FLUCTUATIONS IN A SUPERSYMMETRIC INFLATIONARY UNIVERSE

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It has been demonstrated that fluctuations in the new inflationary universe may be almost scale-invariant, but are unfortunately too large. We show that supersymmetric inflationary models allow the fluctuations to be smaller. In a toy supersymmetric model, the perturbations are $O(10^{-4})$ is the Yukawa interactions are $O(10^{-6}\,\mu/m_P)$ where μ is the magnitude of the Higgs vacuum expectation value driving the inflation. It is therefore easier to have small fluctuations if inflation occurs close to the Planck epoch.

There has recently been considerable interest in inflationary scenarios for the very early universe [1–4]. The proposal is that there may have been an epoch of De Sitter exponential expansion driven by the vacuum energy tied up in a scalar field during or before the grand unified theory (GUT) phase transition. The hope [1] has been that such a model would solve in a natural way many of the outstanding cosmologocal problems such as the large-scale homogeneity and isotropy of the present universe, as well as its approximate flatness despite its great longevity ($t_0 \approx 10^{61} t_P$). Inflation was also supposed to explain the low abundances of unseen particles such as monopoles [5] and more recently gravitinos [6]. It was unfortunately difficult to understand how the universe could have made a graceful exit from the original inflationary expansion [7]. A new inflationary scenario was then proposed [2] which evaded this difficulty by having the inflation occur while the scalar field was already "rolling over" into its low temperature global minimum. However, achieving sufficient exponential expansion required adjustments of the parameters of the scalar potential which seemed impossible or at best unnatural in conventional GUTs. These adjustments seem mucl more plausible in a supersymmetric theory [8], but it appears [9] that fine-tuning of the potential parameters is only avoided if the inflation takes place before the GUT epoch and closer to the Planck epoch ("Primordial Inflation").

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As already mentioned, a prime motivation for inflation was the prospect of understanding the large-scale homogeneity and isotropy of the universe [1]. These could for the first time be related, in principle, to considerations of microphysics. What spectrum of density fluctuations should one be seeking? Larger-scale perturbations have a longer time to grow because their scale comes within the horizon at a later epoch. Therefore, in order to ensure that perturbations have the same magnitude $\delta\rho/\rho$ when they enter the horizon, one must require that their initial spectrum $\delta\rho/\rho|_i \sim 1/l^{3n} \sim 1/m^n$ has $n = 2/3$. In order to avoid individual initial perturbations from becoming separate Friedman universes, one needs $n > 2/3 + \epsilon$ (ϵ small) [10,11], while demanding that the initial perturbations be strong enough to form galaxies subsequently implies $n < 2/3 + \epsilon'$ [10,11]. A scale-invariant value of $\delta\rho/\rho = O(10^{-4})$ for the magnitude of perturbations when they enter the horizon, for all scales smaller than the present horizon, would be compatible with upper limits on perturbations in the microwave background radiation and with the birth of galaxies [10,11]. It has been shown [12] that density fluctuations in the context of GUT phase transitions are able to provide the desired scale-invariant spectrum. More recently, the spectrum of density fluctuations in the new inflationary scenario has been studied [13,14], however with rather discouraging results. The good news is that the spectrum is scale-invariant to with slowly-varying logarithmic factors [15] [‡1]. This result follows from the time translation-independence of the inflationary phase during which the scale factor expands exponentially: $a(t_2) = a(t_1)\exp[H(t_2 - t_1)]$. Identical spectra of perturbations at different times t_i get blown up by different exponential factors $\exp[H(t_f - t_i)]$ where t_f is the time when the inflationary epoch finishes. This scale-invariance is maintained until the perturbations come back inside the horizon at a much later epoch. The bad news is that the magnitude of the fluctuations $\delta\rho/\rho$ is much too large [13,14] in the conventional new inflationary scenario [2], and it has been speculated [13] that smaller perturbations might be possible in a supersymmetric theory.

In this paper we investigate the spectrum of perturbations in our toy supersymmetric inflationary model [9]. We show that in this model the spectrum is also approximately scale-invariant, albeit with logarithmic corrections which differ in detail from those found [13,14] in the new inflationary scenario [2]. Fluctuations $\delta\rho/\rho = O(10^{-4})$ can be achieved if the supersymmetric chiral superfield coupling is sufficiently small. This requirement becomes increasingly stringent the longer the onset of inflation is delayed after the Planck epoch. Thus we acquire another motivation for primordial inflation to supplement our previous arguments [9] about the avoidance of fine-tuning.

We first recall relevant features of our toy model for supersymmetric inflation [9]. It is characterized by a tree-level zero temperature potential

$$V(\phi) = \alpha\phi^4 - \beta\phi^3 + \gamma\phi^2 + V(0)\,, \tag{1}$$

which can be obtained from the superpotential

$$W(\phi, X, Y) = aX\phi(\phi - \mu) + bY(\phi^2 - \mu^2)\,, \tag{2}$$

‡1 We understand that a development of this work is in preparation [16]. We have also received private communications on this subject [17].

with the identifications

$$\alpha = a^2 + b^2, \quad \beta = 2a^2\mu, \quad \gamma = (a^2 - 2b^2)\mu^2\,, \tag{3a,b,c}$$

$$V(0) = b^2\mu^4\,. \tag{3d}$$

Notice that the appearance of a ϕ^3 term in the potential $V(\phi)$ (1) is unavoidable in our model if the effective (mass)^2 parameter γ (3c) is to be very small as is required [9] if there is to be sufficient inflation:

$$a^2/b^2 - 2 = O(\tfrac{1}{10}\mu^2/m_P^2)\,. \tag{4}$$

This fine-tuning is technically natural in a supersymmetric theory, but it is unappealing and this was one of our previous motivations [9] for making inflation primordial: $\mu \to m_P$. In the work of Guth and Pi [14] no ϕ^2 term was included in the effective potential. This assumption was technically unnatural, and we see that removing it in a natural way introduces a novel ϕ^3 term in the effective potential (1).

We analyze the spectrum of perturbations along the lines pioneered by Lifshitz [18] and by Olson [19] followed by Guth and Pi [14]. The homogeneous background field $\phi_0(t)$ evolves according to the equation

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 = -\partial V/\partial\phi|_{\phi=\phi_0}\,. \tag{5}$$

Neglecting the $\ddot{\phi}_0$ term in eq. (5) we find that

$$3H\dot{\phi}_0 = -[4\alpha\phi_0^3 - 3\beta\phi_0^2]\,, \tag{6}$$

and it is easy to verify that the first term on the right-hand side of eq. (6) is also negligible so that

$$\phi_0(t) = -H/\beta t\,, \tag{7}$$

where the time t is measured starting from $t = -\infty$. Position-dependent perturbations $\delta\phi(x, t)$ on the homogenous background $\phi_0(t)$ have the first order effect of producing a position-dependent time delay $\delta\tau(x)$ in the evolution of $\phi_0(t)$ at large times:

$$\delta\tau(x) = -\delta\phi(x, t)/\dot{\phi}_0(t)\,. \tag{8}$$

It has been suggested [14] that the fluctuations $\delta\phi(k, t)$ of wave number k are to be determined by the quantum theory of a free massless scalar field in De Sitter space:

$$\delta\phi(k, t) \approx H/4\pi^{3/2} \tag{9}$$

Strictly speaking, the estimate (9) is only applicable when $t \ll t_*$, where t_* is the time at which the two terms on the right-hand side of the equation

$$\delta\ddot{\phi} + 3H\delta\dot{\phi}$$
$$= -\partial^2 V/\partial\phi^2|_{\phi=\phi_0(t)}\delta\phi + \exp(-2Ht)\partial_i^2\delta\phi \quad (10)$$

are equal:

$$t \ll t_* \approx -\ln(Hk^{-1})/H\,, \quad (11)$$

where the relevant values of Hk^{-1} are very large. On the other hand, the estimate (8) is strictly valid only when $t \gg t_*$. We follow Guth and Pi [14] who boldly assume that the expressions (8) and (9) are both approximately valid when $t = O(t_*)$. From eqs. (7) and (11) we deduce

$$\dot{\phi}_0(t_*) = H/\beta t_*^2 = H^3/\beta \ln^2(Hk^{-1})\,. \quad (12)$$

Using the estimates (9) and (12) we find that

$$\delta\tau \approx \beta \ln^2(Hk^{-1})/4\pi^{3/2}H^2\,. \quad (13)$$

Guth and Pi [14] have shown

$$\delta\rho/\rho \approx 2\sqrt{2}H\delta\tau\,. \quad (14)$$

In our toy potential (1) one deduces from eq. (3d) that

$$H = (\tfrac{8}{3}\pi)^{1/2}(\mu^2/m_P)b \approx (\tfrac{4}{3}\pi)^{1/2}(\mu^2/m_P)a\,, \quad (15)$$

in which case

$$\delta\rho/\rho \approx (2\pi^3)^{-1/2}(\beta/H)\ln^2(Hk^{-1})$$
$$\approx \tfrac{3}{2}(m_P/\mu)(a/\pi^2)\ln^2[(\tfrac{4}{3}\pi)^{1/2}(\mu^2 a/m_P)k^{-1}]\,. \quad (16)$$

It should be noted that this spectrum, like that of Guth and Pi [14], is almost scale-invariant. However, it has a slight difference in the power of the logarithm $\ln(Hk^{-1})$. This is directly traceable to the crucial ϕ^3 term we found in the supersymmetric potential (1)–(3).

Turning now to the numerical evaluation of the perturbations (16), we see that

$$O(10^{-4}) = 0.124(m_P a/\mu)\ln^2[2.05(\mu^2 a/m_P)k^{-1}] \quad (17)$$

if

$$a \approx O(10^{-6})\mu/m_P\,. \quad (18)$$

Thus if μ is of the order of the GUT scale of $O(10^{-3}$ to $10^{-4})\ m_P$, the parameter a in the superpotential must be considerably smaller than any of the "known" chiral couplings, of which the smallest one known is $g_{H\bar{e}e} = O(m_e/m_W) = O(10^{-6})$. On the other hand, a can be considerably larger if one adopts the hypothesis of primordial inflation [9] which we discussed earlier: $\mu = O(m_P)$ requires $a = O(10^{-6})$ which is within the "observed" range. This may be taken as another motivation for primordial supersymmetric inflation ‡2. The values of the superpotential parameters a, b that we find in this analysis are comfortably consistent with the upper bounds we obtained previously [9]. It is amusing to recall at this point that Harrison [10] proposed originally that his scale-invariant perturbations might have been geberated around the Planck epoch.

Thus we have seen that supersymmetric inflation [8,9], unlike the conventional new inflationary scenario [2], can indeed yield an acceptable fluctuation spectrum which is approximately scale-invariant with $\delta\rho/\rho = O(10^{-4})$. We have found that the required values of the superpotential parameters are larger if inflation takes place closer to the Planck epoch. This observation, together with our previous remark about the amount of fine-tuning necessary to ensure sufficient inflation pushes us in the direction of primordial supersymmetric inflation [9].

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‡2 We note in passing that if there had been no β term in the potential (1), then $a = O(10^{-12})$ would have given $\delta\rho/\rho = O(10^{-4})$, independent of the scale at which inflation took place.

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