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Evaluation of the phase of the CP violation parameter η_{+-} and the $K_L - K_S$ mass difference from a correlation analysis of different experiments

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Abstract

The best estimation of φ_{+-} (the phase of the CP violation parameter η_{+-}) and of Δm (the $K_L - K_S$ mass difference) is obtained by averaging the results of different experiments, taking into account the different correlation, existing for most of the experiments, between the measurement of φ_{+-} and Δm. Including the recent measurements, we obtain the average values $\langle \Delta m \rangle = (530.7 \pm 1.3) \times 10^7 \hbar/\mathrm{s}$ and $\langle \varphi_{+-} \rangle = 43.82° \pm 0.63°$. This value of φ_{+-} is in good agreement with the superweak phase $\varphi_{SW} = 43.49° \pm 0.08°$.

1. Introduction

The world's best limits of CPT violation [1,2] are obtained by comparing the phase φ_{+-} of the CP violation parameter η_{+-} with the superweak phase $\varphi_{SW} \equiv \tan^{-1}(2\Delta m/\Delta\Gamma)$. Here $\Delta m(\Delta\Gamma)$ is the mass (total decay width) difference between the K_L and K_S. Since the present experimental results [3-8,10-13] are of comparable precision, the best estimation of the value of φ_{+-} is obtained by averaging the results of all these experiments. Given the different strong correlation of the measurement of φ_{+-} and Δm for most of the experiments, averaging the measurements of φ_{+-} and Δm independently [1] seems not to be the adequate method. A more precise method consists in using all the available experimental information to construct a global likelihood distribution $\mathcal{L}$ depending on the parameters Δm, φ_{+-} (and τ_S, the K_S mean life), as the product of individual likelihood distributions of each experiment. The best estimations for the values of Δm and φ_{+-} are then obtained by maximizing $\mathcal{L}$. The values of the experiments [3-6,10-12] adopted by PDG [1], as well as the values of three recent experiments [7,8,13] are presented in Table 1 (for φ_{+-}), together with their quoted Δm and τ_S dependence, and in Table 2 (for Δm).

A gaussian likelihood distribution taking into account the correlation between φ_{+-} and Δm can be defined with the published information for the experiments [6-8]. The authors of the other experiments [3-5] simply state that their value of φ_{+-} is strongly correlated with Δm and give a linear dependence. In order to define a gaussian likelihood distribution also for these experiments, we have to make some assumptions about the correlation between φ_{+-} and Δm and about the central value of Δm. For the experiment of [3], which gives the most precise value of φ_{+-} among these three experiments, we have been able to deduce that the correlation is larger than 99% [9]. We assume this correlation also holds for the other two experiments [4,5]. For such a strong correlation between φ_{+-} and Δm, the error ellipse of the individual experiment is degenerated to a band (see Fig. 1) within the limit of Δm defined by the other experiments [6-8,10-13]. Consequently the experiments [3-5] contribute to the fit only as one degree of freedom. The systematic uncertainty for the average values due to our assumptions is estimated by varying the correlation between φ_{+-} and Δm and the central value of Δm.

The dependence of φ_{+-} on the value of τ_S is less important, since τ_S is known to a better precision than Δm. For the same reason, the change in the experimental value of Δm is negligible, when varying the value of τ_S within its error (Table 2). The world average value of τ_S [1] is dominated by the experiment of Ref. [6]. The authors assume in their fit $\varphi_{+-} = \varphi_{SW}$ and note that a difference of 1° between φ_{+-} and φ_{SW} would shift the value of τ_S by only 0.0008×10^{-10} s. In our fitting procedure we only take into account the linear dependence of φ_{+-} with τ_S when it is given (Table 1).

Table 1
Measurements of φ_{+-} considered in our fit. For the experiments [6,7] we assume a common systematic error of $\pm 0.3°$ due to regeneration uncertainties

Experiment	φ_{+-} [deg]	Stat. errors		ρ
		φ_{+-} [deg]	Δm [$10^7 \hbar/s$]	
Geweniger [3]	$49.4 \pm 1.0 + 0.565\,(\Delta m - 540.0)$			> 0.99
Carithers [4]	$45.5 \pm 2.8 + 0.224\,(\Delta m - 534.8)$			
Carosi [5]	$46.9 \pm 1.6 + 0.579\,(\Delta m - 535.1) + 303\,(\tau_S - 0.8922)$			
E731[a] [6]	$42.2 \pm 0.9 + 0.189\,(\Delta m - 525.7) - 460\,(\tau_S - 0.8922)$	0.75	4.4	0.74
E773 [7]	$43.53 \pm 0.76 + 0.173\,(\Delta m - 528.2) - 275\,(\tau_S - 0.8926)$	0.58	3.0	0.67
CPLEAR [8]	$42.7 \pm 1.1 + 0.316\,(\Delta m - 527.4) + 30\,(\tau_S - 0.8926)$	0.9	6.7	0.92

[a] The 1994 PDG [1] quotation has an error in the central value for the Δm dependence.

Table 2
Measurements of Δm considered in our fit. The experiments [10–13] measure Δm independently of φ_{+-}, whereas the experiments [6–8] obtain Δm from a fit with floating Δm and φ_{+-}. The experiments [11] and [12] have a common systematic error of $\pm 1.5 \times 10^7 \hbar/s$

Experiment	Δm [$10^7 \hbar/s$]
E731 [6]	525.7 ± 4.9
E773 [7]	529.7 ± 3.7
CPLEAR [8]	529.5 ± 6.7
Cullen [10]	542.0 ± 6.0
Gjesdal [11]	533.4 ± 4.0
Geweniger [12]	$534.0 \pm 3.0 + 12\,(\tau_S - 0.8994)$
CPLEAR [13]	$527.4 \pm 2.9 \pm 83\,(\tau_S - 0.8926)$

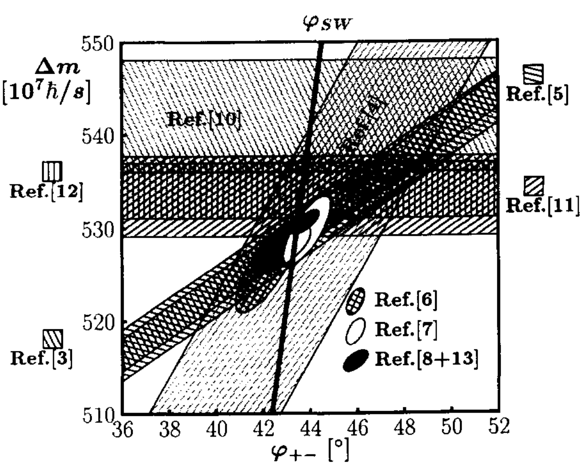

Fig. 1. The plot shows the 1σ contour plots of all measurements listed in Tables 1 and 2. The black ellipse in the center represents the result of our fit. The expected region for the value of φ_{SW} is also shown.

2. Description of the fits

We assume that the measured values $X_i = (\varphi_{+-}{}^i, \Delta m^i)^T$ are distributed around the true value X according to a gaussian likelihood distribution:

$$L_i(X) = k_i e^{-\frac{1}{2}(X_i - X)^T C_i^{-1}(X_i - X)}, \tag{1}$$

where C_i is the corresponding covariance matrix:

$$C_i = \begin{pmatrix} (\sigma_\varphi^i)^2 & c_{\varphi\Delta m}^i \\ c_{\varphi\Delta m}^i & (\sigma_{\Delta m}^i)^2 \end{pmatrix}, \tag{2}$$

and k_i is a normalization factor which generally depends on the elements of C_i. In the case of experiments [10–13], $X_i = \Delta m^i$. The combined likelihood distribution of all measurements is then given by:

$$\mathcal{L}(X) = \prod_i L_i(X). \tag{3}$$

The best estimate of X and its error is given by maximizing $\mathcal{L}(X)$, i.e. minimizing $\chi^2(X) = \sum (X_i - X)^T C_i^{-1}(X_i - X)$ with respect to X. Errors common to different experiments can be taken into account in this procedure by expanding X_i and C_i as follows:

$$X_i \to X_i = \begin{pmatrix} X_k \\ X_l \end{pmatrix}, \quad C_i \to C_i = \begin{pmatrix} C_k & \sigma^2 \\ \sigma^2 & C_l \end{pmatrix}, \tag{4}$$

where σ is the error common to experiment k and l.

Most of the experiments quote the value of φ_{+-} for a fixed value $\Delta m = \Delta\tilde{m}$ together with the linear dependence on Δm. We recall that for a fixed value of Δm, $\Delta\tilde{m}$, the value of φ_{+-} and its error, $\tilde{\varphi}_{+-}$ and $\tilde{\sigma}_\varphi$ respectively, are given by:

$$\tilde{\varphi}_{+-} = \varphi_{+-} + \rho \frac{\sigma_\varphi}{\sigma_{\Delta m}} (\Delta\tilde{m} - \Delta m) , \quad (5)$$

$$\tilde{\sigma}_\varphi = \sigma_\varphi \sqrt{1-\rho^2} , \quad (6)$$

where φ_{+-}, σ_φ and Δm, $\sigma_{\Delta m}$ are the central values and errors of a two-parameter fit and ρ is the correlation coefficient between φ_{+-} and Δm. Therefore we are able to reconstruct the full covariance matrix by using in addition to the value of φ_{+-} and its statistical error, the value of Δm and its error and the linear dependence (Tables 1 and 2). The systematic errors of φ_{+-} and Δm are added in quadrature to the diagonal elements of the covariance matrix.

2.1. Average values for φ_{+-} and Δm

The data used as input to our fits is summarized in Tables 1 and 2 and also presented together with the final result in Fig. 1. Since the values of φ_{+-} depend on the value of τ_S, we leave τ_S as an additional free parameter in the fit. However, its value is constrained by the precision of the world average, $\langle\tau_S\rangle = (0.8926 \pm 0.0012) \times 10^{-10}$s [1].

Using the experimental data available up to 1994 as input to our fit, we find similar results as PDG [1]. However our error for $\langle\Delta m\rangle$ is smaller compared to the PDG result due to the additional information used. For Δm we find $\langle\Delta m\rangle = (532.1 \pm 1.8 \pm 0.1) \times 10^7\hbar$/s and for φ_{+-} the value $\langle\varphi_{+-}\rangle = 44.3° \pm 1.0° \pm 0.1°$, where the second error reflects the uncertainty of the correlation between φ_{+-} and Δm for the experiments [3–5]. Two experiments [6,10] give a large contribution to the χ^2, the first giving a low value and the second a high value of Δm compared to the average value. The χ^2/degree of freedom (dof) for the combined average of φ_{+-} and Δm is 1.0, and in contrast to PDG we do not need to scale the error of Δm by 1.2.

If we include the three recently published measurements [7,8,13] of Δm and φ_{+-} we find the results shown in Table 3 (Fit A). The value of Δm is lower by 1 σ and only one experiment [10] now gives a large contribution to the χ^2. Since this experiment deviates only by 2σ and the total χ^2/dof is 0.8, we have retained it in our fit. By using only the experiments [6–8,10–13], which give the full covariance matrix of their measurements, we obtain the results shown in Table 3 (Fit B), which are in good agreement with Fit A.

Table 3
Results from Fit A and Fit B. Fit A is made using all the experiments whereas Fit B is based only on the experiments [6–8,10–13], which give the full covariance matrix of their measurements. The error for Fit A includes an additional error of $\pm 0.1 \times 10^7\hbar/s$ for Δm and $\pm 0.1°$ for φ_{+-}, obtained by varying the correlation between φ_{+-} and Δm for the older experiments [3–5]

Parameter	Fit A	Fit B
Δm [$10^7\hbar$/s]	530.7 ± 1.3	530.9 ± 1.5
φ_{+-}	43.82° ± 0.63°	43.71° ± 0.66°
τ_S [10^{-10}s]	0.8922 ± 0.0010	0.8923 ± 0.0011
χ^2/dof	0.89	1.02
φ_{SW} [a]	43.49° ± 0.08°	43.50° ± 0.08°

[a] For the K_L mean life we used $\tau_L = (5.15 \pm 0.04) \times 10^{-8}$s [1].

The quoted systematic error of [6,7] concerning their phase measurement which is mainly determined by the knowledge of the regeneration amplitudes has been subject of discussion [14–16]. Increasing the common systematic error of these two experiments to 1° (3°) yields $\varphi_{+-} = 43.94° \pm 0.75°$ (44.06°±0.87°) and $\Delta m = [530.8 \pm 1.4\ (530.9 \pm 1.4)] \times 10^7\hbar$/s. We conclude that the results of our correlated analysis do not change significantly, even with enlarged systematic errors for the measurements reported by [6] and [7], although the precision on φ_{+-} deteriorates.

3. Conclusion

In order to determine the best values of Δm and φ_{+-} using the data available from different experiments, we performed a correlated fit to the data, using when available the individual correlations of these two parameters. Our final result using the experiments [3–8,10–13] is:

$$\langle\varphi_{+-}\rangle = 43.82° \pm 0.63° , \quad (7)$$

$$\langle\Delta m\rangle = (530.7 \pm 1.3) \times 10^7\hbar/\mathrm{s} , \quad (8)$$

with a correlation coefficient of 0.70 between φ_{+-} and Δm, and −0.40 between φ_{+-} and τ_S, i.e.:

$$\varphi_{+-} = 43.82° \pm 0.41° + 0.339\,(\Delta m - 530.7)° - 252\,(\tau_S - 0.8922)° . \quad (9)$$

The value of φ_{+-} is in good agreement with the superweak phase $\varphi_{SW} = 43.49° \pm 0.08°$ as expected from CPT invariance.

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