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STUDY OF THE LEPTON-VIOLATING (μ^-, e^+) REACTION IN MODERN GAUGE THEORIES

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The lepton-violating (μ^-, e^+) reaction has been studied in the context of modern gauge theories. Both left-handed and right–left symmetric models have been examined. Special attention has been paid to the following mechanisms: (i) those mediated by massive neutrinos (light or heavy); (ii) those accompanied by massless or light physical Higgs particles (majoron); (iii) those involving more exotic intermediate Higgs particles, e.g. singly charged isosinglets (Zee model) and doubly charged isotriplets; (iv) right-handed currents. The formalism has been applied to the experimentally interesting process $\mu^- + {}^{58}\mathrm{Ni} \to {}^{58}\mathrm{Fe(gs)} + e^+$. The branching ratio is computed using realistic nuclear wave functions. It is found to be $\leqslant 10^{-27}$ in all models, i.e. too small to be measurable in the foreseeable future. The branching ratio for the (μ^-, e^+) reaction to all nuclear states is estimated to be $\leqslant 10^{-20}$, i.e. beyond the goals of planned experiments.

1. Introduction

All physical processes observed thus far seem to be consistent with the conservation of a lepton number L which in our days is written as

$$L = L_e + L_\mu + L_\tau ,$$

where L_e, L_μ, L_τ are lepton numbers associated with each of the known families (generations) of leptons and are known as electronic muonic, and tauonic lepton numbers respectively. In fact on the basis of present data the numbers L_e, L_μ, L_τ are separately conserved [1–4] and serve as a basis for the classification of the known leptons into three families. The question arises of whether such conservation laws are the consequence of exact symmetries of nature or whether they are broken at some level. In the latter case, which seems to be favored by modern gauge theories and in particular the grand unified versions [5] (GUT), it is interesting to know whether this level is attainable by present day experiments and, in particular, which experiments should be attempted. The answer to this question is crucial since it is well known that conservation of lepton number is intimately related to the nature of the neutrinos and in particular its mass [6–8].

The oldest lepton-violating process studied is the neutrinoless double β decay

$$(A, Z) \to (A, Z+2) + e^- + e^- , \qquad \text{(double negatron decay)} , \tag{1}$$

$$(A,Z) \to (A,Z-2) + e^+ + e^+\,, \qquad \text{(double positron decay)}\,, \tag{2}$$

$$e_b^- + (A,Z) \to (A,Z-2) + e^+\,, \qquad \text{(electron capture)}\,, \tag{3}$$

From such studies* we have learned that the lepton-violating parameter η, which will be given more precise meaning later, must be less than 10^{-5}. It is also known that observation of such processes will imply that the neutrino is a Majorana particle and that the parameter η is related to its mass. The above processes are characterized by small-energy release and they are plagued by radioactivity background problems. Thus more exotic lepton-violating processes have been searched for experimentally e.g.

$$\mu \to e\gamma\,, \tag{4}$$

$$\mu \to e e^+ e\,, \tag{5}$$

$$\mu_b^- + (A,Z) \to e^- + (A,Z)\,, \tag{6}$$

$$\mu_b^- + (A,Z) \to e^+(A,Z-2)\,. \tag{7}$$

Only limits** have thus far been put on the branching ratios for the above reactions which are of order 10^{-10}.

The above reactions, in particular (4)–(6), have also been studied theoretically in the past. In most such investigations the purpose was to extract the parameters of the gauge models from the obtained experimental limit. Process (7) was expected to proceed slower due to nuclear effects***.

In the present work we will focus our attention on reaction (7). We shall show that in a class of models involving light intermediate neutrinos the amplitude associated with process (7) is proportional to the mass of the neutrino while the corresponding one for processes (4)–(6) is proportional to the square of such a mass. It is therefore conceivable that it may be faster than the other three, something which would be very surprising from the pre-gauge theory point of view, which axiomatized that the family lepton number may be broken easier than the total lepton number‡.

* The lower limit on 0ν $\beta\beta$ decay lifetime $T > 10^{22}$ y has been set by the non-observation of the $^{74}Ge \to {}^{74}Se + e^- + e^-$ reaction. See Belliotti et al. [1]. Also recent analysis of the geochemical data of the decays $^{130}Te \to {}^{130}Xe + e^- + e^-$ and $^{128}Te \to {}^{128}Xe + e^- + e^-$ are consistent with no lepton-number-violation, see Kirsten et al. [1].

** Limits on L_μ (and L_e) come from the limits on the branching ratios $R = \Gamma(\mu \to e\gamma)/\Gamma(\mu \to e\nu\bar{\nu})$ which have been put by a number of experimental groups, e.g. see ref. [2]. The most stringent limit is $R < 2\times10^{-10}$. Limits on L_e and L_μ conservation come from the branching ratio $R = \Gamma(\mu^-, e^-)/\Gamma(\mu^-, \nu_\mu)$ on various nuclear targets. The most stringent limit is $R < 7\times10^{-11}$, see e.g. ref. [3]. Limits on total number L come also from the non-observation of (μ^-, e^+) conversion in the presence of nuclear targets. The most stringent limit here is $R < 3\times10^{-11}$ and has recently been established by the (μ^-, e^+) reaction on ^{127}I, see ref. [4].

*** Some of this suppression may be compensated by nuclear physics effects in the case of (μ^-, e^-). Indeed this process does not change the charge of the nucleus and can proceed coherently while (μ^-, e^+) is affected by the nucleons near the surface.

‡ For a discussion of various lepton numbers as well as a review of the older data, see ref. [9].

Reaction (7) has, of course, many similarities with the neutrinoless double β decay. After a careful examination we find, however, that, in addition to the obvious differences resulting from the different energy-momentum transfer, there are some rather surprising features which make (μ^-, e^+) complimentary to neutrinoless $\beta\beta$ decay. These are as follows.

(i) Neutrinoless $\beta\beta$ decay is absolutely forbidden if the intermediate neutrinos are Dirac particles, i.e. the neutrino is different from its antineutrino. On the contrary (μ^-, e^+) can proceed even if the intermediate neutrino is a Dirac particle.

(ii) The amplitudes for neutrinoless $\beta\beta$ decay and (μ^-, e^+) conversion are proportional to some combination of the masses of the neutrino mass eigenstates. The mixing angles and phases are such that the effective mass may be less than the lightest mass. We shall see, however, that when one of the processes is hindered the other is enhanced. Thus assuming non-degenerate neutrino masses such a cancellation cannot account for the non-observation of *both* processes.

The (μ^-, e^+) process has been investigated in the past [10–12]. All calculations, however, have focused on only one gauge model. Furthermore individual branching ratios have not been computed. From the experimental point of view transitions to concrete final nuclear states are more interesting given sufficiently fast rates. The reason is that even though one does not have here formidable radioactivity backgrounds, as in $\beta\beta$ decay, one must cope with the background induced from the reaction

$$\mu_b^- + (A, Z) \rightarrow (A, Z-1) + \gamma + \nu_\mu \qquad \gamma \rightarrow e^- e^+ , \tag{8}$$

in which almost all the energy is carried away by the positron. Since reaction (8) does not violate any conservation law such positrons may overwhelm the experiment. If one considers (μ^-, e^+) transitions leading to the ground state of the final nucleus by a judicious choice of the target nucleus one can arrange so that the maximum positron energy $(E'_e)_{max}$ arising from reaction (8) is less than the well-defined positron energy E_e of reaction (7). In fact one finds that

$$E_e = m_\mu - b + \Delta m - 3m_e , \tag{9}$$

$$(E_e)'_{max} = m_\mu - b + \Delta m' - 3m_e , \tag{10}$$

where b is the binding muon energy (of the order of a few keV) and Δm the suitable (atomic) mass differences. More specifically

$$\Delta m = m(Z) - m(Z-2) , \qquad \Delta m' = m(Z) - m(Z-1) .$$

Thus it is desirable to make the quantity

$$\Delta = E_e - (E'_e)_{max} = \Delta m - \Delta m' = m(Z-1) - m(Z-2)$$

as large as possible. The most favorable situation [9] is achieved by choosing ^{58}Ni as the target nucleus:

$$\Delta \approx 2.5\ \mathrm{MeV}\,, \qquad E_{\mathrm{e}} \approx 106\ \mathrm{MeV}\,.$$

Thus with a resolution better than Δ one can work in a background-free region.

In addition to the ground-state transitions we will also estimate total transition rates using shell-model nuclear wave functions. For comparison we will also present results of such calculations in the simple uniform nuclear density approximation.

The present paper is organized as follows. The essential features of the gauge models we employed and the lepton-violating parameters derived from them are discussed in sect. 2. The structure of the amplitude of (μ^-, e^+) conversion in the various models is exhibited in sect. 3 and a convenient expression for the branching ratio is given in sect. 4. In sect. 5 we present our results and our conclusions.

2. Gauge models

Before we embark on detailed calculations we will briefly discuss the qualitative features of various fashionable gauge models. In particular we will concentrate on situations whereby the parameters of the gauge models can be separated from the nuclear aspects of the problem and thus be given in terms of a single lepton-violating parameter. The models we are going to employ can be grouped into two classes: left-handed models and right–left symmetric models.

2.1. LEFT-HANDED MODELS

Such models are suitable extensions of the standard model [13] so that the weak currents are strictly left-handed but lepton number is broken locally or globally. In such models reactions (1)–(3) and (7) are mediated by (Majorana) neutrinos. We further distinguish two classes of such models:

2.1.1. Models with conventional neutrino masses. We will not elaborate here on how a Majorana mass can be generated. The interested reader is referred to a recent review [14]. We only mention here that the minimal model must be extended to include a right-handed neutrino put in an isospin singlet and/or a weak isospin triplet of Higgs particles. The important thing is that in those situations one should distinguish between two kinds of neutrino eigenstates, as follows.

(i) The weak eigenstates $\nu_0 = (\nu_{\mathrm{e}}, \nu_\mu, \nu_\tau \ldots)$ which appear in weak interactions

$$\begin{pmatrix} \nu_{\mathrm{e}} \\ \mathrm{e}^- \end{pmatrix}_{\mathrm{L}}, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_{\mathrm{L}}, \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_{\mathrm{L}} \cdots. \tag{11}$$

For n flavors there are n additional eigenstates $N_0 = (N_{\mathrm{e}}, N_\mu, N_\tau, \ldots)$ which do not appear in weak interactions.

(ii) The (physical) mass eigenstates symbolically written as $\nu = (\nu_1, \nu_2, \ldots, \nu_n)$ and $N = (N_1, N_2, \ldots, N_n)$. In this grouping the states ν are presumed light and the

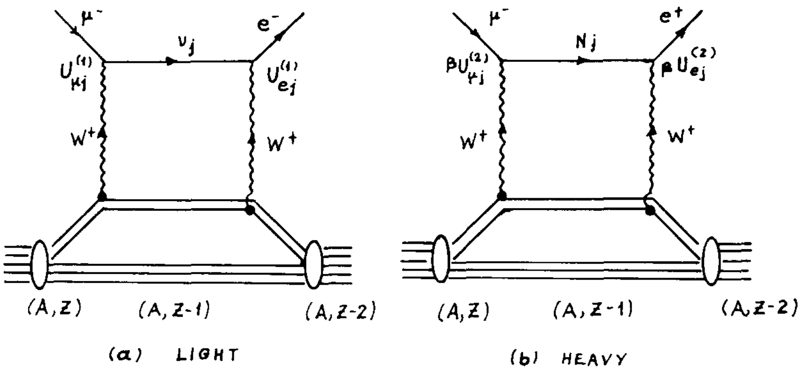

Fig. 1. The Majorana neutrino mediated (μ^-, e^+) conversion when there exist left-handed currents only. (a) corresponds to light and (b) to heavy neutrinos.

states N are heavy. The two sets of states are related by a unitary transformation

$$\begin{pmatrix} \nu_0 \\ N_0 \end{pmatrix} = \left(\begin{array}{c|c} X & Y \\ \hline W & Z \end{array}\right) \begin{pmatrix} \nu \\ N \end{pmatrix}, \tag{12}$$

where (X), (Y), (W), (Z) are $n \times n$ matrices so that the above $2n \times 2n$ matrix is unitary.

We note that if the two mass scales are vastly different the matrices Y and W have elements of order m/M while the matrices X and Z are approximately unitary. The separation of the neutrino states into light and heavy is crucial in view of the approximations which we are going to follow. Specializing into the first two flavors we can write

$$\nu_e = \sum_{j=1}^{n} X_{ej}\nu_j + \sum_{j=1}^{n} Y_{ej}N_j ,$$

$$\nu_\mu = \sum_{j=1}^{n} X_{\mu j}\nu_j + \sum_{j=1}^{n} Y_{\mu j}N_j . \tag{13}$$

Now a typical diagram contributing to (μ^-, e^+) conversion is exhibited in fig. 1. In fig. 1a the intermediate neutrino is one of the ν_j, and in fig. 1b one of the N_j, $j = 1, 2, \ldots, n$. Assuming that the two mass scales are very different and $Z \approx X$ we can write

$$Y \approx \beta Z , \qquad W \approx -\beta X , \tag{14}$$

where β is a sort of mixing angle between the light and the heavy sectors. It is convenient to relate the neutrino mass eigenstate with its charge conjugate partner (Majorana condition) with the relation:

$$\nu^c = (e^{i\alpha})\nu , \qquad N^c = (e^{i\varphi})N , \tag{15}$$

where $(e^{i\alpha})$ and $(e^{i\varphi})$ are diagonal matrices of phases. Then it has recently been shown [14] that with the above definition

$$(X) \to (U^{(1)}), \qquad (Z) \to (U^{(2)}), \tag{16}$$

where $U^{(1)}$ and $U^{(2)}$ are well-known Kobayashi-Maskawa (KM) type $n \times n$ matrices.

From our point of view it is essential to examine the light and heavy neutrino components separately.

If the neutrinos are sufficiently light, e.g. in the eV or even keV region, the process exhibited in fig. 1a depends on the parameters of the gauge model via lepton-violating parameter η'_ν defined by

$$\eta'_\nu = \frac{\langle m_\nu \rangle}{m_e}, \qquad \langle m'_\nu \rangle = \sum_{j=1}^{n} U^{(1)}_{ej} U^{(1)}_{\mu j}\, e^{i\alpha_j} m_j . \tag{17}$$

On the other hand the amplitude associated with the process of fig. 1b is proportional to the lepton-violating parameter η'_N defined by

$$\eta'_N = \beta^2 \frac{m_A}{m'_N}, \qquad \frac{1}{m'_N} = \sum_{j=1}^{n} U^{(2)}_{ej} U^{(2)}_{\mu j}\, e^{i\varphi_j} \frac{1}{M_j}, \tag{18}$$

the quantity $m_A \approx 0.85$ GeV characterizes the size of the nucleon (see sect. 3). The above approximation holds when $M_j > m_p$ (m_p = nucleon mass).

The corresponding lepton-violating parameters entering neutrinoless double* β decay are [15]

$$\eta_\nu = \frac{m_\nu}{m_e}, \qquad \langle m_\nu \rangle = \sum_{j=1}^{n} U^{(1)}_{ej} U^{(1)}_{ej}\, e^{i\alpha_j} m_j , \tag{19a}$$

$$\eta_N = \beta^2 \frac{m_A}{m_N}, \qquad \langle m_\nu \rangle = \sum_{j=1} U^{(2)}_{ej} U^{(2)}_{ej}\, e^{i\alpha_j} m_j . \tag{19b}$$

We thus see that the phases α_j and φ_j are, in principle, observable in those processes in which a neutrino converts to an antineutrino. In the special case of a *CP*-conserving theory the above phases take the values 0 and π and they coincide with the *CP* eigenvalue of the neutrino mass eigenstate to which they correspond.

From the above discussion it is obvious that the neutrino masses $\langle m_\nu \rangle$ and $\langle m'_\nu \rangle$ entering $\beta\beta$ decay and (μ^-, e^+) conversion are average quantities and, in general, different from the mass eigenvalues. We also note that they are different from each other.

In what follows we are going to use the KM matrices available in the literature and treat the phases α_j and φ_j as free parameters. For illustration purposes we will consider the case of two generations [16]. Then the matrix $U^{(1)}$ is characterized by one mixing angle. Since an overall phase is immaterial the problem is characterized by one phase α. Thus

$$\langle m_\nu \rangle = m_1 \cos^2\theta + m_2\, e^{i\alpha} \sin^2\theta, \qquad \langle m'_\nu \rangle = (m_1 - m_2\, e^{i\alpha}) \sin\theta \cos\theta . \tag{20}$$

* We warn the reader that our notation here is slightly different from that of ref. [15].

In a *CP*-invariant theory they become

$$\langle m_\nu \rangle = m_1 \cos^2\theta \pm m_2 \sin^2\theta\,, \qquad \langle m'_\nu \rangle = (m_1 \mp m_2)\sin\theta\cos\theta\,. \tag{21}$$

If one accepts the limits [17] 14 eV $\leqslant m_\nu \leqslant$ 46 eV of the triton decay experiment, which clearly applies in the case of the lightest neutrino, one will tend to attribute the non-observation of neutrinoless $\beta\beta$ decay [15] ($|\langle m_\nu\rangle| < 4$ eV) to cancellations between the various neutrino mass eigenstates. In our example this implies $\alpha = \pi$. As a result in the (μ^-, e$^+$) reaction the various mass eigenstates may enhance each other as indeed happens in our example, $\langle m'_\nu\rangle = \frac{1}{2}(m_1 + m_2)\sin 2\theta$.

In the special case that the neutrinos are Dirac particles, $\beta\beta$ decay is absolute forbidden but (μ^-, e$^+$) may proceed. In our example, $m_1 = m_2 = m$, $\theta = \frac{1}{4}\pi$, $\alpha = \pi$, one finds

$$\langle m_\nu \rangle = 0\,, \qquad \langle m'_\nu \rangle = m\,.$$

i.e. the (μ^-, e$^+$) proceeds maximally.

In our numerical calculations we are going to employ the lepton-violating parameters predicted by the horizontal gauge mixing model recently discussed by Papantonopoulos and Zoupanos [18]. Their KM matrix takes the form

$$U^{(1)} \approx \begin{pmatrix} 1 & \theta_1 & -\theta_3 \\ -\theta_1 & 1 & \theta_2 \\ \theta_3 & -\theta_2 & 1 \end{pmatrix}, \qquad \begin{aligned} &\theta_1 = \sqrt{m_e/m_\mu} = 0.07 \\ &\theta_2 = 5\times 10^{-5} \\ &\theta_3 = 6.2\times 10^{-4}\,. \end{aligned}$$

Their mass eigenvalues are expressed in terms of the lightest i.e.

$$m_2 = \frac{m_e}{m_\mu} m_1\,, \qquad m_3 = \frac{m_\tau}{m_e} m_1\,,$$

thus we find

$$\eta_\nu \approx \frac{m_1}{m_e}\left(1 \pm \frac{m_\mu}{m_e}\theta_1^2\right), \qquad \eta'_\nu \approx \frac{m_1}{m_e}\left(\frac{m_\mu}{m_e} \mp 1\right)\theta_1\,. \tag{22}$$

Taking $m_1 = 30$ eV we obtain

$$\eta_\nu = 1.2\times 10^{-4}(+)\,, \qquad \eta'_\nu = 8.6\times 10^{-4}(+)\,, \qquad \eta_\nu = 0(-)\,,$$
$$\eta'_\nu = 8.8\times 10^{-4}(-)\,, \tag{23}$$

where $+(-)$ refer to the same (opposite) *CP* eigenvalues respectively.

Thus in this model the value of η_ν is very sensitive to the relative *CP* eigenvalues and it is absolutely forbidden if they are opposite. Conversly the (μ^-, e$^+$) reaction is insensitive to such phases.

The above exact cancellation of the $\beta\beta$ process may be an artifact of the way the masses of the neutrinos and the mixing angles scale with the masses of the charged leptons. Since, however, the predictions of the above model are consistent with the limits on the experimental lifetimes of the neutrinoless $\beta\beta$ process we find it suitable for our numerical calculations of the (μ^-, e^+) process.

Following a procedure explained a little below (see subsect. 2.2.1) and assuming [10] that $\beta^2 \approx 10^{-3}$ we get

$$\eta'_N = 8.9 \times 10^{-8}\,(+)\,, \qquad \eta'_N = 3.0 \times 10^{-8}\,(-)\,.$$

2.1.2. Production of Higgs particles in the (μ^-, e^+) process. One can construct gauge theory models which are such that neutral Higgs particles survive the Higgs mechanism as actual physical particles. One such interesting model was recently proposed by Gelmini and Roncadelli which has far reaching implications discussed by Georgi, Glashow and Nussinov [7]. This model in addition to the standard isodoublet of Higgs scalars also contains an isotriplet. Two neutral scalars, one which is massless and the other which is light, survive the Higgs mechanism as physical particles and are designated as χ^0. (The model contains doubly charged scalars as well which however cannot decay to two leptons (see subsect. 2.2.2)). The presence of χ^0 allows lepton-violating processes like that exhibited in fig. 2 in the case of the (μ^-, e^+) reaction. χ^0 does not couple directly with quarks but it does couple to the neutrinos via a coupling of the type

$$L_{\text{eff}} = \sqrt{\tfrac{1}{2}}\, g_{\alpha\beta} \tilde{\nu}_\alpha \nu_\beta \chi^0\,,$$

where α and β are generation indices. This coupling plays the role of the usual mass insertion term in the sense of converting a neutrino of one flavor to an antineutrino of the same or other flavor. The corresponding amplitude for sufficiently light neutrino mass and to leading order is independent of the neutrino mass and it may thus dominate when the neutrino mass is very light or when the cancellation mentioned in sect. 1 is complete. The reaction

$$\mu_b^- + (A, Z) \to (A, Z-2) + e^+ + \chi^0\,, \tag{24}$$

is, of course, distinguishable from reaction (7). Since, however, χ^0 does not interact with ordinary matter it will go undetected (like neutrinos), and it will thus be

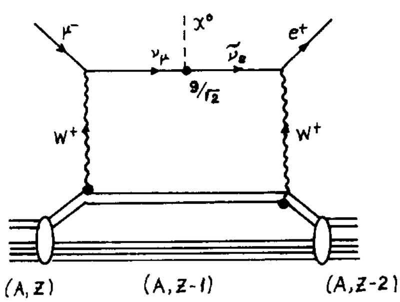

Fig. 2. A possible contribution to (μ^-, e^+) conversion which does not involve the mass of the neutrino, but is accompanied by the emission of a light (massless) Higgs particle.

difficult to experimentally distinguish between positrons originating from (7) or (24). The disadvantage from our point of view is that the positrons originating from eq. (24) are going to have a continuous spectrum. We will therefore limit ourselves to that portion of the available space which is in the background-free region.

If we restrict ourselves in the background-free region the momentum carried away by χ^0 is small. Thus the Fourier integrals required, which in general are complicated, are in this case simple and similar to those entering the mass insertion diagrams. Assuming that $g_{\mu e} \approx g_{ee}$ and using the limits on g_{ee} from neutrinoless $\beta\beta$ decay [15] we get

$$\eta_g' = \sqrt{\tfrac{1}{2}} g_{\mu e} \approx \sqrt{\tfrac{1}{2}} g_{ee} \leqslant 2.8 \times 10^{-3} . \tag{25}$$

2.1.3. The Zee model. This model [19] is interesting in the sense that lepton violation involving different generations is allowed but the neutrinoless $\beta\beta$ decay is forbidden. In this model the lepton-violating amplitude is to leading order independent of the neutrino mass.

The essential ingredient of this model is the existence of a fairly light singly charged Higgs scalar h^- which is a colour and weak isospin singlet. This particle allows for lepton violation which for (μ^-, e^+) is exhibited in fig. 3. The motivation for assuming such a particle surely comes from the fact that the antisymmetric 10-dimensional representation of GUT SU(5) accommodates a particle with such quantum numbers. Following Zee, however, we will follow the implications of the existence of such a particle not in the context of SU(5) but phenomenologically.

The scalar h^- couples of fermions of different generations antisymmetrically. The coupling of interest to us here takes the form

$$\mathscr{L} = f h^+ (\bar{\nu}_{eR}^c \mu_L^- - \bar{e}_R^c \nu_{\mu L}) + \text{h.c.} \tag{26}$$

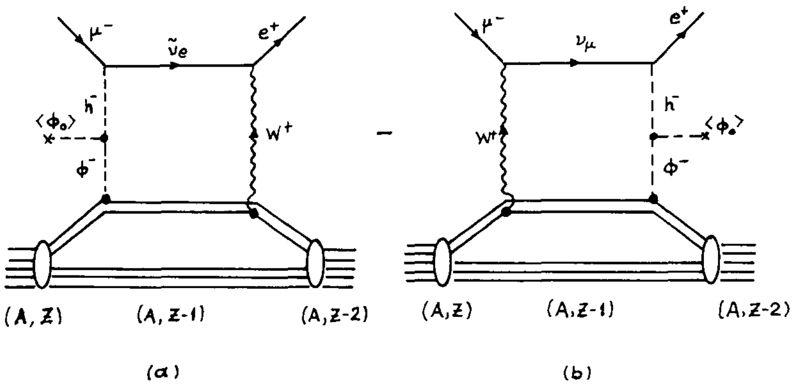

Fig. 3. Schematic representation of the leading diagrams at the one-loop level associated with (μ^-, e^+) reaction in the Zee model. In the hadronic sector two fermions (nucleons or quarks) must participate in the reaction. $|i\rangle$, $|n\rangle$ and $|f\rangle$ stand for the initial, intermediate, and final nuclear states. Note the relative minus sign between the diagrams (a) and (b).

Since h^- does not couple to the quarks directly, communication with the hadronic sector is achieved through the Higgs isodoublets via a coupling of the type

$$\mathscr{L} = M_{\alpha\beta}\varepsilon_{ij}\phi^i_\alpha\phi^j_\beta h^- , \tag{27}$$

where i and j are isospin indices and α, β are generation indices. One needs at least two different isodoublets.

The effective doublet-singlet coupling of fig. 3 takes the form

$$\mathscr{L} = M\langle\phi^0\rangle\phi^- h^- , \tag{28}$$

where M is a specific choice of the parameters of eq. (27).

The above model has been considered in connection with (μ^-, e^+) in a previous publication [20]. In the previous calculation, however, transition rates to all nuclear states have been considered. Here we will consider ground state transitions even though the model would favor $0^+ \to 1^+$ transitions.

Utilizing the results of ref. [20] and proceeding in a way which was outlined in subsect. 2.1.1 we can define an effective lepton-violating parameter η'_z which is related to the masses of the Higgs particles and the couplings given above as follows:

$$\eta'_z = \frac{fMf_{\phi q}\langle\phi_0\rangle}{2\sqrt{2}m^2_\phi m^2_h G_F}, \tag{29}$$

where $f_{\phi q}$ is the coupling of the Higgs doublet to the quarks. We will now give an estimate of η'_z. The quantity f is constrained by an inequality of the type [19]

$$\frac{f^2}{m^2_h} \leqslant 10^{-1}\frac{e^2}{M^2_w}, \qquad \text{or } 10^{-2}\frac{e^2}{M^2_w}, \qquad e = g\sin\theta_w, \qquad \frac{g^2}{M^2_w} = 4\sqrt{2}G_F. \tag{30}$$

Adopting estimates of this type we put $f = \frac{1}{8}g$ and assuming further* that [20]

$$m_\phi \approx M_w, \qquad \langle\phi_0\rangle = \frac{\sqrt{2}}{g}M_w, \qquad f_{\phi q} = \frac{m_q}{\langle\phi_0\rangle} = g\sqrt{2}\frac{m_q}{M_w}, \qquad M_w\langle\phi_0\rangle = 2\frac{M^2_w}{g}, \tag{31}$$

we obtain

$$\eta'_z = \frac{1}{8}\frac{m_q}{M_w}\frac{g}{G_F M^2_w} \approx 1.0 \times 10^{-4}. \tag{32}$$

It is not entirely clear what we should choose for the quark mass [20]. In the above estimate we used $m_q = 10$ MeV. This value may be 30 times larger if the constituent quark mass is used which is not unreasonable [21]. In any case with the above choice of the parameters we see that this mechanism competes with the Majorana mass mechanism discussed in the previous subsections.

* We are here more conservative than in ref. [20]. There the value $m_\phi \approx \frac{1}{8}M_w$ was used.

2.2. LEFT-RIGHT SYMMETRIC MODELS

It is obvious that such models offer more possibilities which can be used in testing their validity. These possibilities arise, as we shall see, from the interference between the leptonic left- and right-handed currents and from the possible decay of doubly charged Higgs particles directly into two leptons.

2.2.1. Neutrino mediated process. In addition to the usual left-handed currents mediated by the ordinary vector bosons one has right-handed currents which are mediated by much heavier vector bosons. The additional currents of interest to us here are

$$\begin{pmatrix} N_e \\ e^- \end{pmatrix}_R, \quad \begin{pmatrix} N_\mu \\ \mu^- \end{pmatrix}_R, \tag{33}$$

where

$$N_e = \sum_{j=1}^{n} W_{ej}\nu_j + \sum_{j=1}^{\eta} Z_{ej}N_j, \qquad N_\mu = \sum_{j=1}^{\eta} W_{\mu j}\nu_j + \sum_{j=1}^{\eta} Z_{\mu j}N_j. \tag{34}$$

Once again it is necessary to consider separately the contributions arising from the light and heavy components.

(*i*) *Light neutrinos.* Since the mixing coefficients are already small the important contribution will come from amplitudes which are independent of the neutrino masses. This is indeed the case when we consider interference between left-handed and right-handed leptonic currents as is shown in fig. 4. The smallness of the mixing

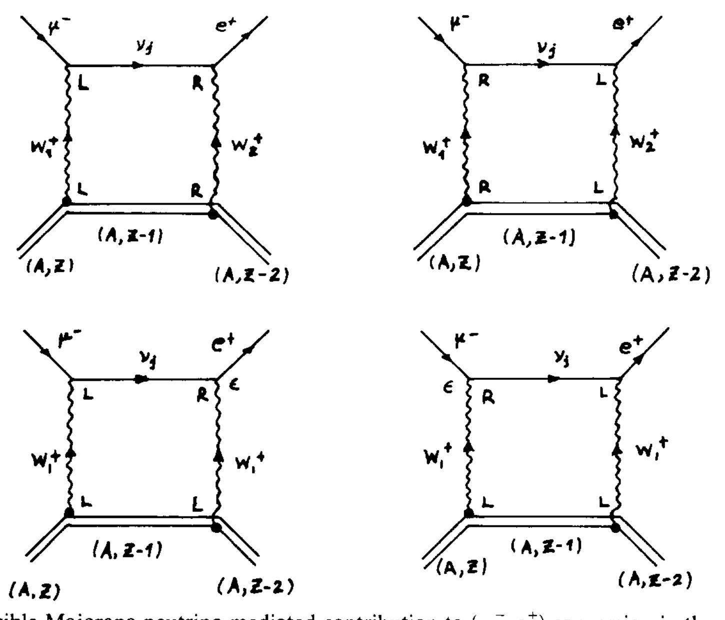

Fig. 4. A possible Majorana neutrino mediated contribution to (μ^-, e^+) conversion in the presence of right-handed interactions. Lepton violation is in this case not explicitly dependent on the neutrino mass. The upper diagrams are proportional to κ and the lower ones to ε (see subsect. 2.2.1).

coefficients may be compensated by the fact that in this case the helicities are such that from the neutrino propagator we pick the momentum term. This term is of the order of an average nucleon momentum (a few MeV/c) that is much bigger than the expected neutrino mass (of a few eV). Clearly the Lorentz structure of the associated amplitude is now very different. Since, however, it has extensively been discussed before in connection with $\beta\beta$ decay we will only give the essential points here.

The weak vector boson eigenstates W_μ^{L} and W_μ^{R} are linear combinations of the mass eigenstates $W_\mu^{(1)}$ and $W_\mu^{(2)}$ with masses approximately $M_{\mathrm{w_L}} = M_{\mathrm{w}}$ and $M_{\mathrm{w_R}}$. More specifically

$$W_\mu^{\mathrm{L}} = cW_\mu^{(1)} - sW_\mu^{(2)}, \qquad W_\mu^{\mathrm{R}} = sW_\mu^{(1)} + cW_\mu^{(2)}, \tag{35}$$

where $c = \cos\theta$ and $s = \sin\theta$ (θ = mixing angle).

The hadronic currents can now be either R-R, in which case the amplitude will be proportional to $\varepsilon = s/c$, or of type L-L in which case the amplitude will be proportional to $\kappa = M_{\mathrm{w}}^2/M_{\mathrm{w_R}}^2$ (see fig. 4). Thus to leading order the lepton-violating parameter associated with this mechanism takes the form

$$\eta'_{\mathrm{RL}} = \tfrac{1}{2}c^4\sqrt{\varepsilon^2+\kappa^2}\sum_{j=1}^{\eta}(X_{\mathrm{e}j}W_{\mu j} + X_{\mu j}W_{\mathrm{e}j}). \tag{36}$$

Loosely speaking this parameter gives the fraction of the right-handed current in the predominantly left-handed weak interaction.

The precise value of η'_{RL} may be difficult to calculate since it depends on so many parameters of the gauge models. We will attempt to provide an estimate in line with what we have done in the previous sections. Using eq. (14) and proceeding in a way analogous to the one that led to eqs. (8a) and (20) we get

$$\eta'_{\mathrm{RL}} = -c^4\sqrt{\kappa^2+\varepsilon^2}\,\beta\sum_{j=1}^{\eta}U_{\mathrm{e}j}^{(1)}U_{\mu j}^{(1)}\,\mathrm{e}^{i\alpha_j}. \tag{37}$$

In the model of Papantonopoulos and Zoupanos [18] we get

$$\eta'_{\mathrm{RL}} = c^4\sqrt{\kappa^2+\varepsilon^2}\,\beta(\theta_1 \mp \theta_1). \tag{38}$$

The quantity $\sqrt{\varepsilon^2+\kappa^2}$ is not accurately known. The estimate $\sqrt{\varepsilon^2+\kappa^2} \approx \frac{1}{10}$ has previously been given [22]. Double β-decay data, however, may be more consistent with a smaller value [15]. We will therefore employ $\sqrt{\varepsilon^2+\kappa^2} = 0.02$ and $c = 1$. Then taking [10, 11] $\beta^2 = 10^{-3}$ and $\theta_1 = 0.07$ we obtain

$$\eta'_{\mathrm{RL}} = 0(+), \qquad \eta'_{\mathrm{RL}} = 2.8\times 10^{-6}(-), \tag{39}$$

the corresponding lepton-violating parameter for $\beta\beta$ decay is $\eta_{\mathrm{RL}} = 10^{-4}$ which is consistent with the available $\beta\beta$-decay data [15].

(*ii*) *Heavy neutrinos*. A contribution arising from the heavy neutrinos is analogous to that of fig. 1b except that β becomes κ and $1-\gamma_5$ becomes $1+\gamma_5$.

This mechanism is thus characterized by a lepton-violating parameter given by

$$\eta'_{\mathrm{RL}} = \kappa^2 \frac{m_{\mathrm{A}}}{m'_{\mathrm{N}}}, \qquad \kappa = \frac{M^2_{\mathrm{wL}}}{M^2_{\mathrm{wR}}}, \tag{40}$$

where m'_{N} is given by eq. (15). If, as is believed, $\kappa^2 > \beta^2$ this mechanism will give a bigger contribution than that associated with η'_{N} of left-handed theories. Using the two-flavor model of ref. [22] we obtain

$$\eta'_{\mathrm{R}} = \frac{m_{\mathrm{A}}}{m_1}\left(1 \mp \frac{m_1}{m_2}\right) \cos\theta' \sin\theta' \kappa^2, \tag{41}$$

Taking [22]

$$\sin\theta' = \sqrt{\frac{m_{\mathrm{e}}}{m_\mu}}, \qquad m_1 = \tfrac{1}{2} m_2 = 10^3\ \mathrm{GeV}, \qquad \kappa = 2\times 10^{-2}, \tag{42}$$

we obtain

$$\eta'_{\mathrm{R}} = 1.2\times 10^{-8}(+), \qquad \eta'_{\mathrm{R}} = 3.5\times 10^{-8}(-), \qquad \eta_{\mathrm{R}} = 8.5\times 10^{-6}(+\text{ or }-). \tag{43}$$

We stress once again that the lepton-violating parameter involving heavy neutrinos is inversely proportional to their mass for masses much greater than the nucleon mass.

2.2.2. Doubly charged Higgs particles. As we have already mentioned in the introduction, in many theories left-handed or right–left symmetric, a possible mechanism for generating a Majorana neutrino mass is the introduction of weak isospin triplets of Higgs scalars. The doubly charged members of such triplets can decay directly into two leptons violating lepton number by two units [6]. These Higgs particles do not couple directly to the quarks. Therefore communication with the hadronic sector is achieved via their coupling to the vector bosons and/or the ordinary isodoublet Higgs particles. It has been shown, however, that such couplings are suppressed in left-handed theories [23] and for this reason we did not discuss such models in sect. 2.1. The contribution of such mechanisms can in principle be sizable in right–left symmetric models.

One such mechanism is exhibited in the Feynman diagram of fig. 5 whereby the doubly charged Higgs particle is coupled to the vector bosons associated with the right-handed currents. Once again the essential ingredients of the gauge model are absorbed in a lepton-violating parameter η'_{H} given by

$$\eta'_{\mathrm{H}} = \frac{1}{G_{\mathrm{F}}\sqrt{2} M^2_{\mathrm{w}}} \frac{\lambda_0 M_0 m_{\mathrm{A}}}{m^2_\Delta} \kappa^2 h_{\mu\mathrm{e}}, \tag{44}$$

where m_Δ is the mass of the triplet in question, $\lambda_0 M_0$ is the effective cubic coupling of the triplet to the vector bosons, κ as given above and $h_{\mu\mathrm{e}}$ represents the coupling of the triplet to the leptons. We will assume that $h_{\mu\mathrm{e}} \approx h_{\mathrm{ee}} = 1$. With "reasonable"

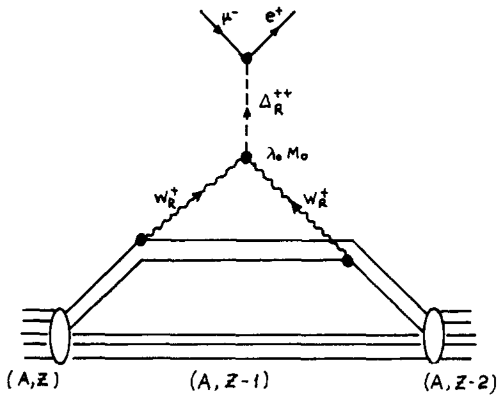

Fig. 5. A possible contribution to (μ^-, e^+) conversion into two leptons of a doubly charged Higgs particle Δ_R^{++}. Communication with the hadronic sector is achieved via the vector bosons associated with the right-handed interaction.

choice of the parameters [24]

$$\lambda_0 M_0/m_\Delta^2 \approx 2\times 10^{-4}\,(\mathrm{GeV})^{-1}\,, \qquad \kappa = 2\times 10^{-2}\,,$$

we obtain

$$\eta'_{\mathrm{H}} \approx 4\times 10^{-6}\,, \tag{45}$$

which may substantially exaggerate the 0ν $\beta\beta$-decay rates [15].

The second mechanism which is exhibited in fig. 6 has been previously discussed in connection with neutrinoless $\beta\beta$ decay [25]. The appropriate lepton-violating parameter now becomes

$$\eta'_\Delta = \frac{\lambda V_{\mathrm{R}} m_{\mathrm{A}}}{m_{\mathrm{H}}^4 m_\Delta^2 G_{\mathrm{F}}^2}\, h_{\mathrm{q}}^2 h_{\mathrm{e}\mu}\,, \tag{46}$$

where $h_{e\mu}$ was given above, h_q is the coupling of the Higgs doublet to the quarks ($h_q^2 \approx 10^{-9}$), m_H is the mass of Higgs doublet ($m_H \approx 100$ GeV), m_Δ is the mass of the Higgs triplet ($m_\Delta \approx 100$ GeV) and V_R is the vacuum expectation value of the neutral member of the right-handed triplet ($\lambda V_R \simeq 1000$). We thus obtain

$$\eta'_\Delta = 10^{-9}\,. \tag{47}$$

We thus see that this mechanism is less important than the one containing η'_H and it will not be discussed further.

We conclude this section by repeating that in the special limit of the small neutrino mass or in the limit of the large mass of all the intermediate particles all the parameters of the gauge model can be separated out and expressed in terms of a lepton-violating parameter. All the uncertainties of the gauge models are contained in these parameters.

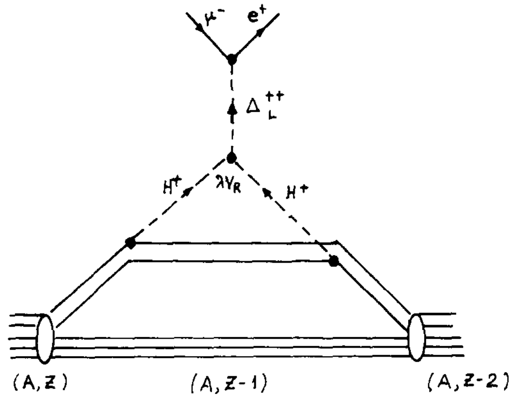

Fig. 6. A similar process with that of the previous figure except that now communication with the hadronic sector is achieved via the Higgs doublets.

3. The amplitude for the (μ^-, e$^+$) conversion

In this section we are going to give the expressions for the invariant amplitude for the various models described in sect. 2. The mathematical details are given elsewhere [15].

(i) *The Majorana mass contribution.* This process is exhibited in fig. 1. The associated amplitude takes the form

$$\mathcal{M} = -\frac{1}{4\pi}\left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{m_e f_A^2}{r_0 A^{1/3}} \sum_{\ell} \eta'_\ell \langle f|\Omega_\ell|i\rangle [\bar{u}^c(p_e)(1-\gamma_5)u(p_\mu)] , \tag{48}$$

(we suppressed for notational convenience the Cabbibo angle). In this equation $R = r_0 A^{1/3}$ ($r_0 = 1.1$ fm) is the nuclear radius, introduced to take into account the gross-dependence on the nuclear mass number, $\ell = \nu$, N corresponds to the light and heavy neutrino case respectively, η'_ν and η'_N are the lepton-violating parameters discussed in sect. 2 [see eqs. (18) and (22)] and Ω_ν and Ω_N are the effective transition operators which for $0^+ \to 0^+$ transitions take the form:

$$\Omega_\ell = \sum_{i \neq j} \tau_-(i)\tau_-(j)\left[\frac{f_V^2}{f_A^2} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right]\frac{R_0}{r_{ij}} F_\ell(r_{ij}) j_0(p_e R_{ij}) j_0(\tfrac{1}{2}p_e r_{ij}) . \tag{49}$$

The quantity $F_\ell(r_{ij})$ depends [15] on the quantities $\delta = \omega r_{ij}$, $\nu = m_\nu r_{ij}$ and $\omega = E_e + E_i - E_{\langle n\rangle} \approx 80$ MeV. In the case of light Majorana neutrino it takes the form

$$F_\nu(r_{ij}) = \frac{2}{\pi}\int_0^\infty \frac{\sin x}{\delta + x}\,dx . \tag{50a}$$

In the case of $\beta\beta$ decay, $\omega \approx 0$ and $F_\nu(r_{ij}) \approx 1$.

For a heavy Majorana neutrino one finds

$$F_{\mathrm{N}}(r_{ij}) = \frac{1}{48}\frac{m_{\mathrm{A}}}{m_{\mathrm{e}}}\alpha(\alpha^2+3\alpha+3), \qquad \alpha = m_{\mathrm{A}} r_{ij}. \tag{50b}$$

The above expression assumed that the nucleon has a size adequately described [15] by a dipole shape form factor with characteristic mass m_{A}. Without such a form factor $F_{\mathrm{N}}(r_{ij})$ is essentially a δ function and results in a negligible contribution.

Of course in the case of heavy Majorana neutrinos processes involving particles other than nucleons, which have a reasonable chance to be found in the nuclear medium, they may also contribute [12–26]. Of such processes for $0^+ \to 0^+$ transitions only the double charge exchange of pions between two nucleons becomes important i.e.

$$\mu^- + \pi^+ \to \pi^- + \mathrm{e}^+ . \tag{51}$$

In the case of neutrinoless $\beta\beta$ decay this process is somewhat smaller but comparable to the one involving only nucleons [15, 26] (fig. 1b). In the case of eq. (51), however, due to the large energy-momentum transfer being involved the effective transition operator is too complicated to be discussed here and we have chosen not to include it. We do not expect this mechanism to substantially alter the conclusions of the present paper.

(ii) *The Majoron emission mechanism.* So long as we limit ourselves to the regime which is free of the reaction background, i.e. so long as χ^0 carries away little momentum, the amplitude is of the same structure as the light neutrino case discussed above except that $\eta'_\nu m_{\mathrm{e}}$ is replaced by $\eta'_{\mathrm{g}} = \sqrt{\tfrac{1}{2}} g_{\mu\mathrm{e}}$ (the dimensionality of the amplitude is different by a power of the mass due to the production of one extra scalar particle).

(iii) *The Zee mechanism.* The amplitude associated with fig. 3, in the case of $0^+ \to 0^+$ when only the vector current contributes, takes the form [20]

$$\mathcal{M} = -\frac{1}{4\pi}\left(\frac{G_{\mathrm{F}}}{\sqrt{2}}\right)^2 \eta'_z \frac{m_{\mathrm{e}} f_{\mathrm{A}}^2}{r_0 A^{1/3}} \langle \mathrm{f}|\Omega_2|\mathrm{i}\rangle \bar{u}^{\mathrm{c}}(p_{\mathrm{e}})\gamma^0 \frac{\boldsymbol{\gamma}\cdot\boldsymbol{p}_{\mathrm{e}}}{9 m_{\mathrm{e}}}(1-\gamma_5)u(p_\mu), \tag{52}$$

$$\Omega_2 = 6\frac{f_{\mathrm{V}}^2}{f_{\mathrm{A}}^2}\sum_{i\neq j}\tau_-(i)\tau_-(\mathrm{j})\frac{R_0}{r_{ij}}\frac{6 j_1(\frac{1}{2} r_{ij} p_{\mathrm{e}})}{p_{\mathrm{e}} r_{ij}} j_0(p_{\mathrm{e}} R_{ij}) k_2(\delta, \nu). \tag{53}$$

Again for light neutrinos we get

$$k_2(\delta, \nu) \approx k_2(\delta, 0) = \frac{2}{\pi}\int_0^\infty \frac{(2\delta + x)\sin x}{(\delta + x)^2}\,\mathrm{d}x . \tag{54}$$

The lepton violating parameter η' is given by eq. (34).

(iv) *Left–right interference term.* This amplitude arises from the Feynman diagrams of fig. 5. It has been extensively discussed in connection with neutrinoless

$\beta\beta$ decay [15] and there is no need to give a detailed derivation here. We find that

$$\mathcal{M} = -\frac{1}{4\pi}\left(\frac{G_F}{\sqrt{2}}\right)^2 \eta'_{RL} \frac{m_e f_A^2}{r_0 A^{1/3}} \{2l_s\langle f|\Omega_s|i\rangle - 2l_A\langle f|\Omega_A|i\rangle\}, \tag{55}$$

where

$$l_s = \bar{u}^c(p_e)\frac{\gamma\cdot p_e}{18m_e}u(p_\mu), \qquad l_A = \bar{u}^c(p_e)\frac{\gamma\cdot p_e}{9m_e}\gamma_5 u(p_\mu), \tag{56}$$

$$\Omega_s = \frac{\varepsilon-\kappa}{\sqrt{\varepsilon^2+\kappa^2}}\Omega_1 + \frac{\kappa}{\sqrt{\varepsilon^2+\kappa^2}}\Omega_2, \qquad \Omega_A = \frac{\varepsilon}{\sqrt{\varepsilon^2+\kappa^2}}\Omega_3, \tag{57}$$

$$\Omega_1 = \sum_{i\neq j}\tau_-(i)\tau_-(j)\frac{R_0}{r_{ij}}\left[3\frac{f_V^2}{f_A^2} - \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + 2\sqrt{\frac{24\pi}{5}}\,Y^2(\hat{\boldsymbol{r}}_{ij})\cdot(\boldsymbol{\sigma}_i\otimes\boldsymbol{\sigma}_j)^2\right]$$
$$\times\frac{6j_1(\frac{1}{2}p_e r_{ij})j_0(p_e R_{ij})}{p_e r_{ij}}k_2(\delta,\nu), \tag{58}$$

$$\Omega_3 = 3\frac{f_V}{f_A}\sum_{i\neq j}\tau_-(i)\tau_-(j)\frac{R_0}{r_{ij}}(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot(i\hat{\boldsymbol{r}}_{ij}\times\hat{\boldsymbol{R}}_{ij})\frac{3j_1(p_e R_{ij})j_0(\frac{1}{2}p_e r_{ij})}{p_e r_{ij}}k_2(\delta,\nu). \tag{59}$$

(The quantity $k_2(\delta, \nu)$ is given by eq. (54). The above effective operators have been normalized is such a fashion so that in the limit $p_e \to 0$ they coincide with those entering 0ν $\beta\beta$ decay [15].

(v) *Heavy particles in right-handed currents.* In the limit of large masses the nucleus cannot distinguish between the mechanisms shown in figs. 1b and 5. Thus the amplitude associated with them will depend on $\eta'_H + \eta'_R$. Proceeding in a fashion analogous to (i) above we obtain

$$\mathcal{M} = -\frac{1}{4\pi}\left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{m_e f_A^2}{r_0 A^{1/3}}(\eta'_R + \eta'_H)\bar{u}^c(p_e)(1-\gamma_5)u(p_\mu)\langle f|\Omega_N|i\rangle, \tag{60}$$

with the operator Ω_N defined in eqs. (49a) and (49b). In both cases the amplitude is inversely proportional to the heavy mass. Restricting ourselves to η'_R we disagree with Riazuddin et al. [22] who predict a quadratic increase with the neutrino mass. Clearly (μ^-, e^+) involves two particles (quarks or nucleons) and one encounters a Fourier rather than a loop integral.

4. Branching ratios

Once the amplitude is known it is fairly straightforward to compute the width for the (μ^-, e^+) conversion. First one finds that

$$\sum_{\text{spins}}|\mathcal{M}|^2 = \left(\frac{G_F}{\sqrt{2}}\right)^4\left(\frac{f_A^2}{4\pi}\right)^2\frac{E_e}{m_e}\frac{m_e^2}{r_0^2 A^{2/3}}|\eta'_{\text{eff}}|^2|\text{ME}|^2, \tag{61}$$

where η'_{eff} is an effective lepton-violating parameter given by

$$|\eta'_{\text{eff}}|^2 = |\eta'_\nu|^2 + |\eta'_N|^2 + |\eta'_z|^2 + |\eta'_H + \eta'_R|^2 + |\eta'_g|^2 , \tag{62}$$

and $|\text{ME}|^2$ is a combination of the various nuclear matrix elements discussed in sect. 3. This expression will not be given here. We only remark that in writing eq. (61) we factored out the energy-dependence characteristic of the purely left-handed (or purely right-handed) neutrino-mediated process which is described by the leptonic current of eq. (48). Additional energy dependence may appear in the quantity $|\text{ME}|^2$. This arises from two sources: (i) the additional momentum dependence of the leptonic currents of eqs. (52) and (56) and (ii) the energy dependence of the nuclear matrix elements themselves which enters via the spherical Bessel functions appearing in the relevant nuclear operators.

We note that if the lepton-violating (μ^-, e^+) process proceeds via only one of the mechanisms described in sect. 2, η'_{eff} coincides with the lepton-violating parameter associated with that mechanism alone and $|\text{ME}|^2$ is, except for a possible kinematical factor, the square of the nuclear matrix element. In general what we have is interference between the various mechanisms, which is not discussed in detail here. Only for illustration purposes we will exhibit the effect of all these mechanisms combined assuming that the lepton-violating parameters are real (as given by the relevant expression of sect. 2).

Using eq. (61) it is easy to find the relevant width i.e.

$$\Gamma\begin{pmatrix}\mu^- \to e^+ \\ |i\rangle \to |f\rangle\end{pmatrix} = \frac{1}{\pi} m_e p_e \langle \phi_\mu^2 \rangle \sum_{\text{spins}} |\mathcal{M}|^2 , \qquad \langle \phi_\mu^2 \rangle = \frac{(\alpha m_\mu)^3}{\pi} \frac{Z_{\text{eff}}^4}{Z} .$$

$\langle \phi_\mu^2 \rangle$ is the average of the muon wave function at the center of the nucleus. Z_{eff} is different from Z in order to account for possible distortions of the muon wave function compared with the hydrogenic one. We thus obtain

$$\Gamma\begin{pmatrix}\mu^- \to e^+ \\ |i\rangle \to |f\rangle\end{pmatrix} = \frac{1}{4} \frac{m_\mu}{(2\pi)^4} (G_F m_\mu^2)^4 \alpha^3 \frac{Z_{\text{eff}}^4}{Z} \left(\frac{m_e}{m_\mu}\right)^2 \frac{E_e p_e}{m_\mu^2} \frac{|\eta'_{\text{eff}}|^2 |\text{ME}|^2}{A^{2/3} (m_\mu r_0)^2} .$$

The corresponding width for Majorana emission (eq. (24)) with energy between 0 and $\Delta = 2.5$ MeV (background-free region) must be multiplied by the kinematical scale factor $\xi = (1/2\pi^2) E_e \Delta / m_e^2 \approx 50$.

The total muon capture rate is given by [27]

$$\Gamma(\mu^- \to \nu_\mu) = m_\mu \frac{\alpha^3}{2\pi^2} Z_{\text{eff}}^4 (G_F m_\mu^2)^2 f(A, Z) [F_\nu^2 + 3F_A^2 + F_p^2 - 2F_A F_p] ,$$

where F_V, F_A and F_p are the vector, axial vector and pseudoscalar form factors such that $F_V^2 + 3F_A^2 + F_p^2 - 2F_A F_P \approx 5.9$. The function $f(Z, A)$ takes into account the two-nucleon correlations given by [28]

$$f(A, Z) = 1 - 0.03 \frac{A}{2Z} + 0.25 \left(\frac{A}{2Z} - 1\right) + 3.24 \left(\frac{Z}{2A} - \frac{1}{2} - \left|\frac{1}{8Z} - \frac{1}{4A}\right|\right)$$

which is 0.16 for $A = 2Z$.

Thus the branching ratio $R = \Gamma(\mu^-, |i\rangle \to e^+|f\rangle)/\Gamma(\mu^-, \nu_\mu)$ takes the form

$$R = \rho|\eta'_{\rm eff}|^2 \frac{E_e p_e}{m_\mu^2} \frac{|\mathrm{ME}|^2}{A^{2/3} Z f(A,Z)},$$

with $\rho = 1.5 \times 10^{-21}$. The smallness of ρ is due to the fact that the process (μ^-, e^+) is a second-order weak process while the ordinary muon capture is a first-order one. We expect, of course, additional suppression since, as we have seen, the lepton-violating parameters are small compared to unity.

5. Results and discussion

The general formalism described in the previous sections can now be applied in the specific transition $^{58}\mathrm{Ni} \to {}^{58}\mathrm{Fe}$ (gs). The required nuclear matrix elements are: 0.202, 21, −0.360, 0.153, −0.034 for the operators Ω_ν, Ω_N, Ω_1, Ω_2 and Ω_3 respectively. These matrix elements were obtained using the nuclear model described in detail in ref. [29]. The relevant branching ratios are presented in table 1 together with their corresponding lepton-violating parameters. All branching ratios are extremely small, the largest being $\sim 10^{-27}$ and beyond the goals of any

TABLE 1

The lepton-violating parameters and the corresponding branching ratios and half-lives entering (μ^-, e^+) conversion are presented.

$\mu^- \to e^+$				$e^- \to e^+$	
$\eta'_{\rm eff}$	$R(0^+ \to 0^+$ (g.s.))	$(0^+ \to 0^+$ (g.s.)) $T_{1/2}$ (y)	$R(0^+ \to \text{all})$	$\eta_{\rm eff}$	$T_{1/2}$ (y)
$\eta'_\nu = 8.4 \times 10^{-4}$	6.2×10^{-31}	5.3×10^{15}	8.0×10^{-26}	$\eta_\nu = 1.2 \times 10^{-4}$	8.6×10^{24}
$\eta'_\nu = 8.7 \times 10^{-4}$	6.7×10^{-31}	4.9×10^{15}	8.5×10^{-26}	$\eta_\nu = 0$	∞
$\eta'_N = 8.9 \times 10^{-8}$	7.9×10^{-35}	4.2×10^{19}	1.3×10^{-29}	$\eta_N = 8.9 \times 10^{-7}$	8.4×10^{24}
$\eta'_N = 3.0 \times 10^{-8}$	8.9×10^{-36}	3.7×10^{19}	1.5×10^{-30}	$\eta_N = 3.0 \times 10^{-7}$	7.4×10^{25}
$\eta'_g = 2.8 \times 10^{-3}$	3.4×10^{-28}	9.7×10^{12}	4.5×10^{-23}	$\eta_g = 2.8 \times 10^{-3}$	1.6×10^{23}
$\eta'_z = 10^{-4}$	2.6×10^{-30}	1.3×10^{15}	4.5×10^{-23}	$\eta_z = 0$	∞
$\eta'_H = 4 \times 10^{-6}$	1.6×10^{-31}	2.0×10^{16}	2.6×10^{-26}	$\eta_H = 4 \times 10^{-6}$	4.1×10^{23}
$\eta'_R = 3.5 \times 10^{-8}$	1.1×10^{-35}	3.0×10^{20}	2.0×10^{-30}	$\eta_H = 8.5 \times 10^{-6}$	9.0×10^{22}
$\eta'_R = 1.2 \times 10^{-8}$	1.3×10^{-36}	2.5×10^{21}	2.3×10^{-31}		
$\eta_{RL} = 0$	0	∞	0	$\eta_{RL} = 2 \times 10^{-5}$	1.3×10^{27}
$\eta_{RL} = 2.8 \times 10^{-6}$	8.0×10^{-33}	4×10^{17}	1.1×10^{-26}		

For comparison the corresponding quantities for (e^-, e^+) are also given. Whenever a double entry appears the upper row refers to CP eigenvalues of the same sign and the lower one to those of opposite sign. The total rates shown in the table are those obtained in the shell-model treatment. For further explanations see text.

experiment. The smallness of these branching ratios can be attributed to the following three factors.

(i) The smallness of the parameter ρ discussed in sect. 4, which has to do with the fact that the lepton-violating processes are second order in weak interactions.

(ii) The smallness of the lepton-violating parameters which are smaller than 10^{-4}.

(iii) The suppression of the ground-state to ground-state transition due to the large (by nuclear standards) momentum transfer $|\boldsymbol{q}| \approx m_\mu$. Indeed the nuclear matrix elements involved in double β decay (small momentum transfer) computed in exactly the same nuclear model in the above order are [29] 1.17, 60, −1.77, 0.79, −1.17.

The branching ratios can, of course, be improved if one considers the sum of the transition strengths to all final nuclear states, which as we have seen introduces background problems. Such calculations are of course complicated and model dependent. An estimate of the sum rule is possible if one makes two assumptions, which are spelled out in some detail in refs. [12] and [15]. These are the following:

(i) the energy dependence of the matrix elements can be treated in an average way, and this makes it possible to invoke closure over the final states;

(ii) the 4-body pieces of the operator $\Omega^+\Omega$ can be neglected.

With the above assumptions it is easy to compute the matrix element of the two-body piece of the operator which is pretty much independent of the nuclear model used. The reason for this is that now all protons and not only the ones near the nuclear surface contribute. This is demonstrated by computing this matrix element in two models, the nuclear shell model (s.m.) and the uniform density (u.d.) approximation. It turns out that the spin singlet of the two-nucleon wave function dominates. Thus in the u.d. approximation we included the singlet piece i.e. we multiplied our result with the probability of finding two nucleons in a singlet state ($P_0 = 0.25$). The s.m. treatment included both singlets and triplets. We thus obtained:

$$\langle \mathrm{i}|\Omega_\nu^+\Omega_\nu|\mathrm{i}\rangle = 5.3\times 10^3\,, \qquad \langle \mathrm{i}|\Omega_\mathrm{N}^+\Omega_\mathrm{N}|\mathrm{i}\rangle = 7.5\times 10^7\,, \qquad \text{(s.m.)}$$

$$\langle \mathrm{i}|\Omega_\nu^+\Omega_\nu|\mathrm{i}\rangle = 5.5\times 10^3\,, \qquad \langle \mathrm{i}|\Omega_\mathrm{N}^+\Omega_\mathrm{N}|\mathrm{i}\rangle = 1.1\times 10^7\,. \qquad \text{(u.d.)}$$

We see that the matrix element associated with Ω_ν is almost the same in both cases. The matrix element of Ω_N is larger in the (s.m.) treatment since the harmonic oscillator wave functions peak closer to the origin something that favors the somewhat short-ranged operator Ω_N.

The sum rules associated with the Zee-model have previously been discussed and we will not elaborate them here. We only mention that in ref. [20] the rates have been overestimated by a factor of $\sim 10^4$ since m_φ was there taken to be 10 GeV. The spin structure associated with the R-L interference term has also been found previously [12]. On the basis of such estimates this contribution is expected to be approximately 300 bigger than that associated with $\langle \mathrm{i}|\Omega_\nu^+\Omega_\nu|\mathrm{i}\rangle$. The branching ratio ($R \sim 10^{-29}$) obtained by considering heavy Majorana neutrinos with mass $\sim 10^3$ GeV is in agreement with the results of the simplistic model of ref. [12] and much smaller than the estimate of ref. [22].

From table 1 we see that even if we compute transitions to all final nuclear states we are far from the goals of currently planned experiments ($R \sim 10^{-12}$). The largest computed branching ratios are $R \sim 10^{-22}$ and arise from unconventional models involving extended Higgs sectors.

The smallness of the branching ratios associated with ground-state transitions may come as a surprise to most people. In hindsight, however, this appears reasonable. If we convert the branching ratios to life-times for all mechanisms considered we find that the (μ^-, e^+) reaction is just about 10^{10} faster than its sister reaction (e^-, e^+). This is pretty much what one expects on the basis of simple kinematics ($\sim (m_\mu/m_e)^5$). (The half-lifes of the (e^-, e^+) reaction have been included in table 1 for the reader's convenience. For details see ref. [15]). Thus it is not so much the slowness of the reaction which makes it undetectable but the short time the muon lives that makes it unable to contribute to our understanding of lepton non-conservation.

We should stress that our results depend on gauge parameters about which little is known. Sometimes our lepton-violating parameter depends on large powers of such parameters making it less reliable. This is the reason why we have absorbed all the parameters of the gauge models into one lepton-violating parameter. One can, if necessary, put one's own choice of gauge parameters and apply our formalism.

From the above discussion, barring a complete cancellation among the various neutrino mass eigenstates or a world in which only mechanisms similar to the Zee model remain effective, 0ν double β decay appears to be more promising in unraveling the mysteries of lepton non-conservation. After all, there are many more nuclei in a gram of matter than the largest number of stopped muons ($\sim 10^{11}$) and they live much longer. This, of course, should not deter the experimental searches for rare muon decays. One hopes that such searches will continue. After all, our conclusions depend on not so well understood aspects of gauge theory models. There may be unforseen mechanisms which could invalidate our conclusions.

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