Greybody factors in a rotating black-hole background-II : fermions and gauge bosons.

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Abstract

We study the emission of fermion and gauge boson degrees of freedom on the brane by a rotating higher-dimensional black hole. Using matching techniques, for the near-horizon and far-field regime solutions, we solve analytically the corresponding field equations of motion. From this, we derive analytical results for the absorption probabilities and Hawking radiation emission rates, in the low-energy and low-rotation case, for both species of fields. We produce plots of these, comparing them to existing exact numerical results with very good agreement. We also study the total absorption cross-section and demonstrate that, as in the non-rotating case, it has a different behaviour for fermions and gauge bosons in the low-energy limit, while it follows a universal behaviour – reaching a constant, spin-independent, asymptotic value – in the high-energy regime.

1 Introduction

The leading motivation for studying higher-dimensional theories [1, 2] is that they provide a framework for the unification of gravitation with the rest of the fundamental forces. Gravity, and possibly scalar fields, propagate in a $(4 + n)$-dimensional spacetime (the *Bulk*), while ordinary matter is confined in a four-dimensional hypersurface, the *Brane*. In models with large extra dimensions, the traditional Planck scale is an effective scale, while the fundamental Planck scale of gravitation is related to it in terms of the number and size of the extra dimensions. In such a framework higher-dimensional black holes can be created in trans-planckian collisions [3]. This opens the possibility of seeing them in ground-based colliders [4], or in cosmic ray interactions [5]. These higher-dimensional black holes and their properties have been the subject of a number of articles in the last few years – for a summary of their properties and phenomenological implications, see [6, 7, 8].

A black hole created in a high energy collision is expected to evaporate through Hawking radiation [9] both in the bulk, through the emission of gravitons and scalar fields, and on the brane through the emission of fermions and gauge bosons. The black hole is expected to undergo a number of phases: First, the *balding* phase, in which the black hole emits mainly gravitational radiation, losing all "hair" inherited from the original particles. Then, in the *spin-down* phase, the black hole loses all its angular momentum, through the emission of Hawking radiation. Third is the *Schwarzschild* phase, in which the black hole loses its actual mass due to the emission of Hawking radiation. Finally, the *Planck* phase – where the black hole's mass or temperature reach the characteristic scale of gravity – which needs a quantum gravity theory in order to be studied. There have been both numerical and analytical studies of the Hawking radiation emitted from such a higher-dimensional black hole, for the Schwarzschild phase [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22], as well as for the spin-down phase [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34].

In the present work we focus on the evaporation of a rotating $(4 + n)$-dimensional black hole through Hawking radiation in the form of fermions and gauge bosons on the brane. Numerical studies of fermion and gauge boson emission by a rotating black hole on the brane already exist in the literature [28, 29, 30]. In this article, we continue our effort, initiated in [34] with the study of scalar fields, to derive complementary analytic results for higher-spin fields using an analytical approach. In section 2, we consider the gravitational background corresponding to a higher-dimensional rotating black hole, and write down the equations, describing all radiative components of fermion and gauge boson fields propagating on the brane, in the form of a single master equation. In section 3, we solve this equation analytically by using a well-known solution matching technique: we first derive the solution at the near horizon regime, and then at the far field regime. Next, we stretch and match the two solutions at the intermediate zone, in the low-energy and low-rotation limit, thus producing a smooth solution for all spacetime. We then use this solution in order to compute, for every type of field, the absorption probability, a quantity that characterizes the Hawking radiation. In section 4 we produce plots for the absorption coefficient and compare them with existing numerical results. In sections 5 and 6, we study the asymptotic behaviour of the corresponding cross-section and the

profile of the energy emission rates, respectively. In section 7, we state our conclusions.

2 Master equation on a brane embedded in a rotating $(4+n)D$ black hole background

The background around a $(4+n)$-dimensional rotating black hole is given by the Myers-Perry solution [35]. As is usual, here we focus on the case where the black hole, being created by the collision of brane-localised particles, is characterised by only one non-zero angular momentum component parallel to our brane. Then, the line-element takes the form

$$ds^2 = \left(1 - \frac{\mu}{\Sigma \, r^{n-1}}\right)dt^2 + \frac{2a\mu \sin^2\theta}{\Sigma \, r^{n-1}} \, dt \, d\varphi - \frac{\Sigma}{\Delta} \, dr^2 - \Sigma \, d\theta^2$$

$$- \left(r^2 + a^2 + \frac{a^2\mu \sin^2\theta}{\Sigma \, r^{n-1}}\right)\sin^2\theta \, d\varphi^2 - r^2\cos^2\theta \, d\Omega_n^2, \tag{1}$$

where

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{n-1}}\,, \qquad \Sigma = r^2 + a^2\,\cos^2\theta\,, \tag{2}$$

and $d\Omega_n^2$ is the line-element on a unit n-sphere. The mass M_{BH} and angular momentum J of the black hole are then given in terms of the parameters a, μ

$$M_{BH} = \frac{(n+2)A_{n+2}}{16\pi G}\,\mu\,, \qquad J = \frac{2}{n+2}\,M_{BH}\,a\,, \tag{3}$$

with G being the $(4+n)$-dimensional Newton's constant, and A_{n+2} the area of a $(n+2)$-dimensional unit sphere given by

$$A_{n+2} = \frac{2\pi^{(n+3)/2}}{\Gamma[(n+3)/2]}\,. \tag{4}$$

The corresponding line-element on the brane can be found by projecting out the angular variables that parametrize the extra dimensions. In that case, the factor $d\Omega_n^2$ disappears and the 4-dimensional brane background is described by the line-element

$$ds^2 = \left(1 - \frac{\mu}{\Sigma \, r^{n-1}}\right)dt^2 + \frac{2a\mu \sin^2\theta}{\Sigma \, r^{n-1}} \, dt d\varphi - \frac{\Sigma}{\Delta} \, dr^2 - \Sigma \, d\theta^2$$

$$- \left(r^2 + a^2 + \frac{a^2\mu \sin^2\theta}{\Sigma \, r^{n-1}}\right)\sin^2\theta \, d\varphi^2\,. \tag{5}$$

In the above $0 < \varphi < 2\pi$, $0 < \theta < \pi$ and n stands for the number of extra, spacelike dimensions that exist transverse to the brane $(D = 4 + n)$.

We would like to study the emission of gauge bosons and fermions by the aforementioned projected black-hole background. We assume that the emitted particle modes couple only minimally to the gravitational background and have no other interactions,

therefore, they satisfy the corresponding free equations of motion. By using the Newman-Penrose formalism [36, 37], and assuming the factorized ansatz

$$\Psi_s(t, r, \theta, \varphi) = e^{-i\omega t}\, e^{im\varphi}\, R_s(r)\, S_{s,j}^m(\theta)\,, \tag{6}$$

where $S_{s,j}^m(\theta)$ are the so-called spin-weighted spheroidal harmonics [38, 39, 40, 41, 42, 43], the free equations of motion for particles with spin $s = 0, \frac{1}{2}$ and 1 may be combined to form the following complete "master" equation [29], satisfied by the radial part of all radiative components of the field,

$$\Delta^{-s} \frac{d}{dr}\left(\Delta^{s+1} \frac{dR_s}{dr}\right) + \left[\frac{K^2 - isK\Delta'}{\Delta} + 4is\omega r + s\left(\Delta'' - 2\right)\delta_{s,|s|} - \Lambda_{sj}\right] R_s = 0\,, \tag{7}$$

where

$$K = (r^2 + a^2)\,\omega - am\,, \qquad \Lambda_{sj} = \lambda_{sj} + a^2\omega^2 - 2am\omega\,. \tag{8}$$

In the above, λ_{sj} is the angular eigenvalue appearing in the equation satisfied by the spheroidal harmonics, namely

$$\frac{1}{\sin\theta} \frac{d}{d\theta}\left(\sin\theta \frac{dS_{s,j}^m}{d\theta}\right) + \left(a^2\omega^2 \cos^2\theta - 2a\omega s \cos\theta - \frac{(m + s\cos\theta)^2}{\sin^2\theta} + \lambda_{sj} + s\right) S_{s,j}^m = 0\,. \tag{9}$$

From Eq. (7), it is clear that the radial parts of the radiative components with $s = |s|$ and $s = -|s|$ satisfy different equations due to the presence of the term that is multiplied by the $\delta_{s,|s|}$ factor. This obstacle can be overcome by redefining

$$R_{+|s|} \equiv \Delta^{-|s|}\, P_{+|s|}\,, \qquad R_{-|s|} \equiv P_{-|s|}\,. \tag{10}$$

In terms of the new radial functions, the master radial equation on the brane takes the simplified form

$$\Delta^{|s|} \frac{d}{dr}\left(\Delta^{1-|s|} \frac{dP_s}{dr}\right) + \left(\frac{K^2 - isK\Delta'}{\Delta} + 4is\omega r - \tilde{\Lambda}_{sj}\right) P_s(r) = 0\,, \tag{11}$$

where the Δ''-term has now disappeared, and $\tilde{\Lambda}_{sj} = \Lambda_{|s|j} + 2|s|$. The angular eigenvalue $\lambda_{|s|j}$ cannot be expressed in closed form, however it can be written in terms of a power series with respect to $a\omega$ [38, 40, 44, 45] as

$$\lambda_{|s|j} = -|s|(|s| + 1) + \sum_k f_k\,(a\omega)^k = j(j+1) - |s|(|s|+1) - \frac{2ms^2}{j(j+1)}a\omega + \dots\,. \tag{12}$$

By solving Eq. (11) one can compute the absorption coefficient $|A_{sjm}|^2$ for the propagation of the field on the specific gravitational background. This quantity is needed to compute the Hawking radiation of the black hole on the brane. For example, the differential energy emission rate is given by

$$\frac{d^2 E^{(s)}}{dt\, d\omega} = \frac{1}{2\pi} \sum_{j,m} \frac{\omega}{\exp\left[k/T_\mathrm{H}\right] \pm 1}|\mathcal{A}_{sjm}|^2\,, \tag{13}$$

3

with k and T_H given by

$$k \equiv \omega - m\Omega = \omega - \frac{ma}{r_H^2 + a^2}\,, \qquad T_{\mathrm{H}} = \frac{(n+1) + (n-1)a_*^2}{4\pi(1 + a_*^2)r_H}\,, \qquad (14)$$

with Ω the angular velocity, T_H the temperature of the black hole and r_H the black-hole horizon radius, defined through the relation $\Delta(r_H) = 0$.

3 Analytical Solution

In this section, we will derive analytic solutions to the radial master equation valid at the two asymptotic regimes of the black-hole horizon $(r \simeq r_H)$ and far-field $(r \gg r_H)$. We will then demand that the two solutions are smoothly connected at an intermediate radial zone in order to construct a complete solution, valid at all radial regimes, for a field with arbitrary spin s.

3.1 The Near-Horizon Regime

In order to bring Eq. (11) in the form of a known differential equation, we apply the following transformation [34]

$$r \to f(r) = \frac{\Delta(r)}{r^2 + a^2} = \frac{r^2 + a^2 - \mu/r^{n-1}}{r^2 + a^2} \implies \frac{df}{dr} = (1 - f)\,r\,\frac{A(r)}{r^2 + a^2}\,, \qquad (15)$$

where $A(r) = (n+1) + (n-1)\,a^2/r^2$. Then, we may rewrite Eq. (11) near the horizon $(r \simeq r_H)$ – keeping s in the master equation as an arbitrary parameter – as

$$f\,(1-f)\,\frac{d^2 P_s}{df^2} + (1 - |s| - B_* f)\,\frac{dP_s}{df} + \left[\frac{K_*^2 - isK_*\Delta_*'}{A_*^2\,f\,(1-f)} + \frac{(4is\omega_* - \tilde{\Lambda}_{sj})(1 + a_*^2)}{A_*^2\,(1-f)} \right] P_s(r) = 0\,, \qquad (16)$$

where we have defined $\omega_* = \omega r_H$, $a_* = a/r_H$. Also $\Delta_*' = \Delta'(r_H) = A_*$, B_* is now

$$B_* \equiv 1 - |s| + \frac{2|s| + n\,(1 + a_*^2)}{A_*} - \frac{4a_*^2}{A_*^2}\,, \qquad (17)$$

while A_* and K_* are given by

$$A_* = n + 1 + (n-1)a_*^2\,, \qquad (18)$$

$$K_* = (1 + a_*^2)\omega_* - a_* m\,. \qquad (19)$$

If we make the redefinition: $P(f) = f^\alpha(1 - f)^\beta F(f)$, Eq. (16) takes the form of a hypergeometric equation [46]

$$f\,(1-f)\,\frac{d^2 F}{df^2} + [c - (1 + a + b)\,f]\,\frac{dF}{df} - ab\,F = 0\,, \qquad (20)$$

4

with

$$a = \alpha + \beta + B_* - 1\,, \qquad b = \alpha + \beta\,, \qquad c = 1 - |s| + 2\alpha\,. \tag{21}$$

The power coefficients α and β can be determined by solving the modified second-order algebraic equations

$$\alpha^2 - |s|\,\alpha + \frac{K_*^2}{A_*^2} - \frac{isK_*}{A_*} = 0 \tag{22}$$

and

$$\beta^2 + \beta\,(B_* + |s| - 2) + \frac{K_*^2}{A_*^2} - \frac{isK_*}{A_*} + \frac{(4is\omega_* - \tilde{\Lambda}_{sj})\,(1 + a_*^2)}{A_*^2} = 0\,, \tag{23}$$

respectively. The first of these two equations leads to the following solutions for the parameter α

$$\alpha_\pm = \frac{|s|}{2} \pm \left(\frac{iK_*}{A_*} + \frac{s}{2} \right)\,, \tag{24}$$

while the second equation for β admits the solutions

$$\beta_\pm = \frac{1}{2}\left[(2 - |s| - B_*) \pm \sqrt{(B_* + |s| - 2)^2 - \frac{4K_*^2 - 4isK_*A_*}{A_*^2} - \frac{4(4is\omega_* - \tilde{\Lambda}_{sj})\,(1 + a_*^2)}{A_*^2}} \right]. \tag{25}$$

The general solution of the master equation near the horizon is then given by

$$\begin{aligned} P_{NH}(f) = A_- f^\alpha\,(1 - f)^\beta\,F(a, b, c; f) \\ + A_+\,f^{-\alpha}\,(1 - f)^\beta\,F(a - c + 1, b - c + 1, 2 - c; f)\,. \end{aligned} \tag{26}$$

We must now impose the boundary condition that no outgoing modes exist near the horizon of the black hole. Using the solutions (24), in the limit $r \to r_H$, or $f(r) \to 0$, we obtain either

$$P_{NH}(f) \simeq A_-\,f^{\frac{|s|+s}{2}}\,f^{iK_*/A_*} + A_+\,f^{-\frac{|s|+s}{2}}\,f^{-iK_*/A_*}\,, \tag{27}$$

for $\alpha = \alpha_+$, or

$$P_{NH}(f) \simeq A_- f^{\frac{|s|-s}{2}}\,f^{-iK_*/A_*} + A_+ f^{-\frac{|s|-s}{2}}\,f^{iK_*/A_*}\,, \tag{28}$$

for $\alpha = \alpha_-$. We may now use the tortoise-like coordinate $y = r_H(1 + a_*^2)\ln(f)/A_*$ that, in the limit $r \to r_H$, becomes identical [34] to the usual tortoise coordinate r_*, defined by $dr_*/dr = (r^2 + a^2)/\Delta(r)$. Then, the factors $f^{\pm iK_*/A_*}$ reduce to $e^{\pm iky}$ describing an outgoing and incoming free wave, respectively. According to Teukolsky's classic analysis [47], the correct boundary condition at the horizon of the black hole, for a field with non-zero spin, is

$$R_s \sim \Delta^{-s}\,e^{-ikr_*}\,, \tag{29}$$

which, in our case, translates to

$$P_{+|s|} \sim e^{-iky} = f^{-iK_*/A_*}\,, \qquad P_{-|s|} \sim f^{|s|}\,e^{-iky} = f^{|s|}\,f^{-iK_*/A_*}\,. \tag{30}$$

Demanding that our asymptotic near-horizon (NH) solution obeys the above boundary condition, leads to the selection $\alpha = \alpha_-$ and $A_+ = 0$, bringing the NH solution to the final form

$$P_{NH}(f) = A_-\,f^\alpha\,(1 - f)^\beta\,F(a, b, c; f)\,. \tag{31}$$

The criterion for the convergence of the hypergeometric function $F(a, b, c; f)$, i.e. $Re(c - a - b) > 0$, can finally be applied, and leads to the choice $\beta = \beta_-$. All the above results reduce to the ones for scalar fields [34] if we set $s = 0$. They also reduce to the ones valid for general spin-s fields in a non-rotating black hole background [7] if we set $a = 0$.

For the purpose of matching the near-horizon and far-field (FF) solutions at an intermediate radial zone, we need to extrapolate ('stretch') our NH solution to values of the radial coordinate that are much larger than the horizon radius. We will do that by changing first the argument of the hypergeometric function from f to $1 - f$ by using the following relation [46]

$$
\begin{aligned}
P_{NH}(f) &= A_- f^\alpha (1 - f)^\beta \left[\frac{\Gamma(c)\,\Gamma(c - a - b)}{\Gamma(c - a)\,\Gamma(c - b)} F(a, b, a + b - c + 1; 1 - f) \right. \\
&+ \left. (1 - f)^{c-a-b} \frac{\Gamma(c)\,\Gamma(a + b - c)}{\Gamma(a)\,\Gamma(b)} F(c - a, c - b, c - a - b + 1; 1 - f) \right].
\end{aligned}
$$
(32)

The function $f(r)$ may be alternatively written as

$$
f(r) = 1 - \frac{\mu}{r^{n-1}} \frac{1}{r^2 + a^2} = 1 - \left(\frac{r_H}{r} \right)^{n-1} \frac{(1 + a_*^2)}{(r/r_H)^2 + a_*^2} ,
$$
(33)

where we have used the horizon equation $\Delta(r_H) = 0$ in order to eliminate μ from the above relation. In the limit $r \gg r_H$, and for $n \geq 0$, the above expression goes to unity.

By using the above, the argument of the "stretched" hypergeometric function goes again to zero, and the "stretched" near-horizon solution takes the form

$$
P_{NH}(f) \simeq A_- (1 - f)^\beta \frac{\Gamma(c)\,\Gamma(c - a - b)}{\Gamma(c - a)\,\Gamma(c - b)} + A_- (1 - f)^{-\beta + 2 - B_* - |s|} \frac{\Gamma(c)\,\Gamma(a + b - c)}{\Gamma(a)\,\Gamma(b)} .
$$
(34)

For $r \gg r_H$, the quantity $(1 - f)$ can be accurately approximated by

$$
1 - f \simeq (1 + a_*^2) \left(\frac{r_H}{r} \right)^{n+1} ,
$$
(35)

thus, bringing Eq. (34) to a simpler power-law form

$$
P_{NH}(r) \simeq A_1 \, r^{-(n+1)\beta} + A_2 \, r^{(n+1)(\beta + |s| + B_* - 2)} ,
$$
(36)

with

$$
A_1 = A_- \left[(1 + a_*^2) \, r_H^{n+1} \right]^\beta \frac{\Gamma(c)\Gamma(c - a - b)}{\Gamma(c - a)\Gamma(c - b)} ,
$$
(37)

$$
A_2 = A_- \left[(1 + a_*^2) \, r_H^{n+1} \right]^{-(\beta + |s| + B_* - 2)} \frac{\Gamma(c)\Gamma(a + b - c)}{\Gamma(a)\Gamma(b)} .
$$
(38)

6

3.2 The Far-Field Regime

We now turn to the far-field regime, where the radial master equation (11) becomes

$$\frac{d^2 P_s}{dr^2} + \frac{2(1-|s|)}{r}\frac{dP_s}{dr} + \left(\omega^2 + \frac{2is\omega}{r} - \frac{\lambda_{|s|j} + 2|s| + a^2\omega^2}{r^2}\right) P_s = 0. \tag{39}$$

By redefining $P_s = e^{-i\omega r} r^{\frac{1}{2}(2|s|-1+Z)} \tilde{P}_s$, with

$$Z = \sqrt{(2|s|-1)^2 + 4(\lambda_{|s|j} + 2|s| + a^2\omega^2)}, \tag{40}$$

Eq. (39) takes the form of a confluent hypergeometric differential equation whose solution can be expressed in terms of the Kummer functions M and U, i.e. [46]

$$P_{FF}(r) = e^{-i\omega r} r^{\frac{1}{2}(2|s|-1+Z)} \times$$

$$\left[B_1 M\left(\tfrac{1}{2} - s + \tfrac{Z}{2}, 1 + Z, 2i\omega r\right) + B_2 U\left(\tfrac{1}{2} - s + \tfrac{Z}{2}, 1 + Z, 2i\omega r\right)\right]. \tag{41}$$

As in the case of the NH solution, the FF one also needs to be "stretched", this time towards small values of the radial coordinate. In order to do this, we take the $r \to 0$ limit in (41), which gives [46]

$$P_{FF}(r) \simeq B_1 r^{\frac{1}{2}(2|s|-1+Z)} + B_2 r^{\frac{1}{2}(2|s|-1-Z)} \frac{\Gamma(Z)}{\Gamma(\frac{1}{2} - s + \frac{Z}{2})} (2i\omega)^{-Z}. \tag{42}$$

In analogy with the stretched NH solution (36), the stretched FF solution also has a power-law form. In order to construct a complete radial solution, we have to match these two expressions at an intermediate zone. To this end, we note that, for $\omega r_H \ll 1$ and $a_* \ll 1$, we get, from Eq. (17), $B_* \simeq 2 - |s| + \frac{2|s|-1}{n+1}$, and

$$\beta \simeq \frac{1}{2(n+1)}\left(1 - 2|s| - \sqrt{(2|s|-1)^2 + 4\tilde{\Lambda}_{sj}}\right). \tag{43}$$

In this case, the stretched near-horizon solution (36) takes the form

$$P_{NH}(r) = A_1 r^{\frac{1}{2}\left(2|s|-1+\sqrt{(2|s|-1)^2+4\tilde{\Lambda}_{sj}}\right)} + A_2 r^{\frac{1}{2}\left(2|s|-1-\sqrt{(2|s|-1)^2+4\tilde{\Lambda}_{sj}}\right)}. \tag{44}$$

From the definitions (8) and (40) for the constants $\Lambda_{|s|j}$ and Z, respectively, one can see that, within our approximation, the powers of r in the two stretched solutions (42) and (44) become identical. Then, a smooth matching is realized provided that we identify the corresponding coefficients in front of the same power of r. From this, we obtain

$$\frac{B_1}{B_2} = \frac{\Gamma(Z)}{\Gamma(\frac{1}{2} - s + \frac{Z}{2})} (2i\omega)^{-Z} \frac{A_1}{A_2}, \tag{45}$$

with A_1/A_2 following from Eq. (38)

$$\frac{A_1}{A_2} = \frac{\Gamma(c-a-b)\Gamma(a)\Gamma(b)}{\Gamma(c-a)\Gamma(c-b)\Gamma(a+b-c)}\left[(1+a_*^2)\, r_H^{n+1}\right]^{2\beta+|s|+B_*-2}. \tag{46}$$

Equation (45) ensures the smooth matching of the two asymptotic solutions, and thus the existence of a complete solution to the radial master equation describing the propagation of a field with arbitrary spin s in the gravitational background induced on the brane.

3.3 Computing the Absorption Probability

The derivation of the complete solution for the radial function $P_s(r)$ opens the way for the calculation of the corresponding absorption probability $|A_{sjm}|^2$. The latter appears in the expression of the various emission rates for the Hawking radiation emitted on the brane from the higher-dimensional black hole given in Eq. (1). In order to derive this quantity, we expand the far-field solution (41) in the limit $r \to \infty$, and obtain [46]

$$
\begin{aligned}
P_{FF}(r) \;\simeq\; & \left(\frac{B_1 \, e^{i\pi\left(\frac{1}{2}-s+\frac{Z}{2}\right)}}{(2i\omega)^{\left(\frac{1}{2}-s+\frac{Z}{2}\right)}} \frac{\Gamma\left(1+Z\right)}{\Gamma\left(\frac{1}{2}+s+\frac{Z}{2}\right)} + \frac{B_2}{(2i\omega)^{\left(\frac{1}{2}-s+\frac{Z}{2}\right)}} \right) \frac{e^{-i\omega r}}{r^{1-s-|s|}} \\
& + \; \frac{B_1}{(2i\omega)^{\left(\frac{1}{2}+s+\frac{Z}{2}\right)}} \frac{\Gamma\left(1+Z\right)}{\Gamma\left(\frac{1}{2}-s+\frac{Z}{2}\right)} \frac{e^{i\omega r}}{r^{1+s-|s|}} \\
\equiv \; & Y_s^{(in)} \frac{e^{-i\omega r}}{r^{1-s-|s|}} + Y_s^{(out)} \frac{e^{i\omega r}}{r^{1+s-|s|}} \,.
\end{aligned}
\tag{47}
$$

Let us focus first on the case of fields with spin 1/2: from the master radial equation (11) one may easily derive, similarly to the 4-dimensional case [37, 48, 49, 50], that

$$
\frac{d}{dr}\left(|P_{\frac{1}{2}}|^2 - |P_{-\frac{1}{2}}|^2 \right) = 0 \,.
\tag{48}
$$

The conserved – for arbitrary values of r – quantity inside the parenthesis is proportional to the radial component of the particle current produced by the black hole. Then, the absorption probability is defined as the ratio of the flux of particles at the black hole horizon over the one at infinity

$$
|A_{\frac{1}{2}jm}|^2 = \frac{F_{in}^{(H)}}{F_{in}^{(\infty)}} = 1 - \frac{F_{out}^{(\infty)}}{F_{in}^{(\infty)}} \,,
\tag{49}
$$

where in the second part of the equation, we have used the conservation of the total flux. The flux of fermions at infinity may be found by integrating the radial component of the conserved current over a 2-dimensional sphere at asymptotic infinity. Applying this and using Eq. (47), we find

$$
|A_{\frac{1}{2}jm}|^2 = 1 - \frac{|P_{\frac{1}{2}}^{(out)}|^2 - |P_{-\frac{1}{2}}^{(out)}|^2}{|P_{\frac{1}{2}}^{(in)}|^2 - |P_{-\frac{1}{2}}^{(in)}|^2} = 1 + \frac{|Y_{-\frac{1}{2}}^{out}|^2}{|Y_{\frac{1}{2}}^{(in)}|^2} \,.
\tag{50}
$$

Using, finally, the explicit expressions for $Y_{\pm\frac{1}{2}}^{(out)}$, as these are defined in Eq. (47), we find that they are related through the equation

$$
Y_{-\frac{1}{2}}^{(out)} = \frac{2i\omega}{\sqrt{\lambda_{1/2j} + 1 + a^2\omega^2}} Y_{\frac{1}{2}}^{(out)} \,,
\tag{51}
$$

which, therefore, leads to the following expression for the absorption probability for fermions

$$
|A_{\frac{1}{2}jm}|^2 = 1 - \frac{4\omega^2}{\lambda_{1/2j} + 1 + a^2\omega^2} \left| \frac{Y_{\frac{1}{2}}^{(out)}}{Y_{\frac{1}{2}}^{(in)}} \right|^2 \,.
\tag{52}
$$

For fields with spin $s = 1$, there is no conserved particle current. In order to compute the absorption probability, a technique, introduced in [51], may be followed in which the radial master equation is transformed, through a radial function redefinition and the use of the tortoise coordinate r_*, to an alternative one with real, short-range potential. Then, the asymptotic solution at infinity for the gauge field is given, in terms of the new radial function, by the following expression [51, 52]

$$X_{jm\omega} \sim e^{-i\omega r_*} + A_{jm\omega}^{(in)} e^{i\omega r_*} , \tag{53}$$

i.e. by the sum of outgoing and incoming plane waves with constant amplitudes, from which the expression of A_{1jm} may easily follow

$$|A_{1jm}|^2 = 1 - |A_{jm\omega}^{(in)}|^2 . \tag{54}$$

Although the exact analysis is quite cumbersome [51, 52], it yields relations connecting the constant amplitude $A_{jm\omega}^{(in)}$ in Eq. (53) with the constant coefficients $Y_s^{(in,out)}$ appearing in our Eq. (47) and leading finally to

$$|A_{1jm}|^2 = 1 - \frac{16\omega^4}{B_{jm\omega}^2} \left| \frac{Y_1^{(out)}}{Y_1^{(in)}} \right|^2 . \tag{55}$$

The constant $B_{jm\omega}$ is defined as the coefficient appearing in the differential equation [37]

$$\Delta \, D_0^\dagger D_0^\dagger P_{+1} = B_{jm\omega} \, P_{-1} , \tag{56}$$

where $D_0^\dagger = \partial_r + iK/\Delta$, or, equivalently, through the relation

$$Y_{-1}^{(out)} = -\frac{4\omega^2}{B_{jm\omega}} Y_1^{(out)} \tag{57}$$

holding for the asymptotic solution (47) when substituted in Eq. (56). By using the explicit expressions of $Y_{\pm 1}^{(out)}$ from Eq. (47), we obtain

$$Y_{-1}^{(out)} = -\frac{4\omega^2}{\lambda_{1j} + 2 + a^2\omega^2} Y_1^{(out)} , \tag{58}$$

which leads to $B_{jm\omega} = \lambda_{1j} + 2 + a^2\omega^2$, and, thus, to the final expression for the absorption probability for brane-localised fields with spin 1

$$|A_{1jm}|^2 = 1 - \frac{16\omega^4}{(\lambda_{1j} + 2 + a^2\omega^2)^2} \left| \frac{Y_1^{(out)}}{Y_1^{(in)}} \right|^2 . \tag{59}$$

4 Plotting our Analytic Results

We can now use Eqs. (52) and (59) – our main analytic results – to produce plots for the absorption probabilities for spin-1/2 and spin-1 particles, respectively, propagating in
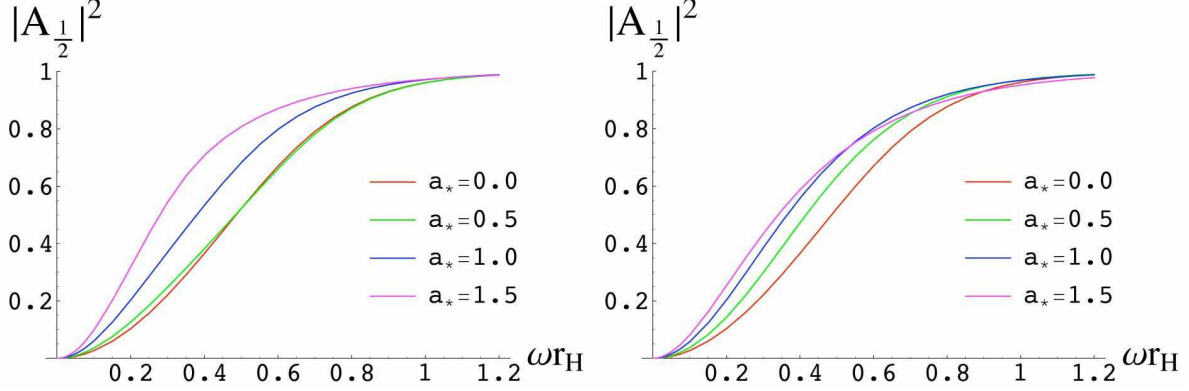
9

Figure 1: Absorption probability $|\mathcal{A}_{1/2}|^2$ for brane spinor particles, for the modes $j = 1/2$ and $m = 1/2, -1/2$, from left to right, for $n = 6$ and $a_* = 0.0, 0.5, 1.0, 1.5$.
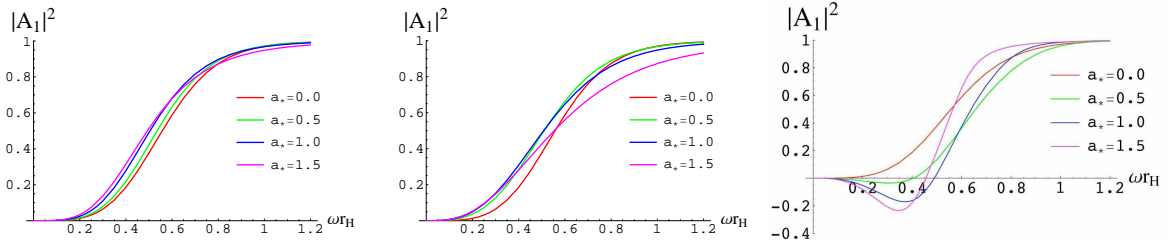

Figure 2: Absorption probability $|\mathcal{A}_1|^2$ for brane boson particles, for the modes $j = 1$ and $m = 0, -1, 1$, from left to right, for $n = 6$ and $a_* = 0.0, 0.5, 1.0, 1.5$.

the induced-on-the-brane gravitational background. Upon selection of particular values of the angular momentum numbers (j, m), the values of A_{sjm} will be plotted as a function of the energy parameter ωr_H, and then compared with existing numerical results from the literature [28, 29, 30].

We remind the reader that in the process of producing a complete analytic solution to the radial master equation, we were forced to assume that $\omega r_H \ll 1$ and $a_* \ll 1$, therefore, strictly speaking, our results are valid only in the low-energy and low-rotation limit. However, at times, our plots will extend beyond this range of validity to exhibit the remarkably good agreement we obtain even outside the assumed range of validity of our approximations. Another point that we would like to stress is that, in general, an increase in the number of extra dimensions n improves the validity of our approximation: as n increases, the assumed behavior of $f(r)$ at infinity in Eq. (15) becomes more accurate, and terms that were neglected during the matching of the asymptotic solutions, such as K_*/A_*, become even more suppressed.

In Figs. 1 and 2, we plot the absorption probability for fermions and gauge bosons, respectively, for the lowest partial waves in each case, i.e. $(j = 1/2, m = \pm 1/2)$ and $(j = 1, m = 0, -1, 1)$, in the case of $n = 6$ and for various values of a_*. From Fig. 1, we see that, for fermions with $m > 0$, the absorption probability is monotonically increasing

with the angular momentum parameter over most of the energy regime, while, for $m < 0$, $|A_{1/2}|^2$ increases with a_* in the low-energy regime and is suppressed at the high-energy regime. For bosons, and $m = 0$, the absorption probability is increasing with the angular momentum of the black hole for small values of a_* but this increase is decelerated (and eventually reversed) for higher values of a_*; for $m < 0$, the absorption probability is enhanced with a_* in the low-energy regime but suppressed in the high-energy one; finally for $m > 0$, an increase in a_* leads to more negative values of $|A_1|^2$ in the superradiant regime [53] (where the quantity $\omega - m\Omega$, as well as $|A_1|^2$, becomes negative signalling the enhancement of the incoming wave's amplitude by a rotating black hole) and to more positive values in the non-superradiant one. Our results are in excellent agreement with the exact numerical ones [28, 29, 30] in the low-energy and low-angular-momentum regime, and remain in very good agreement even beyond this range of values – a deviation from the exact results appears only for values of the angular momentum parameter larger than unity.

A similar agreement with the exact behaviour appears in the dependence of the absorption probabilities for both fermions and gauge bosons on the number of extra dimensions n. To avoid repetition of known results, we do not present here any plots for the dependence of $|A_{1/2}|^2$ and $|A_1|^2$ on n, but only comment on the derived behaviour. For fermions, a monotonic decrease is observed for $m < 0$, while for $m > 0$, an enhancement in the low-energy regime is followed by a suppression in the high-energy one. For bosons, a monotonic suppression with n is found for the non-superradiant modes ($m \leq 0$) over the whole energy regime, while for the superradiant ones ($m > 0$) a similar suppression in the non-superradiant regime is replaced by an enhancement (i.e. more negative values) in the superradiant one.

5 Asymptotic Values and Absorption Cross-Section

By expanding further our analytic expressions (52) and (59) for the absorption probabilities for fermions and gauge fields, respectively, one may obtain simplified, compact expressions that reveal more clearly the low-energy asymptotic behaviour in each case as well as potential differences in the asymptotic values of the corresponding absorption cross-sections.

In the limit $\omega \to 0$, we obtain, from Eq. (40), $Z \simeq 2j + 1$, and then, from the expression of B_1/B_2, Eq. (45), the result

$$\frac{B_1}{B_2} \simeq \frac{\Gamma(2j+1)}{\Gamma(1+j-s)} (2i\omega)^{-2j-1} \left(\frac{A_1}{A_2}\right)_{\omega=0} \equiv M_{sjm} (2i\omega)^{-2j-1}, \qquad (60)$$

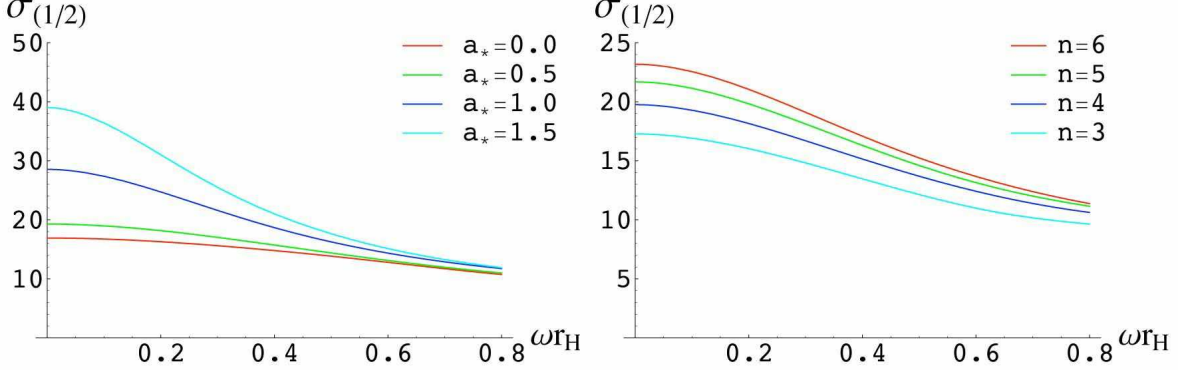with M_{sjm} a complex constant independent of the energy ω. Starting from the fermions

Figure 3: Total absorption cross-section for spinor fields, in r_H^2 units, for the cases (a) $n = 6$ and various a_*, and (b) $a_* = 0.5$ and various n, in low energies.

and Eq. (52), we set $s = 1/2$ and find

$$
|A_{\frac{1}{2}jm}|^2 \simeq 1 - \left| \frac{M_{\frac{1}{2}jm}\,\Gamma(2j+2)}{M_{\frac{1}{2}jm}\Gamma(2j+2)\,e^{i\pi(j+1/2)} + \Gamma(j+3/2)\,(2i\omega)^{2j+1}} \right|^2
$$

$$
\simeq \frac{(2\omega)^{2j+1}\,\Gamma(j+3/2)}{\Gamma(2j+2)} \left(\frac{1}{M_{\frac{1}{2}jm}} + \frac{1}{M^*_{\frac{1}{2}jm}} \right) + \,.....\,. \tag{61}
$$

The corresponding absorption cross-section, defined as $\sigma_{sjm} = \pi\,|A_{sjm}|^2/\omega^2$ for each partial wave [54], will then have the form

$$
\sigma_{\frac{1}{2}jm} = \frac{2^{2j+1}\pi\,\omega^{2j-1}\,\Gamma(j+3/2)}{\Gamma(2j+2)} \left(\frac{1}{M_{\frac{1}{2}jm}} + \frac{1}{M^*_{\frac{1}{2}jm}} \right) + \,....\,. \tag{62}
$$

From the above result, we may easily read that, similarly to the case of a non-rotating higher-dimensional black hole [10, 12], the absorption cross-section of the lowest fermionic mode, with $j = 1/2$, assumes a non-zero asymptotic value, namely

$$
\sigma_{\frac{1}{2}\frac{1}{2}m} = 2\pi \left(\frac{1}{M_{\frac{1}{2}\frac{1}{2}m}} + \frac{1}{M^*_{\frac{1}{2}\frac{1}{2}m}} \right) , \tag{63}
$$

while all higher fermionic partial waves, with $j > 1/2$, will have zero absorption cross-section as $\omega \to 0$. The quantity $M_{\frac{1}{2}\frac{1}{2}m}$ depends both on the number of extra dimensions n and on the angular momentum parameter a_* of the black hole. In Figs. 3(a,b), we depict the dependence of $\sigma_{\frac{1}{2}}$ – summed over j and m – on both a_* and n, respectively. We observe that the asymptotic value of the absorption cross-section for fermions in the low-energy regime is enhanced in terms of both parameters of the gravitational background.

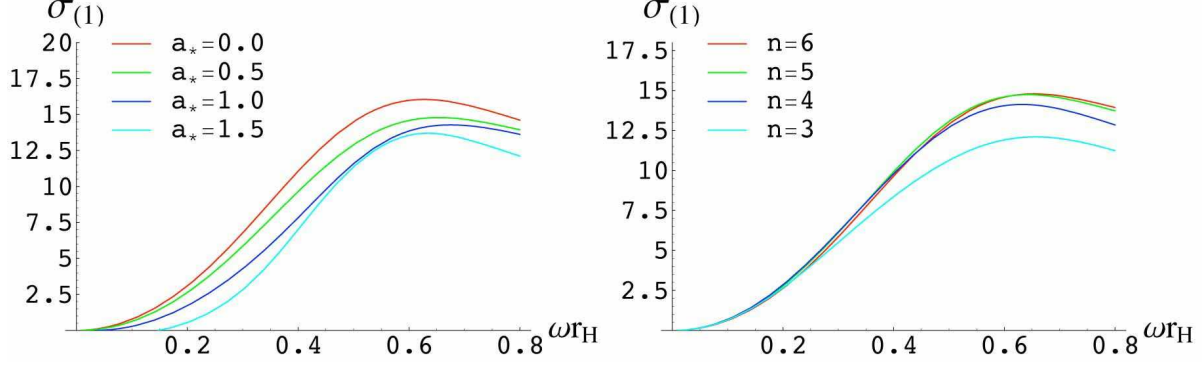For gauge fields, with $s = 1$, Eq. (59) leads to a similar result for the absorption

12

Figure 4: Total absorption cross-section for vector fields, in r_H^2 units, for the cases (a) $n = 6$ and various a_*, and (b) $a_* = 0.5$ and various n, in low energies.

probability, namely

$$
\begin{aligned}
|A_{1jm}|^2 &\simeq 1 - \left| \frac{M_{1jm}\,\Gamma(2j+2)}{M_{1jm}\Gamma(2j+2)\,e^{i\pi j} + \Gamma(j+2)\,(2i\omega)^{2j+1}} \right|^2 \\
&\simeq \frac{(2\omega)^{2j+1}\,\Gamma(j+2)}{\Gamma(2j+2)}\,\frac{i(M_{1jm}^* - M_{1jm})}{|M_{1jm}|^2} + \dots .
\end{aligned}
\tag{64}
$$

The corresponding absorption cross-section will similarly have the form

$$
\sigma_{1jm} = \frac{2^{2j+1}\pi\,\omega^{2j-1}\,\Gamma(j+2)}{\Gamma(2j+2)}\,\frac{i(M_{1jm}^* - M_{1jm})}{|M_{1jm}|^2} + \dots .
\tag{65}
$$

In this case, all gauge fields modes, including the lowest one with $j = 1$, have zero asymptotic value of absorption cross-section – this is in agreement with the corresponding results derived in the non-rotating case [10, 12]. The dependence of σ_1 – summed again over j and m – on a_* and n is now given in Figs. 4(a,b). Again, an increase in both parameters causes an enhancement in the absorption cross-section for gauge fields in the low-energy regime.

As we saw in the previous section, the absorption probability $|A_{sjm}|^2$ for both species of fields, fermions and gauge bosons, approaches unity for large values of the energy parameter ωr_H. Therefore, the absorption cross-section for each partial wave will start from either a zero or non-zero low-energy value – depending on the spin of the field and its partial wave numbers, will reach a maximum value and then asymptote to zero as $\mathcal{O}(1/\omega^2)$. Nevertheless, we expect the behaviour of the total cross-section σ_s – summed over j and m – to be radically different. Analyses in the case of a non-rotating black hole [12, 55] have demonstrated that the total cross-section for all brane-localised species of fields – scalar, spinor and gauge fields – reach asymptotically a universal constant high-energy value that depends only on the number of extra dimensions. In the case of a rotating black hole, it has been similarly shown, both in the 5-dimensional [33] and $(4 + n)$-dimensional case [34] that, in the case of scalar fields, a similar constant asymptotic value is reached in the high-energy regime. In the latter work [34], it was
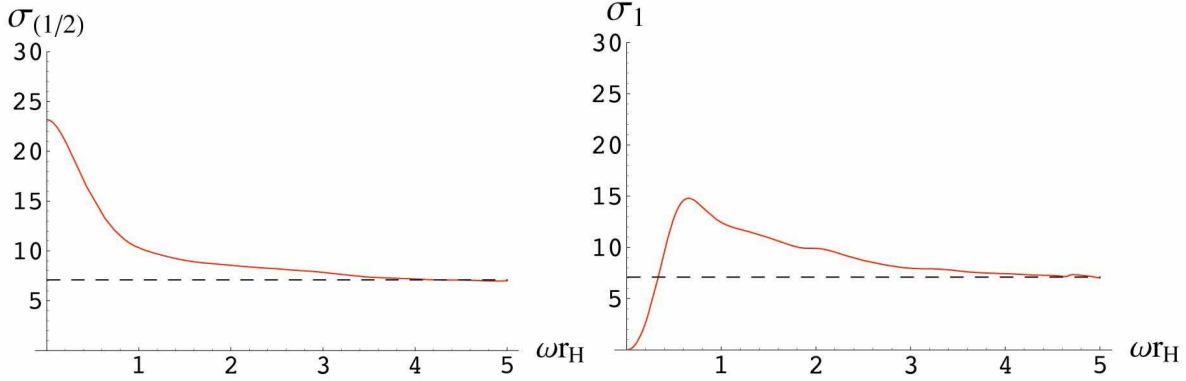
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Figure 5: Total cross section, in r_H^2 units, for spinor and vector fields, for the case $n = 6$ and $a_* = 0.5$. The dashed line denotes the theoretical asymptotic value of σ, as computed in [34].

shown in detail that the absorption cross-section can be described in the high-energy regime by the geometrical optics limit and that the exact asymptotic value of σ_s can be approximated by an analytic expression giving the cross-section for a particle approaching the black hole parallel to the rotation axis.

Thus, it remains to be seen whether the absorption cross-section, in the case of the remaining species of fields – i.e. fermions and gauge bosons, approaches again the same asymptotic value at high energies, as in the non-rotating case. By using our analytic expressions (52, 59) for the absorption probabilities, we have computed the partial wave cross-sections and, by taking the sum over j and m in each case, the total absorption cross-sections for fermions and gauge fields, respectively. Their behaviour over the whole energy regime is shown in Figs. 5(a,b). Although our results are expected to hold only in the low-energy limit and therefore not to yield any reliable results in the high-energy regime, we have nevertheless attempted to compute the high-energy asymptotic value of σ_s. Surprisingly enough, our results do not break down at high ωr_H and successfully yield a constant high-energy asymptotic value for the absorption cross-sections $\sigma_{1/2}$ and σ_1. In addition, the asymptotic values are identical to the one for scalar fields computed analytically in [34] – and denoted here with the horizontal dashed line. Our results, therefore, prove the universality in the behaviour of fields with arbitrary spin in the high-energy regime, and the dependence of their absorption cross-section only on parameters of the gravitational background even in the case of a higher-dimensional rotating black hole.

6   Energy Emission Rates

As is well known, the absorption probabilities $|A_{sjm}|^2$ can be used in order to determine the various emission rates for the Hawking radiation. In the present case, our derived analytic expressions (52) and (59), together with the expression for the temperature (14), can lead to the emission rates for a higher-dimensional rotating black hole directly on
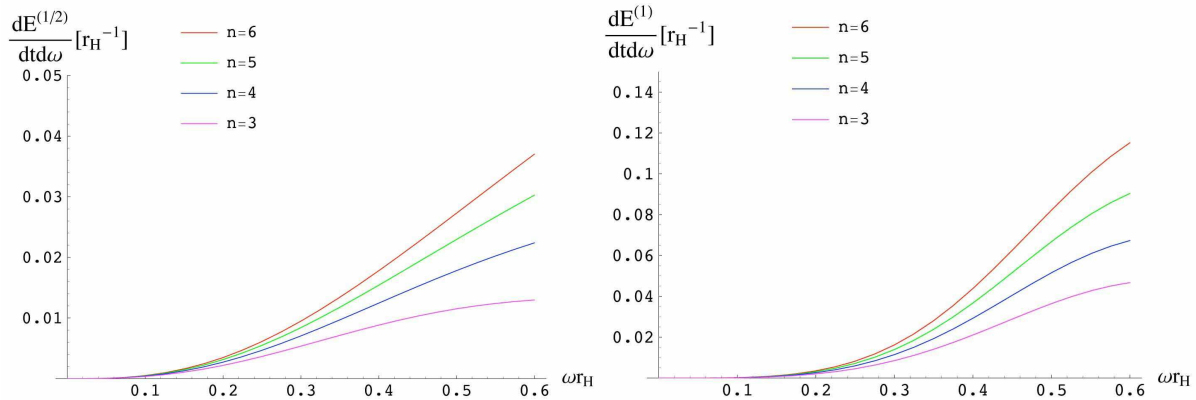
Figure 6: Energy emission rates for spinor and vector fields, for $a_* = 0.5$ and various n.
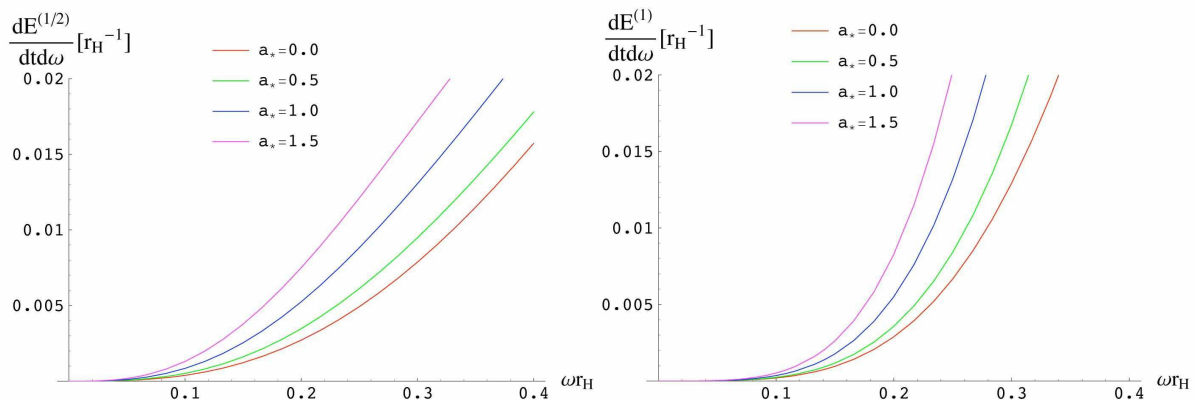
our brane, in the form of fermions and gauge bosons.

Figure 7: Energy emission rates for spinor and vector fields, for $n = 6$ and various a_*.

Since our analytic expressions are, strictly speaking, valid in the low-energy and low-angular-momentum limit, we first focus our attention on these particular regimes. In the context of our analysis, we have studied all emission rates, associated with the Hawking radiation, namely the flux (number of particles), power (energy) and angular momentum emission rates, and they were all found to exhibit the same behaviour in terms of the number of extra dimensions n and the angular momentum parameter a_*. Therefore, as an indicative example, we display here our results for the energy emission rates following from Eq. (13). In Figs. 6 and 7, we present the energy emission rate, i.e. the energy emitted by the black hole per unit time and unit frequency, in the form of fermions and gauge fields, as a function of n and a_*, respectively. One may easily observe that the energy flux is indeed enhanced with both topological parameters, in accordance with the exact numerical results of [28, 29, 30].

Even by looking at the low-energy regime, one can accurately conclude that the emission of gauge bosons dominates over the one for fermions, for the same values of the parameters a_* and n. This feature is also in accordance with the results of the exact
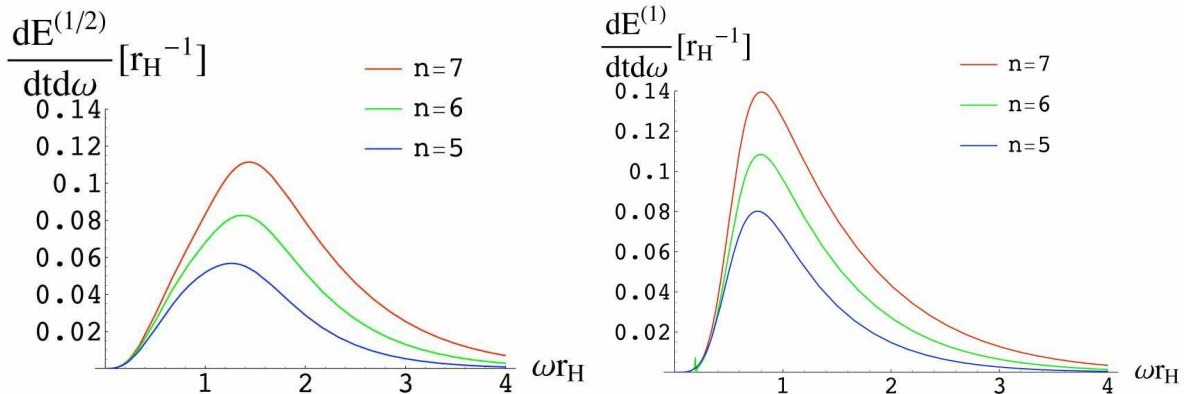
Figure 8: Energy emission rates for spinor and vector fields, for $a_* = 0.2$ and various n.

numerical analysis. In addition, the magnitudes of the corresponding energy emission rates are also correctly reproduced by our analytic results. Motivated by this, in Fig. 8, we plot the energy emission rates, for fermions and gauge bosons, for higher values of the energy parameter ωr_H and various configurations of n, while a_* is kept fixed. The restricted validity of our analytic results are bound to lead to deviations from the exact numerical results: indeed, our curves tend to reach their maximum point and their tail region sooner than the exact ones. Nevertheless, apart from the above, the agreement is remarkable: not only is the general profile of the energy emission rates and their dependence on the number of extra dimensions reproduced but the magnitudes at the peak of their curves – reached at $\omega r_H \simeq 1$, or even well beyond this – are also accurately derived.

7   Conclusions

In the present article, we studied the emission of Hawking radiation by higher-dimensional rotating black holes into fermionic and gauge boson degrees of freedom on the brane. The same topic has been studied before in the literature, however, those studies were either purely numerical or focused on specific cases like 5-dimensional space-times. In contrast, here, we carried out an analytical study of the problem deriving results valid for a black hole in a spacetime with arbitrary numbers of extra dimensions.

As a starting point of our analysis, we formulated the radial master equation, describing the propagation of degrees of freedom with spin s on the brane, in such a way that both radiative components of the given fields, i.e. with $s = \pm|s|$, are described through the same equation. As the complexity of the problem forbids an all-energies analytical treatment, we followed an approximate method that allows us to find a solution in the low-energy and low-rotation regime. In Section 3, we used a well-known matching technique, which consists of solving the field equations in the near-horizon and far-field regime, and then matching the two solutions to construct a complete, smooth solution for the radial part of the field. This allowed us to compute the absorption probabilities

for brane-localised fermions and gauge bosons through two purely analytic expressions, namely Eqs. (52) and (59), respectively.

Using these expressions, the corresponding absorption probabilities were plotted as a function of the energy-parameter ωr_H, for various values of the number of extra dimensions n and angular momentum parameter a_*. These plots were presented in Section 4. Comparing our results to numerical results existing in the literature, we found them to be in very good agreement. The dependence of $|A_{sjm}|^2$ on both n and a_* was correctly reproduced as well as additional effects such as the superradiance in the case of gauge bosons. This agreement extended beyond the low-ω and low-a_* regime, even for intermediate values of a_* or values of ωr_H larger than unity. In section 5, we presented compact, simplified expressions for the absorption probabilities valid in the limit $\omega \to 0$. As in the non-rotating case, the behaviour was found to be different with the absorption probability for gauge bosons reducing to zero faster than the one for fermions. This resulted in a different behaviour, in the very low-energy regime, of the corresponding total absorption cross-sections: while the one for gauge bosons goes always to zero, the one for fermions assumes a non-zero, constant value that depends on both a_* and n. The extended validity of our results allowed us to compute the dependence of the total cross-section over the whole energy regime. We were then able to derive the high-energy asymptotic values, that turned out to be the same for fermions and gauge bosons and to coincide with the one for scalar fields computed in a previous work of ours via the use of the geometrical optics limit.

Finally, in section 6, we computed the energy emission rates for Hawking radiation by a rotating, higher-dimensional black hole on the brane in the form of fermions and gauge bosons. These were also found to be in remarkable agreement with the exact numerical results in the low-energy and low-angular-momentum regime, and to lead also to fairly accurate results beyond those limits.

In conclusion, our study shows that analytical treatment of the propagation of fields with spin s on a brane embedded in a higher-dimensional, rotating black hole background is indeed possible, and that the derived results, although they cannot offer the level of accuracy and completeness of the exact numerical ones, can certainly reproduce quite accurately the dependence of the various quantities on the topological parameters of spacetime as well as a good estimate of their actual magnitudes.

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