GAUGE HIERARCHY GENERATION AND COSMOLOGY IN LOCALLY SUPERSYMMETRIC GUTS

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We discuss a locally supersymmetric GUT with no unnaturally small couplings and just two input mass scales: $M_P \approx 10^{19}$ GeV and $\mu \approx 10^{11}$ GeV. We get out both the weak scale $M_W \approx \mu^2/M_P$ and the GUT scale $M_X \approx (\mu M_P)^{1/2}$. The model produces an acceptable cosmological scenario similar to the one we have previously discussed for globally supersymmetric GUTs.

Global supersymmetry was introduced into grand unified theories to ameliorate the gauge hierarchy problem [1]. Originally, one expected to get a low energy theory in which supersymmetry was spontaneously broken at energies comparable to the weak interaction scale. This led to enormously complicated models, as it is not easy to break global supersymmetry: if there is a supersymmetric state in the Hilbert space, it is automatically the ground state.

Recently, however, various authors have considered locally supersymmetric models [2–22]. Breaking local supersymmetry is much easier than breaking global supersymmetry. Furthermore, it is possible to break local supersymmetry at an intermediate scale [‡1] $\mu \simeq 10^{11}$ GeV with fields that have only gravitational couplings to the fields which describe our world. This leads to theories which, at low energies, look like globally supersymmetric models with explicit soft breaking terms [5,7,10]. The scale of soft breaking is the gravitino mass, $m_{3/2}$, which is of order $\mu^2/M_P \simeq 100$ GeV. All superpartners (squarks, sleptons, gauginos) can get masses of order $m_{3/2}$ from the soft breaking terms, and the spontaneous breakdown of SU(2) $\times$ U(1) can be induced.

Thus, the models proposed so far can account for the weak interaction scale ($M_W \simeq m_{3/2} \simeq \mu^2/M_P$). However, they still suffer from some of the problems of ordinary grand unified theories. The GUT scale M_X is put in by hand. Many unnaturally small couplings must be introduced in the superpotential. In particular, getting a reasonable cosmology [23] out of these models requires some couplings to be as small as 10^{-12} but not zero [24].

In this paper we will exhibit a class of locally supersymmetric GUTs in which such small couplings arise in a completely natural way. Furthermore, the grand unification scale M_X is not put in by hand. Rather, M_X turns out to be related to μ and M_P by

$$M_X^4 \simeq \mu^2 M_P^2\ , \quad M_X^4 \simeq m_{3/2} M_P^3\ , \qquad (1,2)$$

which is well satisfied by $M_X \simeq 10^{15}$ GeV, $\mu \simeq 10^{11}$ GeV, $M_P \simeq 10^{19}$ GeV. The small coupling we need turns out to be proportional to $(\mu/M_P)^{3/2} \simeq 10^{-12}$. We obtain these remarkable results by including nonrenormalizable interactions in the superpotential, suppressed by inverse powers of M_P. The only other scale in the lagrangian is the scale of supersymmetry breaking, $\mu \simeq 10^{11}$ GeV.

[‡1] Any quoted numerical values for mass scales should be considered uncertain by at least an order of magnitude.

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We note that it is perfectly natural to include non-renormizable interactions in the superpotential, since $N = 1$ supergravity is not renormalizable to begin with. Of course, we do not know how to compute divergent radiative corrections, and so we take the point of view advocated in ref. [9]. We consider $N = 1$ supergravity to be an effective theory valid at energies below M_P, and suppose that all effects of divergent radiative corrections are already contained in the specified non-renormalizable terms.

We start with three chiral superfields with complex scalar components Σ, X and Z. Σ is an SU(5) **24**, while X and Z are SU(5) singlets. The superpotential is

$$W = M_P^{-1}\lambda_1 X^4 + M_P^{-2}\lambda_2 X^2 \operatorname{tr}(\Sigma^3) + \mu^2(Z + \Delta)\,, \quad (3)$$

where Δ is a constant which must be adjusted to cancel the cosmological constant. The terms involving Z and Δ are responsible for the spontaneous breaking of local supersymmetry. If the other fields were not present, and we set $\Delta = (2 - \sqrt{3})M, M \equiv M_P/(8\pi)^{1/2}$, Z would get a VEV of $(\sqrt{3} - 1)M$, the gravitino would get a mass $m_{3/2} = \exp(2 - \sqrt{3})\mu^2/M$, and the cosmological constant would be zero [2]. This picture will not be disturbed if X and Σ get VEVs which are much less than M_P. In this case, we can construct a low energy effective potential which is [apart from a trivial rescaling of W by a factor $\exp(2 - \sqrt{3})$] [7]

$$V_{\text{eff}} = |\partial W/\partial X|^2 + \operatorname{tr}|\partial W/\partial \Sigma|^2$$
$$+ m_{3/2}[(A + 1)\lambda_1 M_P^{-1} X^4$$
$$+ (A + 2)\lambda_2 M_P^{-2} X^2 \operatorname{tr}(\Sigma^3) + \text{h.c.}]$$
$$+ m_{3/2}^2[|X|^2 + \operatorname{tr}|\Sigma|^2] + \tfrac{1}{2} D^\alpha D^\alpha\,,$$
$$\partial W/\partial X = 4\lambda_1 M_P^{-1} X^3 + 2\lambda_2 M_P^{-1} X \operatorname{tr}(\Sigma^3)\,,$$
$$\partial W/\partial \Sigma = 3\lambda_2 M_P^{-2} X^2 [\Sigma^2 - \tfrac{1}{5}\operatorname{tr}(\Sigma^2)]\,,$$
$$D^\alpha = g \operatorname{tr} \lambda^\alpha [\Sigma, \Sigma^*]\,. \quad (4)$$

Here A is a constant which depends on the details of local supersymmetry breaking [10]; in our case, $A = 3 - \sqrt{3}$ at tree level, but this result is subject to potentially large gravitational radiative corrections [25].

We have analyzed V_{eff} in detail. Rather than present that analysis here, we will make some order-of-magnitude estimates which indicate the main results. If we set λ_1, λ_2, and all numerical factors to unity, ignore the matrix character of Σ, and define $x \equiv X/M_P$, $\sigma \equiv \Sigma/M_P$, and $\epsilon \equiv m_{3/2}/M_P \simeq (\mu/M_P)^2 \simeq 10^{-16}$, we get

$$V_{\text{eff}}/M_P^4 = |x^3 + x\sigma^3|^2 + |x^2\sigma^2|^2$$
$$+ \epsilon(x^4 + x^2\sigma^3 + \text{h.c.}) + \epsilon^2(|x|^2 + |\sigma|^2)\,. \quad (5)$$

At $x = \sigma = 0$, V_{eff} has a local minimum with $V_{\text{eff}} = 0$. Our detailed analysis shows that this is the global minimum of V_{eff} unless $A > 3$. From now on, we assume $A > 3$. There are now three other minima, all with $V_{\text{eff}} < 0$. One of them is

$$\sigma = 0\,, \quad x \simeq \epsilon^{1/2}\,, \quad V_{\text{eff}} \simeq -\epsilon^3 M_P^4\,. \quad (6)$$

The others are found by setting $x^2 \simeq -\sigma^3$, which, to an excellent approximation, minimizes V_{eff} with respect to x. Then we get

$$V_{\text{eff}}/M_P^4 \simeq |\sigma|^{10} - \epsilon(\sigma^6 + \text{h.c.}) + \epsilon^2|\sigma|^2\,, \quad (7)$$

which yields a minimum at

$$\sigma \simeq \epsilon^{1/4}\,, \quad x \simeq \epsilon^{3/8}\,, \quad V_{\text{eff}} \simeq -\epsilon^{5/2} M_P^4\,. \quad (8)$$

We see that the minimum of eq. (8) is much deeper than that of eq. (6). Furthermore, our detailed analysis reveals that the matrix Σ must be of the form diag(2,2,2,−3,−3) or diag(1,1,1,1,−4), and that *the SU(3) × SU(2) × U(1) symmetric minimum is the lowest for all values of* λ_1 *and* λ_2.

What do these results mean? First, since the VEV of Σ sets the scale of SU(5) breaking, and $\sigma = \Sigma/M_P \simeq \epsilon^{1/4} \simeq (\mu/M)^{1/2}$, we find that the GUT scale M_X satisfies

$$M_X^2 \simeq \mu M_P\,. \quad (9)$$

Second, the SU(3) × SU(2) × U(1) symmetric minimum is lower in energy density than the SU(5) symmetric minimum $X = \Sigma = 0$ by an amount $\epsilon^{5/2} M_P^4 \simeq \mu^5/M_P$. Third, the barrier between these two minima is never larger than the largest term in V_{eff} for $0 < |x| < \epsilon^{3/8}$ and $0 < |\sigma| < \epsilon^{1/4}$, that is, the barrier is no larger than $\epsilon^{5/2} M_P^4 \simeq \mu^5/M_P$, the same as the splitting between the states.

Why this is so can be seen by noting that if we replace X by its VEV in the superpotential, the effective, renormalizable self-coupling of Σ is $\lambda_2 M_P^{-2}\langle X\rangle^2 \times \operatorname{tr}(\Sigma^3) \simeq 10^{-12} \operatorname{tr}(\Sigma^3)$. Thus we have generated a small renormalizable coupling for Σ from our starting point of only non-renormalizable interactions among X and Σ. This small coupling suppresses the barriers between the SU(5), SU(4) × U(1), and SU(3) × SU(2) × U(1) phases.

It should be clear that the basic result – small renormalizable couplings arising from non-renormalizable ones suppressed only by inverse powers of M_P – is quite general and does not depend on the detailed form of the superpotential. For example, we would get very similar results from any superpotential of the form

$$W = \lambda_1 M_P^3 (X/M_P)^{2n} + \lambda_2 M_P^3 (X/M_P)^n \operatorname{tr}(\Sigma/M_P)^p \,, \tag{10}$$

with $p > n$. We would find $M_X^{2p-2} \simeq m_{3/2} M_P^{2p-3}$, and $\langle X \rangle \simeq \epsilon^{p/2n(p-1)} M_P$. We chose $n = 2$ and $p = 3$ because $M_X^4 \simeq m_{3/2} M_P^3$ seems approximately correct, and because $\langle X \rangle \simeq \epsilon^{3/8} M_P \simeq 10^{-6} M_P$ is useful in what follows.

The small barrier between the SU(5) and SU(3) × SU(2) × U(1) phases is exactly what we need to realize the cosmological scenario we have previously discussed for globally supersymmetric GUTs [23,24]. Thermal effects keep the universe in the SU(5) symmetric phase until the temperature drops to the SU(5) confinement scale, $\Lambda \simeq 10^9$–10^{10} GeV. Then the number of massless degrees of freedom in this phase changes, and the universe tunnels to the now preferred SU(3) × SU(2) × U(1) phase, provided that the barrier between the phases is no larger than about Λ^4. In the model in this paper, this condition is satisfied. Since the transition is delayed to temperatures as low as 10^9 GeV, the number density of magnetic monopoles produced in the transition does not exceed, and may be quite close to, the observed upper limit [23].

We still need a mechanism to create the cosmological baryon number asymmetry (CBA). In the second paper of ref. [24], the CBA was created after the transition by decays of weakly coupled SU(5) singlet particles. The needed superpotential was

$$W = (10^{-6})\, \mathrm{Y H \bar{H}} + (10^{10}\ \mathrm{GeV})\, \bar{\mathrm{H}}_3 \mathrm{H}_3 + (10^{10}\ \mathrm{GeV})\, \mathrm{Y}^2 \,, \tag{11}$$

where Y is the SU(5) singlet, and H_3 and $\bar{H}_3$ are the colour triplet components of H and $\bar{H}$, a **5** and $\bar{\mathbf{5}}$. (Actually we need two sets of **5** + $\bar{\mathbf{5}}$ Higgs fields; see ref. [26].) After the phase transition, the Y particles decay, out of equilibrium (because of the small couplings) to Higgs colour triplets, which then decay, out of equilibrium, to quarks and leptons. *CP* and baryon number violation in these decays produces the CBA.

We can reproduce the superpotential of eq. (11) via

$$W = M_P^{-1} \lambda_3 \mathrm{XYH\bar{H}} + M_P^{-1} \lambda_4 \mathrm{X\bar{H}\Sigma H} + \lambda_5 \mu \mathrm{\bar{H}H} + \lambda_6 \mu \mathrm{Y}^2 . \tag{12}$$

We have introduced no new mass scales beyond μ and M_P. All the λ's are of order one. Yet when we replace X and Σ by their VEVs, we recover eq. (11), provided we fine tune λ_4 and λ_5 to give the SU(2) doublet components of H and $\bar{H}$ masses much less than μ. (Recall that $\langle X \rangle \simeq \epsilon^{3/8} M_P \simeq 10^{-6} M_P$.) So, once again, we do not need to put any very small (i.e. 10^{-6}) couplings into the original lagrangian. Even the $\mathrm{HQ\bar{Q}}$ couplings needed for quark and lepton masses can be made naturally small by using terms like $M_P^{-1}\mathrm{H\Sigma Q\bar{Q}}$ or $M_P^{-2} \times \mathrm{H\Sigma^2 Q\bar{Q}}$ instead [21].

At low energies, our model looks like a globally supersymmetric Weinberg–Salam model with softly broken supersymmetry. There are, in addition, some other chiral superfields: an SU(3) octet, an SU(2) triplet, and an SU(3) × SU(2) × U(1) singlet, all with masses of order $m_{3/2}$; originally, they were part of Σ. All have couplings to the Higgs doublets in the effective, low energy superpotential of order $\langle X \rangle / M_P \simeq 10^{-6}$. The singlet is potentially dangerous since its interactions beyond tree level can destabilize the hierarchy between M_X and $m_{3/2}$ [15,16]. However, in our case the singlet couples so weakly that this is not a problem. We have not attempted a full analysis of the low energy potential; we hope that radiative corrections will induce the spontaneous breakdown of SU(2) × U(1) [9].

Our model, as it stands, is not quite perfect. It requires additional light fields to obtain acceptable predictions for $\sin^2\theta_W$ and Λ_{QCD} [27,28]. Also, we must ensure that our 10^{10} GeV colour triplet Higgs particles do not lead to rapid proton decay. Various means of avoiding this potential problem have been discussed in ref. [26].

Also, the SU(5) to SU(3) × SU(2) × U(1) phase transition does not result in a long period of exponential expansion (inflation). However, we can still solve the horizon, flatness, and rotation problems with inflation at temperature of order M_P [20,28–30]. We have discussed elsewhere [20] how this can occur in $N = 1$ supergravity (again making use of non-renormalizable couplings), while producing a spectrum of density fluctuations which can lead to galaxy formation.

It is not clear whether or not primordial inflation solves the gravitino problem [28,31,32]. In any case, a gravitino mass of 10^4 GeV avoids the problem altogether [31], without inflation. It is also possible that our model will result in some modest expansion just before the SU(5) to SU(3) × SU(2) × U(1) transition. A change in the scale factor of the universe by three or four orders of magnitude would permit a gravitino mass of 100 to 1000 GeV, which is just what we want.

In conclusion, let us summarize. Starting from a locally supersymmetric lagrangian with *no* unnaturally small coupling constants and only two mass scales, $M_P \simeq 10^{19}$ GeV and $\mu \simeq 10^{11}$ GeV, we have generated both the weak scale $M_W \simeq m_{3/2} \simeq \mu^2/M_P$, and the GUT scale $M_X \simeq (\mu M_P)^{1/2}$. We have also reproduced previous scenarios [23,24] for early cosmology in supersymmetric GUTs without problems. Coupled with primordial inflation [20,28], we have no horizon, flatness, rotation, monopole, gravitino, baryon number, or galaxy formation [30] problems. The density of monopoles in the universe today may be close to the observed upper limit.

Note added in the proof. We believe that the text of this paper does not sufficiently emphasize the fact that the SU(5) to SU(3) × SU(2) × U(1) transition will not occur without non-perturbative effects in the SU(5) phase. The barrier between phases, though much less than M_X^4, is not small enough to allow quantum mechanical tunnelling at a reasonable rate. However, if (for example) supersymmetry is broken non-perturbatively in the SU(5) phase, the free energy density near $\Sigma = 0$ will be of order Λ^4, as high or higher than the barrier between phases. This will cause a second-order (or weakly first-order) SU(5) to SU(3) × SU(2) × U(1) transition at a critical temperature of order Λ. Even if supersymmetry is not broken non-perturbatively, other confinement effects in the SU(5) phase (which obscure the physics at $\Sigma \lesssim \Lambda$) may drive the transition. None of these effects have consequences for physics in the SU(3) × SU(2) × U(1) phase.

References

[1] L. Maiani, in: Proc. Summer School of Gif-sur-Yvette (1979) p. 3;
E. Witten, Nucl. Phys. B188 (1981) 513.

[2] E. Cremmer et al., Nucl. Phys. B147 (1979) 105;
E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Phys. Lett. 116B (1982) 231; CERN preprint TH.3448.

[3] R. Barbieri, S. Ferrara, D.V. Nanopoulos and K.S. Stelle, Phys. Lett. 113B (1982) 219.

[4] H.P. Nilles, Phys. Lett. 115B (1982) 193; Proc. Johns Hopkins Workshop on Current problems in particle theory 6, eds. G. Domokos et al. (Johns Hopkins U.P., Baltimore, 1982); CERN preprint Th.3398 (1982).

[5] J. Ellis and D.V. Nanopoulos, Phys. Lett. 116B (1982) 133.

[6] R. Arnowitt, A.H. Chamseddine and P. Nath, Phys. Rev. Lett. 49 (1982) 970; Northeastern Univ. preprint NUB-2565.

[7] R. Barbieri, S. Ferrara and C. Savoy, Phys. Lett. 119B (1982) 343.

[8] L.E. Ibañez, Phys. Lett. 118B (1982) 73.

[9] J. Ellis, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 121B (1983) 123.

[10] H.P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. 120B (1982) 346.

[11] L.E. Ibañez, Madrid preprint FTUAM/82-8 (1982).

[12] N. Ohta, Tokyo preprint UT-388 (1982).

[13] E. Cremmer, P. Fayet and L. Girardello, Ecole Normale Supérieure preprint LPTENS 82/30 (1982).

[14] M. Srednicki, CERN preprint TH. 3459 (1982).

[15] H.P. Nilles, M. Srednicki and D. Wyler, preprint CERN TH.3461 (1982).

[16] A.B. Lahanas, preprint CERN TH 3467, to be published in Phys. Lett. 124B (1983).

[17] B.A. Ovrut and S. Raby, IAS preprint (1982).

[18] U. Ellwanger, Oxford preprint (1982).

[19] S. Ferrera, D.V. Nanopoulos and C.A. Savoy, Phys. Lett. 123B (1983) 214.

[20] D.V. Nanopoulos, K.A. Olive, M. Srednicki and K. Tamvakis, Phys. Lett. 123B (1983) 41.

[21] D.V. Nanopoulos and M. Srednicki, Phys. Lett. 124B (1983) 37.

[22] S. Ferrara, L. Girardello and H.P. Nilles, CERN preprint TH.3494 (1982).

[23] D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 110B (1982) 449;
M. Srednicki, Nucl. Phys. B202 (1982) 327.

[24] M. Srednicki, Nucl. Phys. B206 (1982) 132;
D.V. Nanopoulos, K.A. Olive and K. Tamvakis, Phys. Lett. 115B (1982) 15.

[25] R. Barbieri and S. Cecotti, Pisa University preprint SNS 82.9 (1982).

[26] D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 114B (1982) 235.

[27] A. Masiero, D.V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. 115B (1982) 298.

[28] J. Ellis, D.V. Nanopoulos, K.A. Olive and K. Tamvakis, CERN preprint TH.3404 (1982).

[29] J. Ellis, D.V. Nanopoulos, K.A. Olive and K. Tamvakis, Phys. Lett. 118B (1982) 335.

[30] J. Ellis, D.V. Nanopoulos, K.A. Olve and K. Tamvakis, Phys. Lett. 120B (1983) 331.

[31] S. Weinberg, Phys. Rev. Lett. 48 (1982) 1303.

[32] J. Ellis, A.D. Linde and D.V. Nanopoulos, Phys. Lett. 118B (1982) 59.