A three-generation SU(4)×O(4) string model

I. Antoniadis
Centre de Physique Theorique, Ecole Polytechnique, F-91128 Palaiseau, France

G.K. Leontaris and J. Rizos
Theoretical Physics Division, University of Ioannina, GR-45110 Ioannina, Greece

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We present a four-dimensional string model whose effective field theory has an O(6)×O(4) gauge symmetry which is broken to the standard model at a very high scale. The fermions of the third generation receive masses from trilinear couplings, retaining the mass relation $m_b=m_\tau$ at the GUT scale. All the dangerous color triplets become superheavy while a see-saw mechanism provides the right-handed neutrino with a superheavy mass. The model also predicts a number of "exotic" states with fractional electric charges.

A main issue in string theories is to derive a model which at low energies leads to the theory of strong and electroweak interactions with all its observed properties. This turns out to be a non-trivial game despite the large degeneracy of four-dimensional string vacua. Several such constructions have appeared in the literature the last few years [1–4], usually based on gauge groups larger than the standard model $SU(3)_C\times SU(2)_L\times U(1)_Y$, but not many of them seem to be phenomenologically viable. One of the main difficulties is that of obtaining the right Yukawa couplings which forbid fast proton decay and flavor changing neutral currents and give rise to the observed fermion mass hierarchies. These properties look completely ad hoc at the standard model and must be imposed by making use of several discrete symmetries since there are no continuous symmetries left over in string theories.

Since many of these properties appear naturally in grand unified theories (GUTs), an appealing idea is to proceed through a grand unified scale, combining automatically the benefit of the successful properties of GUTs. The main problem in this approach is the absence of adjoint or higher self-conjugate matter representations (usually needed to break the GUT group) in theories where the gauge symmetries are realized at a level $k=1$ Kac–Moody algebra. Other more complicated constructions based on higher values of k seem to be problematic [5], mainly due to the appearance of "exotic" representations, like SU(3) octets or $SU(2)_L$ triplets, etc. A nice prediction of all $k=1$ constructions is the absence of such high representations, but the price to pay is the unavoidable existence of color neutral states with fractional electric charges (unless one deals with unattractively small values for the weak angle at the GUT scale) [6]. Such particles would have severe phenomenological consequences unless they are confined by some additional "hidden" gauge group to form integrally charged states in analogy with QCD [3,7].

The only promising example of a three-generation GUT-string model is based on the flipped SU(5)×U(1) gauge symmetry which is broken to the standard model using only $10+\overline{10}$ Higgs representations [2]. Another example has been suggested in ref. [8], based on the $SU(4)\times SU(2)_L\times SU(2)_R$ Pati–Salam type left–right symmetric model [9]. The aim of this work is to derive this model from four-dimensional strings. Since the gauge group is isomorphic to a product of orthogonal groups O(6)×O(4), it can in principle be constructed, in

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the fermionic formulation [10,11], using only periodic or antiperiodic boundary conditions for the world-sheet fermions.

It is worth reviewing at this point, the minimal supersymmetric GUT version of the model [8], where its elegance and economy in the Higgs sector is more transparent. The quark and lepton fields are accommodated in the following representations of the $SU(4)\times SU(2)_L\times SU(2)_R$ symmetry:

$$F_L(4,2,1)=q(3,2,\tfrac{1}{3})+\ell(1,2,-1)\,,$$
$$\bar{F}_R(\bar{4},1,2)=u^c(\bar{3},1,-\tfrac{4}{3})+d^c(\bar{3},1,\tfrac{2}{3})+e^c(1,1,2)+N^c(1,1,0)\,, \tag{1}$$

where the quantum numbers on the right-hand side are with respect to the $SU(3)_C\times SU(2)_L\times U(1)_Y$ group. Note that $F_L+\bar{F}_R$ make up the 16 spinorial representation of SO(10). In addition, the model employs the following superfields:

$$H(4,1,2)=\bar{u}^c_H(3,1,\tfrac{4}{3})+\bar{d}^c_H(3,1,-\tfrac{2}{3})+\bar{e}^c_H(1,1,-2)+\bar{N}^c_H(1,1,0)\,,$$
$$\bar{H}(\bar{4},1,2)=u^c_H(\bar{3},1,-\tfrac{4}{3})+d^c_H(\bar{3},1,\tfrac{2}{3})+e^c_H(1,1,2)+N^c_H(1,1,0)\,,$$
$$h(1,2,2)=h(1,2,-1)+h^c(1,2,1)\,,$$
$$D(6,1,1)=D(3,1,-\tfrac{2}{3})+D^c(\bar{3},1,\tfrac{2}{3})\,,$$
$$\Phi_i(1,1,0)\quad i=0,1,2,3\,. \tag{2}$$

The charge operator is defined as a linear combination of the three diagonal generators of the SU(4), $SU(2)_L$ and $SU(2)_R$ groups:

$$Q=\tfrac{1}{6}T_{15}+\tfrac{1}{2}T_{3L}+\tfrac{1}{2}T_{3R}\,,\quad T_{15}=\mathrm{diag.}(1,1,1,-3)\quad T_{3L}=T_{3R}=\mathrm{diag}(1,-1)\,, \tag{3a,b}$$

so that all particles acquire the correct charge assignment.

We now discuss in brief the pattern of gauge symmetry breaking. The H, $\bar{H}$ fields break the SU(4) and $SU(2)_R$ symmetry at a high scale when they develop VEVs along their neutral directions $\langle H\rangle=\langle\bar{N}^c_H\rangle$ and $\langle\bar{H}\rangle=\langle N^c_H\rangle$. Note that both of them are incomplete multiplets of the 16 and $\overline{16}$ representations of the SO(10) group. The fields $h(1,2,2)$ and $D(6,1,1)$ arise both from the 10 of SO(10). After the first stage of symmetry breaking $D(6,1,1)$ decomposes to a triplet and an antitriplet. Both of them combine with the uneaten d^c_H and $\bar{d}^c_H$ fields of $\bar{H}$ and H and form massive states of order M_{GUT}, so that the proton does not decay fast. The field $h(1,2,2)$ decomposes to the usual two electroweak doublets which realize the subsequent breaking of the standard model gauge group by the Higgs mechanism. Finally, one of the four singlets Φ_i acquires a non-vanishing VEV. The superpotential of the model in its minimal version reads [8]

$$W=\lambda_1F_L\bar{F}_Rh+\lambda_2\bar{F}_RH\Phi_0+\lambda_3HHD+\lambda_4\bar{H}\bar{H}D+\lambda_5hh\Phi_0+\lambda_6\Phi^3+\ \dots\,. \tag{4}$$

In (4), coupling λ_1 provides all the sixteen fermions of the model with masses, λ_2 generates a see-saw mechanism providing the right-handed neutrino with a superheavy mass while the λ_3 and λ_4 terms give masses of order M_{GUT} to the color triplets, as we already mentioned above. Finally, λ_5 realizes the Higgs mixing preventing the appearance of the unwanted electroweak axion.

Next, we derive the model from the fermionic formulation of four-dimensional strings of ref. [10]. Our string model is generated by the following nine vectors of boundary conditions for all the world-sheet fermions:

$$
\begin{aligned}
\zeta &= \{ \quad ; \bar{z}^{12} \quad \bar{\Phi}^{1\ldots 6} \}, \\
S &= \{\psi^\mu, \chi^{1\ldots 6} \quad ; \quad \}, \\
b_1 &= \{\psi^\mu, \chi^{12}, y^{3456}\bar{y}^{3456} \quad ; \bar{\psi}^{1\ldots 5}\bar{\eta}^1 \}, \\
b_2 &= \{\psi^\mu, \chi^{34}, y^{12}\bar{y}^{12} \quad \omega^{56}\bar{\omega}^{56} \quad ; \bar{\psi}^{1\ldots 5}\bar{\eta}^2 \}, \\
b_3 &= \{\psi^\mu, \chi^{56}, \quad \omega^{1234}\bar{\omega}^{1234} ; \bar{\psi}^{1\ldots 5}\bar{\eta}^3 \}, \\
b_4 &= \{\psi^\mu, \chi^{12}, y^{36}\bar{y}^{36} \quad \omega^{45}\bar{\omega}^{45} \quad ; \bar{\psi}^{1\ldots 5}\bar{\eta}^1 \}, \\
b_5 &= \{\psi^\mu, \chi^{34}, y^{26}\bar{y}^{26} \quad \omega^{15}\bar{\omega}^{15} \quad ; \bar{\psi}^{1\ldots 5}\bar{\eta}^2 \}, \\
b_6 &= \{ \quad y^6\bar{y}^6 \quad \omega^6\bar{\omega}^6 \quad ; \bar{\psi}^{1\ldots 5}\bar{\eta}^{123} \quad \bar{\Phi}^{123}\bar{z}^1 \}, \\
\alpha &= \{ \quad y^{46}\bar{y}^{46} \quad \omega^{46}\bar{\omega}^{2346} \quad ; \bar{\psi}^{123}\bar{\eta}^{12}\bar{z}^{12} \quad \}.
\end{aligned} \tag{5}
$$

In our notation the world sheet fermions appearing in a certain vector are periodic, while the remaining are antiperiodic. The semicolon separates real from complex world sheet fermions and those with a "bar" are right movers. ψ^μ, χ^i, y^i, ω^i, $(i=1, ..., 6)$ are the left real fermions among which world-sheet supersymmetry is non-linearly realized [10]. In order to obtain the desired low energy spectrum we have made a specific choice of the projection coefficients $c\binom{b_i}{b_j}$ which we summarize in the following matrix form:

$$
\begin{array}{c|ccccccccc}
 & \zeta & S & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & \alpha \\
\hline
\zeta & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\
S & & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\
b_1 & & & -1 & -1 & -1 & -1 & 1 & -1 & -1 \\
b_2 & & & & -1 & -1 & 1 & -1 & -1 & 1 \\
b_3 & & & & & -1 & 1 & 1 & -1 & 1 \\
b_4 & & & & & & -1 & -1 & 1 & -1 \\
b_5 & & & & & & & -1 & 1 & -1 \\
b_6 & & & & & & & & 1 & -1 \\
\alpha & & & & & & & & & -1
\end{array} \tag{6}
$$

The upper (lower) element $b_i(b_j)$ of the coefficient $c\binom{b_i}{b_j}$, corresponds to the b_ith row (b_jth column) of the projection coefficient matrix (6). The missing entries of the latter can be calculated easily using the modular invariance constraint equations [10].

We proceed now to the description of the symmetry breaking by the GSO projection mechanism. The basis vectors $\{S, \zeta, \mathbb{1}=b_1+b_2+b_3+\zeta\}$, define an $N=4$ space–time supersymmetric model with an $SO(28)\times E_8$ gauge group. The element S with exactly eight left movers (including the ψ^μ), plays the role of the supersymmetry generator. b_1 reduces the supersymmetries to $N=2$ and b_2 leads to the required $N=1$. Furthermore, SO(28) breaks down to $SO(10)\times SO(6)^3$ with six chiral families $(16, 4)+(16, \bar{4})$, two from each of the sectors b_1, b_2, b_3. Finally the rest of the vectors (i.e. $b_{4,5,6}$, and α) break the group $SO(10)\times SO(6)^3\times E_8$ down to $SO(6)\times SO(4)\times U(1)^4\times U(1)'\times SU(8)_{\mathrm{hidden}}$ and project out half of the chiral families. The $SO(6)\times SO(4)$ symmetry corresponds to $\bar{\psi}^{123}$ and $\bar{\psi}^{45}$ world sheet fermions, the four U(1)'s to the world sheet currents $\bar{\eta}^1\bar{\eta}^{1*}$, $\bar{\eta}^2\bar{\eta}^{2*}$, $\bar{\eta}^3\bar{\eta}^{3*}$ and $\bar{\omega}^2\bar{\omega}^3$, while the fifth U(1)′ to the linear combination $\bar{z}^1\bar{z}^{1*}-\bar{z}^5\bar{z}^{5*}$.

We are now ready to present the massless spectrum of the model. The sectors b_1, ..., b_5 produce the three chiral families of quarks and leptons, as in eq. (1), plus two Higgs multiplets in the $(4, 1, 2)+(\bar{4}, 1, 2)$ representations of $SU(4)\times SU(2)_L\times SU(2)_R$, as in eq. (2):

$$b_1: \bar{F}_{1R}=(\bar{4},1,2)_{(1/2,0,0,0)}, \quad F_{1L}=(4,2,1)_{(1/2,0,0,0)},$$
$$b_2: \bar{F}_{2R}=(\bar{4},1,2)_{(0,1/2,0,0)}, \quad \bar{F}'_{2R}=(\bar{4},1,2)_{(0,-1/2,0,0)},$$
$$b_3: \bar{F}_{3R}=(\bar{4},1,2)_{(0,0,-1/2,1/2)}, \quad F_{3L}=(4,2,1)_{(0,0,-1/2,-1/2)},$$
$$b_4: F_{4R}=(4,1,2)_{(1/2,0,0,0)}, \quad F_{4L}=(4,2,1)_{(-1/2,0,0,0)},$$
$$b_5: \bar{F}_{5R}=(\bar{4},1,2)_{(0,-1/2,0,0)}, \quad F_{5R}=(4,1,2)_{(0,-1/2,0,0)}. \tag{7}$$

In (7) the states appear only with their charges under $U(1)^4$, as they are neutral under $U(1)'$. From the Neveu–Schwarz sector, in addition to the graviton, dilaton and the antisymmetric tensor, one also gets the following fields:

(a) Two L–R Higgs doublets:

$$h_3=(1,2,2)_{(0,0,1,0)}, \quad \bar{h}_3=(1,2,2)_{(0,0,-1,0)}, \tag{8a}$$

which are obtained by acting on the vacuum with the fermionic oscillators of $\chi^5+i\chi^6$, with $\bar{\psi}^\alpha$ or $\bar{\psi}^{\alpha*}$ ($\alpha=4,5$) and $\bar{\eta}^3$ or $\bar{\eta}^{3*}$.

(b) Five singlets with zero $U(1)$ charges:

$$\Phi_1=(1,1,1)_{(0,0,0,0)} \quad \chi^1+i\chi^2, \bar{y}^1, \bar{\omega}^1, \quad \Phi_2=(1,1,1)_{(0,0,0,0)} \quad \chi^3+i\chi^4, \bar{y}^4, \bar{\omega}^4,$$
$$\Phi_3=(1,1,1)_{(0,0,0,0)} \quad \chi^5+i\chi^6, \bar{y}^2, \bar{y}^3, \quad \Phi_4=(1,1,1)_{(0,0,0,0)} \quad \chi^5+i\chi^6, \bar{y}^5, \bar{\omega}^5,$$
$$\Phi_5=(1,1,1)_{(0,0,0,0)} \quad \chi^5+i\chi^6, \bar{y}^6, \bar{\omega}^6, \tag{8b}$$

where on the right of each of the above fields we indicate the fermionic oscillators used to obtain the states.

(c) Four singlets with non-trivial $U(1)^4$ charges:

$$\Phi_{12}=(1,1,1)_{(1,1,0,0)}, \quad \bar{\Phi}_{12}=(1,1,1)_{(-1,-1,0,0)}, \quad \Phi_{\bar{1}\bar{2}}=(1,1,1)_{(1,-1,0,0)}, \quad \bar{\Phi}_{\bar{1}\bar{2}}=(1,1,1)_{(-1,1,0,0)} \tag{8c}$$

obtained using the oscillators of $\chi^5+i\chi^6$ with $\bar{\eta}^1$ or $\bar{\eta}^{1*}$ and $\bar{\eta}^2$ or $\bar{\eta}^{2*}$.

(d) The following four sextet fields:

$$D_1=(6,1,1)_{(1,0,0,0)}, \quad \bar{D}_1=(6,1,1)_{(-1,0,0,0)}, \chi^1+i\chi^2,$$
$$D_2=(6,1,1)_{(0,1,0,0)}, \quad D_2=(6,1,1)_{(0,-1,0,0)}, \chi^3+i\chi^4. \tag{8d}$$

Finally, the sector $S+b_4+b_5$ gives the following states:

$$h_{12}=(1,2,2)_{(1/2,1/2,0,0)}, \quad \bar{h}_{12}=(1,2,2)_{(-1/2,-1/2,0,0)}, \bar{\psi}^\alpha \text{ or } \bar{\psi}^{\alpha*},$$
$$\zeta_1=(1,1,1)_{(1/2,-1/2,0,0)}, \quad \bar{\zeta}_1=(1,1,1)_{(-1/2,1/2,0,0)}, \bar{y}^5,$$
$$\zeta_2=(1,1,1)_{(1/2,-1/2,0,0)}, \quad \bar{\zeta}_2=(1,1,1)_{(-1/2,1/2,0,0)}, \bar{\omega}^5,$$
$$\xi_1=(1,1,1)_{(1/2,1/2,1,0)}, \quad \bar{\xi}_1=(1,1,1)_{(-1/2,-1/2,-1,0)}, \bar{\eta}^3 \text{ or } \bar{\eta}^{3*},$$
$$\xi_2=(1,1,1)_{(1/2,-1/2,0,1)}, \quad \bar{\xi}_2=(1,1,1)_{(-1/2,1/2,0,-1)}, \bar{\omega}^{23},$$
$$\xi_3=(1,1,1)_{(-1/2,1/2,0,1)}, \quad \bar{\xi}_3=(1,1,1)_{(1/2,-1/2,0,-1)}, \bar{\omega}^{23},$$
$$\xi_4=(1,1,1)_{(1/2,1/2,-1,0)}, \quad \bar{\xi}_4=(1,1,1)_{(-1/2,-1/2,1,0)}, \bar{\eta}^3 \text{ or } \bar{\eta}^{3*}. \tag{9}$$

The fields described so far, belong to the observable sector (i.e. they transform trivially under the SU(8) hidden group) and they all have zero charge under the fifth $U(1)'$. There are two more classes of massless states in our model. The first class contains fields which are SU(8) singlets but have U(1)′ charges and transform also non-trivially under the observable symmetry. These are ten left and ten right doublets plus one (4, 1, 1)+ ($\bar{4}$, 1 ,1), listed in table 1. The second class contains fields which are singlets under the observable gauge symmetry but transform non trivially under U(8)×U(1)′; they are listed in table 2. As mentioned above, the main phenomenological headache in all $k=1$ constructions is the appearance of "exotic" states with fractional electric charges. In the case of the SU(4)×O(4) group the only allowed "exotic" representations [12] are exactly those appearing in table 1, i.e. (4, 1, 1), ($\bar{4}$, 1, 1), (1, 2, 1), and (1, 1, 2). More precisely, $h^{\pm}_{\rm L,R}$ contain particles with charges $\pm\frac{1}{2}$ while H and $\bar{H}$ contain color triplets with charges $\pm\frac{1}{6}$ and exotic leptons with charges $\pm\frac{1}{2}$. As shown in ref. [6], it could be possible to avoid these fractional charges at the price of dealing with unattractively small values for $\sin^2\theta_{\rm w}$. For instance in our model, one could modify the charge operator (3) by including the U(1)′ generator Q':

$$Q_{\rm new}=Q_{\rm old}-\tfrac{1}{2}Q' . \tag{3'}$$

In this case the doublet fields $h^{\pm}_{\rm L,R}$ acquire integer electric charges (±1, 0), while the SU(4) tetraplets contain color triplets with charges $\frac{2}{3}$ and neutral color singlets. Unfortunately (3′) implies $\sin^2\theta_{\rm w}=\frac{3}{14}$ at $M_{\rm GUT}$!

The superfields appearing in (7)–(9) and in tables 1 and 2, constitute the massless spectrum of our model. Comparing (1), (2) with (7)–(9) we realize that in the string model all the fields participating in the phenomenological analysis of ref. [8] appear. We found three families accommodated as in (1), and we obtained the tetraplet Higgs fields with the right transformation properties as in (2). There are also enough singlets to realize the see-saw mechanism which will provide the right-handed neutrinos with superheavy masses. In addition, we

Table 1
Analysis under SU(4)×SU(2)$_{\rm L}$×SU(2)$_{\rm R}$×U(1)^4×U(1)′.

$b_1+\alpha$	$h^{+}_{1\rm L}=(1,2,1)_{(0,-1/2,0,1/2)(-1)}$ $h^{+}_{1\rm R}=(1,1,2)_{(0,1/2,0,1/2)(-1)}$	$h'^{+}_{1\rm L}=(1,2,1)_{(0,-1/2,0,-1/2)(-1)}$ $h'^{+}_{1\rm R}=(1,1,2)_{(0,1/2,0,-1/2)(-1)}$
$b_1+b_2+b_4+\alpha$	$h^{-}_{2\rm L}=(1,2,1)_{(1/2,0,0,1/2)(+1)}$ $h^{-}_{2\rm R}=(1,1,2)_{(1/2,0,0,-1/2)(+1)}$	$h'^{-}_{2\rm L}=(1,2,1)_{(-1/2,0,0,1/2)(+1)}$ $h'^{-}_{2\rm R}=(1,1,2)_{(-1/2,0,0,-1/2)(+1)}$
$b_2+b_3+b_5+\alpha$	$h^{+}_{3\rm L}=(1,2,1)_{(1/2,-1/2,1/2,0)(-1)}$ $h^{-}_{3\rm R}=(1,1,2)_{(-1/2,1/2,1/2,0)(+1)}$	$h^{-}_{3\rm L}=(1,2,1)_{(1/2,-1/2,-1/2,0)(+1)}$ $h^{+}_{3\rm R}=(1,1,2)_{(-1/2,1/2,-1/2,0)(-1)}$
$b_4+\alpha$	$h^{+}_{4\rm L}=(1,2,1)_{(0,1/2,0,1/2)(-1)}$ $h^{-}_{4\rm R}=(1,1,2)_{(0,1/2,0,-1/2)(+1)}$	$h^{-}_{4\rm L}=(1,2,1)_{(0,-1/2,0,1/2)(+1)}$ $h^{+}_{4\rm R}=(1,1,2)_{(0,-1/2,0,-1/2)(-1)}$
$b_1+b_4+b_5+\alpha$	$h^{-}_{5\rm L}=(1,2,1)_{(1/2,0,0,1/2)(+1)}$ $h^{+}_{5\rm R}=(1,1,2)_{(-1/2,0,0,1/2)(-1)}$	$h^{+}_{5\rm L}=(1,2,1)_{(1/2,0,0,1/2)(-1)}$ $h^{-}_{5\rm R}=(1,1,2)_{(-1/2,0,0,-1/2)(+1)}$
$S+b_2+b_4+\alpha$	$H_4=(4,1,1)_{(0,0,0,1/2)(-1)}$	$\bar{\rm H}_4=(\bar{4},1,1)_{(0,0,0,-1/2)(+1)}$

Table 2
Analysis under SU(8)×U(1)′×U(1)^4.

$b_1+b_6\ (+\zeta)$	$\bar{Z}_1=(\bar{8},\frac{1}{2})_{(0,-1/2,-1/2,0)}$	$Z_1=(8,-\frac{1}{2})_{(0,-1/2,-1/2,0)}$
$b_2+b_6\ (+\zeta)$	$Z_2=(8,-\frac{1}{2})_{(1/2,0,-1/2,0)}$	$Z'_2=(8,-\frac{1}{2})_{(-1/2,0,-1/2,0)}$
$b_4+b_6\ (+\zeta)$	$\bar{Z}_3=(\bar{8},\frac{1}{2})_{(0,1/2,-1/2,0)}$	$\bar{Z}'_3=(\bar{8},\frac{1}{2})_{(0,-1/2,1/2,0)}$
$b_5+b_6\ (+\zeta)$	$\bar{Z}_4=(\bar{8},\frac{1}{2})_{(1/2,0,1/2,0)}$	$Z_4=(8,-\frac{1}{2})_{(1/2,0,-1/2,0)}$
$b_2+b_3+b_5+b_6\ (+\zeta)$	$Z_5=(8,-\frac{1}{2})_{(1/2,-1/2,0,1/2)}$	$\bar{Z}_5=(\bar{8},\frac{1}{2})_{(1/2,-1/2,0,-1/2)}$

got a number of "exotic" representations presented in tables 1 and 2. As we already mentioned, in addition to the observable $SO(6)\times SO(4)$ gauge group there are four extra $U(1)$ symmetries under which the observable fields carry non-zero charges. One linear combination of them has been found to be anomalous:

$$U(1)_A = U(1)_1 - U(1)_2 - U(1)_3\,, \quad \mathrm{Tr}\, Q_A = 72\,. \tag{10a}$$

The remaining combinations

$$U(1)'_1 = U(1)_1 + U(1)_2\,, \quad U(1)'_2 = U(1)_1 - U(1)_2 + 2U(1)_3\,, \quad U(1)_4 \tag{10b}$$

are free from gauge and gravitational anomalies. The cancellation of all mixed anomalies among the three $U(1)$'s constitutes a non-trivial consistency check for the massless spectrum of the model. Next, we proceed to the calculation of the superpotential. The tree level non-vanishing terms are

$$\begin{aligned} W = {} & F_{4L}\bar{F}_{5R}h_{12} + (1/\sqrt{2})F_{4R}\bar{F}_{5R}\bar{\zeta}_2 + \bar{F}_{3R}F_{3L}h_3 + D_1 D_2 \bar{\Phi}_{12} + D_1 \bar{D}_2 \bar{\Phi}_{\overline{12}} + \bar{D}_1 D_2 \Phi_{\overline{12}} + \bar{D}_1 \bar{D}_2 \Phi_{12} \\ & + (F_{1L}F_{1L} + \bar{F}_{1R}\bar{F}_{1R} + F_{4R}F_{4R})\bar{D}_1 + F_{4L}F_{4L}D_1 + (\bar{F}'_{2R}\bar{F}'_{2R} + F_{5R}F_{5R} + \bar{F}_{5R}\bar{F}_{5R})D_2 + \bar{F}_{2R}\bar{F}_{2R}\bar{D}_2 \\ & + \tfrac{1}{2}(h_{12}\bar{h}_{12} + \zeta_1\bar{\zeta}_1 + \zeta_2\bar{\zeta}_2 + \xi_1\bar{\xi}_1 + \xi_2\bar{\xi}_2 + \xi_3\bar{\xi}_3 + \xi_4\bar{\xi}_4)\Phi_3 + (\bar{\zeta}_1\zeta_2 + \bar{\zeta}_2\zeta_1)\Phi_4 \\ & + (h_{12}h_{12} + \xi_1\xi_4)\bar{\Phi}_{12} + (\bar{h}_{12}\bar{h}_{12} + \bar{\xi}_1\bar{\xi}_4)\Phi_{12} + (\zeta_1\zeta_1 + \zeta_2\zeta_2 + \xi_2\bar{\xi}_3)\bar{\Phi}_{\overline{12}} + (\bar{\zeta}_1\bar{\zeta}_1 + \bar{\zeta}_2\bar{\zeta}_2 + \bar{\xi}_2\xi_3)\Phi_{\overline{12}} \\ & + (\xi_4\bar{h}_{12}h_3 + \bar{\xi}_1 h_{12}h_3 + \bar{\xi}_4 h_{12}\bar{h}_3 + \xi_1\bar{h}_{12}\bar{h}_3) + H_4\bar{H}_4\Phi_3 + \bar{F}_{1R}h'_{\bar{2}R}H_4 + \bar{F}'_{2R}h_{\bar{4}R}H_4 \\ & + h^{+}_{3L}h_{\bar{3}R}\bar{h}_3 + h_{\bar{3}L}h^{+}_{3R}h_3 + h_{\bar{3}L}h^{+}_{3L}\bar{\Phi}_{\overline{12}} + \Phi_{\overline{12}}h_{\bar{3}R}h^{+}_{3R} + h^{+}_{1L}h_{\bar{5}R}h_{12} + h'^{+}_{1R}h_{\bar{5}L}\bar{h}_{12} \\ & + \bar{\xi}_2 h^{+}_{1L}h_{\bar{5}L} + \xi_2 h'^{+}_{1R}h_{\bar{5}R} + (h'^{+}_{1L}h_{\bar{5}L}\bar{\zeta}_1 + h^{+}_{1R}h_{\bar{5}R}\zeta_1)/\sqrt{2} + Z_5\bar{Z}_5\Phi_{\overline{12}} + Z_3\bar{Z}_4\bar{\xi}_4 + (\bar{Z}'_3 Z_4\bar{\zeta}_2)/\sqrt{2}\,, \end{aligned} \tag{11}$$

where a common overall normalization factor $g/\sqrt{2}$ is assumed. One can easily notice that (11) contains all the terms appearing in the simple GUT version in (4), plus the Yukawa coupling for the "exotic" states. The symmetry breaking proceeds here in an analogous manner with the minimal case described previously. Special analysis should be carried out however for the anomalous $U(1)_A$ factor. The latter is known to be broken by the Dine–Seiberg–Witten mechanism [13] in which the anomalous D-term generated by a VEV of the dilaton field is cancelled by VEVs that break some of the non-anomalous gauge symmetries so that supersymmetry is preserved. This condition reads

$$\sum Q_{Ai}|\phi_i|^2 = -3g^2/8\pi^2\,. \tag{12}$$

Thus, a proper flat direction should be found in our scalar potential $\Phi = \sum a_i\phi_i$, where $\phi_i = \{\zeta_i, \xi_i, ...\}$, so that its VEV will cancel the D-term and stabilize the vacuum.

It is easy to see that there is such a choice of VEVs which breaks all four surplus $U(1)$ factors together with the breaking of $SU(4)\times SU(2)_R$ to $SU(3)_C\times U(1)_Y$ at a high scale of the order of 10^{16}–10^{17} GeV. Considering for instance non-vanishing VEVs for the singlet fields Φ_{12}, $\bar{\Phi}_{\overline{12}}$, ξ_1, $\bar{\xi}_2$ we break all four $U(1)$'s. Furthermore, the breaking of $SU(4)\times SU(2)_R$ will require in general VEVs for the two Higgs fields defined by F_{4R}, F_{5R}, together with two linear combinations of $\bar{F}_{1R}$, $\bar{F}_{2R}$, $\bar{F}'_{2R}$, $\bar{F}_{3R}$. Note that the fifth $U(1)'$ can be broken by giving a VEV to at least one of the states of table 2: $\langle Z_i\rangle = \langle\bar{Z}_j\rangle \neq 0$. The above choice leads to the following picture of the spectrum of our model:

(1) With $\langle\xi_1\rangle$, $\langle\Phi_{12}\rangle \neq 0$, all components of $\bar{h}_{12}$ and $\bar{h}_3$ Higgs fields become superheavy. h_{12} then plays the role of the two electroweak Higgs doublets which provide masses to the fermions of the "third" generation $F_{4L}+\bar{F}_{5R}$ through the first superpotential term. As in the GUT version of the model [8] one gets the mass relations $m_b = m_\tau$ and $m_t = m_{\nu 4}$. The first relation is the successful prediction of old GUTs but the second gives too large a value for the neutrino mass. However the second superpotential term $\langle F_{4R}\rangle\bar{F}_{5R}\bar{\zeta}_2$ provides a see-saw mechanism which leads to a massless linear combination of ν_{4L} and $\bar{\zeta}_2$ which is identified as the left-handed neutrino ν_τ.

The third superpotential term could in principle provide masses to the fermions of one more generation

$\bar{F}_{3L}+F_{3L}$ through non vanishing VEVs of two additional electroweak Higgs doublets in h_3,

$$h_3 \equiv \begin{pmatrix} h_3^0 \\ h_3^- \end{pmatrix} + \begin{pmatrix} h_3^+ \\ \bar{h}_3^0 \end{pmatrix} . \tag{13}$$

Because of the superpotential term $\langle h_{12} \rangle \bar{\xi}_1 h_3$ only a linear combination of h_3^0 and $\bar{h}_3^0$ is massless and can acquire a VEV. One thus obtains $\langle h_3^0 \rangle \langle \bar{h}_{12}^0 \rangle = \langle \bar{h}_3^0 \rangle \langle h_{12}^0 \rangle$ which implies the mass relation $m_t/m_b = m_c/m_s$ at the GUT scale. This leads to a definite prediction for the top mass of the right order of magnitude.

However there are two obstacles in this interpretation: on the one hand we are left with four light Higgs doublets; on the other hand $\bar{F}_{3R}$ is forced to acquire a nonvanishing VEV to satisfy the D-flatness conditions (unless $\langle \bar{Z}_5 \rangle \neq 0$). In this case, the $SU(2)_L$ doublet in F_{3L} (see eq. (1)) is combined with the second doublet in (13) and becomes superheavy via the third superpotential term $\langle \bar{F}_{3R} \rangle F_{3L} h_3$. The first doublet in (13) is now identified with the left handed leptons of this generation. The resulting accommodation of quarks and leptons is similar to the one proposed in ref. [14] for the top quark generation. In fact the right handed up-quark is a linear combination of u_3^c and the $\lambda = (\bar{3}, 1, 1)$ piece of the SU(4) gauginos, as one finds by simple inspection of the gauge interaction terms:

$$\langle N_3^c \rangle u_3^c \bar{\lambda} + M_{\rm GUT} \lambda \bar{\lambda} . \tag{14}$$

Furthermore, the left-handed neutrino corresponds to a massless combination of h_3^0 in (13) and the singlets Φ_3 and ξ_4 as one concludes from the following superpotential terms:

$$\langle h_{12} \rangle \bar{\xi}_1 h_3 \langle \Phi_{12} \rangle \bar{\xi}_1 \xi_4 + \tfrac{1}{2} \langle \xi_1 \rangle \bar{\xi}_1 \Phi_3 . \tag{15}$$

The right-handed neutrino N_3^c, which remains massless at this stage, is expected to receive a mass at least of the order of the supersymmetry breaking scale.

(2) At the first stage of symmetry breaking, following the Higgs field decomposition (2), there remain two triplet d_{Hi}^c and two anti-triplet $\bar{d}_{Hi}^c$ combinations ($i=1, 2$) which mediate proton decay. However, they are combined with the four triplets and antitriplets arising from the decomposition of D_i, $\bar{D}_i$ and form six superheavy massive states via the superpotential terms:

$$\langle \bar{\Phi}_{\bar{1}\bar{2}} \rangle D_1 \bar{D}_2 + \langle \Phi_{12} \rangle \bar{D}_1 \bar{D}_2 + (\langle \bar{F}_{1R} \rangle \bar{F}_{1R} + \langle F_{4R} \rangle F_{4R}) \bar{D}_1$$
$$+ 2(\langle \bar{F}'_{2R} \rangle \bar{F}'_{2R} + \langle F_{5R} \rangle F_{5R}) D_2 + 2 \langle \bar{F}_{2R} \rangle \bar{F}_{2R} \bar{D}_2 . \tag{16}$$

Thus the proton does not decay fast since there are no dimension-five operators, as all masses in (16) are of Dirac type. There are also two linear combinations u_H^c and $\bar{u}_H^c$ which remain uneaten by the Higgs mechanism; although they cannot mediate proton decay, they are expected to receive masses from non-renormalizable superpotential terms.

(3) The VEVs of the singlet fields $\langle \bar{\Phi}_{\bar{1}\bar{2}} \rangle$, $\langle \bar{\xi}_2 \rangle$ provide also masses for four of the "exotic" left doublets, already from the trilinear superpotential terms. Moreover, $\langle \bar{F}_{1R} \rangle$ and $\langle \bar{F}'_{2R} \rangle$ make superheavy another combination of the right-handed exotic doublets and the corresponding component of H_4. The rest remain massless at the present stage and this seems to constitute the main phenomenological problem of our model. A few more of these exotic states are expected to receive masses from non-renormalizable superpotential terms. For this purpose, a detailed analysis of the higher order terms and the possible allowed VEVs is required. Let us point out that one should also take into account the effects of the condensation of the remaining "hidden" group G ($SU(8) \supset G$) which most likely takes place. In a preliminary analysis we checked that at least most of the right-handed doublets become superheavy assuming non-zero VEVs for $\langle Z_i \bar{Z}_j \rangle$ fermion bilinears. The inclusion of non-renormalizable terms is also necessary to generate possible contributions to the fermion mass matrices and in particular to provide the lighter generations with masses.

In conclusion, we derived a three-generation string model with an $SU(4) \times SU(2)_L \times SU(2)_R$ "observable" symmetry and a "hidden" SU(8) group. The observable gauge symmetry is broken down to the standard model at a high energy scale $M_{\rm GUT} \approx (10^{16}\text{–}10^{17})$ GeV within $\sin^2\theta_{\rm w} = \frac{3}{8}$ (at $M_{\rm GUT}$) via the VEVs of two Higgs fields in $(4, 1, 2) + (\bar{4}, 1, 2)$ representations. The latter are combined with VEVs for some of the singlet fields and break simultaneously four surplus U(1) forces (one of them being anomalous) along the D and F flat directions of the scalar potential. All dangerous color triplets mediating proton decay become superheavy at the GUT scale. The second stage of symmetry breaking is realized by the VEVs of two $SU(2)_L$ doublet Higgses providing masses to the fermions of the heaviest generation. The successful relation $m_b = m_t$ at the GUT scale is obtained, while a see-saw mechanism provides a superheavy mass for the right-handed neutrino, leaving the left-handed massless. The light spectrum of the model contains also a number of states in $(4, 1, 1)$, $(\bar{4}, 1, 1)$, $(1, 2, 1)$ and $(1, 1, 2)$ representations. These states carry fractional electric charges and might have dramatic phenomenological consequences. We have discussed possible scenarios which could make most of them superheavy and a more detailed analysis is currently under investigation.

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