart from the GUT scale M_X of $SU(6)$ breaking, the model is also characterized by the extra scale of

$SU(2)_R$ breaking M_R. Below the scale M_R the spectrum contains an extra pair of coloured triplets of mass $M_I \leq M_R$. The $SU(6)$ and $SU(2)$ gauge couplings always meet at energies above M_X introducing an additional scale which can always be taken to be the string unification scale $M_s \sim 5 \times 10^{17}$ GeV. In addition these models have some other very interesting features. The electroweak Higgses, together with extra charge-1/3 colored triplets, participate in the same representations that contain quarks and leptons. These isodoublets and colour-triplets also come in family replicas. Despite that, the triplet-doublet splitting can be achieved naturally without the need of any numerical fine tuning. Quark and lepton masses arise from a single Yukawa coupling, a cubic interaction of matter superfields, just as in E_6, despite the fact that matter fields come in a reducible representation. Thus, $SU(6) \times SU(2)$ displays Yukawa coupling unification. In the case of one pair of Higgs isodoublets the mass relations $m_t \cot\beta = m_b = m_\tau$ are true and imply strong constraints on m_t, $\tan\beta$ [36] and the various supersymmetry breaking parameters [37].

2. Symmetry breaking and model building

Decomposing the **15** representation under $SU(4) \times SU(2) \times U(1) \subseteq SU(6)$ we obtain

$$\mathbf{15} = (\mathbf{1},\mathbf{1},4) + (\mathbf{6},\mathbf{1},-2) + (\mathbf{4},\mathbf{2},1)$$

Thus, it is clear that the GUT symmetry breaking $SU(6) \times SU(2) \to SU(4) \times SU(2) \times SU(2)$ can be achieved with a non-zero v.e.v. of $(\mathbf{15},\mathbf{1}) + (\overline{\mathbf{15}},\mathbf{1})$ in the D-flat direction $\langle(\mathbf{1},\mathbf{1},4)\rangle = \langle(\mathbf{1},\mathbf{1},-4)\rangle$. The decomposition of the adjoint **35**of $SU(6)$ under $SU(4) \times SU(2) \times U(1)$ takes the form $\mathbf{35} = (\mathbf{1},\mathbf{1},0) + (\mathbf{1},\mathbf{3},0) + (\mathbf{15},\mathbf{1},0) + (\mathbf{4},\mathbf{2},-3) + (\overline{\mathbf{4}},\mathbf{2},3)$. This helps to see that the tetraplets in **15**will be absorbed while the sextets $(\mathbf{6},\mathbf{1},-2)$, $(\mathbf{6},\mathbf{1},2)$ and the singlet combination $(\mathbf{1},\mathbf{1},4) + (\mathbf{1},\mathbf{1},-4)$ will survive the Higgs phenomena. Nevertheless, cubic superpotential couplings

$$(\mathbf{15})^3 = (\mathbf{6},\mathbf{1},-2)^2(\mathbf{1},\mathbf{1},4) + (\mathbf{4},\mathbf{2},1)^2(\mathbf{6},\mathbf{1},-2)$$

while not spoiling F-flatness, can give the sextets a mass. The first stage of symmetry breaking being as described, two possibilities arise as we move further down.

a) An $SU(6) \times SU(2)_L$ model.

The desired breaking pattern down to the standard model gauge group is

$$SU(6) \times SU(2)_L \to M_X SU(4) \times SU(2)_R \times SU(2)_L$$
$$\to M_R SU(3)_C \times U(1)_Y \times SU(2)_L$$

As we described above, the first stage of symmetry breaking requires a pair of Higgses [2]

$$\mathscr{H}(\overline{\mathbf{15}},\mathbf{1}) + \overline{\mathscr{H}}(\mathbf{15},\mathbf{1}) = (N_H)_{(\mathbf{1},\mathbf{1},\mathbf{1})} + (\Delta_H^c + \Delta_H)_{(\overline{\mathbf{6}},\mathbf{1},\mathbf{1})} + (E_H^c + N_H^c + D_H^c + U_H^c)_{(\overline{\mathbf{4}},\mathbf{2},\mathbf{1})} + \text{conj. reps.} \quad (1)$$

A D-flat v.e.v. $\langle N_H \rangle = \langle \overline{N}_H \rangle = V_X$ breaks $SU(6) \times SU(2)_L$ to $SU(4) \times SU(2)_R \times SU(2)_L$. The surviving states are the sextets $(\Delta_H^c + \Delta_H)_{(\mathbf{6},\mathbf{1},\mathbf{1})}$, $(\overline{\Delta}_H^c + \overline{\Delta}_H)_{(\mathbf{6},\mathbf{1},\mathbf{1})}$ plus a singlet combination $N_H + \overline{N}_H$.

The second stage of symmetry breaking down to the standard model gauge group can be achieved introducing an additional pair

$$H(\overline{\mathbf{15}},\mathbf{1}) + \overline{H}(\mathbf{15},\mathbf{1}) = (n_H)_{(\mathbf{1},\mathbf{1},\mathbf{1})} + (\delta_H^c + \delta_H)_{(\mathbf{6},\mathbf{1},\mathbf{1})} + (e_H^c + \nu_H^c + d_H^c + u_H^c)_{(\overline{\mathbf{4}},\mathbf{2},\mathbf{1})} + \text{conj. reps.} \quad (2)$$

A D-flat v.e.v. $\langle \nu_H^c \rangle = \langle \overline{\nu}_H^c \rangle = V_R$ breaks $SU(4) \times SU(2)_R$ down to $SU(3)_C \times U(1)_Y$. The nine surviving states are the sextets $(\delta_H^c + \delta_H)_{(\mathbf{6},\mathbf{1},\mathbf{1})}$, $(\overline{\delta}_H^c + \overline{\delta}_H)_{(\mathbf{6},\mathbf{1},\mathbf{1})}$ and the singlets $(n_H)_{(\mathbf{1},\mathbf{1},\mathbf{1})}$, $(\overline{n}_H)_{(\mathbf{1},\mathbf{1},\mathbf{1})}$, which do not participate in the symmetry breaking anyway, a pair of colour-triplets d_H^c, $\overline{d}_H^c$ and a combination of singlets $\nu_H^c + \overline{\nu}_H^c$.

The Higgs sector superpotential, suitable for the desired symmetry breaking pattern will be taken to

[2] We introduce here a mixed notation where the $SU(4) \times SU(2)_R \times SU(2)_L \subset SU(6) \times SU(2)$ quantum numbers are shown explicitly while the symbols for the fields indicate Standard Model quantum numbers in the usual notation: $\Delta^c, D^c, \delta^c \to (\overline{\mathbf{3}},\mathbf{1},\frac{1}{3})$, $U^c, u^c \to (\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})$, $E^c, e^c \to (\mathbf{1},\mathbf{1},1)$, $N, n, \nu^c \to (\mathbf{1},\mathbf{1},0)$, $h, L, \eta \to (\mathbf{1},\mathbf{2},-\frac{1}{2})$ etc.

be the cubic superpotential (invariant under the discrete symmetry $H \to -H$)

$$W_H = \lambda \mathscr{H} H^2 + \overline{\lambda \mathscr{H} H}^2 \tag{3}$$

which leads to the Higgs mass terms

$$(W_H)_{\text{mass}} = \lambda V_X \delta_H^c \delta_H + \lambda V_R d_H^c \Delta_H + \bar{\lambda} V_X \overline{\delta_H^c \delta_H} + \bar{\lambda} V_R \overline{d_H^c \Delta_H} + \ldots \tag{4}$$

Two pairs of coloured triplets, namely δ_H, δ_H^c and $\overline{\delta_H}$, $\overline{\delta_H^c}$, obtain a mass of $O(M_X)$ and two pairs, namely d_H^c, Δ_H and $\overline{d_H^c}$, $\overline{\Delta_H}$, obtain a mass of $O(M_R)$. Below M_R, apart from the four singlet states n_H, $\overline{n_H}$, $N_H + \overline{N_H}$ and $\nu_H^c + \overline{\nu_H^c}$, the renormalizable superpotential interactions in (3) leave the pair of coloured triplets Δ_H^c, $\overline{\Delta_H^c}$ massless. Effective non-renormalizable terms could be added to (3) which would generate masses for these states. These could arise due to the exchange of massive states present in all string constructions. Their exact form depends on the details of the specific string model. A necessary constraint on these terms is the requirement that they should not spoil F-flatness. Note however that violations of F-flatness that lead to scalar masses of $O(\text{TeV})$ can be tolerated since the supersymmetry breaking is of that order. This implies $\langle F \rangle / M \leq O(\text{TeV})$ or $\langle F \rangle \leq (10^{11}\ \text{GeV})^2$. High order non-renormalizable terms of the form $(\mathscr{H}\overline{\mathscr{H}})^n / M^{2n-3}$ lead to $F \sim (M_X/M)^{2n-1} M^2$ and satisfy this constraint if $(M_X/M)^{2n-1} \leq 10^{-14}$. For $M_X/M \sim 0.01$ this corresponds to $n \geq 4$. Writing down the contents of such a term, gives

$$\Delta W_H = (\mathscr{H}\overline{\mathscr{H}})^n / M^{2n-3} = \frac{(V_X)^{2n-2}}{M^{2n-3}} (\Delta_H \overline{\Delta_H} + \Delta_H^c \overline{\Delta_H^c} + \ldots) \tag{5}$$

This corresponds to an intermediate mass $M_I \sim (M_X/M)^{2n-2} M$ for the triplets Δ_H^c, $\overline{\Delta_H^c}$ and the singlet $N_H + \overline{N_H}$. The generated intermediate mass is constrained to be $M_I \sim (M_X/M)^{2n-2} M \leq O(10^{-14}) M^2/M_X$. For $M_X/M \sim 0.01$, this implies $M_I \leq O(10^6\ \text{GeV})$. For $M_X/M \sim 0.1$, $M_I \leq O(10^5\ \text{GeV})$. Note also that, although the product in (5) can arise either as $(15)_{ij} \overline{(15)}^{ij}$ or as $(15)_{ij}(15)_{kl}(15)_{mn} \epsilon^{ijklmn} \overline{(15)}^{pq} \overline{(15)}^{rs} \overline{(15)}^{tz} \epsilon_{pqrstz}$, only the first case gives rise to masses. It should be remarked however that the exact form of the non-renormalizable terms depends on the details of the, possibly, underlying string model and the above presented mechanism that supplies the remaining colour-triplets with an intermediate mass, although perfectly consistent in the framework of the GUT, serves as an existence proof of phenomenologically desirable scenaria that can be realized within the string model. Summarizing, we see that introducing the Higgses $\mathscr{H}$, $\overline{\mathscr{H}}$, H, $\overline{H}$ we can achieve the successive breaking $SU(6) \times SU(2)_L \to SU(4) \times SU(2)_R \times SU(2)_L \to SU(3)_C \times U(1)_Y \times SU(2)_L$. Below M_X the two pairs of Higgses contribute to the spectrum of the model with two sextets $(\Delta_H + \Delta_H^c)_{(\bar{6},1,1)}$ and *conjugate states*, two tetraplets $(u_H^c + d_H^c + e_H^c + \nu_H^c)_{(\bar{4},2,1)}$ and *conjugate states*, and the three singlets n_H, $\overline{n_H}$, $N_H + \overline{N_H}$. Below M_R only the coloured triplets Δ_H^c, $\overline{\Delta_H^c}$ survive, together with four singlets n_H, $\overline{n_H}$, $N_H + \overline{N_H}$, $\nu_H^c + \overline{\nu_H^c}$. The coloured triplets obtain a mass at the lower scale M_I.

Quarks and leptons can be accommodated in the $(\overline{\mathbf{15}},\mathbf{1}) + (\mathbf{6},\mathbf{2})$ representation in three family replicas

$$\phi(\overline{\mathbf{15}},\mathbf{1}) = N_{(1,1,1)} + (\delta^c + \delta)_{(6,1,1)} + (e^c + \nu^c + d^c + u^c)_{(\bar{4},2,1)} \tag{6}$$

$$\psi(\mathbf{6},\mathbf{2}) = (h + h^c)_{(1,2,2)} + (q + l)_{(4,1,2)} \tag{7}$$

Notice that together with quarks and leptons we have pairs of isodoublets, suitable as electroweak Higgses h, h^c and coloured triplets δ, δ^c in three family replicas too. We next introduce the matter self coupling

$$W_M = Y_{ijk} \phi_i \psi_j \psi_k = \tilde{Y}_{ijk} \left(q_j d_i^c h_k + q_j u_i^c h_k^c + l_j e_i^c h_k + l_j \nu_i^c h_k^c + \tfrac{1}{2} \delta_i q_j q_k + \delta_i^c q_j l_k + N_i h_j h_k^c \right) \tag{8}$$

where $\tilde{Y}_{ijk} = Y_{ijk} + Y_{ikj}$. Quark and lepton masses arise exclusively from the single Yukawa coupling $\tilde{Y}_{ijk}$. The fifth and sixth term in (8), together with a $\delta\delta^c$ mass term for the extra colour-triplets, give rise to the $D = 5$ operators $qqql$ that violate baryon number and can induce proton decay [35]. These operators are controllable if the $\delta_i \delta_j^c$ mass is of $O(M_X)$. This can be achieved if we introduce the couplings

$$\Delta W_M = \tfrac{1}{2} \lambda_{ij} \mathscr{H} \phi_i \phi_j + \tfrac{1}{2} \lambda'_{ij} \mathscr{H} \psi_i \psi_j \tag{9}$$

which give the terms

$$\lambda_{ij}\left(N_H\delta_i\delta_j^c + e_j^c u_j^c \Delta_H + \nu_i^c d_j^c \Delta_H + u_i^c d_j^c \Delta_H^c + \ldots\right)$$
$$+ \lambda'_{ij}\left(N_H h_i h_j^c + q_i q_j \Delta_H + q_i l_j \Delta_H^c\right) \quad (10)$$

Masses of $O(\lambda V_X)$ and $O(\lambda' V_X)$ correspondingly can now arise for the coloured triplets and isodoublets. Note that both λ_{ij} and λ'_{ij} couplings are symmetric and λ'_{ij} must have at least one zero eigenvalue. Baryon number violating operators of $D=5$ of the type $e^c u^c u^c d^c$, $\nu^c u^c d^c d^c$ and $qqql$ would also appear if a mass term Δ_H-Δ_H^c were present. This is not the case however since Δ_H^c mixes only with $\overline{\Delta_H^c}$ which does not mix with matter fields. Nevertheless, $D=6$ operators of the form $\lambda_{ij}\lambda'_{km}\left(u_i^c d_j^c q_k^\dagger l_m^\dagger\right)$, generated by the exchange of a scalar triplet Δ_H or Δ_H^c, are still possible and can be dangerous in the case of an intermediate triplet mass. The presence of these operators depends on the structure of the coupling matrices $\lambda_{ij}\lambda'_{km}$. For instance, in the case of diagonal couplings, a simple condition like $\lambda_{1j}=0, j=1,2$ would be sufficient to render these operators harmless without affecting the massiveness of δ_i, δ_i^c. In a general situation this problem can be evaded by requiring a more complicated texture structure for λ_{ij} and/or λ'_{km}. Another more drastic solution is to set $\lambda'_{km}=0$. In this case no such operators exist but all three pairs of isodoublets are left massless. This is not a big problem since their masses can be generated by non-renormalizable terms. For example the term $\frac{\lambda''_{ij}}{M^3}H^4\psi_i\psi_j$ leads to a $\lambda''_{ij}\frac{<\nu_H^c>^4}{M^3}h_i h_j^c$ isodoublet mass matrix without introducing any unwanted couplings.

Expressing the Yukawa interactions in terms of the Higgs mass eigenstates, we obtain

$$Y_{ij}^a\left(q_i d_j^c \tilde{h}_a + l_i e_j^c \tilde{h}_a + q_i u_j^c \tilde{h}_a^c + l_i \nu_j^c \tilde{h}_a^c\right),$$
$$a = 1,2,3 \quad (11)$$

It is clear that in the case of only one pair of massless Higgses, since there is a single, generation dependent, Yukawa coupling, we have the fermion mass relations $m_b = m_t \cot\beta = m_\tau$ which imply the prediction of m_t [36] as well as strong constraints on $\tan\beta$ ($\tan\beta \geq 40$) and the susy-breaking parameters [37]. In the case that a second pair of isodoublets is massless and obtain v.e.v.'s, there is no simple mass relation unless there is v.e.v. alignment. Depending on the details of the specific string model or possibly existing family symmetries, the Yukawa couplings for the light generations could arise as effective non-renormalizable terms with a suppressing factor $(\langle S\rangle/M)^n$ involving the v.e.v. of a singlet field S [41]. Nevertheless, an alternative to such a scenario would be to keep more doublets light and impose a hierarchy on the available many v.e.v's. Non-renormalizable terms can also generate the required large Majorana mass for the right-handed neutrino. Introducing

$$Y''_{ij}\phi_i\phi_j\overline{H}^2/M \quad (12)$$

we are only led to

$$Y''_{ij}\nu_i^c\nu_j^c\langle\overline{\nu}_H^c\rangle^2/M = \frac{Y''_{ij}V_R^2}{M}\nu_i^c\nu_j^c \quad (13)$$

Depending on the effective non-renormalizable coupling Y''_{ij} this term can lead to acceptable neutrino masses through a see-saw mechanism.

b) An SU(6) × SU(2)$_R$ model.

The first breaking $SU(6)\times SU(2)_R \to SU(4)\times SU(2)_L \times SU(2)_R$ occurs exactly as in the previous model using a pair of Higgses $(\mathbf{15},\mathbf{1}) + (\overline{\mathbf{15}},\mathbf{1})$. Nevertheless, due to the different position of $SU(2)_L$ their content looks different in terms of standard model quantum numbers. Again we introduce

$$\mathscr{H}(\mathbf{15},\mathbf{1}) + \overline{\mathscr{H}}(\overline{\mathbf{15}},\mathbf{1})$$
$$= (N_H)_{(1,1,1)} + (\Delta_H + \Delta_H^c)_{(6,1,1)}$$
$$+ (L_H + Q_H)_{(4,2,1)} + \text{conj. reps.} \quad (14)$$

A D-flat v.e.v. $\langle N_H\rangle = \langle\overline{N}_H\rangle = V_X$ breaks the gauge group to $SU(4)\times SU(2)_L\times SU(2)_R$. The coloured triplets Δ_H, Δ_H^c, $\overline{\Delta}_H^c$, $\overline{\Delta}_H$ and a combination of singlets, namely $N_H + \overline{N}_H$, survive the Higgs phenomena.

The second stage of symmetry breaking down to the Standard Model group is achieved introducing

$$H(\mathbf{6},\mathbf{2}) + \overline{H}(\mathbf{6},\mathbf{2}) = (\eta_H + \eta_H^c)_{(1,2,2)}$$
$$+ (d_H^c + u_H^c + e_H^c + \nu_H^c)_{(\overline{4},1,2)}$$
$$+ \text{conj. reps.} \quad (15)$$

A D-flat v.e.v. $\langle\nu_H^c\rangle = \langle\overline{\nu}_H^c\rangle = V_R$ breaks the gauge group down to $SU(3)_C\times SU(2)_L\times U(1)_Y$. In addition to η_H, η_H^c, $\overline{\eta}_H$, $\overline{\eta}_H^c$ which are not affected by the

Higgs phenomena, d_H^c, $\overline{d}_H^c$ and the singlet $\nu_H^c + \overline{\nu}_H^c$ survive.

The Higgs sector superpotential will be taken to be again a cubic one of the form

$$W_H = \lambda \mathscr{H} H^2 + \overline{\lambda \mathscr{H} H}^2 \tag{16}$$

which leads to the Higgs mass terms

$$(W_H)_{\text{mass}} = \lambda V_X \eta_H \eta_H^c + \lambda V_R \Delta_H d_H^c + \text{conj. reps.} \tag{17}$$

Two pairs of Higgs isodoublets become massive at M_X and two pairs of coloured triplets, namely Δ_H, d_H^c and $\overline{\Delta_H}$, $\overline{d_H^c}$ become massive at M_R. Below M_R we are left with a massless pair of colour-triplets Δ_H^c, $\overline{\Delta_H^c}$. Following the philosophy of the previous model we can expect that non-renormalizable terms $(\mathscr{H}\overline{\mathscr{H}})^n/M^{2n-3}$ will give rise to a mass of intermediate value $M_I \sim 10^6$ GeV or smaller for the pair Δ_H^c, $\overline{\Delta_H^c}$ as well as for the singlet $N_H + \overline{N_H}$.

The matter fields are introduced as

$$\phi(\mathbf{15},\mathbf{1}) = N_{(1,1,1)} + (\delta + \delta^c)_{(6,1,1)} + (l + q)_{(4,2,1)} \tag{18}$$

$$\psi(\overline{\mathbf{6}},\mathbf{2}) = (h + h^c)_{(1,2,2)} + (d^c + u^c + e^c + \nu^c)_{(\overline{4},1,2)} \tag{19}$$

in three family replicas. The Yukawa terms are obtained from

$$W_M = Y_{ijk}\phi_i\psi_j\psi_k \tag{20}$$

$$= \tilde{Y}_{ijk}\big(l_i e_j^c h_k + l_i \nu_j^c h_k^c + q_i d_j^c h_k + q_i u_j^c h_k^c + N_i h_j h_k^c + \delta_i d_j^c \nu_k^c + \delta_i u_j^c e_k^c + \delta_i^c d_j^c u_k^c\big) \tag{21}$$

Let us now introduce the Higgs-matter interactions

$$\Delta W_M = \lambda_{ij}\mathscr{H}\phi_i\phi_j + \lambda'_{ij}\mathscr{H}\psi_i\psi_j = \lambda_{ij}\big(N_H\delta_i\delta_j^c + \Delta_H q_i q_j\big) + \lambda'_{ij}\big(N_H h_i h_j^c + \Delta_H d_i^c \nu_j^c + \Delta_H u_i^c e_j^c + \Delta_H^c d_i^c u_j^c\big) \tag{22}$$

Note that the couplings λ_{ij}, λ'_{ij} are symmetric. These couplings are sufficient to render all triplets δ_i, δ_i^c massive with a mass of $O(M_X)$. Of course λ'_{ij} has to be restricted in family space in order to obtain the desired number of massless pairs of electroweak Higgses. The $D=5$ operators that violate baryon number and can arise from W_M are going to be sufficiently suppressed if δ_i, δ_i^c have a mass of $O(M_X)$. On the other hand, since Δ_H and Δ_H^c do not mix, there is no danger of any $D=5$ operators arising from ΔW_M. Note that Δ_H obtains its mass from d_H^c which does not mix with matter. $D=6$ operators involving the exchange of a Δ_H scalar are also possible and could be dangerous for low M_R. The related discussion in the previous model applies also here. If the condition $\lambda'_{ij}=0$ is imposed in order to circumvent this problem, the non-renormalizable term $\frac{\lambda''_{ij}}{M^3}H^4\psi_i\psi_j$ can be invoked to generate $\frac{<\nu_H^c>^4}{M^3}$ masses for the unwanted doublets.

Finally the Yukawa interactions are identical to those of the previous model and all points concerning Yukawa coupling unification and mass relations are the same. Again, in order to obtain acceptable neutrino masses, the non-renormalizable interactions

$$Y''_{ij}\psi_i\psi_j\overline{HH}/M$$

have to be invoked. This term generates a Majorana mass matrix for the right-handed neutrinos of order $Y''_{ij}V_R^2/M$.

3. Renormalization group analysis

As follows from the previous analysis the $SU(2)_R$ scale M_R is a free parameter while the triplet mass scale M_I depends on the details of the model and in particular of the non-renormalizable contributions in the superpotential. In addition the $SU(6)\times SU(2)_L$ model, denoted as model (I) from now on, and the $SU(4)\times SU(2)_R$ model, denoted as model (II), are indistinguishable bellow M_R where we have an $SU(3)_C\times SU(2)_L\times U(1)_Y$ theory with an extra charge-1/3 colour-triplet pair of intermediate mass M_I. In what follows we shall assume both M_R and M_I as free parameters and derive the constraints imposed on them when demanding $SU(6)$ and $SU(2)$ coupling unification at the string scale $M_S = 5.27\times g_S\times 10^{17}$ GeV. For simplicity we consider here only the case of one or two ($N_H = 1,2$) massless isodoublet pairs below M_R and leave the more complicated analysis of extra intermediate mass doublets for a future publication [38]. Integrating the renormalization group equations from M_Z up to M_R, we obtain

$$\alpha_i^{-1}(M_R) = \alpha_i^{-1} + \frac{b_i}{2\pi}\ln\left(\frac{M_Z}{M_R}\right) + \frac{\tilde{b}_i}{2\pi}\ln\left(\frac{M_I}{M_R}\right) \tag{23}$$

Where $i = 1,2,3$ and α_1, α_2, α_3 stand for the values of the three gauge couplings at M_Z. The RG coefficients take up the values $b_3 = -3$, $b_2 = N_H$, $b_1 = 6 + \frac{3N_H}{5}$, $\tilde{b}_3 = 1$, $\tilde{b}_2 = 0, \tilde{b}_1 = \frac{2}{5}$ At an energy scale $\mu \geq M_R$ we have an $SU(4) \times SU(2)_R \times SU(2)_L$ theory in both cases (I) and (II). Note that the particle content for (I) and (II) is the same. The three gauge couplings are given by

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_R) + \frac{b_i}{2\pi}\ln\left(\frac{M_R}{\mu}\right) \tag{24}$$

where $i = 4, 2R, 2L$. The coefficients take up the values $b_4 = -2$, $b_{2L} = N_H$, $b_{2R} = 4 + N_H$. The matching conditions at M_R are

$$\alpha_4(M_R) = \alpha_3(M_R) \tag{25}$$

$$\alpha_{2R}^{-1}(M_R) = \tfrac{5}{3}\alpha_1^{-1}(M_R) - \tfrac{2}{3}\alpha_3^{-1}(M_R) \tag{26}$$

$$\alpha_{2L}(M_R) = \alpha_2(M_R) \tag{27}$$

As μ moves away from M_R two possibilities arise. If α_{2R} meets first with α_4, we have model (I). In the opposite case of α_{2L} meeting first with α_4, we have model (II). From the point of view of the renormalization group analysis, the low energy data as well as the free parameters of the theory, M_R and M_I, determine which case is realized.

Let us first study the case of model (I) which is defined by

$$\alpha_6(M_X) = \alpha_{2R}(M_X) = \alpha_4(M_X) \geq \alpha_{2L}(M_X) \tag{28}$$

M_X stands for the $SU(6) \times SU(2)$ breaking scale. Manipulating all previously displayed equations, we obtain

$$\frac{1}{2\pi}\ln\left(\frac{M_X}{M_Z}\right) = \frac{5}{3(6+N_H)}\left(\alpha_1^{-1} - \alpha_3^{-1}\right) - \frac{4}{\pi(6+N_H)}\ln\left(\frac{M_R}{M_Z}\right) - \frac{1}{2\pi(6+N_H)}\ln\left(\frac{M_I}{M_Z}\right) \tag{29}$$

The inequality in (28) defining model (I) is equivalent to

$$F = \tfrac{5}{3}(2+N_H)\alpha_1^{-1} + \tfrac{2}{3}(4-N_H)\alpha_3^{-1} - (6+N_H)\alpha_2^{-1} + \frac{2}{\pi}\ln\left(\frac{M_I}{M_Z}\right) - \frac{4}{\pi}(2+N_H)\ln\left(\frac{M_R}{M_Z}\right) \leq 0 \tag{30}$$

At a scale $\mu \geq M_X$ the $SU(6) \times SU(2)_L$ gauge couplings are

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_X) + \frac{b_i}{2\pi}\ln\left(\frac{M_X}{\mu}\right) \tag{31}$$

Where $i = 6, 2L$. The coefficients take up the values $b_6 = -1$, $b_{2L} = 3$. The fact that $b_6 < 0$ and $b_{2L} > 0$ together with (28) imply that there exists always a scale $\mu = M_S > M_X$ at which the $SU(6)$ and $SU(2)$ couplings become equal

$$\alpha_6(M_S) = \alpha_{2L}(M_S) \equiv \alpha_S \tag{32}$$

Solving for it, we obtain

$$\frac{1}{2\pi}\ln\left(\frac{M_S}{M_Z}\right) = \tfrac{1}{4}\left(\alpha_2^{-1} - \alpha_3^{-1}\right) - \frac{1}{8\pi}\ln\left(\frac{M_I}{M_Z}\right) + \frac{(2-N_H)}{8\pi}\ln(M_X/M_Z) \tag{33}$$

while the value of the unified gauge coupling is given by

$$\alpha_S^{-1} = \alpha_3^{-1} + \frac{1}{2\pi}\ln\frac{M_S}{M_Z} + \frac{1}{2\pi}\ln\frac{M_X}{M_Z} + \frac{1}{2\pi}\ln\frac{M_I}{M_Z} \tag{34}$$

The case of the model (II) is obtained for

$$\alpha_6 = \alpha_{2L}(M_X) = \alpha_4(M_X) \geq \alpha_{2R}(M_X) \tag{35}$$

and the inequality (28) amounts to $F \geq 0$. The GUT scale is given by

$$\frac{1}{2\pi}\ln\left(\frac{M_X}{M_Z}\right) = \frac{1}{(N_H+2)}\left(\alpha_2^{-1} - \alpha_3^{-1}\right) - \frac{1}{2\pi(N_H+2)}\ln\left(\frac{M_I}{M_Z}\right) \quad (36)$$

The $SU(6) \times SU(2)_R$ gauge couplings at a scale $\mu \geq M_X$ are given by the corresponding expressions for model (I) with the replacements $b_6 \to b_6' = -3$ and $b_{2L}' \to b_{2R}' = 9$. These couplings always meet at a scale M_S defined by

$$\alpha_6(M_S) = \alpha_{2R}(M_S) \equiv \alpha_S \quad (37)$$

The scale M_S is obtained to be

$$\frac{1}{2\pi}\ln\left(\frac{M_S}{M_Z}\right) = \tfrac{5}{36}\left(\alpha_1^{-1} - \alpha_3^{-1}\right) - \frac{1}{24\pi}\ln(M_I/M_Z) + \frac{(6-N_H)}{24\pi}\ln(M_X/M_Z) - \frac{1}{3\pi}\ln(M_R/M_Z) \quad (38)$$

while the formula for the unified gauge coupling takes the form

$$\alpha_S^{-1} = \alpha_3^{-1} + \frac{3}{2\pi}\ln\frac{M_S}{M_Z} + \frac{1}{2\pi}\ln\frac{M_I}{M_Z} - \frac{1}{2\pi}\ln\frac{M_X}{M_Z} \quad (39)$$

Given M_I and M_R and the values of the low energy parameters we can now check whether $F < 0$ or $F > 0$. In the former case we are in Model (I) and we can use (29), (33), (34) in order to solve for M_X and M_S as well as α_S. In the latter we are in model (II) and we must use (37), (39), (40) instead. If we further assume that the unification scale M_S is the *string scale* $M_S = 5.27 \times g_S \times 10^{17}$ GeV we can express M_I, and subsequently M_X and α_S, as a function of M_R.

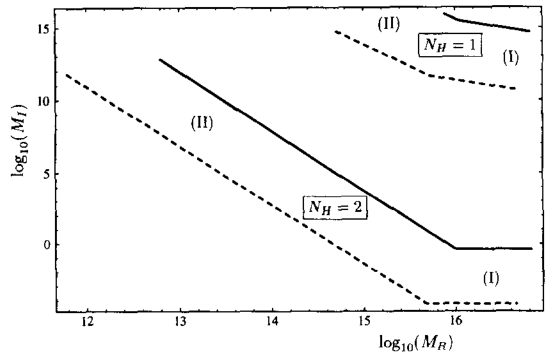

Fig. 1. The intermediate scale $\log_{10}(M_I)$ in the case $N_H = 1,2$ as function of $\log_{10}(M_R)$, for $\alpha_3 = .11$ (dashed line) and $\alpha_3 = .13$ (continuous line).

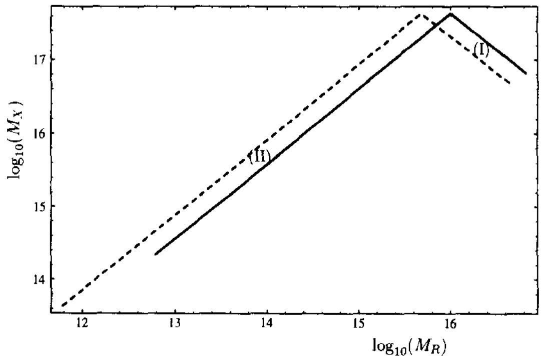

Fig. 2. The GUT scale $\log_{10}(M_X)$ in the case $N_H = 2$ as function of $\log_{10}(M_R)$, for $\alpha_3 = .11$ (dashed line) and $\alpha_3 = .13$ (continuous line).

In Figs. 1, 2 and 3 we give plots of M_I and M_X as functions of $\log(M_R)$ for $M_S = 5.27 \times g_S \times 10^{17}$ GeV in the cases $N_H = 1,2$ for both models (I) and (II). As indicative input values we have taken those of [39], while for $\alpha_3(M_Z)$ we have utilized the range 0.11–0.13. We shall not worry about the uncertainty in these values since we intend here to investigate the generic dependence on the parameter M_R instead of obtaining a precise determination. The allowed region is the one between the dashed and the full line. As it can be seen in the figures there is a wide range of M_R and M_I values for which both versions of the model can be realized. Namely, for $N_H = 1$ model (I) requires [3] $10^{15.8}$ GeV $< M_R < 10^{16.3}$ GeV and model (II) $10^{14.6}$ GeV $< M_R < 10^{16.1}$ GeV while 10^{11} GeV $< M_I < 10^{15}$ GeV. For $N_H = 2$, model (I) demands $10^{15.4}$ GeV $< M_R < 10^{16.4}$ GeV and $M_I \sim \mathcal{O}$(GeV) and model (II) 10^{14} GeV $< M_R < 10^{15.8}$ GeV with 10^3 GeV $< M_I < 10^8$ GeV. From Fig. 1 it can be seen that the intermediate scale M_I in the case of model (I) is either very small ($N_H = 2$) or very large

[3] We have taken into account proton decay constraints on M_X.

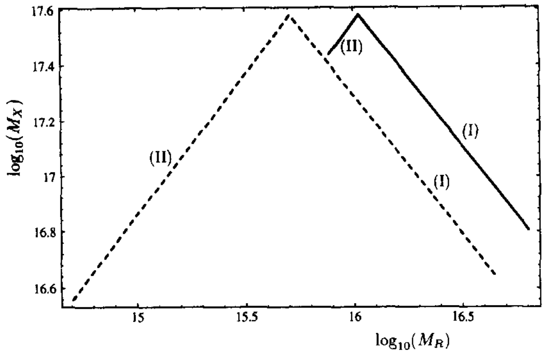

Fig. 3. The GUT scale $\log_{10}(M_X)$ in the case $N_H = 1$ as function of $\log_{10}(M_R)$, for $\alpha_3 = .11$ (dashed line) and $\alpha_3 = .13$ (continuous line).

($N_H = 1$). The previously proposed mechanism for generation of M_I through the specific non-renormalizable terms (5), can be realized for $M_R \sim 10^{14}$–10^{15} GeV, $M_I \sim 10^6$ GeV in the case of model (II). Note however that we have left for future study [38] the more complicated case of an intermediate scale mass for the second pair of Higgs isodoublets which could lead to acceptable M_I within the range of the proposed mechanism for both models.

4. Brief discussion and conclusions.

We have constructed and analyzed $SU(6) \times SU(2)$ GUT. Depending on the identification of the electroweak $SU(2)_L$ gauge group with the GUT $SU(2)$ factor or with the $SU(2) \subset SU(6)$ two possible models, $SU(6) \times SU(2)_L$ and $SU(6) \times SU(2)_R$, arise. Both models can accommodate the MSSM particles, as well as the gauge symmetry breaking Higgses, in representations allowed by the $k = 1$ superstring embedding. The models possess several characteristic features of their own. From a group theoretic point of view they are unique since only **(15,1)** and $(\overline{\mathbf{6}},\mathbf{2})$ representations are sufficient to accommodate matter as well as all the Higgses required for symmetry breaking. In contrast, flipped $SU(5)$ or minimal $SU(5)$ need **(10,1)**, $(\overline{\mathbf{5}}, -3)$ and $(\mathbf{5}, -2)$ or **10**, **5** and **24**. The models are characterized by an intermediate, nevertheless high, scale M_R. The gauge couplings above the *unification scale* M_X always intersect at a scale M_S which for a range of values of M_R can play the role of the *string unification scale*. Thus, for these models there is no unification mismatch problem. The particle content of the resulting $SU(3)_C \times SU(2)_L \times U(1)_Y$ theory bellow M_R is that of MSSM with an extra charge-1/3 colour-triplet pair of mass M_I and, possibly, an extra pair of electroweak isodoublets. Remarkably the intermediate scale M_I can be relatively small as it can be explicitly seen in the framework of the proposed mechanism for the extra triplet mass generation.

The question of the generation of the scale M_R and M_X, a general question addressing all scales that arise from flat directions, could in principle be studied in the framework of the softly broken theory. There, the supersymmetry breaking in conjunction with non-renormalizable terms can stabilize the related vevs and give rise to intermediate scales. However, this is not an easy problem since the susy breaking parameters run appreciably with the renormalization scale. This study should necessarily involve the renormalization group equations for the soft parameters [40] and it is beyond the scope of this article.

An interesting property of these models, inherited from $E(6)$, and shared by $SO(10)$, is the fact that they are characterized by *Yukawa coupling unification*, since all fermion masses result from a common generation-dependent Yukawa coupling. Note however that only in the case of one pair of electroweak v.e.v.s we obtain simple fermion mass relations. On the other hand, the Higgs isodoublets and the extra pair of right-handed d-quarks contained in the matter representations can receive masses separately and can be split without the need for any numerical fine tuning. In order to maintain one or more massless isodoublet pairs in a truly natural sense a family-dependent symmetry would be required. Nevertheless, if, as it happens in specific string constructions, the Yukawa coupling family hierarchy comes about as a hierarchy in the order of non-renormalizable terms, such a family symmetry could arise in the form of a selection rule [41].

Let us now say a few things about the features of possible embedding of these models in $k = 1$ superstring constructions. Assuming a gauge group $G = \prod_{i \in I} G_i$, where the group factors are realized at the level k_i of the *Affine world sheet algebra*, the conformal weight of a representation $(r_1, \ldots, r_n)$ of G is given by the formula

$$h_{KM} = \sum_{i=1}^{n} \frac{\dim(G_i)}{\dim(r_i)} \frac{T(r_i)}{(k_i + T(A_i))}$$

where $T(r_i)$ is the index of the representation r_i normalized as $T = \frac{1}{2}$ for the vector of $SU(n)$. A_i is the adjoint of G_i. Applying this formula for $G = SU(6) \times SU(2)$ and $k_i = 1$, we obtain that the conformal weights of the representations **(6,1)**, **(6,2)**, **(15,1)**, **(15,2)**, **(20,1)**, **(20,2)** are 5/12, 2/3, 2/3, 11/12, 3/4 and 1 correspondingly and thus all these states could be generically massless in a $k = 1$ construction. Some of these representations e.g **(6,1)** will lead to exotic fractional charge states. In a fully realistic superstring derived model the appearing exotic representations should circumvent conflicts with phenomenology either due to supermassiveness or due to the confining properties of the hidden sector gauge group. For the moment no ''realistic'' $k = 1$ $SU(6) \times SU(2)$ superstring model has been constructed [4]. However, the GUT analysis provides encouraging results in order to proceed towards this direction.

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